Fusion Plasma Theory II. 2019

Week 14

(*) For electrons,

$$\frac{1}{\omega - kv} \approx -\frac{1}{kv} + \cdots \qquad (23.24)$$

then

$$\int_{-\infty}^{\infty} \frac{\frac{\partial f_0}{\partial v}}{\omega - kv}\, dv \approx \frac{n}{k v_{Te}^2} \qquad (23.25)$$

$$\Rightarrow \boxed{D(k,\omega) \equiv 1 + \frac{\omega_{pe}^2}{k^2 v_{Te}^2} - \frac{\Omega_{pi}^2}{\omega^2}} \qquad (23.27)$$

where $\omega_{pe}^2 = \frac{ne^2}{m_e \epsilon_0}$, $\Omega_{pi}^2 = \frac{ne^2}{M_i \epsilon_0}$ (23.28)

For $k\lambda_{De} \equiv \frac{k v_{Te}}{\omega_{pe}} \ll 1$, 2nd and 3rd term dominate in Eq(27),

$$\Rightarrow \boxed{\omega = k C_s}$$ ion acoustic wave

$$C_s^2 \equiv T_e / M_i$$

24. Kinetic Effects on Plasma Waves

Landau's treatment.

(*) Previously, we have solved the d.r. of plasma wave keeping only the real part (Vlasov's treatment).

~~Eq.(23.12)~~)

$$D(k,\omega) \equiv 1 + \frac{e^2}{mk\epsilon_0}\int_{-\infty}^{\infty} \frac{\frac{\partial f_0}{\partial v}}{\omega - kv}\,dv = 0 \qquad (23.12).$$

(*) As, we have learned previously, more precise treatment of the integral is according to Landau.

$$\boxed{\int_{-\infty}^{\infty} \frac{\frac{\partial f_0}{\partial v}}{\omega - kv}\,dv = P_r\int_{-\infty}^{\infty} \frac{\frac{\partial f_0}{\partial v}}{\omega - kv}\,dv - \frac{\pi i}{k}\left.\frac{\partial f_0}{\partial v}\right|_{v=\frac{\omega}{k}}}$$

(24.18).

* The Principal part has been evaluated before.

$$D \simeq 1 - \frac{\omega_{pe}^2}{\omega^2} + \cdots$$

* The resonant part (imaginary) has to be kept as well;

$$\Rightarrow \quad D \equiv 1 - \frac{\omega_{pe}^2}{\omega^2} + i\left(\frac{\pi}{2}\right)^{1/2} \frac{\omega_{pe}^2 \omega}{k^3 v_{Te}^3} \exp\left(-\frac{\omega^2}{2k^2 v_{Te}^2}\right) \qquad (24.21)$$

* Solving this d.r. iteratively for $Re(\omega) \gg Im(\omega)$,

$$\omega \simeq \omega_{pe} - \frac{i}{2}\left(\frac{\pi}{2}\right)^{1/2} \frac{\omega_{pe}^4}{k^3 v_{Te}^3} \exp\left[-\frac{\omega_{pe}^2}{2k^2 v_{Te}^2}\right] \qquad (24.22)$$

(electron) Landau damping of plasma waves:

— This is exponentially small for $\frac{\omega}{k} \gg v_{Te}$.

(*) The wave damps in an entirely collisionless system!

24.5. Ion Acoustic Waves: Ion Landau Damping

* Similar procedure for ion acoustic waves with $v_{Ti} \ll \frac{\omega}{k} \ll v_{Te}$;

$$D(k,\omega) \equiv 1 + \sum_\sigma \frac{e_\sigma^2}{m_\sigma k \epsilon_0} \left\{ P_r \int_{-\infty}^{\infty} \frac{\frac{\partial f_{0\sigma}}{\partial v}}{\omega - kv} dv - \frac{\pi i}{k} \frac{\partial f_0}{\partial v}\bigg|_{v=\frac{\omega}{k}} \right\}. \quad (24.28)$$

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$$\boxed{D(k,\omega) \equiv 1 + \frac{\omega_{pe}^2}{k^2 v_{Te}^2} - \frac{\Omega_{pi}^2}{\omega^2} + i\left(\frac{\pi}{2}\right)^{\frac{1}{2}} \left[\frac{\omega_{pe}^2 \omega}{k^3 v_{Te}^3} + \frac{\Omega_{pi}^2 \omega}{k^3 v_{Ti}^3} e^{-\frac{\omega^2}{2k^2 v_{Ti}^2}} \right]} \quad (24.32).$$

electron Landau damping

ion Landau damping

Solving Eq. (24.32) approximately for $k\lambda_{De} \ll 1$ and $T_i \ll T_e$, we obtain $Re(\omega) \approx k C_s$ and

$$Im(\omega) = \frac{1}{2}\left(\frac{\pi}{2}\right)^{1/2}\left[\frac{\omega^2}{k v_{Te}} + \frac{T_e}{T_i}\left(\frac{\omega}{k v_{Ti}}\right)^2 \exp\left(-\frac{\omega^2}{2k^2 v_{Ti}^2}\right)\right]$$

$$\simeq \frac{1}{2}\left(\frac{\pi}{2}\right)^{1/2} k C_s \left[\underbrace{\left(\frac{m_e}{M_i}\right)^{1/2}}_{\text{electron LD}} + \underbrace{\left(\frac{T_e}{T_i}\right)^{3/2} e^{-T_e/2T_i}}_{\text{ion LD}}\right] \tag{24.33}$$

electron LD — very small because $\frac{m_e}{M_i} \ll 1$.

ion LD. — Smaller as $\frac{T_e}{T_i} \gg 1$,