



# Lecture 2

# Fluid Measurements





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- 2.1 Density measurement
- 2.2 Viscosity measurement
- 2.3 Pressure measurement
- 2.4 Surface elevation
- 2.5 Velocity measurement
- 2.6 Shear measurements
- 2.7 Flowrate measurement
- 2.8 River Topography
- 2.9 Large scale measurement



## Objectives

- Learn how to measure properties of diverse fluids
- Introduce flow measuring devices for both laboratory and field experiments based on the fluid mechanics theory



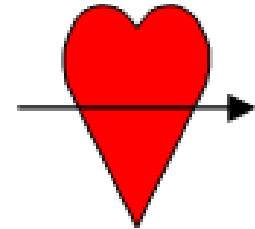


## 2.1 Density measurement

- There are many properties of fluids, but we only pay attention to the **density and viscosity** since other properties are measured by other disciplines (physics, chemistry).

- Density measurements

$$\rho = \frac{M}{V}$$



- 1) Weighing a known volume: pycnometer (비중병)
- 2) Hydrostatic weighing: plummet, Westphal balance
- 3) Hydrometer (비중계)



## 1) Weighing method

- Pycnometer (specific gravity bottle; 비중병)
  - Equipment to weigh accurately a known volume of liquid.
  - The specific weight of the liquid is calculated as

$$\gamma_t = \frac{W_2 - W_1}{V_l} \quad (2.1)$$

- $W_2$  = total weight of the pycnometer,
  - $W_1$  = weight of empty pycnometer
  - $\gamma_t$  = specific weight of the liquid
  - $V_l$  = volume of the liquid
- density depends on the temperature



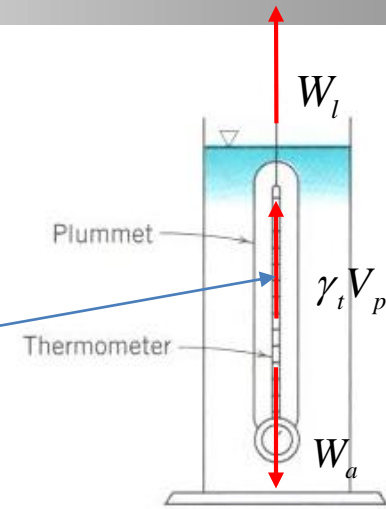


## 2) Plummet (or density ball)

- Based on the Archimedes principle

$$W_l + \gamma_t V_p = W_a \quad (2.2)$$

Buoyant force



$W_a$  = weight of the plummet in air

$W_l$  = weight of the plummet when suspended in the liquid

$V_p$  = volume of the plummet

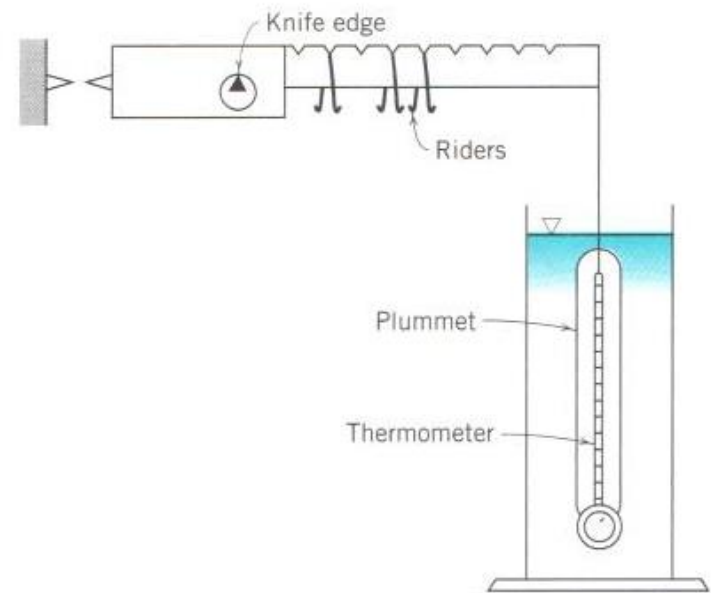
$\gamma_t$  = specific weight of the liquid





## Westphal balance

- Utilize the buoyant force on a plummet



(b) Westphal balance.



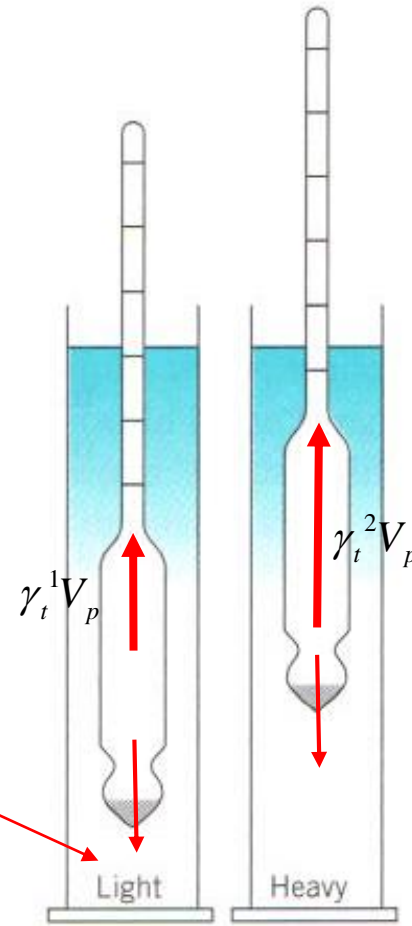


### 3) Hydrometer (비중계)

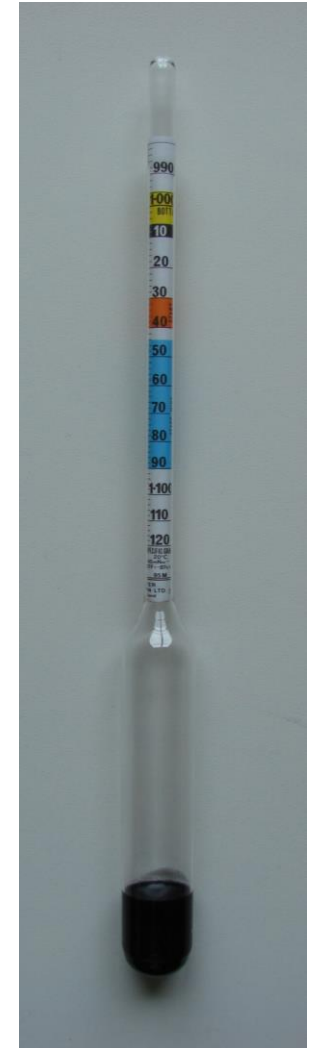
- The most common method
- Float with different immersions in liquids of different densities

Light liquid

$$\gamma_t^1 < \gamma_t^2$$



(c) Hydrometers.





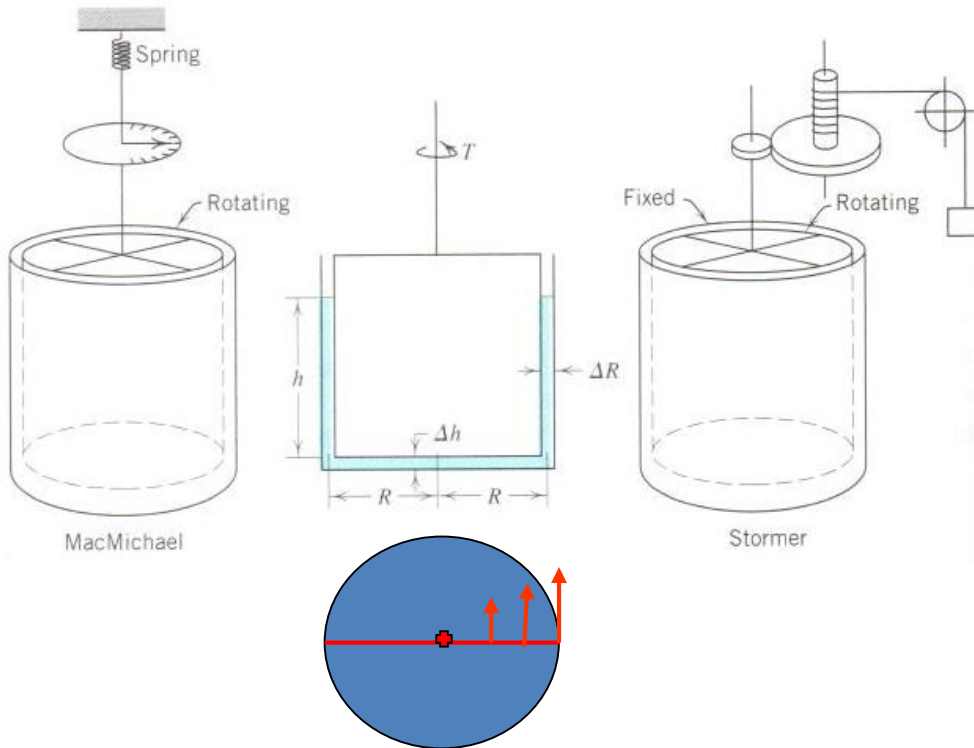


## 2.2 Viscosity measurement

- Viscosity measurements are made with devices known as viscosimeters or viscometers
  - 1) Rotational viscometer
  - 2) Tube viscometer
- The operation of these viscometers depends on the existence of laminar flow.
- Viscosity depends on the temperature and when it is measured, done in constant temperature.



# 1) Rotational Viscometers



Outer cylinder is rotating at constant angular velocity (각속도),  $\omega$   
 Measure torque (회전모멘트) on the inner cylinder



- Rotational Viscometers – based on Couette (laminar) flow
- Measure torque,  $T$  on the inner tank (side + bottom)

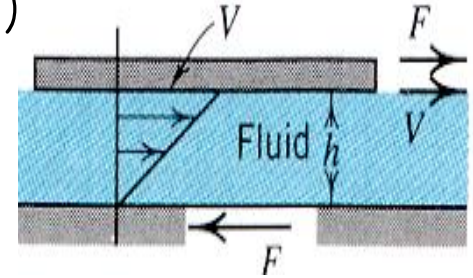
$$T = F r = \tau A r$$

$$T = T_1 + T_2$$

$$\tau = \mu \frac{dv}{dy}$$

$$V = \omega R$$

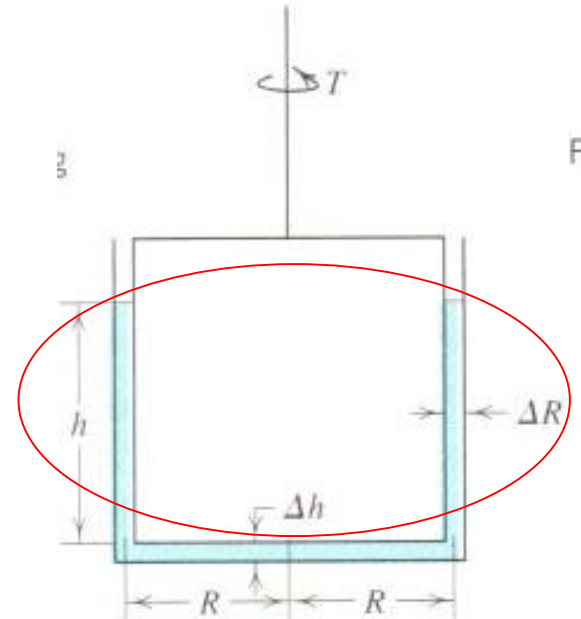
$$\tau = \mu \frac{V}{h}$$



$$T_1 = F_1 r_1 = \tau_1 A_1 r_1$$

$$\tau_1 = \mu \frac{dV}{dy} = \mu \frac{V}{\Delta R}; A_1 = 2\pi R_i h; r_1 = R_i$$

$$T_1 = \frac{2\pi\mu R_i^2 h V}{\Delta R} \quad (2.3)$$





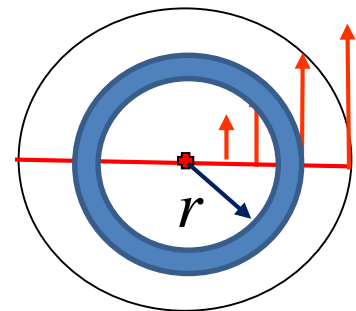
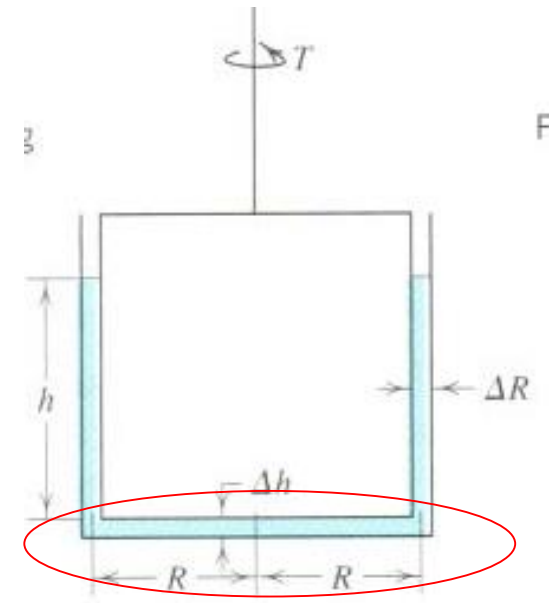
$$T_2 = \int_0^R dT = \int_0^R \tau dA \cdot r$$

$$\tau = \mu \frac{dv}{dy} = \mu \frac{V}{\Delta h} = \mu \frac{\omega r}{\Delta h}$$

$$T_2 = \int_0^R \left( \mu \frac{\omega r}{\Delta h} \right) (2\pi r \cdot dr) \cdot r = \frac{2\pi\mu\omega}{\Delta h} \int_0^R r^3 dr$$

$$= \frac{\pi\mu\omega}{2\Delta h} R^4 = \frac{\pi R^3 \mu V}{2\Delta h} \quad (2.4)$$

$$T = \frac{2\pi R^2 h \mu V}{\Delta R} + \frac{\pi R^3 \mu V}{2\Delta h} \quad (2.5)$$



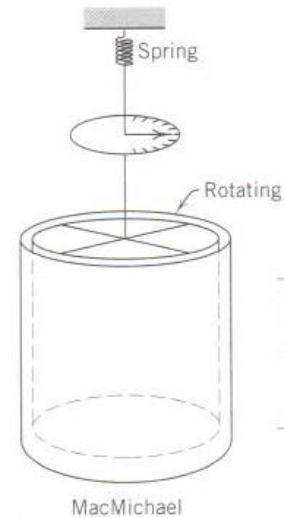


■ MacMichael viscometer

- Torque is proportional to the torsional deflection  $\theta$

$$T = K_1 \theta$$

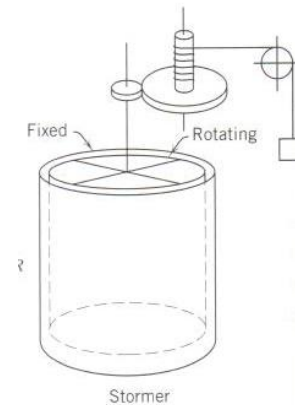
$$\mu = \frac{K_1 \theta}{KN} \quad (2.6)$$



■ Stormer viscometer

- Torque is constant since it is proportional to the weight  $W$ , and the time  $t$  required for a fixed number of revolutions is inversely proportional to  $N$

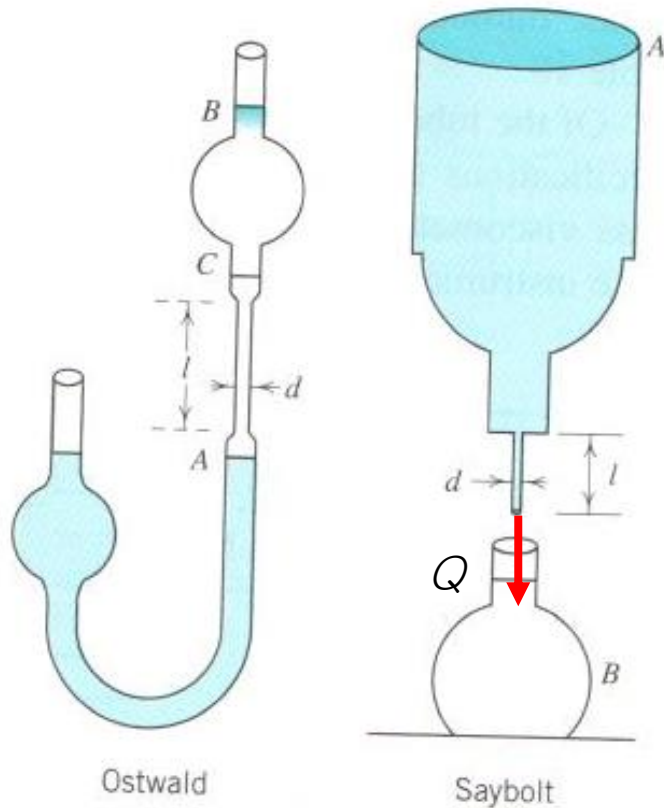
$$\mu = \frac{T}{KN} = \frac{T}{KK_2} t \quad (2.7)$$





## 2) Tube-type viscometer

- Ostwald – low viscosity
- Saybolt instrument – medium to high viscosity



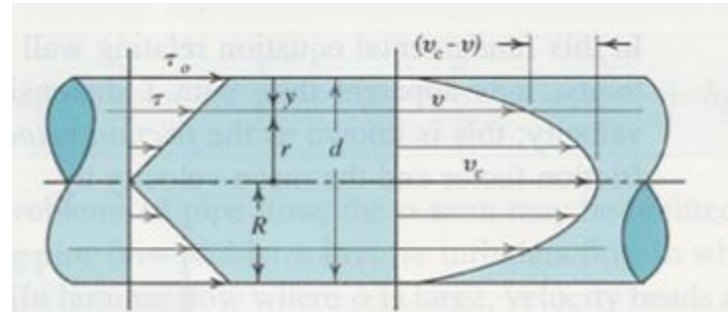


- The tube-type viscometers involve the unsteady laminar flow of a fixed volume of liquid through a small tube under standard head conditions.
- The time for the quantity of liquid to pass through the tube becomes a measure of the kinematic viscosity of the liquid.
- Apply the Hagen-Poiseuille law for laminar flow in a circular tube

$$Q = \frac{\pi d^4 \gamma h_L}{128 \mu l}$$

$$Q = \frac{V_l}{t}$$

$$\mu = \left( \frac{\pi d^4 h_L}{128 V_l l} \right) \gamma t \quad (2.8)$$



$V_l$  is volume of liquid passed in time  $t$

$d, l, V_l, h_L$  are constants of the instrument





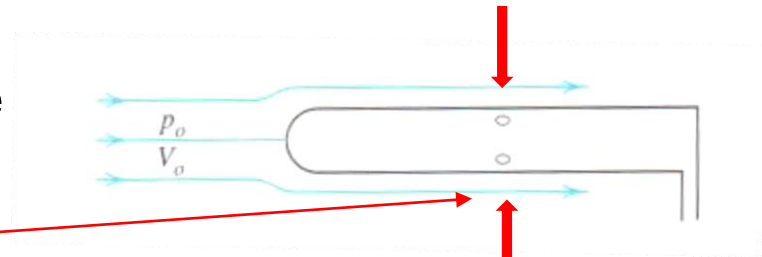
## 2.3 Pressure measurement

- Static pressure
  - The accurate measurement of static pressure in a fluid at rest may be accomplished with comparative ease.
  - manometer, pressure gages
  - To measure static pressure in a flowing fluid a device is required within fits the streamline picture and causes no flow disturbance.
  - It should contain a small hole whose axis is normal to the direction of motion, which usually is not known in advance.



- Static tube

- Assuming that it is aligned to the direction

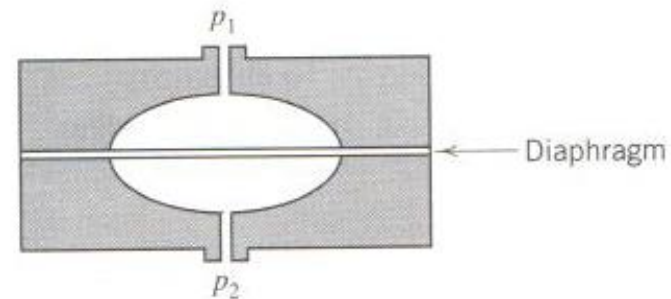


$$V_{measured} > V_0 \rightarrow p_{measured} < p_0$$

- This error is minimized by making the tube as small as possible.

- Transducers (connect to the manometer)

- Transducer (diaphragm gage) convert pressure differential to an electric output.

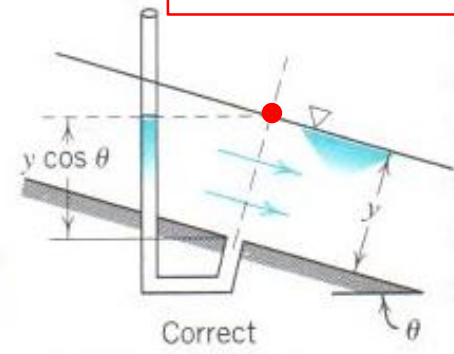
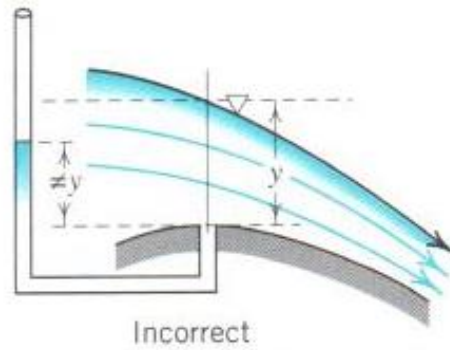
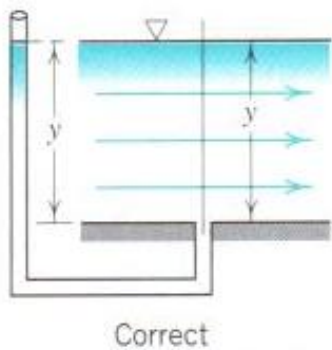




## 2.4 Surface elevation

### 1) Manometer, piezometer column, pressure-gage:

- The elevation of the surface of a liquid at rest may be determined
- The same methods may be applied to flowing liquids.
- The piezometer method is used only where the streamlines are essentially straight and parallel.



$$\begin{aligned} \text{수심} &= y \\ \text{수위} &= y \cos \theta \end{aligned}$$

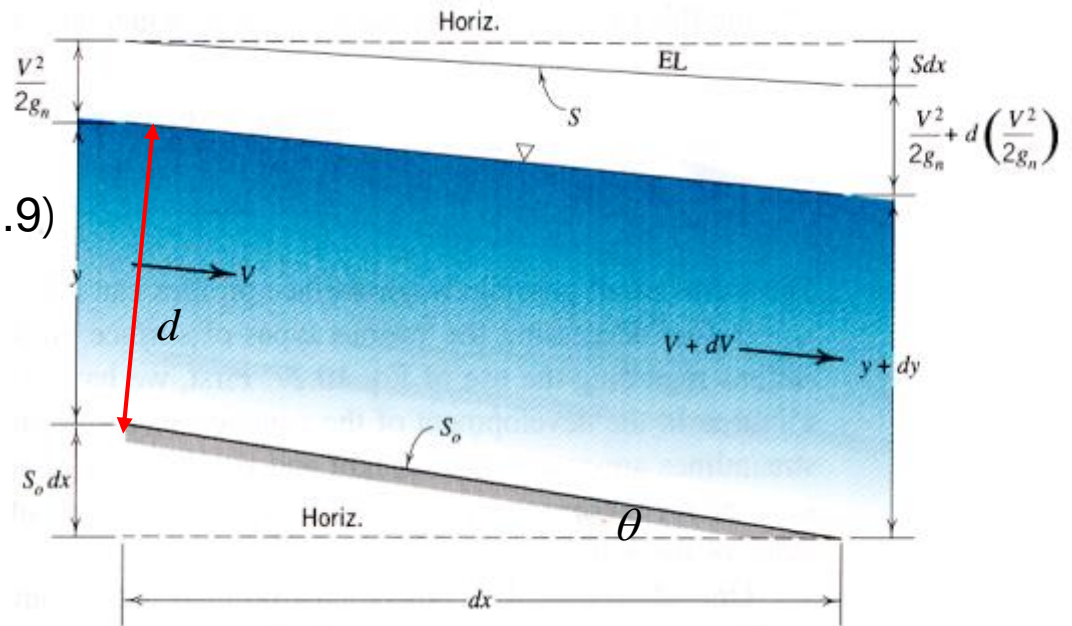


[Re] Derivation of gradually varied flow equation

- $dx$  = length of a channel segment
- $y \sim d$  = depth of flow (varies by  $dy$ );
- $V$  = average velocity (varies by  $dV$ )
- $S_0$  = bed slope
- $S$  = energy (friction) slope

$$H = z + d \cos \theta + \frac{V^2}{2g} \approx z + y + \frac{V^2}{2g} \quad (2.9)$$

when  $\theta$  is small  $\rightarrow d \cos \theta \approx y$





2) *Floats:*

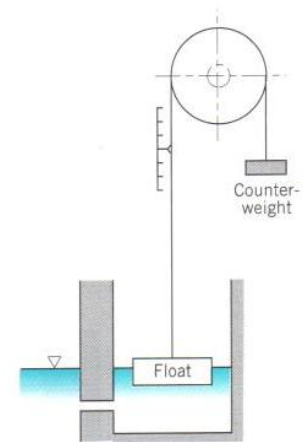
used for measuring the surface

3) *Staff gages:*

crude but direct measurements of liquid-surface elevation

4) *Wading rod:*

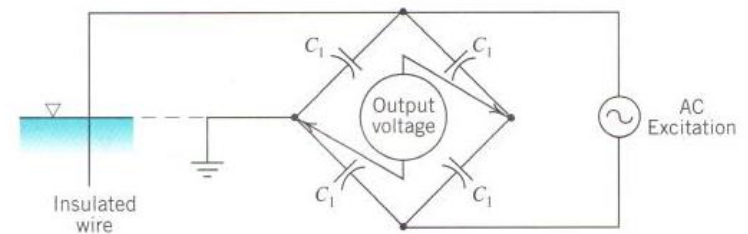
- Depth is shallow enough or measuring from a low footbridge
- Wading rod to be placed firmly on the streambed



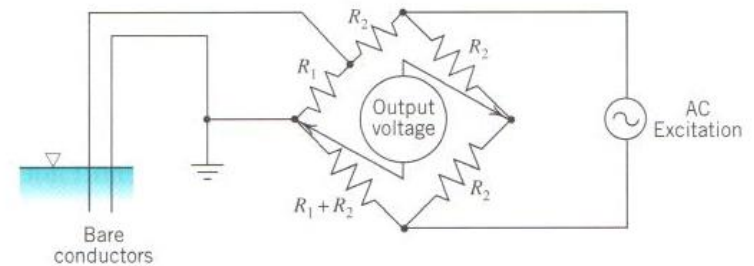


### 5) Insulated wire and bare conductors (전도체)

- output bridge voltage is proportional to the change in depth



(a) Capacitive sensor.



(b) Resistive sensor.



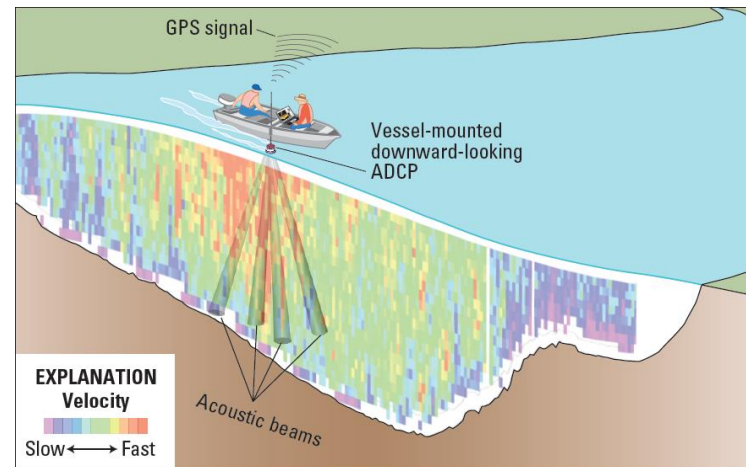
## 6) Sonic devices

- Measure the time necessary for a sound pulse to travel to the surface and return to its source

- Echo sounder (MBES/ADCP)

Measuring depth when making a moving boat measurement

Records a continuous trace of the streambed on a digital or analog chart.







# 2.5 Velocity measurement

## 1) Pitot-static tube (피토관)

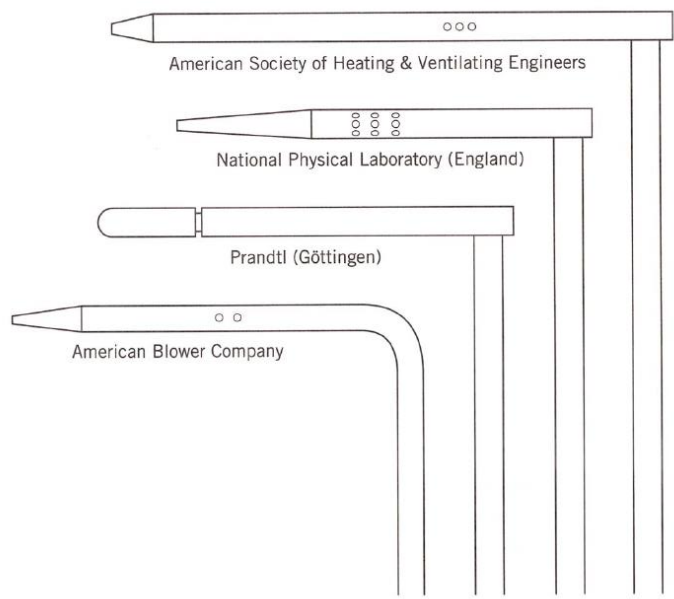
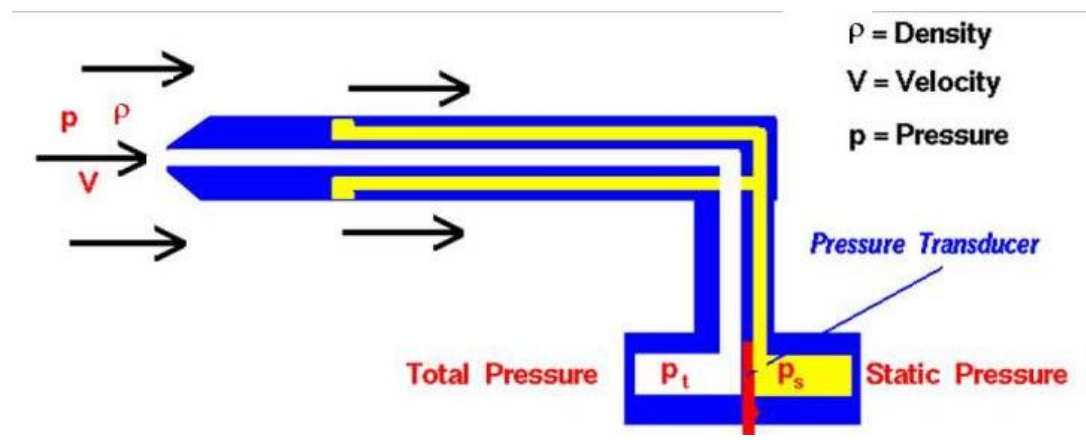


Fig. 14.12 Pitot-Static tubes (to scale).





- Stagnation pressure (total pressure) 정체압력
  - Aligned to the flow direction
  - Apply Bernoulli equation

$$\frac{p_s}{\gamma} = \frac{p_0}{\gamma} + \frac{V_0^2}{2g} \quad (2.10)$$

<p>↑</p> <p>Total pressure 정체압력</p>	<p>↑</p> <p>Static pressure 정압력</p>	<p>↑</p> <p>Dynamic pressure 동압력</p>
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$$V_0 = \sqrt{\frac{2g(p_s - p_0)}{\gamma}} \quad (2.11)$$

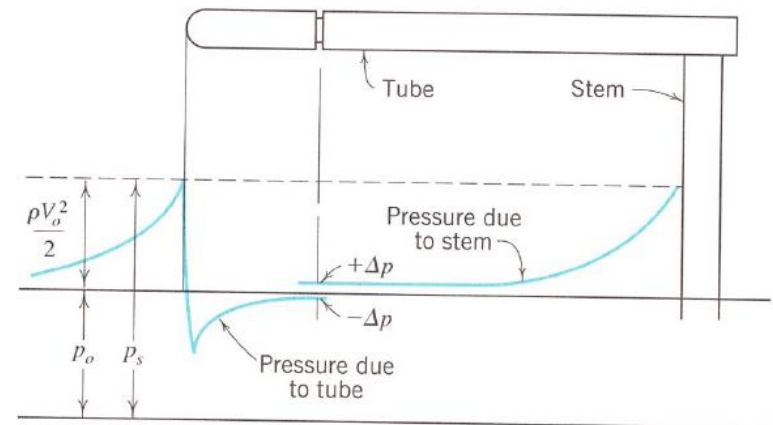
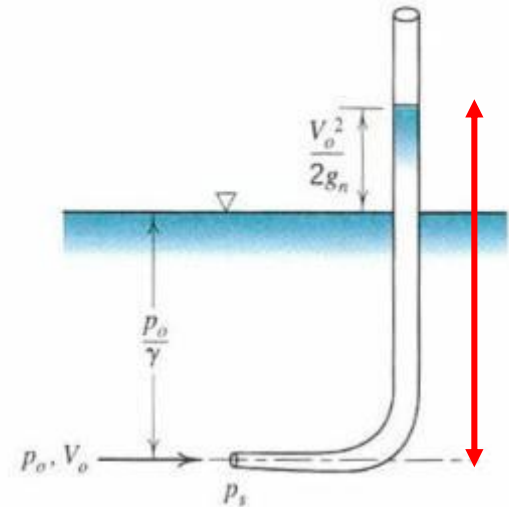


Fig. 14.13 Prandtl's pitot-static tube.



- Airbus-380
- Helicopter





## 2) Current meters

- Anemometers and current meters

$$V = aN + b$$

$N$  = 회전수



Current meters –  
propeller type



Current meters –  
cup type



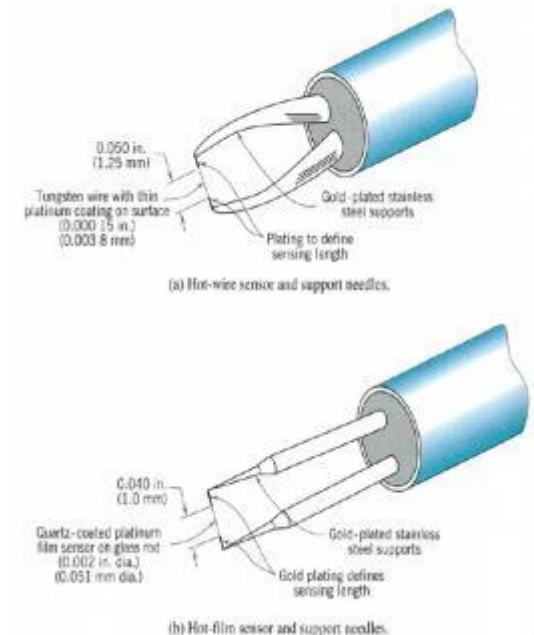
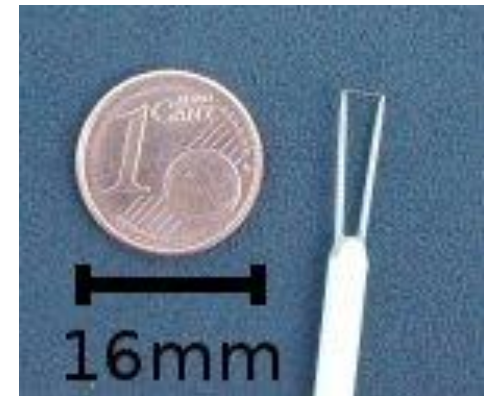
Anemometers -air





### 3) Hot wire anemometer

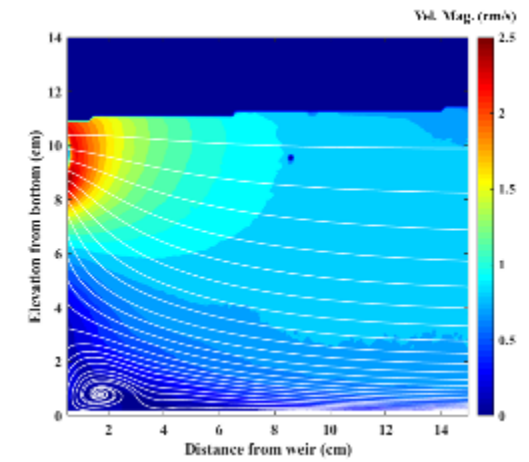
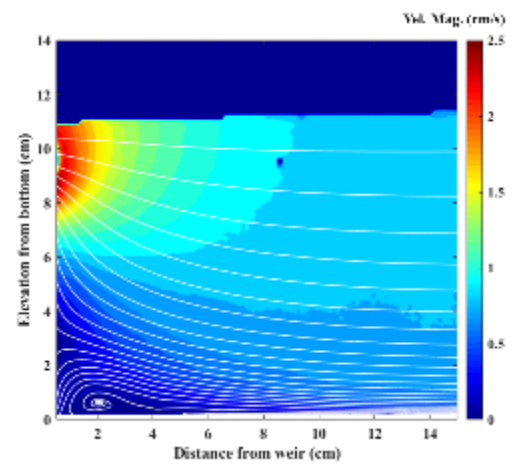
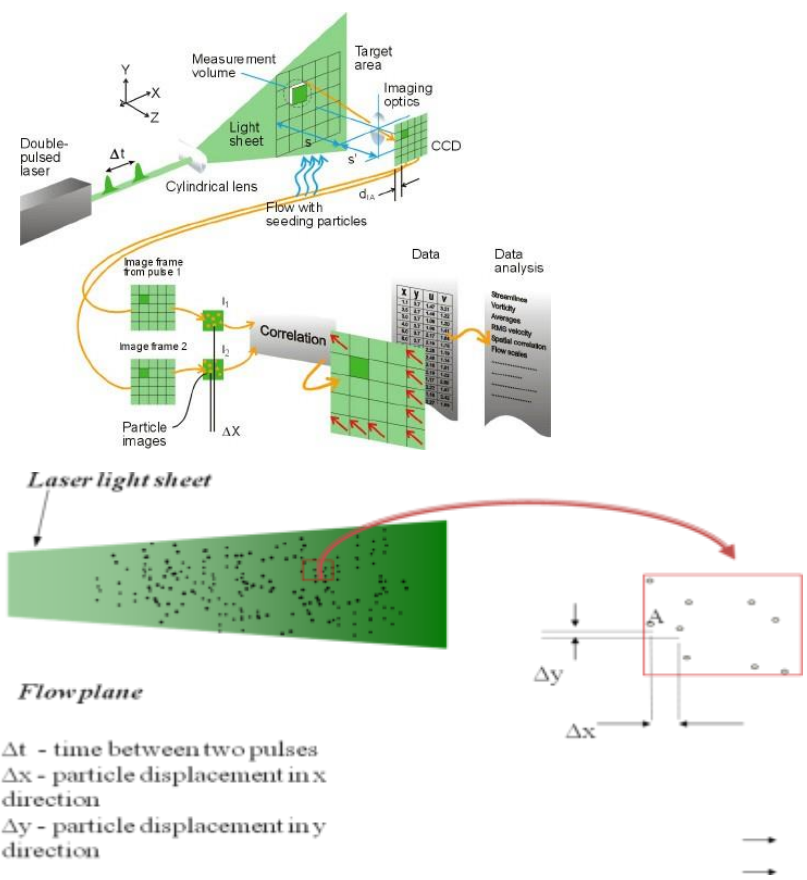
- Change of temperature affects the electric current flow or voltage drop through wire.
- Fine platinum wire (film) is heated electrically by a circuit that maintains voltage drop constant.
- When inserted into the stream, the cooling, which is a function of the velocity, can be detected as variations in voltage.





# 4) Laser techniques

## 1) PIV (Particle Image Velocimetry)





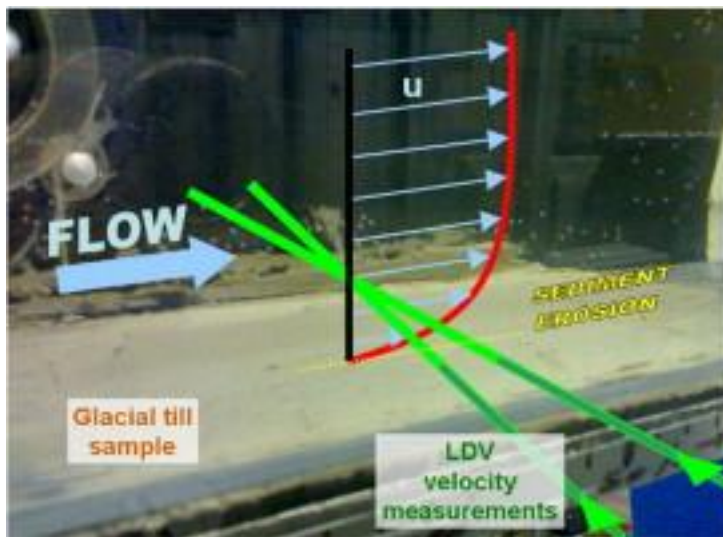
➤ Weir tests







## 2) LDV (Laser Doppler Velocimeter)





- ~ use Doppler effect
- ~ A laser (ultrasonic) beam transmitted into the fluid will be reflected by impurities or bubbles in the fluid to a receiving sensor at a different frequency.
- The transmitted and reflected signals are then compared by electronic means to calculate the Doppler shift which is proportional to the velocity.
- ~ non-intrusive sensing (immersible LDA)
- ~ sampling frequency is up to 20,000 Hz

$$F_{doppler} = -F_{source} \frac{V}{C} \quad (2.12)$$

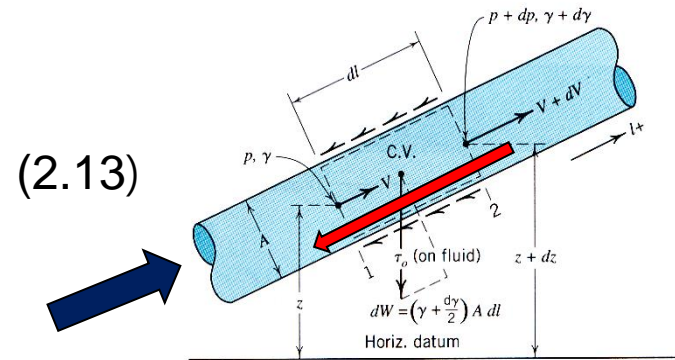


## 2.6 Shear measurements

- Shear determined by inference
  - No device has yet been invented which is capable of measuring the stress between moving layers of fluid.
  - measure wall shear  $\tau_0$
  - From  $\tau_0$  the shear between moving layers may be deduced using shear stress equations.
  - For a cylindrical pipe of uniform roughness, the wall shear can be obtained from pressure measurements as given below:

$$\tau_0 = \frac{\gamma d}{4l} \left( \frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2 \right) = \frac{\gamma d}{4l} h_L$$

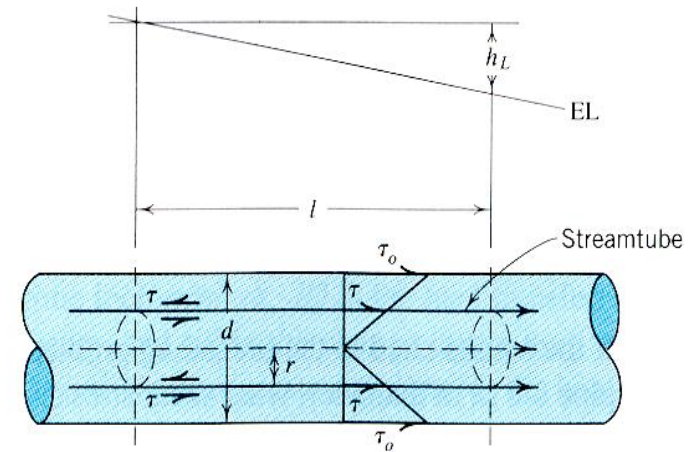
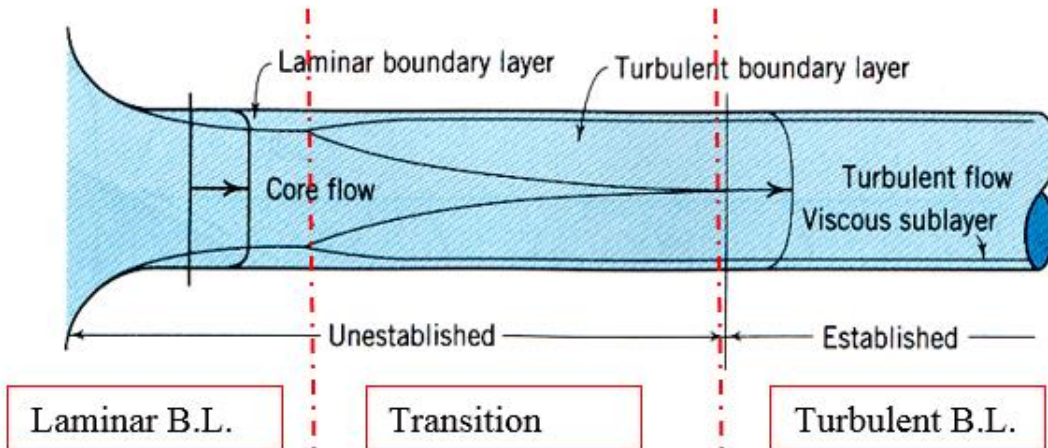
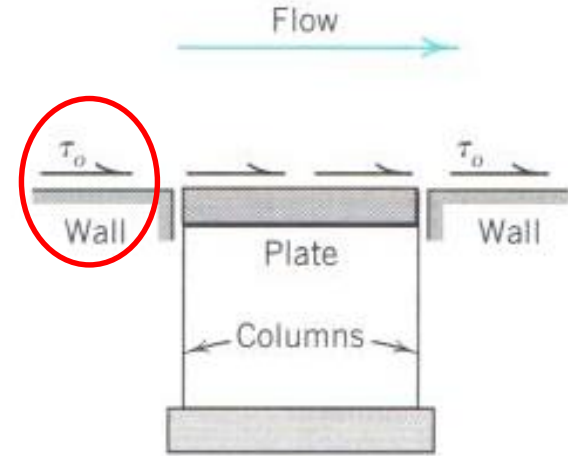
Head loss





# 1) Wall probes

- A movable plate mounted on elastic columns fastened to a rigid support
- The columns are deflected slightly by the shearing force of fluid on plate.
- This small deflection measured by strain gages, and the wall shear stress deduced from this deflection.



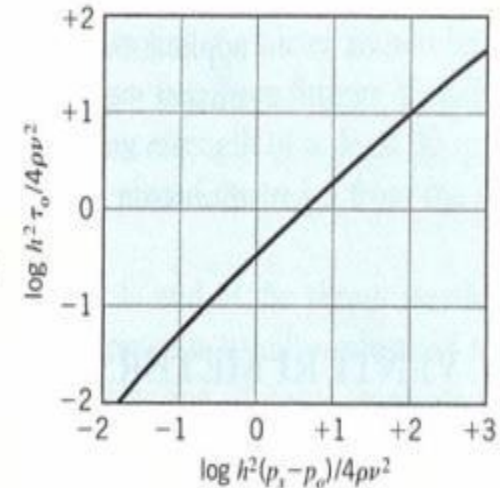
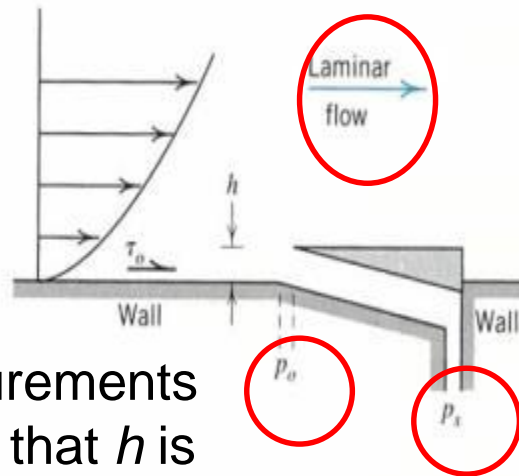


## 2) Wall Pitot tubes

- Stanton tube

- To measure wall shear, Stanton invented the wall pitot tube of which the wall forming one side of the pitot tube.
- Calibration in laminar flow and in the viscous sublayer of the turbulent pipe flow showed that

$$\frac{\tau_0 h^2}{4\rho\nu^2} = fn\left(\frac{(p_s - p_0)h^2}{4\rho\nu^2}\right) \quad (2.14)$$

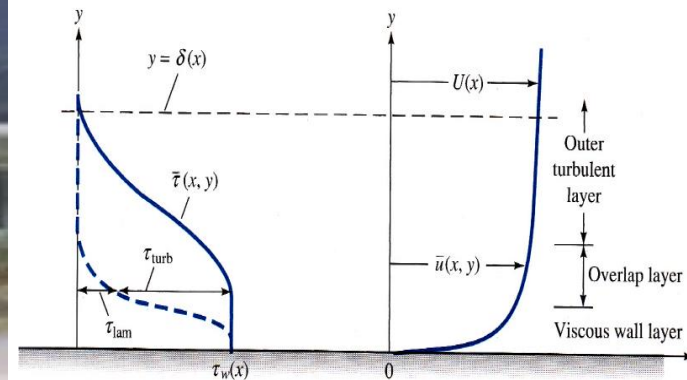
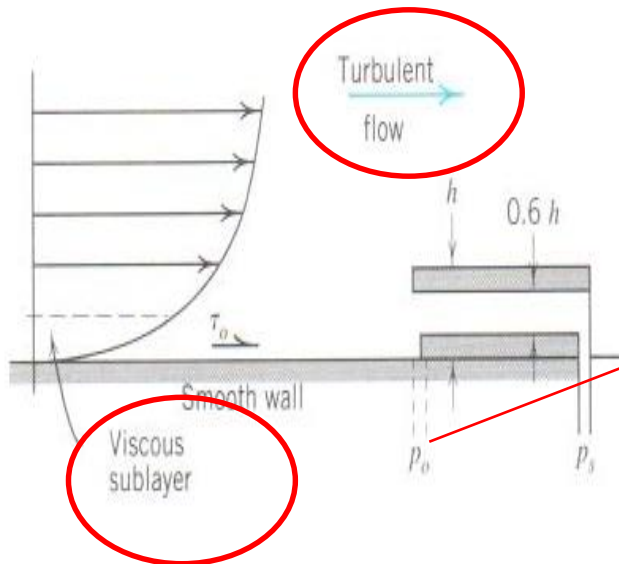


$\tau_0$  can be calculated from measurements of  $(p_s - p_0), \rho, \nu, h$  providing that  $h$  is smaller than the thickness of viscous region.



## - Preston tubes

- Preston applied idea to turbulent flow over smooth surface
- This tube is not submerged in the viscous sublayer
- Its performance depends on the similarity of the velocity profiles through the buffer zone between the viscous sublayer and the turbulent region.



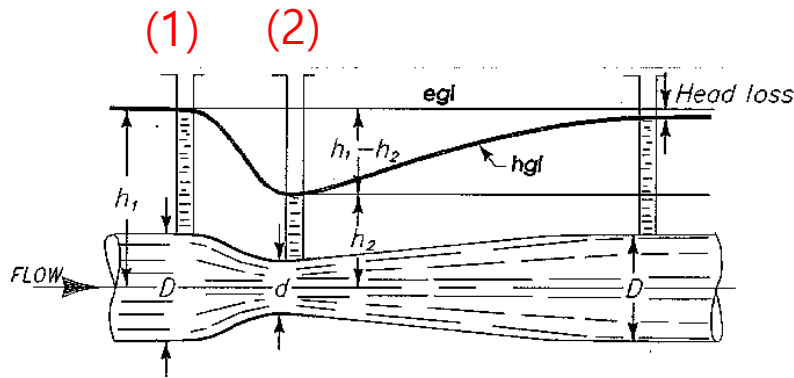




## 2.7 Flowrate measurement

### 1) Differential measurement system

#### i) Venturi meter



- Consider head loss at the constriction using Bernoulli equation

$$Q = \frac{C_v A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \sqrt{2g \left( \frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2 \right)} \quad (2.15)$$

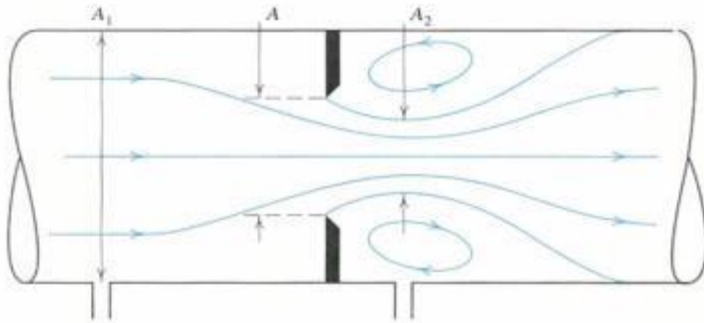
$C_v$  = coefficient for Venturi meter (Fig. 14.24)





## ii) Pipe orifice meter

- The minimum section occurs at section 2 (Vena contracta)



$$Q = \frac{C_v A_2}{\sqrt{1 - C_c^2 \left( \frac{A_2}{A_1} \right)^2}} \sqrt{2g \left( \frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2 \right)} = CA \sqrt{2g \left( \frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2 \right)} \quad (2.16)$$

$$C = \frac{C_v C_c}{\sqrt{1 - C_c^2 \left( \frac{A}{A_1} \right)^2}}$$

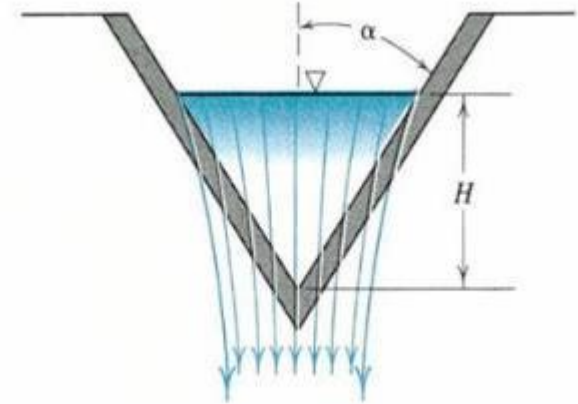
$C$  – Fig. 14.27



### 3) Weir

Weir- a regular obstruction over which flow occurs

- Spillway – overflow section of dam
- Sharp-crested weir
- Broad-crested weir
  - critical-depth meter





Consider energy equation through the strip

$$H + \frac{V_1^2}{2g} = (H - h) + \frac{v_2^2}{2g}$$

$$v_2 = \sqrt{2g \left( h + \frac{V_1^2}{2g} \right)}$$

$$q = \int_0^H v_2 dh = \frac{2}{3} \sqrt{2g} \left[ \left( H + \frac{V_1^2}{2g} \right)^{3/2} - \left( \frac{V_1^2}{2g} \right)^{3/2} \right]$$

$\frac{V_1^2}{2g}$  can be neglected since  $P \gg H$  and  $V_1$  is small

$$q \cong \frac{2}{3} \sqrt{2g} H^{3/2}$$

$$q = C_w \frac{2}{3} \sqrt{2g} H^{3/2}$$

(2.17)

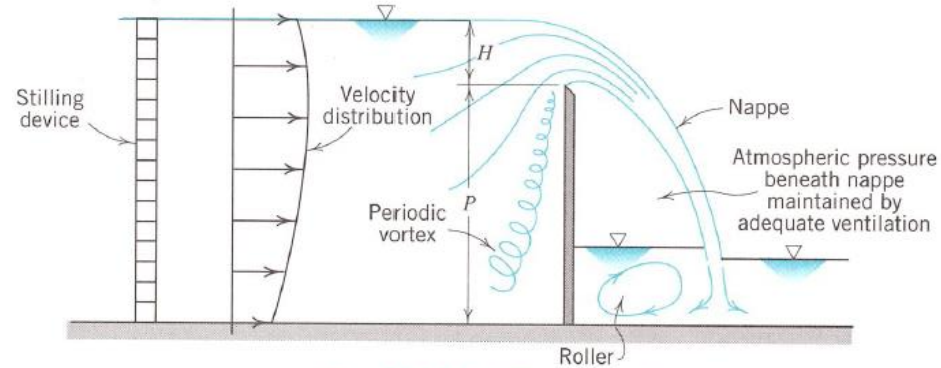
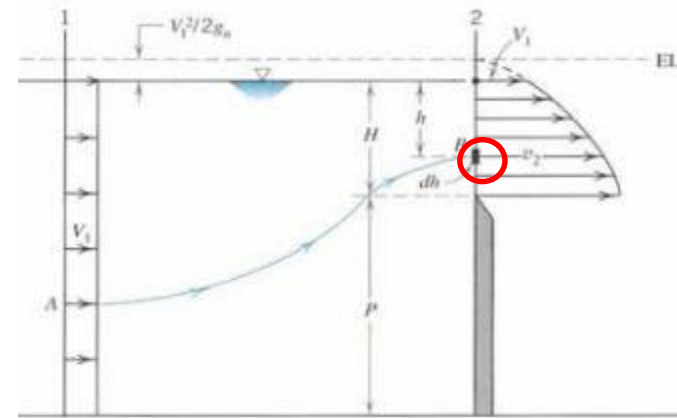


Fig. 14.35 Weir flow (actual).

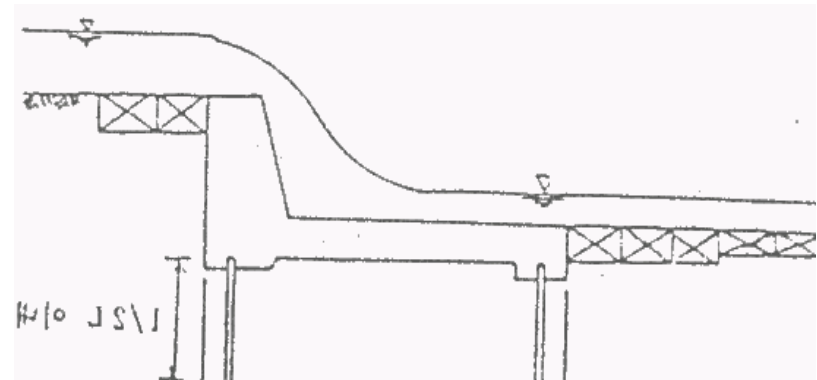
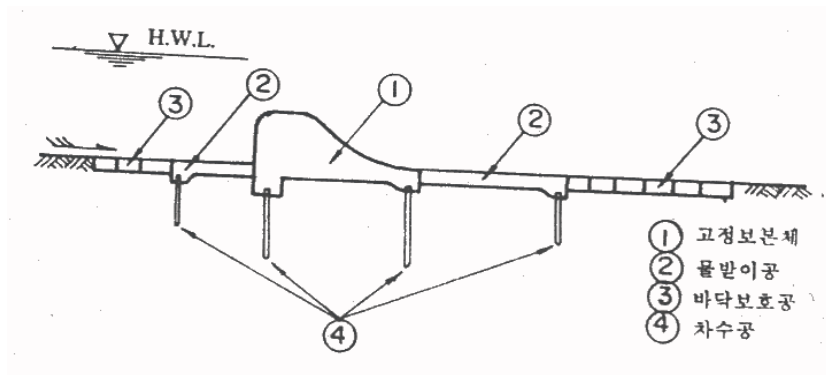


$C_w$  = weir coefficient



# Dam and Weir

- Large dam 대댐
- Small dam 소댐:  $H < 15m$
- Navigation dam 주운댐
- Weir: 수위 상승
- Drop structure 하상유지공(낙차공)





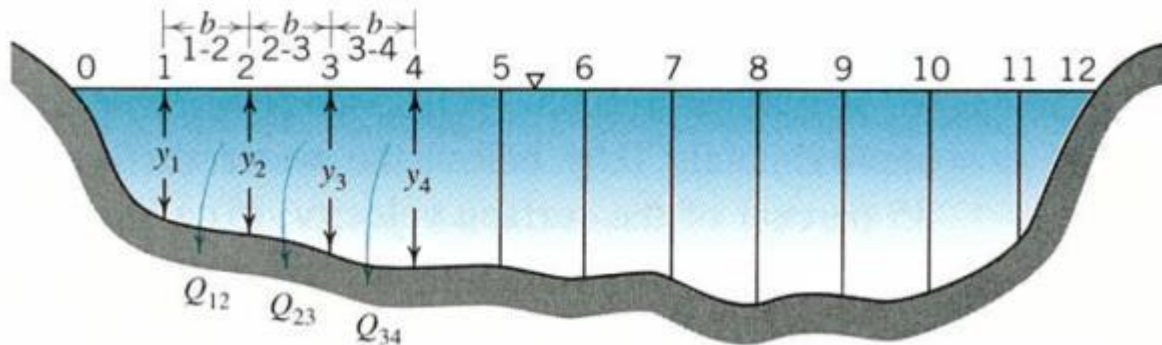
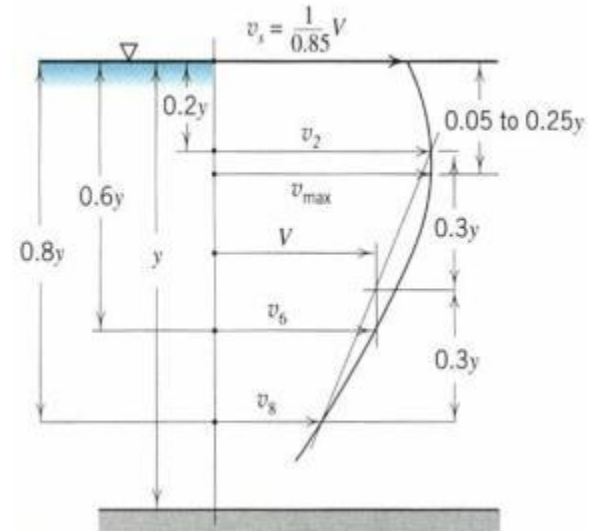


# Velocity-area method

The standard method of river flow measurement is to measure the velocity by means of a current meter and integrate the depth-averaged velocity to obtain the flowrate.

$$V = \frac{v_{0.2} + v_{0.8}}{2} \quad (2.18)$$

$$Q_{12} = b_{12} \left( \frac{y_1 + y_2}{2} \right) \left( \frac{V_1 + V_2}{2} \right) \quad (2.19)$$



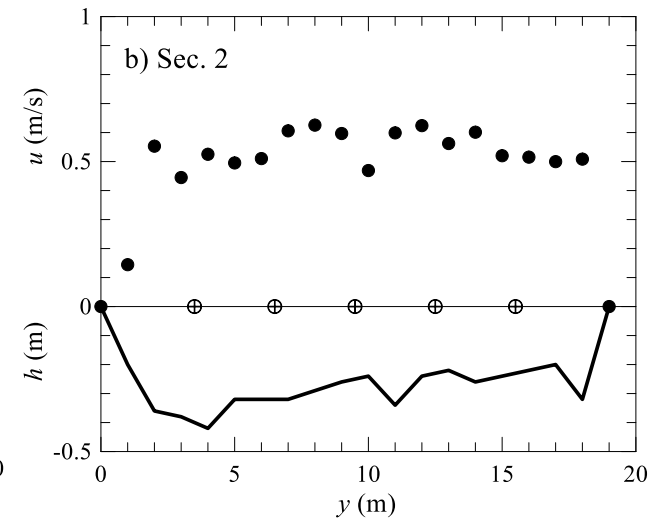
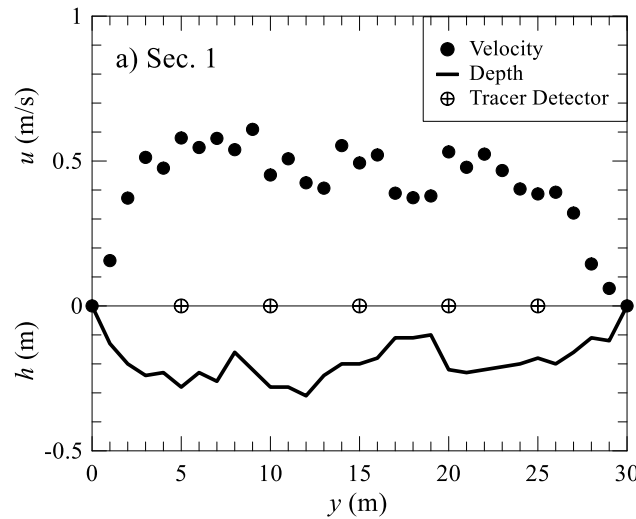


## Velocity profile

- measured by wading with ADV or by dragging boat-mounted ADCP
- Lateral distribution of velocity is obtained



Velocity measurements with ADV



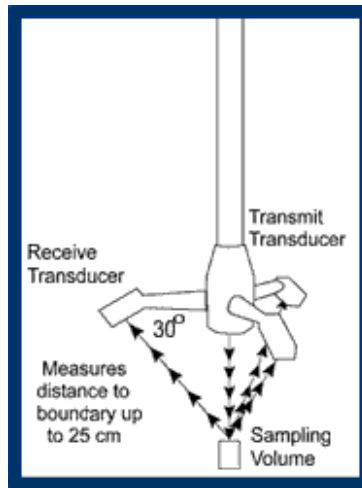
<Velocity profiles>



Acoustic Doppler Velocimeter

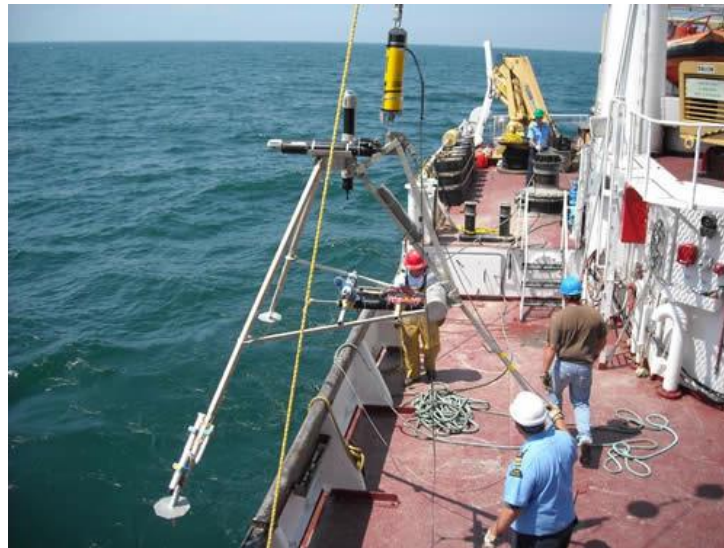
$$V = \frac{F_{Doppler}}{F_{source}} \times \frac{c}{2} \quad (2.20)$$

C = 물에서의 음속





- Acoustic Doppler Velocimeter





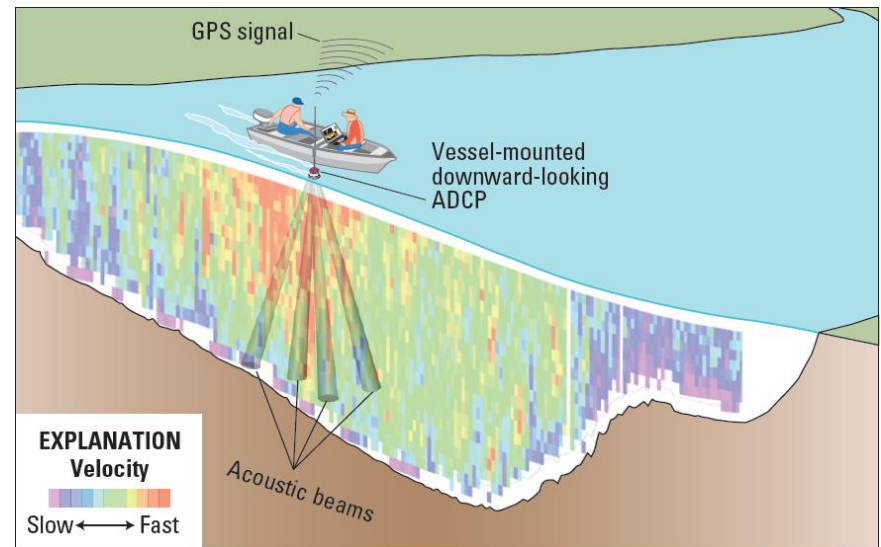


# ADCP (Acoustic Doppler Current Profiler)

ADCPs measure non-intrusively 3D velocity and bathymetry

~ applied Doppler principle by bouncing a pulse off particles

$$F_d = 2F_s \frac{V}{C_s} \quad (\text{two Doppler shift})$$





# ADCP



Figure 1.--StreamPro acoustic Doppler current profiler.

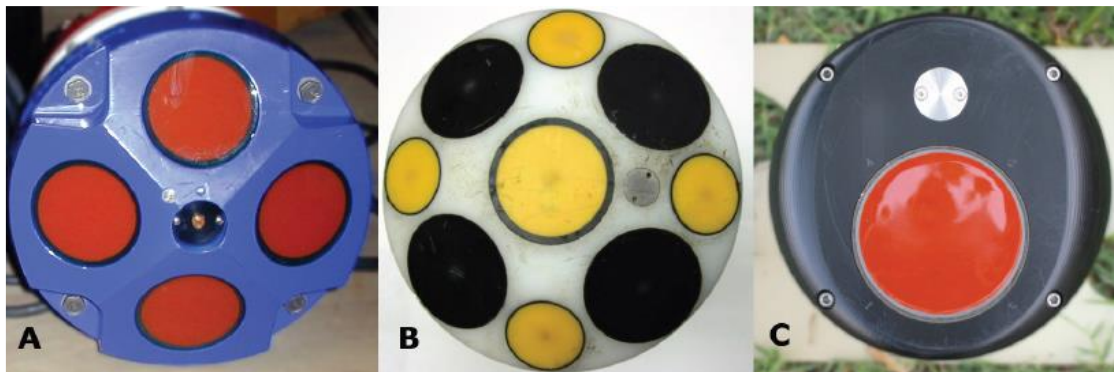


ADCP

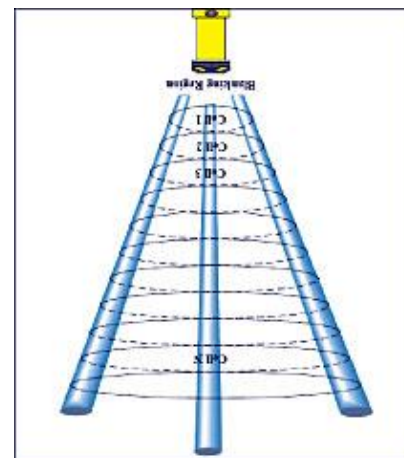
Global Positioning System (GPS) Antenna



# ADCP



**A: TRDI Rio Grande; B: Sontek M9; C: TRDI RiverRay**





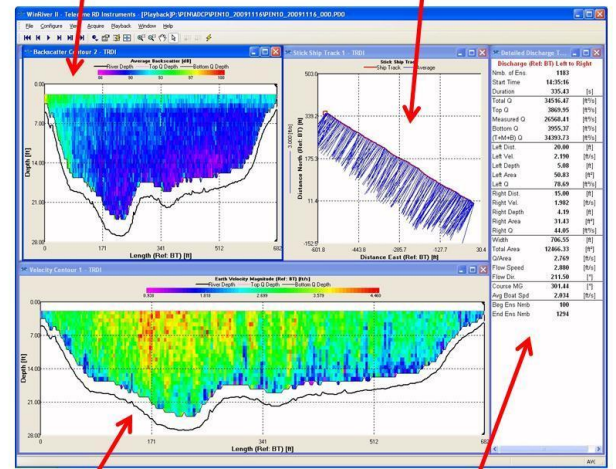


# ADCP



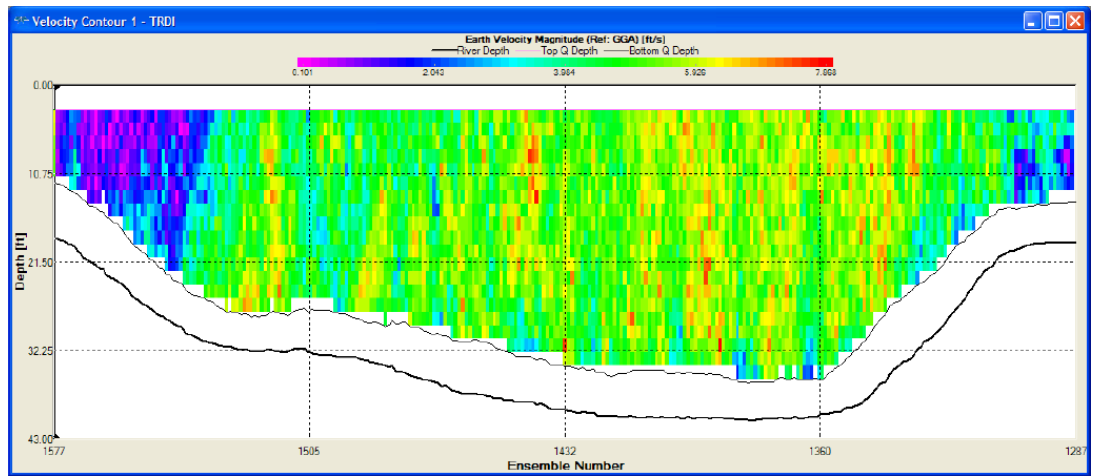
Acoustical backscatter contour plot.

Ship track with average velocity bars



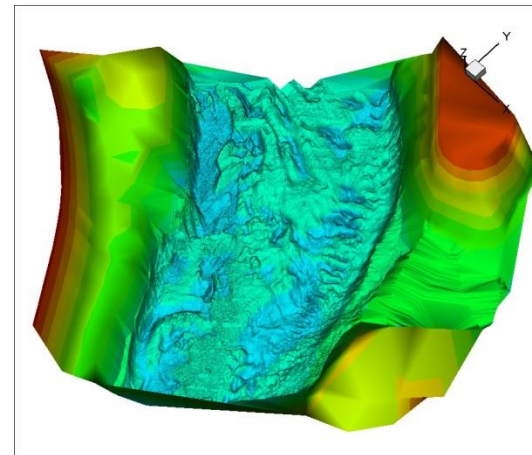
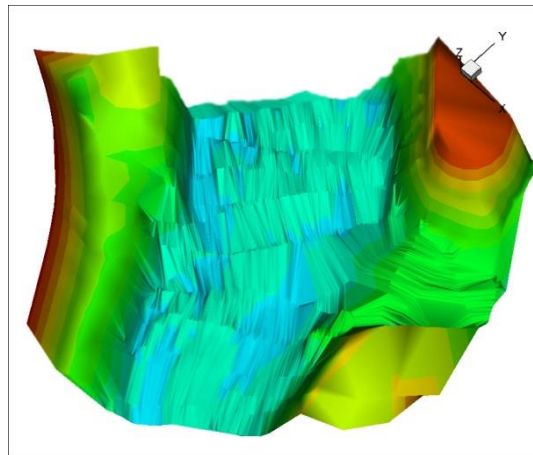
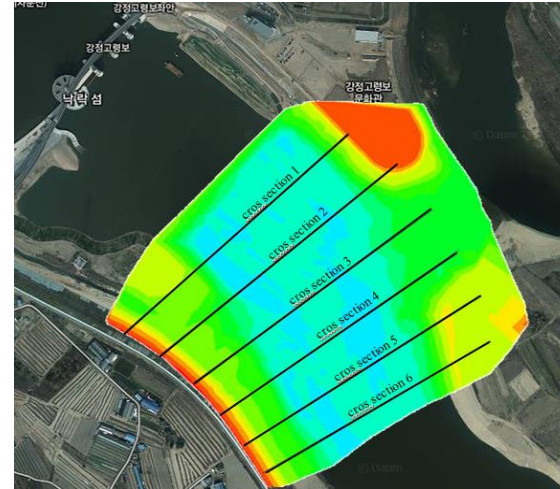
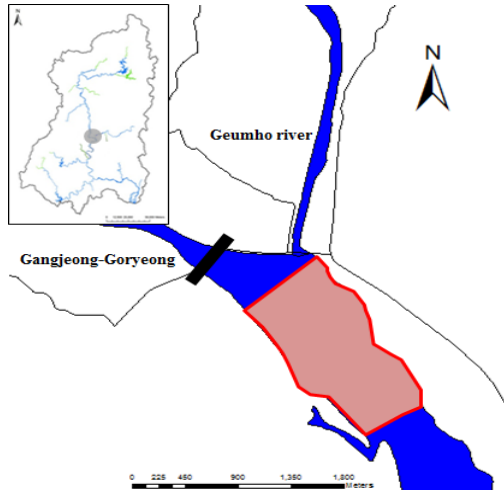
Water velocity magnitude contour plot

Tabular discharge summary table





# 2.8 River Topography

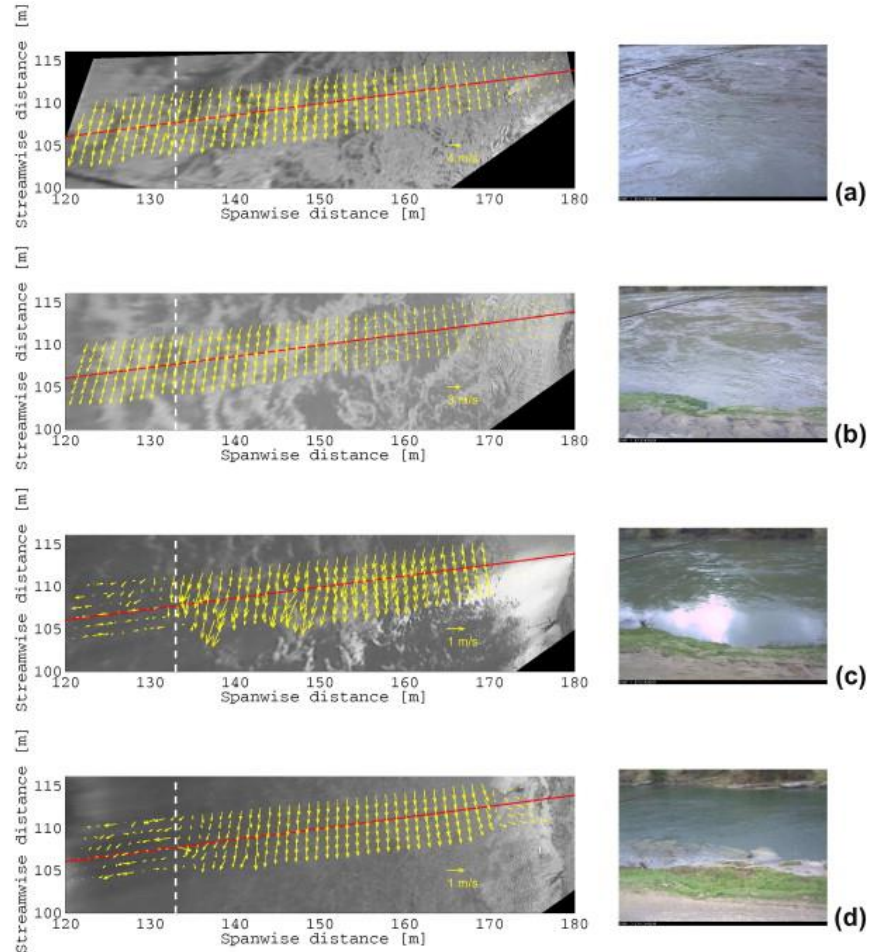
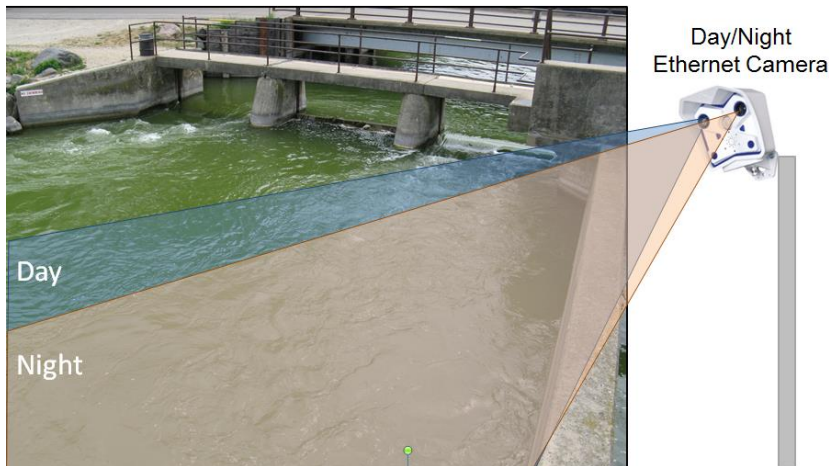


Channel topography measured by ADCP and MBES



## 2.9 Large scale measurement

- LSPIV
- ~ use CCD camera in rivers







# Remote measurements

- UAV (Unmanned Aerial Vehicle)
- Drone

~ aerial survey

~ river topography, velocity, water quality





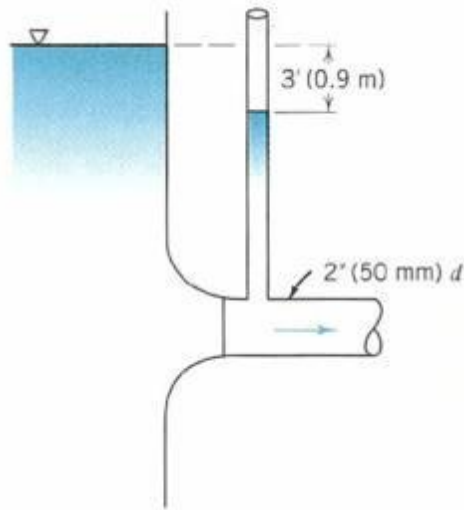


**Homework Assignment No. 1**  
**Due: 1 week from today**  
**Answer questions in Korean or English**

1. (14-8) If the torque required to rotate the inner cylinder of problem 1.69 at a constant speed of  $4 \text{ r/min}$  is  $2.7 \text{ N}\cdot\text{m}$ , calculate the approximate viscosity of the oil.
  
2. (14-37) A Preston tube of  $12.7 \text{ mm}$  outside diameter is attached to the hull of a ship to measure the local shear. When the ship moves through freshwater ( $20 \text{ }^\circ\text{C}$ ) the pressure difference is found to be  $3.6 \text{ kPa}$ . Calculate the local shear.



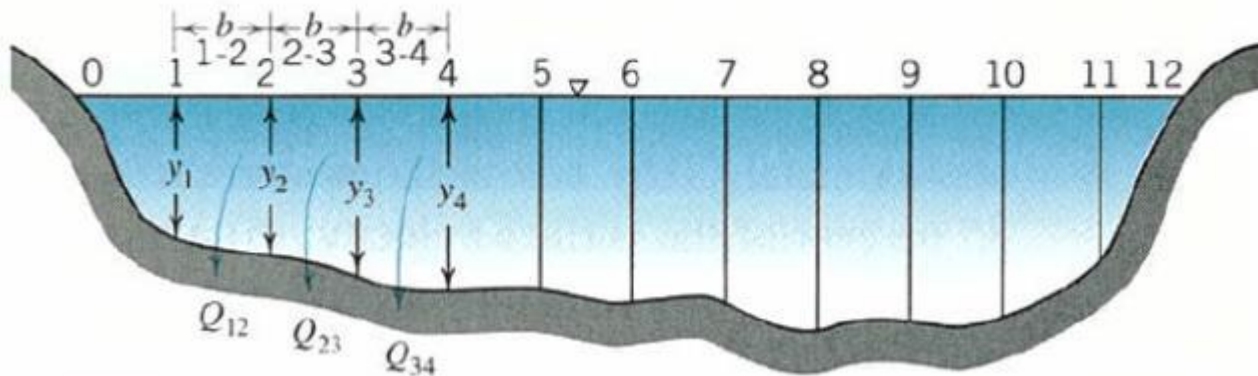
3. (14-61) Calculate the flowrate if  $c_v$  for this entrance nozzle is 0.96.



4. (14-82) A rectangular channel 5.4 m wide carries a flowrate of  $1.4 \text{ m}^3/\text{s}$ . A rectangular sharp-crested weir is to be installed near the end of the channel to create a depth of 0.9 m upstream from the weir. Calculate the necessary weir height.



5. (14-103) The following data are collected in a current-meter measurement at the river cross section of Fig. 14.44, which is 18 m wide at the water surface. Assume  $V[m/s] = 0.677 \times (r/s)$  and calculate the flowrate in the river.





Sec. No.	0	1	2	3	4	5	6	7	8	9	10	11	12
Depth (m)	0.0	0.9	0.96	1.05	1.08	1.11	1.17	1.20	1.32	1.32	1.26	1.05	0.0
rpm@ 0.2y	-	40.0	53.5	58.6	63.0	66.7	61.5	56.3	54.0	52.6	50.0	45.0	-
rpm@ 0.8y	-	30.7	42.8	50.0	54.2	58.8	53.3	49.4	46.5	43.2	40.1	32.5	-