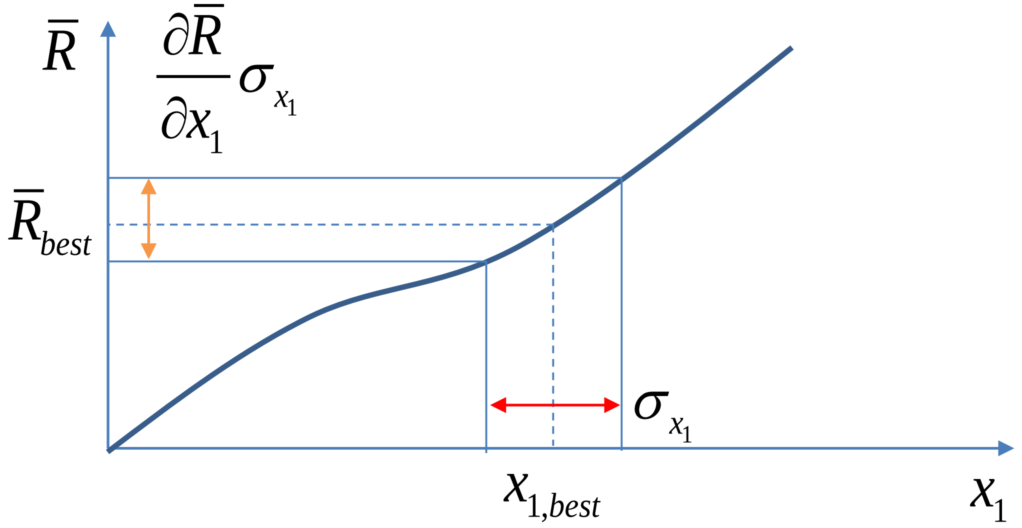
Ch. 3 Error Analysis



Contents

Today's objectives

- Understanding errors and uncertainties in experiments and measurements
- Introduce error analysis
- How to write technical report

3.1 Definition

- The measurement error always occurs in the experiments
- Potential impact of the measurement error in experiments is called "***uncertainty***" (the range in which true value exist)
- When applied to engineering, estimate uncertainty by comparing measured values with exact solution from theory
- Error (uncertainty) analysis consists of steps of
  - estimating each step's measurement error
  - estimating uncertainty propagated into final results

- Measured value
  - Measure physical quantities several times
  - Best estimate would be the average of measured results

$$x_{mea} = x_{best} \pm \delta$$

불확실도 = 오차의 범위

x_{best} = best estimate (최량 추정값)

δ = margin of error, uncertainty

  - True value cannot be obtained

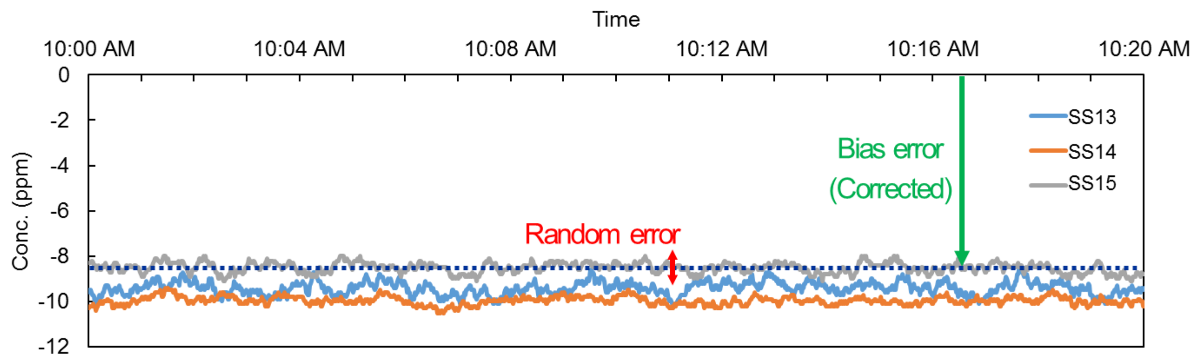
$$x_{best} - \delta < x_{mea} < x_{best} + \delta$$

- Total error (δ) :
  - Difference between measured and true value
  - Summation of system error and random error

$$\delta_i = \beta + \epsilon_i$$

β = system (bias) error (계통오차) → Calibration

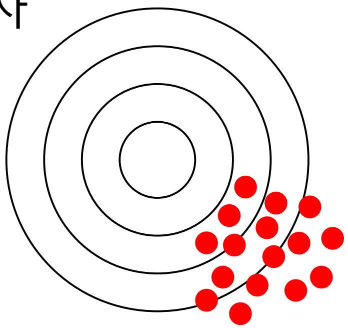
ϵ_i = random (precision) error (랜덤오차) → Statistical approach



▪ Accuracy

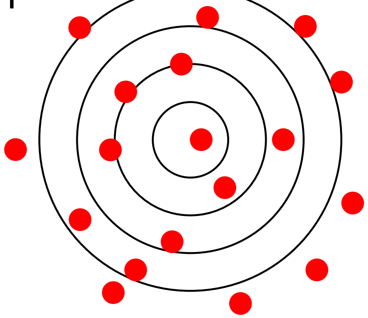
- Biased, Precise → Inaccurate

계통오차



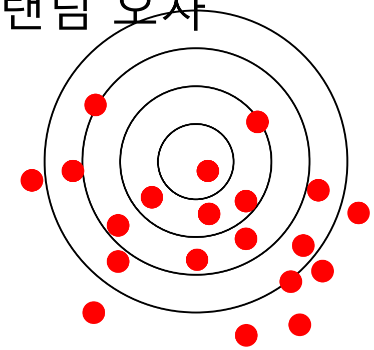
- Unbiased, imprecise → Inaccurate

랜덤오차

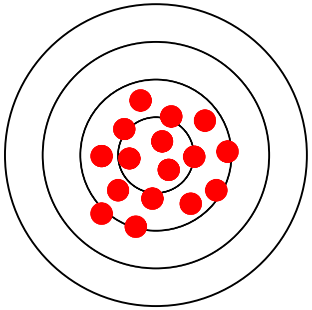


- Biased, imprecise → Inaccurate

계통+랜덤 오차



- Unbiased, Precise → Accurate

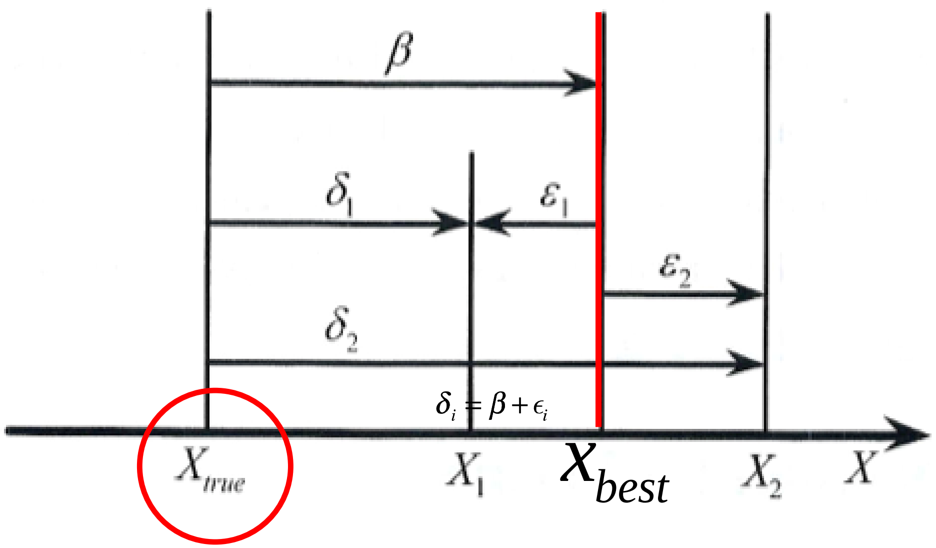


- System (bias) error
  - Determinate error, systematic error, deviation
  - Constant of total error
  - Causes: ① Errors in calibration, ② Data acquisition, and ③ Data removal process
  - ① Most of equipment undergoes calibration process before measurement, which can lead to bias errors
  - ② In the acquisition of data, environmental factors such as temperature and humidity are different from the calibration process, resulting in bias errors
  - ③ In the process of fitting, rounding, and discarding are used to reduce the size of the data, numerical errors occur

- Random (precision) error

  - Indeterminate error
  - Random error is caused by the fact that the random noise, resistance or friction of the electricity behaves differently, and that the steady state cannot be maintained
  - Also, measurement results of repeated runs under the same experimental conditions are slightly different
  - Because of its random nature, it expresses precision errors in a stochastic or statistical method

- True value



- In order to obtain X_{true}, measurements are conducted
- There are two results, X_1 *and* X_2.
- Two results have the same bias errors (β)
- Random errors (ε_i) can be different
- Total error is summation of bias and random error

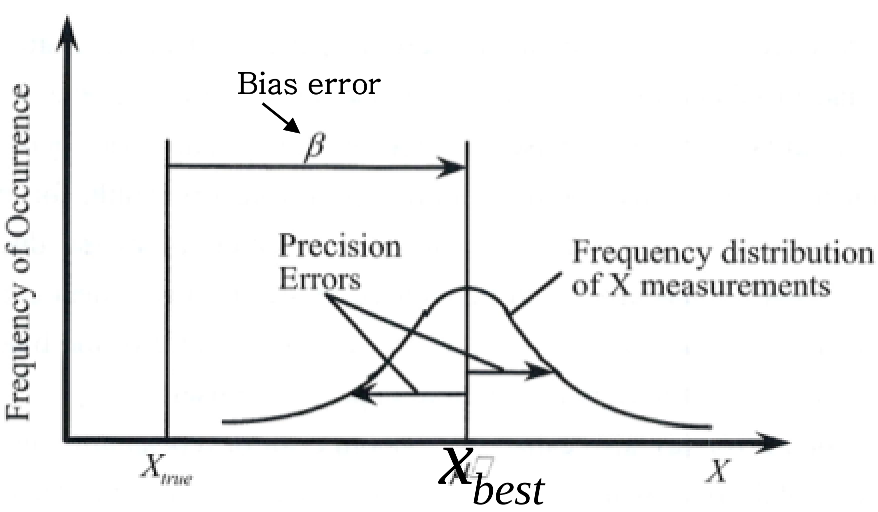
$$\delta_i = \beta + \epsilon_i$$

- Best estimate

– When the number of measurement increases into *N*, bias error is difference between <u>average of measured values</u> (x_{best}) and true value (x_{true})

– Random errors are distributed around the mean value

– However, true value (x_{true}) and bias error are unknown, we estimate <u>confidence interval</u> in which measured value is existing between



$$x_{best} \pm u_x$$

where x_{best} is average of *N* measured value; u_x is summation of bias and precision error under <u>C % confidence level (믿음한계)</u>

Distribution of random error

Distribution of N measurements → Normal distribution

$$G_{X,\sigma}(x) = \frac{1}{\sigma_x\sqrt{2\pi}}\exp(-(x-X)^2/2\sigma_x{}^2)$$

$$\sigma_x = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2}$$

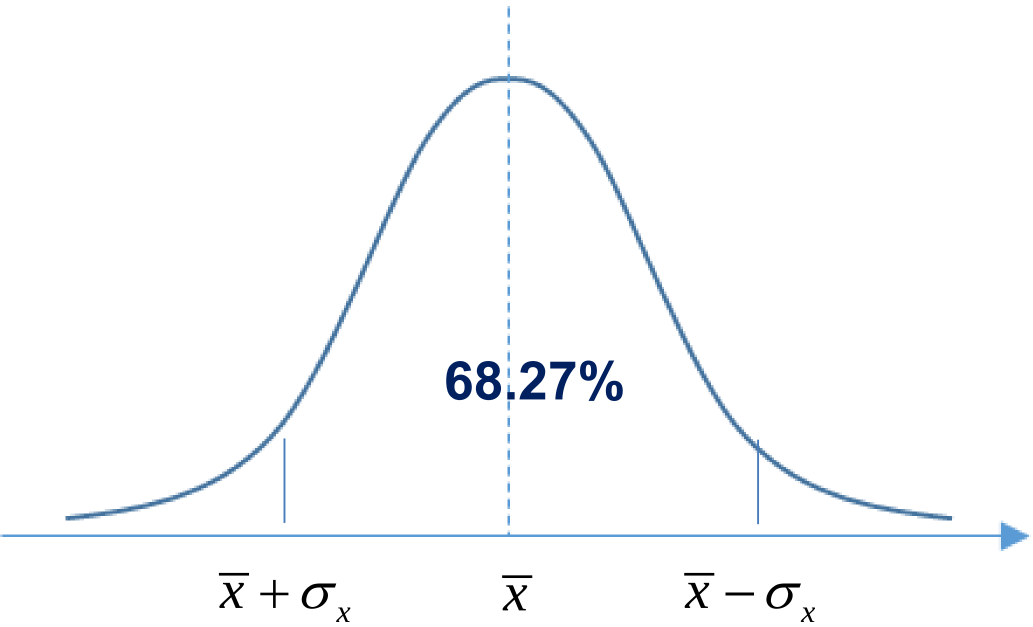
X = **true value**

$$x = x_{best} \pm \delta = \bar{x} \pm \sigma_x$$

→ 68% of measured values are in this range

$$x = x_{best} \pm \delta = \bar{x} \pm 2\sigma_x$$

→ 95% of measured values are in this range



3.2 Analysis of Error

- In experiment, we estimate values we want by measuring several quantities which can be measurable and then calculating values we want using measured variables
- For example, it is difficult to measure viscosity directly from experiment. On the other hand, discharge and diameter and length of pipe can be easily measured. Then, we can use Hagen-Poiseuille law for laminar equation to estimate viscosity.

$$Q = \frac{\pi d^4 \gamma h_L}{128 \mu l} \qquad \mu = \frac{\pi d^4 \gamma h_L}{128 Q l}$$

- Uncertainty of estimated density occurs since uncertainty of measured quantities propagate into the estimated viscosity ➡ Uncertainty analysis

- Suppose that a set of data is obtained and a result *R* is calculated from the data.
- The result *R* is a function of all the bits of measured data x_i, and is written as

$$R = f(x_1, x_2, x_3, \ldots., x_N)$$

- The *N* bits of measured data are designated by x_i, and the best estimate of a bit of data is $\overline{x}_i$, and the estimated uncertainty bound is σ_{x_i} , then

$$x_i = \overline{x}_i + \sigma_{x_i}$$

Error propagation 오차의 전파

- The relative uncertainty in the *i*th bit of data is $\sigma_{x_i} / \overline{x_i}$

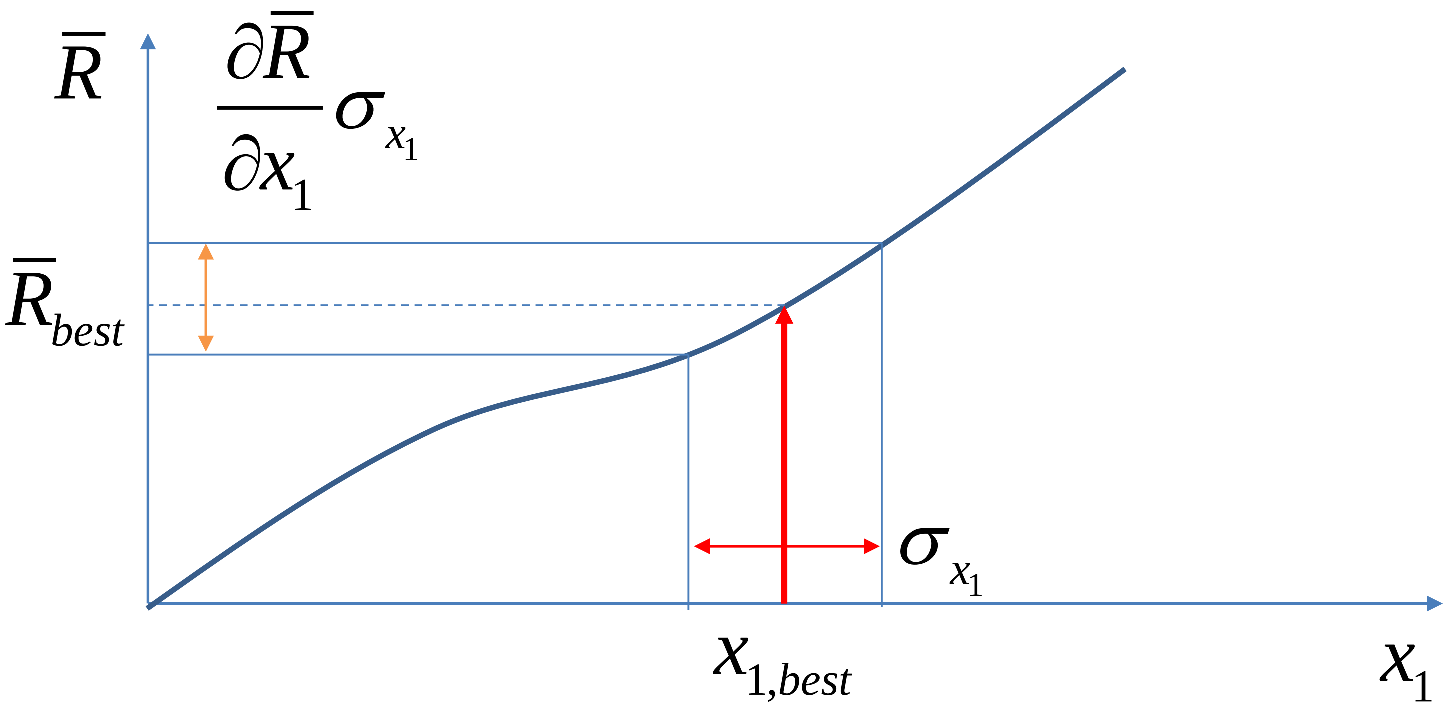
- The effect on *R* of the uncertainty in the *i*th bit of data is $\frac{\partial R}{\partial x_i}\sigma_{x_i}$

- The uncertainty in *R*, σ_R, is given by

$$\sigma_R = \pm\left\{\left(\frac{\partial R}{\partial x_1}\sigma_{x_1}\right)^2 + \left(\frac{\partial R}{\partial x_2}\sigma_{x_2}\right)^2 + \left(\frac{\partial R}{\partial x_3}\sigma_{x_3}\right)^2 \cdots + \left(\frac{\partial R}{\partial x_N}\sigma_{x_N}\right)^2\right\}^{1/2}$$

- The relative uncertainty (상대불확실도)

$$U_R = \frac{\sigma_R}{\overline{R}}$$


$\bar{R}$
$\frac{\partial \bar{R}}{\partial x_1}\sigma_{x_1}$
$\bar{R}_{best}$
σ_{x_1}
$x_{1,best}$
x_1

▪ Example (IP 14.2)

- For the U-tube water and mercury (Hg) manometer setup shown in the figure, find the absolute and relative uncertainties of the pressure measured at A, if the following uncertainties are known:

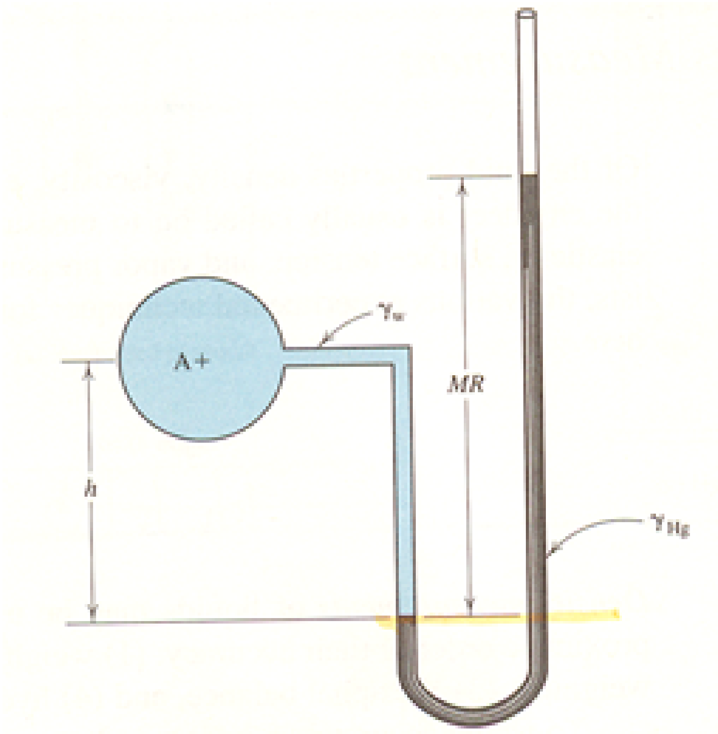
$\gamma_w = 9.798 \pm 0.002$ kN/m^3

$\gamma_{Hg} = 132.97 \pm 0.0245$ kN/m^3

$MR = 0.300 \pm 0.005$ m

$h = 1.200 \pm 0.0025$ m

$\bar{x}_i$ $\quad$ σ_{x_i}



Solution

- Based on the approach to manometry shown in Chapter 2,

$$p_A + \gamma_W h - \gamma_{Hg} MR = 0;$$

$$p_A = \gamma_{Hg} MR - \gamma_W h.$$

- Thus,

$$\frac{\partial p_A}{\partial \gamma_W} = -h; \quad \frac{\partial p_A}{\partial \gamma_{Hg}} = MR; \quad \frac{\partial p_A}{\partial MR} = \gamma_{Hg}; \quad \frac{\partial p_A}{\partial h} = -\gamma_W$$

- Now, we have

$$\sigma_{p_A} = \pm \left\{ \left(\frac{\partial p_A}{\partial \gamma_W} \sigma_{\gamma_W} \right)^2 + \left(\frac{\partial p_A}{\partial \gamma_{Hg}} \sigma_{\gamma_{Hg}} \right)^2 + \left(\frac{\partial p_A}{\partial MR} \sigma_{MR} \right)^2 + \left(\frac{\partial p_A}{\partial h} \sigma_h \right)^2 \right\}^{1/2}$$

$$= \pm \left\{ \left(-h\sigma_{\gamma_W} \right)^2 + \left(MR\sigma_{\gamma_{Hg}} \right)^2 + \left(\gamma_{Hg}\sigma_{MR} \right)^2 + \left(-\gamma_W \sigma_h \right)^2 \right\}^{1/2}$$

- Introducing the numerical values produces

$$\sigma_{p_A} = \pm\{(-1.2\times 0.002)^2 + (0.300\times 0.0245)^2 + (132.97\times 0.005)^2 + (-9.798\times 0.0025)^2\}^{1/2}$$
$$= \pm 0.665 \text{ kPa}$$

$$p_A = 132.97\times 0.3 - 9.798\times 1.2 = 39.891 - 11.758 = 28.133 \pm 0.665 \text{ kPa}$$

$$U_{p_A} = \pm\frac{\sigma_{p_A}}{p_A} = \pm 0.024 \approx 2.4\%$$

유효숫자 또는 소수점 자리를 동일하게 함

3.3 Technical Writing

Contents of Report

(1) Title

(2) Abstract (including keywords)

(3) Introduction – necessity and objective of research

(4) Body- research trends, theoretical backgrounds, experiments, numerical simulations

(5) Conclusion – summary, findings

(6) Reference

(7) Appendix

- Remarks:

1) Spacing between lines should be 1-1/2 space (160%).
2) Introduce only significant governing equation and BC& IC.
3) Introduce only significant pictures and graphs.
4) Unimportant pictures and graphs should be included in the appendix.
5) Show your solutions or results with three significant figures.

Homework Assignment No. 2
Due: 1 week from today
Answer questions in Korean or English

1. (14-1) The viscosity of a fluid may be determined by measuring the head loss over a section of a tube in which a laminar flow is occurring. If the head loss h_L and the flowrate Q are known to within $\pm 5\%$ at a 95% Confidence level and the specific weight γ, diameter d, and section length l are known within $\pm 1\%$, what is the error bound on the viscosity μ?

2. (14-32) What is the error bound for the velocity measured by a sonic anemometer, according to Eq. 14.12, if the distance d is known to within $\pm 1\%$, the speed of sound to within $\pm 2\%$ and the travel-time difference to within $\pm 0.5\%$?