Fusion Plasma Theory II. 2019

Week 9

Furthermore, more detailed linear theory addressing physical effects beyond the description of resistive MHD, which include

i) long mean free path of less collisional plasmas ($\nu_{*e} < 1$) and kinetic effects. This is usually a stabilizing influence.

ii) finite ion Larmor radius effects. This is also usually a stabilizing effect.

These effects can modify the instability criterion.

Now, one can get $\Delta' > \Delta^{Th}$, where $\Delta^{Th} > 0$ comes from detailed layer calculations including the aforementioned effects.

If one considers a modification of the Ohm's law, in particular, including the effects which come from low collisionality, and long mean free path, it can lead to the "Neoclassical MHD".

One crucial modification is the effect of "bootstrap current", in banana (or plateau) collisionality regime.

The bootstrap current requires both the
low collsionallity such that banana orbits are relevant, and
the density (pressure) gradient, so that there are nonuniform distribution of trapped particles.

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$\Rightarrow$ Consider a modification of the Ohm's law, in particular.
Include effects which come from low collisionality, and long mean free path $\Rightarrow$ "Neoclassical MHD".
One crucial modification is the effect of "bootstrap current", in banana (or plateau) collisionality regime.

Bootstrap current requires:

1. low collsionallity $\Rightarrow$ banana orbits are relevant
2. density (pressure) gradient $\Rightarrow$ nonuniform distribution of trapped ptls.

This consideration:

$$\left(v_{\parallel,\text{typical trapped ptls}} \Delta_b \frac{\partial}{\partial r} n_{\text{trapped ptls}} \right) \Rightarrow j_{\text{bs}}$$

Neoclassical Tearing Mode

(Ref. Wesson page 359)

$$\frac{\partial\psi}{\partial t} = \frac{\eta}{\mu_o}\nabla_\perp^2\psi \simeq \frac{\eta}{\mu_0}\frac{\partial^2}{\partial r^2}\psi$$

Now

$$\frac{\partial\psi}{\partial t} = E_h = \eta(j_h - \delta j_b)$$

Bootstrap current exist for $\partial p/\partial r \neq 0$, but in the presence of island there's a local deficiency of bootstrap current due to the flattening of $T_e \Rightarrow \delta j_b = -j_b$.

Ignoring details of the order of one coeffcient in front,

$$j_b \simeq -\frac{\epsilon^{1/2}}{B_\theta}\frac{\partial p}{\partial r}.$$

What is Bootstrap current?
(Ref. Miyamoto, page 224 -)
Making an analogy to the diamagnetic current due to $\partial p/\partial r \neq 0$,

$$
\begin{aligned}
j_b = (ev_\parallel)\left(-\frac{dn_{\text{trapped}}}{dr}\right)\Delta_{\text{banana}} &\simeq e\,\epsilon^{1/2} v_{\mathrm{T,e}}\,\epsilon^{1/2}\frac{\partial n}{\partial r}\frac{1}{\epsilon^{1/2}}\frac{rB}{RB_p}\rho_e \\
&\simeq -\epsilon^{3/2}\frac{1}{B_p}\frac{\partial p}{\partial r}
\end{aligned}
$$

This heuristic derivation is an underestimation of bootstrap current.
More systematic derivation comes from the electron momentum evolution:

$$
\frac{d}{dt}(n_e m_e u_\parallel) = -n_e e B_p\, u_r + f_{ei}
$$

where

$$
f_{ei} = -n_e m_e u_\parallel \nu_{ei} \;\text{ and }\; n_e u_r = -D_{\text{banana}}\frac{\partial n_e}{\partial r}.
$$

For a simple derivation of bootstrap current, let's review single particle dynamics and scalings of transport in banana collisionality regime.

- $\mu = \frac{1}{2}mv_\perp^2/B$ is conserved
- L_ϕ, canonical angular momentum is conserved
- E, single particle energy is conserved

$$\begin{aligned}
L_\phi &= (mv_\| + \frac{e}{c}A_\phi)R \\
&= 0 + \frac{e}{c}A_\phi R \quad \text{at tips} \\
&= mv_\|(0) + \frac{e}{c}A_\phi(0)R \quad \text{at the mid plane}
\end{aligned}$$

$\Rightarrow$

$$\frac{e}{c}R\Delta A_\phi = mv_\parallel(0), \quad \Delta A_\phi = B_\theta \Delta r,\, \Delta\psi = RB_\theta\Delta r$$

$$\Rightarrow \Delta r = \frac{mc}{eB_\theta}v_\parallel(0) = \frac{q}{\epsilon}\frac{v_\parallel(0)}{\Omega_{ce}} = \frac{q}{\sqrt{\epsilon}}\rho_e$$

$$E = \mu B(r,\theta) + \frac{1}{2}mv_\parallel^2 = \mu B_{\max}(r,\theta_0)$$

$\Rightarrow$

$$\begin{aligned} v_\parallel(\theta)^2 &= \frac{2\mu}{m}(B_{\max} - B(r,\theta)) \\ &= \frac{2\mu}{m}B_0\,\epsilon\,(\cos\theta_0 - \cos\theta) \leq 0 \quad \text{for some } \theta \\ &\Rightarrow \text{ trapped particles} \end{aligned}$$

Effective collisionality is enhanced over 90 degree collision frequency. ($\because$ a particle does not have to be scatterd all the way to 90 degree to get significantly deviated from the original orbit)

$$\Delta\theta^2 \propto \nu_{\text{eff}} t$$

$$\Rightarrow \quad \nu_{\text{eff}} = \frac{\nu_{90}}{\sqrt{\epsilon}^2}, \quad \text{if } \nu_{\text{eff}} < \omega_{\text{be}} = \frac{\sqrt{\epsilon} v_{\text{th,e}}}{qR}$$

Plasma is in the banana collisionality regime if $\nu_* = \nu_{\text{eff}}/\nu_{\text{be}} < 1$ i.e., a typical trapped electron can execute a few banana excursions before it gets detrapped by collision.

$$\Delta_{\text{banana}} \simeq \frac{q}{\epsilon^{1/2}} \rho_e$$

Fraction of trapped particles $\sim \sqrt{\epsilon}$. Particle diffusion coefficient in banana collisionality regime (Neoclassical...)

$$D \sim \frac{\Delta x^2}{\Delta t}.$$

Here, we need to take into account the fact that trapped particles and passing particles have different characteristic Δx and Δt for collisional diffusive process,

$$\Rightarrow D_{\text{banana}} \simeq (\text{fraction of trapped ptls}) \cdot \nu_{\text{eff}} \cdot \Delta^2_{\text{banana}} + (\text{fraction of passing ptls}) \cdot \nu_{90} \cdot \Delta r^2_{\text{passing}}$$

$$D = \sqrt{\epsilon} \cdot \left(\frac{\nu_{ei}}{\epsilon}\right) \cdot \left(\frac{q}{\sqrt{\epsilon}}\rho_e\right)^2 + (1 - \cancel{\sqrt{\epsilon}}) \cdot \nu_{ei}(q\rho_e)^2$$
$$\sim \frac{q^2}{\epsilon^{3/2}}(\nu_{ei}\rho_e^2)$$

$\therefore$ From $n_e u_r = -D_{\text{banana}}\frac{\partial n_e}{\partial r}$, and $f_{ei} = -n_e m_e u_\parallel \nu_{ei}$, we obtain

$$j_b \simeq -\frac{\epsilon^{1/2}}{B_p}\frac{dp}{dr},$$

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which is larger than our previous result from a heuristic consideration. Now, we know the contribution of bootstrap current (or its deficiency) for magnetic island evolution.

$$\frac{\partial \psi}{\partial t} = \eta\left(\frac{1}{\mu_0}\frac{\partial^2}{\partial r^2}\psi - \underline{\frac{\epsilon^{1/2}}{B_p}\frac{\partial p}{\partial r}}\right)$$

⇒ We'll do the same derivation done for the classical tearing mode.
⇒ modified Rutherford Equation for magnetic island.

Neoclassical Tearing Mode

$$\frac{\partial \psi}{\partial t} = \eta(j_b - \delta j_b)$$

Bootstrap current

$$j_b \simeq -c_1\frac{\epsilon^{1/2}}{B_\theta}\frac{\partial p}{\partial r}$$