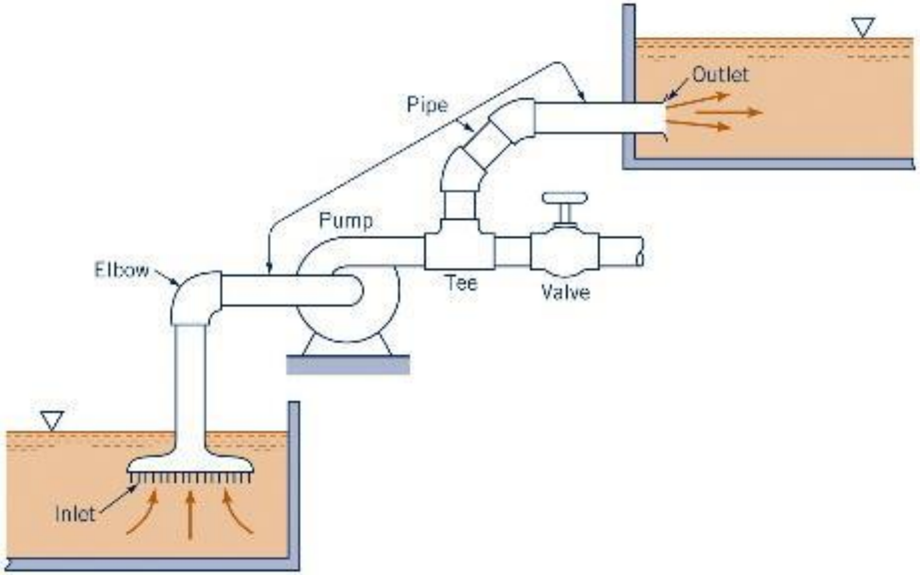
Lecture 7

Pipe Problems (1)



Contents

Objectives

- Analyze practical problems of pipe flow
- Extend the single pipe problem to the pumped pipeline

Review

- Fundamental equations
  - Based on impulse-momentum equation for incompressible flow, we got the relationship between shear stress and head loss
- Laminar flow
  - We derived equations of velocity profile and friction factor for laminar flow
- Turbulent flow
  1) We derived velocity profile for all pipe from Prandtl's theory and Nikuradse's experimental data.
  2) We learned smooth pipes and got velocity profiles near pipe wall.
  3) Using velocity profile for smooth pipes, we derived friction factor which only can be solved by trial-errors.
  4) We study velocity profile for rough pipe cases considering roughness height, and we also derive the friction factor equation.
  5) We learned how to figure out whether a pipe has smooth or rough wall.
- Engineering problem
  - To solve more practical problems with commercial pipes, we learned Moody diagram, and methods for non-circular pipes and empirical formula.

7.1 Classification of Pipeline Problems

- All steady-flow pipe problems may be solved by application of the work-energy equation for head loss and continuity equation.

→ Most effectively by the construction of the energy and hydraulic grade lines

- **Engineering pipe-flow problems** – commercial pipe
  1) Calculation of **head loss, h_L,** and pressure variation
  2) Calculation of **flow rate, Q**
  3) Calculation of **size of pipe, d**

Single pipe-flow problems

1. Type 1: Find h_L & Δp for given Q, d, e → IP. 9.10

   Explicit solution is possible

   Find f using Moody diagram and then use Darcy-Weisbach equation to calculate h_L

$$h_L = f \frac{l}{d} \frac{V^2}{2g_n}$$

2. Type 2: Find Q for given d, e, H

   Explicit solution is not possible → trial & error method

   friction factor $f = fn(\mathrm{Re}) = fn(Q)$

3. Type 3: Find d for given Q, Δp

   Explicit solution is not possible → trial & error method

   friction factor $f = fn(\mathrm{Re}) = fn(d)$

7.2 Two Reservoirs Problems

Apply energy equation

$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g_n} = z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g_n} + h_L$$

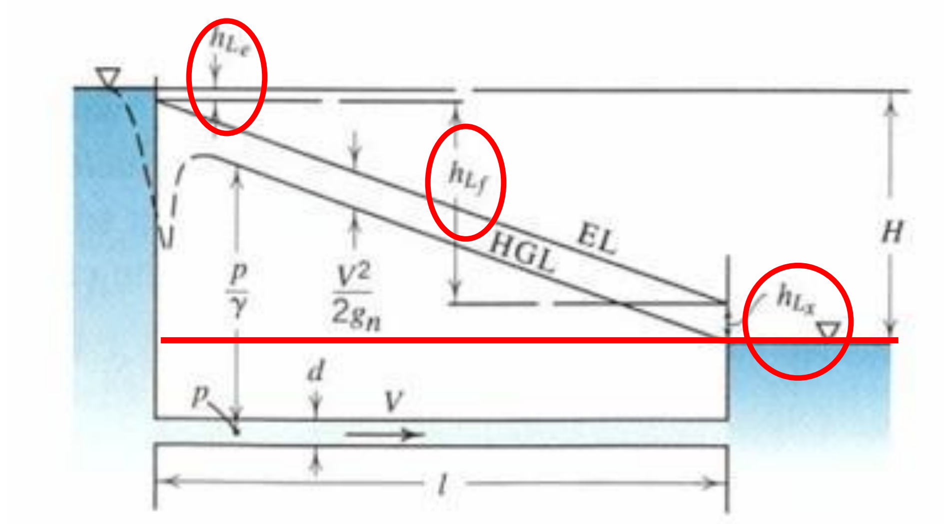
$$z_1 = H; z_2 = 0; p_1 = p_2 = 0; V_1 = V_2 \approx 0$$

$$h_L = h_{L_e} + h_{L_f} + h_{L_x}$$

h_{L_e} = entrance loss; h_{L_x} = exit loss; h_{L_f} = pipe friction loss

$$\boxed{H = h_L = \left(0.5 + f\frac{l}{d} + 1\right)\frac{V^2}{2g_n}}$$

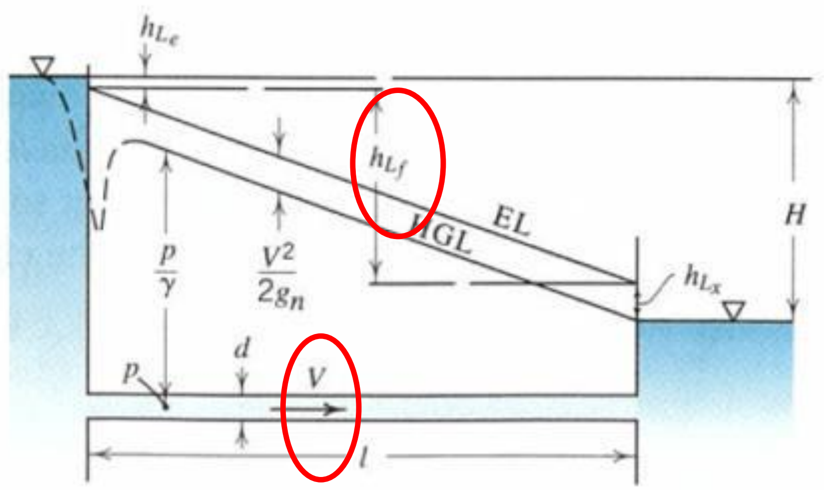
(7.1)



- Usually, *l* is much longer than *d*, *l/d* = 1,000, in this case, we can assume *f* to be 0.03. In such case, *f* ·*l/d* ~30, and this is almost 30 times the sum of other terms.

- Therefore, as length of pipe increases, we can ignore the effect of local losses.
- Then we can simplify the equation

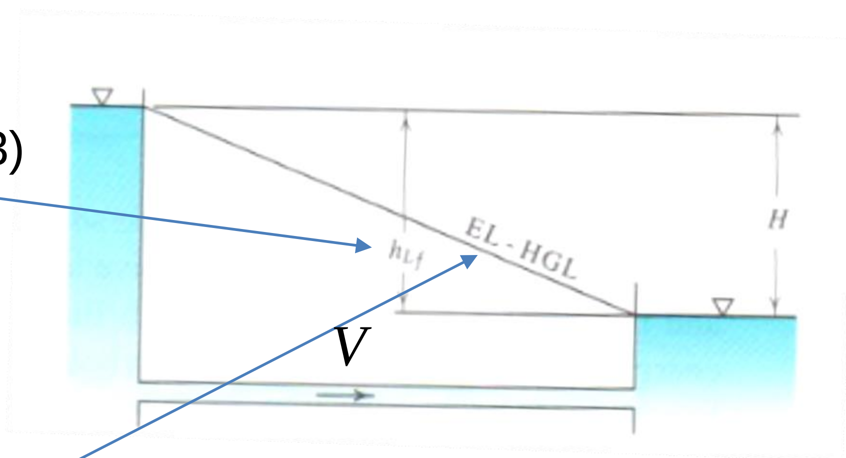
$$H = h_L \approx h_{Lf} = f\frac{l}{d}\frac{V^2}{2g_n} \quad (7.2)$$



- As length of pipe increases, we can ignore the velocity head in the energy equation.

→ Making energy and hydraulic grade lines coincident

$$h_L \approx h_{Lf} = f\frac{l}{d}\frac{V^2}{2g_n} \qquad (7.3)$$



Ignore $\dfrac{V^2}{2g_n}$

IP 9.15 (p.368-9)

- A clean cast iron pipeline 0.30 m in diameter and 300 m long connects two reservoirs having surface elevations 60 m and 75 m. Calculate the flowrate through this pipe, assuming water at 10°C and a square-edged entrance.

IP 9.15 (p.368-9)

- Calculation Procedure for Type 2 problem:
  1. Calculate Reynolds number with assumed value of velocity in pipe (V_1)
  2. Determine *e/d* and friction factor (use **Moody diagram**)
  3. Calculate the local losses and whole friction loss
  4. Then apply work-energy eq.
  5. From this equation, determine pipe velocity (V_2).
  6. If (V_2- V_1) is larger than error limit, recalculate Reynolds number.
  7. Then, obtain new friction factor from **Moody diagram using** V_2
  8. With new *f* value, apply work-energy eq.to calculate again velocity (V_3).
  9. Repeat this procedure until $\Delta V < \varepsilon$

$$H = h_L = \left(0.5 + f\frac{l}{d} + 1\right)\frac{V^2}{2g_n}$$

$$\mathrm{Re} = \frac{Vd}{\nu} = \frac{V \times 0.3}{1.306 \times 10^{-6} m^2 / s} = 229,000V$$

Guess $V \approx 2m/s,$

$$\mathrm{Re} = 458,000$$

- From Fig. 9.11, for clean cast iron pipe, *e/d* = 0.00083.
- *e/d* and Reynolds number give friction factor as 0.02 from Moody diagram.
- Now apply work-energy eq. with energy loss term consisting of friction loss and minor losses

$$z_1 + \frac{p_1}{g} + \frac{V_1^2}{2g_n} = z_2 + \frac{p_2}{g} + \frac{V_2^2}{2g_n} + h_L$$

$$z_1 + \frac{p_1}{g} + \frac{V_1^2}{2g_n} = z_2 + \frac{p_2}{g} + \frac{V_2^2}{2g_n} + \left(0.5 + f\frac{l}{d} + 1\right)\frac{V^2}{2g_n}$$

$$75 + 0 + 0 = 60 + 0 + 0 + \left(0.5 + 0.02\frac{300}{0.3} + 1\right)\frac{V^2}{2 \times 9.81} = 1.096V^2$$

$$V = 3.70\,m/s$$

- Now with this velocity, recalculate Reynolds number (847,250) and *f* (0.0193). Then we get *V=3.76m/s*.

$$\Delta V = 3.76 - 3.70 = 0.06 \sim small\, enough$$

$$Q = AV = \frac{\pi}{4}(0.3)^2(3.76) = 0.266\,m^3/s$$

IP 9.16 (p. 370)

- A smooth PVC pipeline 200 ft long is to carry a flowrate of 0.1 ft^3/s between two water tanks whose difference in surface elevation is 5 ft. If a square-edged entrance and water 50°F are assumed, what diameter of pipe is required.

  - Type 3 problem represents the most difficult to solve because the unknown diameter renders both the Reynolds number and the relative roughness as unknown.
  - But, in case of smooth pipe, the relative roughness is irrelevant, thus we can use the smooth pipe line on the Moody diagram.

- Sol)

$$\mathrm{Re} = \frac{Vd}{\nu} = \frac{Qd}{A\nu} = \frac{0.1d}{\pi/4\, d^2 (1.41\times10^{-5}\ ft^2/s)} = \frac{9,020}{d} \qquad \text{(A)}$$

- Apply work-energy eq. with energy loss term consisting of friction loss and minor losses

$$5 = \left(0.5 + f\frac{200}{d} + 1\right)\frac{V^2}{2g_n} \qquad \text{(B)}$$

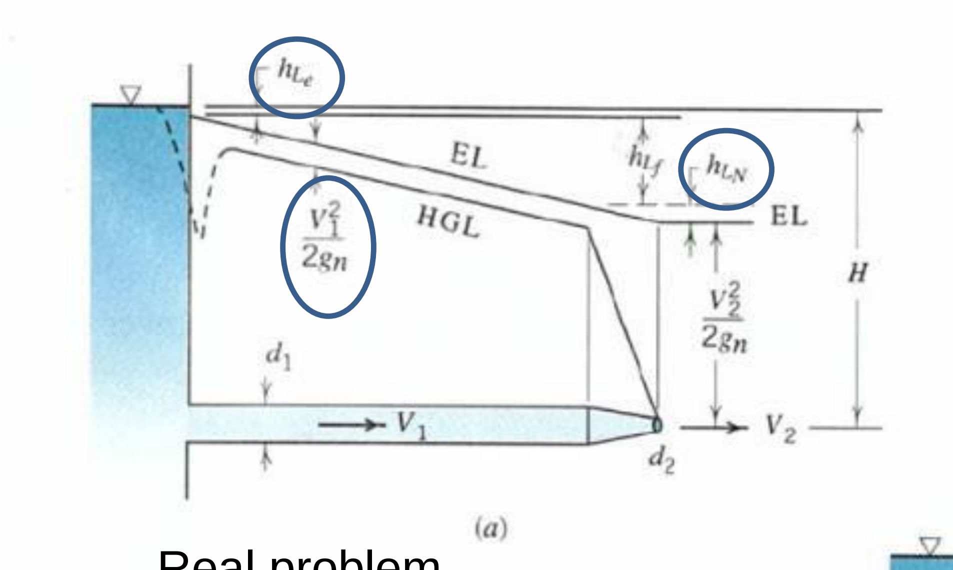
- Assume *d* = 0.25 ft, then from (A) we get *Re* = 36,080
- Use Moody diagram to get f = 0.022
- And, use $V = Q/A$ to get V = 2.04; $V^2/2g$ = 0.0644
- Then, RHS of (B) is 1.23 which is not close to LHS of (B).

- Repeat this process

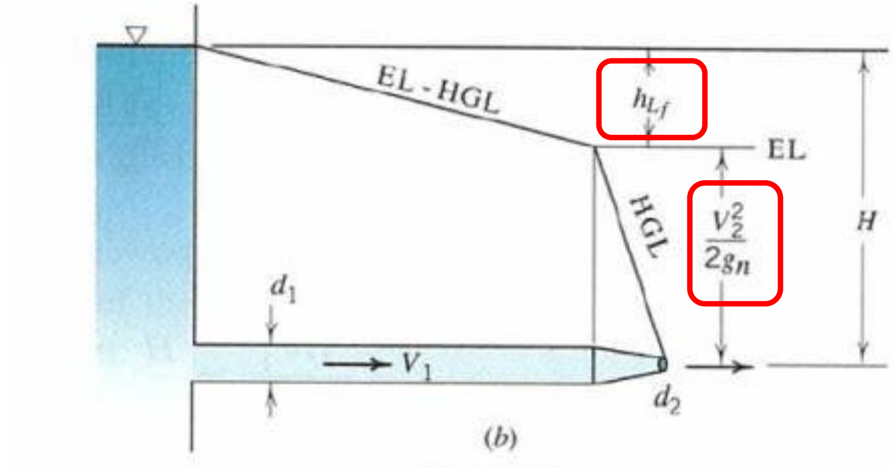
Moody diagram

d	*Re*	*f*	*V*	*V²/2g*	*RHS of (B)*
0.25	36,000	0.0220	2.04	0.0644	1.23
0.20	45,100	0.0212	3.18	0.157	3.56
0.18	50,100	0.0208	3.93	0.240	5.90
0.187	48,200	0.0210	3.64	0.206	4.94

- *Pipeline from a reservoir terminating in a nozzle*



Real problem



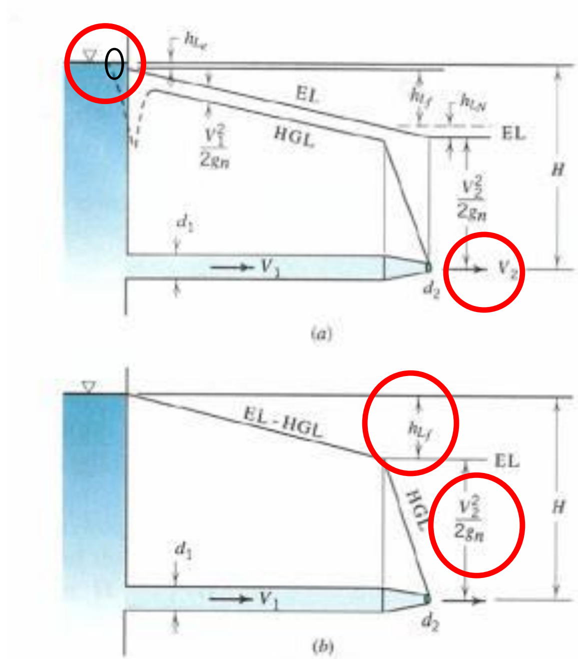
Simplified problem

- In this case, mainline velocity head is either significant or negligible but, nozzle's velocity head need to be considered.

$$z_0 + \frac{p_0}{g} + \frac{V_0^2}{2g_n} = z_2 + \frac{p_2}{g} + \frac{V_2^2}{2g_n} + h_L$$

$$p_0 = 0; V_0 \approx 0; z_2 = 0; p_2 = 0$$

- Exit loss can be ignored.



$$h_L = \left(K_{L_e} + f\frac{l}{d_1}\right)\frac{V_1^2}{2g_n}$$

Nozzle velocity head

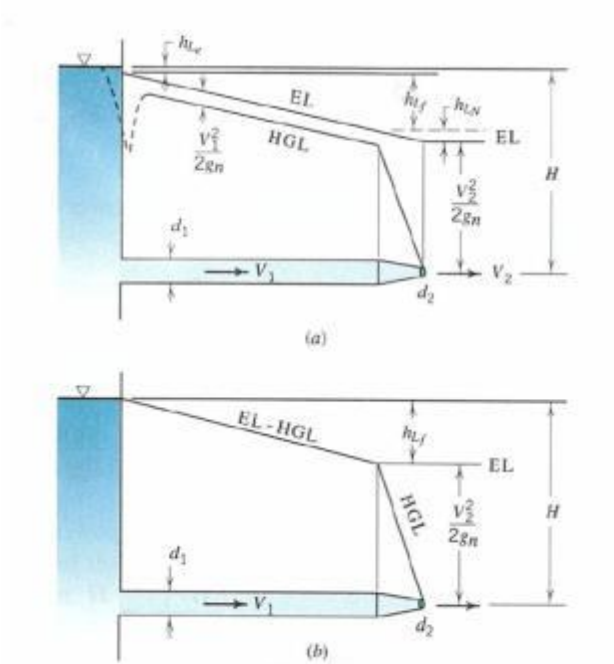
$$H = \frac{V_2^2}{2g_n} + \left(K_{L_e} + f\frac{l}{d_1}\right)\frac{V_1^2}{2g_n} \qquad (7.4)$$

Head loss due to pipe velocity

- Using continuity equation

$$A_1V_1 = A_2V_2 \qquad V_2 = \left(\frac{d_1}{d_2}\right)^2 V_1$$

$$\boxed{H = \left(\frac{d_1^4}{d_2^4} + K_{L_e} + f\frac{l}{d_1}\right)\frac{V_1^2}{2g_n}} \qquad (7.5)$$

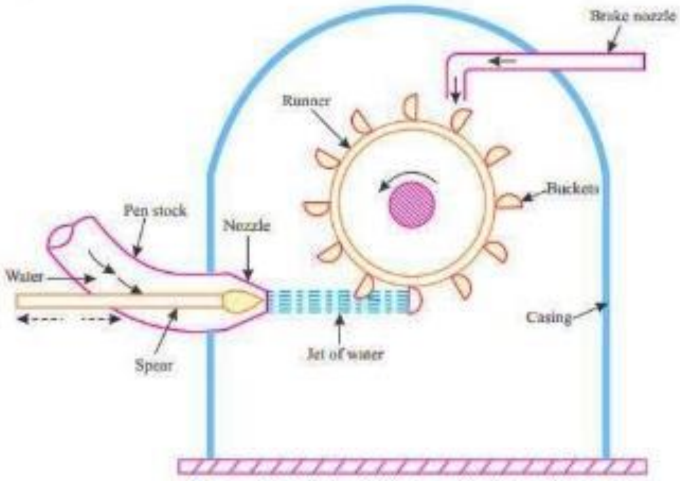


Generation of electricity by jet

- Power is work done in given time or product of pressure and flow rate.

$$Power = Energy\ \ flow\,rate = weight\ \ flow\,rate \times energy\ \ per\ unit\ weight$$

$$\boxed{P = (Q\gamma)H_e = (Q\gamma)\left(\frac{V_2^{\,2}}{2g_n}\right)} \qquad (7.6)$$



- Rearrange (1) neglecting local loss

$$H_e = \frac{V_2^2}{2g_n} = H - f\frac{l}{d_1}\frac{V_1^2}{2g_n} = H - f\frac{l}{d_1}\frac{Q^2}{2g_n A_1^2}$$

H_e = effective head

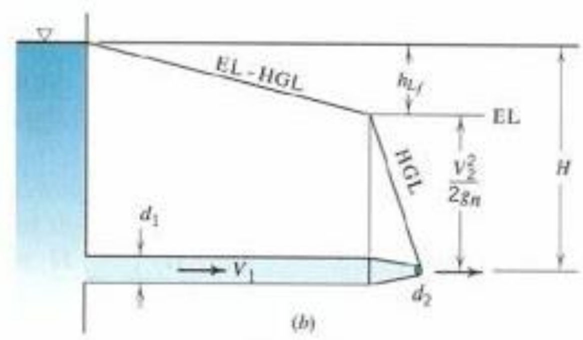
- Substitute this into (3)

$$P = \gamma Q\left(H - \frac{flQ^2}{2g_n d_1 A_1^2}\right)$$

- Find <u>maximum jet power</u> by differentiating; $\dfrac{\partial P}{\partial Q} = 0$

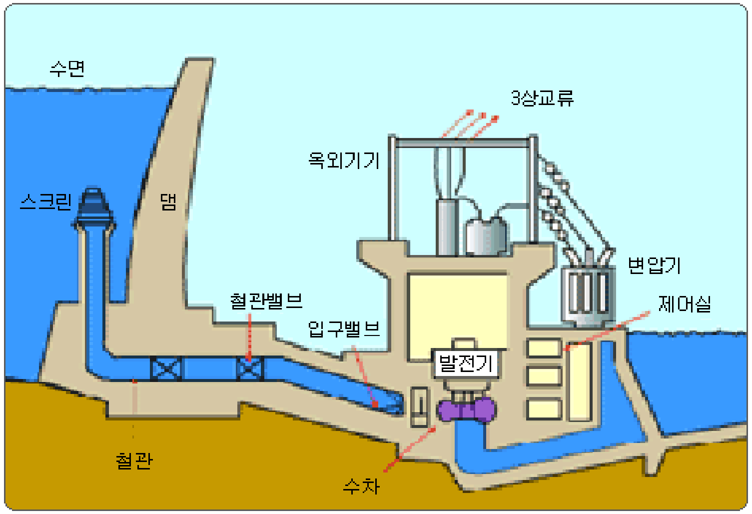
$$\frac{flQ^2}{2g_n d_1 A_1^2} = \frac{H}{3}$$

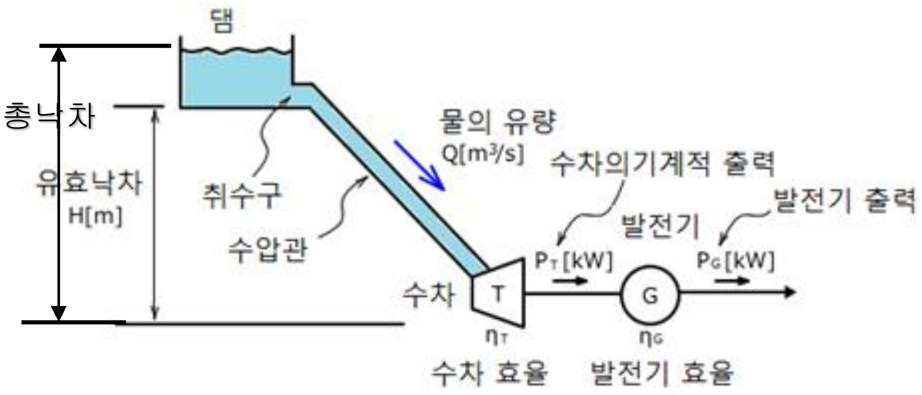
$$\Rightarrow \boxed{\frac{V_2^2}{2g_n} = \frac{2}{3}H} \quad and \quad h_{Lf} = f\frac{l}{d_1}\frac{V_1^2}{2g_n} = \frac{1}{3}H \qquad (7.7)$$



→ Actual limitation of the flow rate.

Hydropower





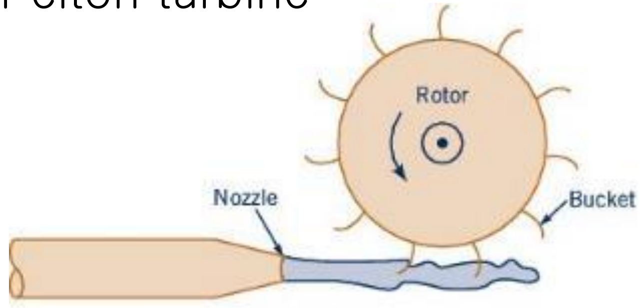
양수발전

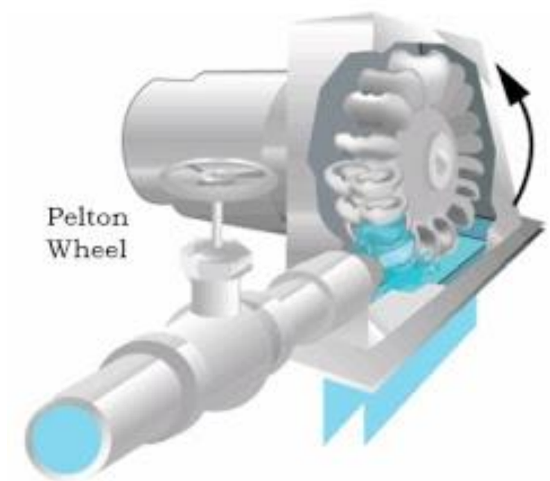
소수력발전 (< 3,000 kW)

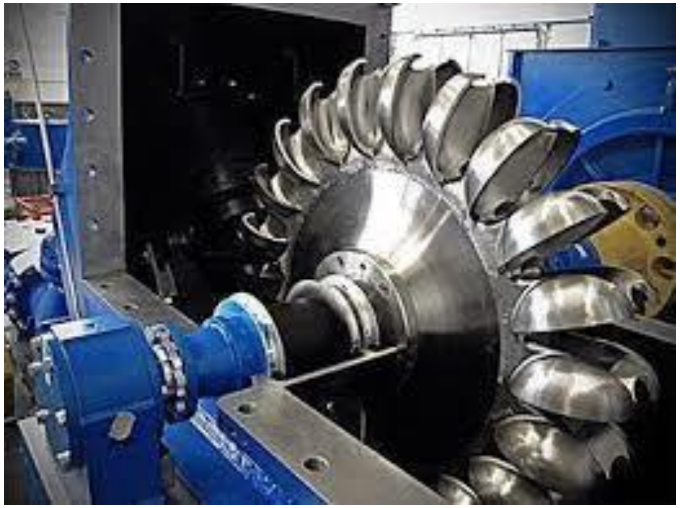
▪ Turbomachine

High head (impulse turbine): Pelton turbine
Low head (reaction turbine): Francis turbine, Kaplan turbine

1) Pelton turbine

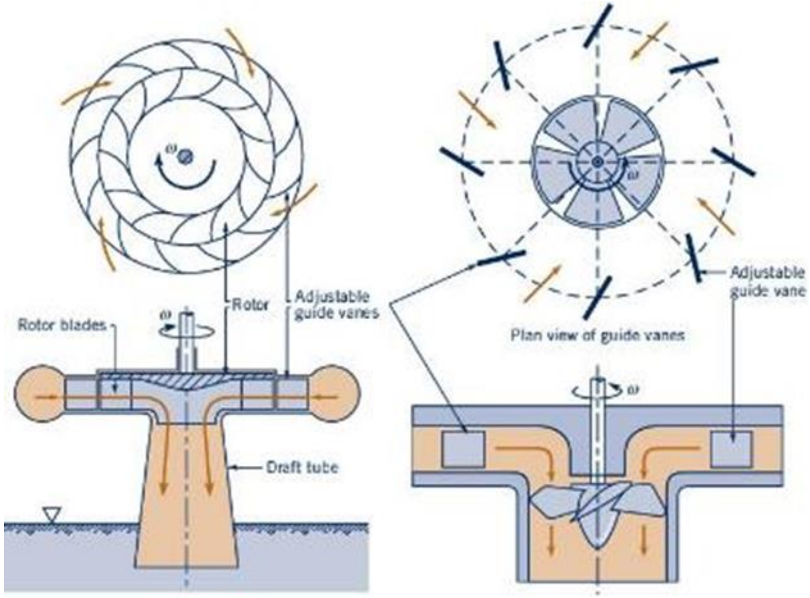






2) Francis turbine

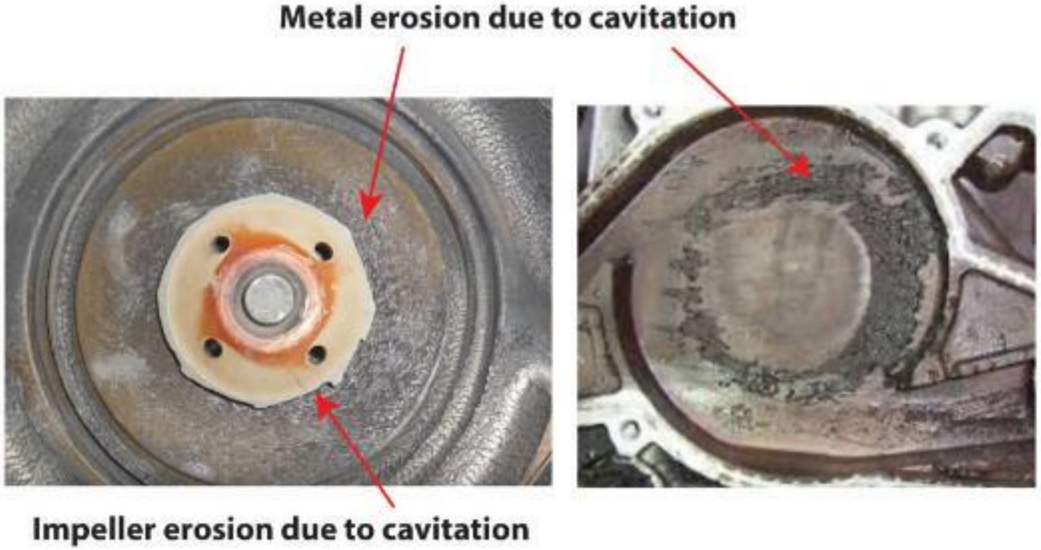
– radial flow



3) Kaplan turbine

– axial flow

Cavitation

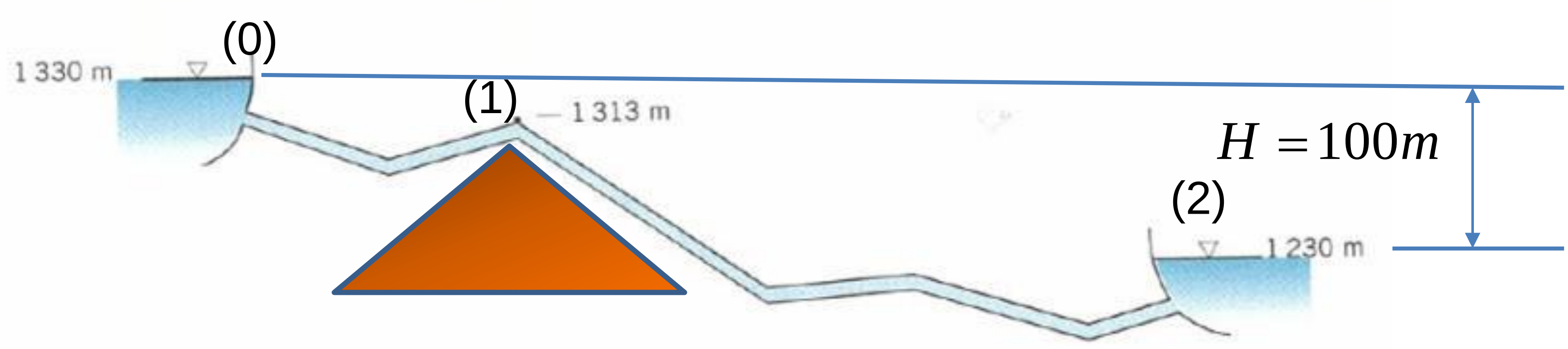


- When flow is too fast, the absolute pressure may fall to the vapor pressure of the liquid, at which point cavitation sets in.
- Large negative pressure in pipe should be avoided, if happens, then prevent from exceeding about two thirds of the difference between barometric (101.3 kPa) and vapor pressures (2.34 kPa for water at 20℃).
- Most engineering liquids contain dissolved gases which will come out of solution well before the cavitation point is reached.

IP 9.17 (p. 372~3)

A pipeline is being designed to convey water between two reservoirs whose elevations are shown below. The pipeline is 20km long and the preliminary pipeline profile has the line passing over a ridge where the pipeline elevation is 1,313 m at a distance of 4km from the upstream reservoir.

There is concern that the ridge is too high and will create an unacceptably low pressure in the pipeline. What is your recommendation on as to the feasibility of the proposed location of the pipeline?



Solution:

- Neglect the local losses and assume that the energy line and the hydraulic grade line to be coincident (**neglect velocity head**)
- Then EL-HGL will fall uniformly 100 m over 20km. $\rightarrow h_f = 100$
- Since the ridge is 4 km, then head loss at the ridge will be

$$h_{f1} = 100 \times 1/5 = 20$$

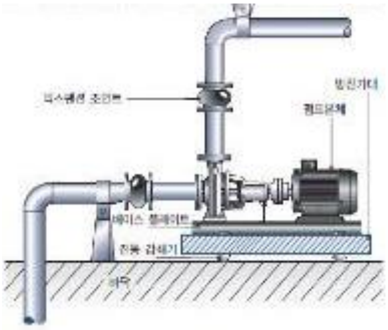
- Since we assume EL=HGL, it means that velocity head at the pipe is negligible. Therefore, the difference of elevation and EL height will result in pressure drop.

$$z_0 = z_1 + \frac{p_1}{\gamma} + \cancel{\frac{v_1^2}{2g_n}} + h_{f1}$$

$$1330 = 1313 + \frac{p_1}{\gamma} + h_{f1}(=20) \qquad \frac{p_1}{\gamma} = -3\,m$$

This is smaller than 2/3 of water vaporized pressure (−10 m of water), so seems to be OK.

7.3 Pumped Pipeline



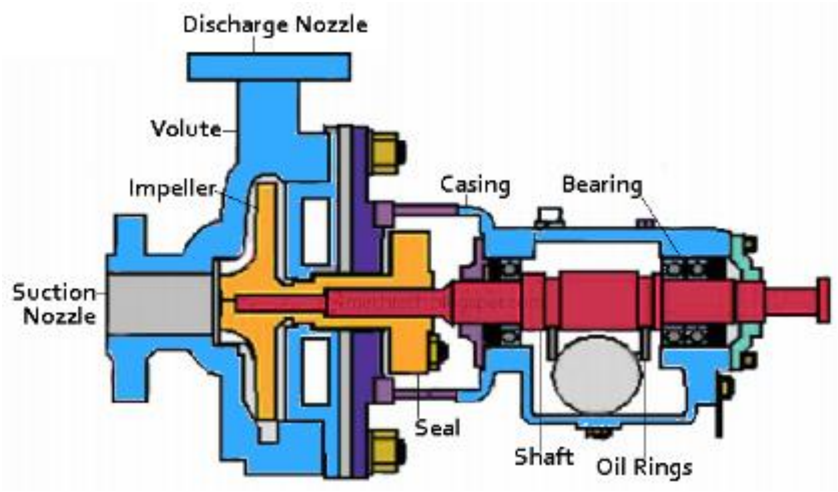
- Gravity-flow pipelines: use gravity
- Pumped pipeline
  - Source pump: located at the upstream end of the pipeline
  - Booster pump: located at intermediate point in the pipeline
- To determine the power required to meet flow rate (*Q*) and pressure demands (*H*), use work-energy equation

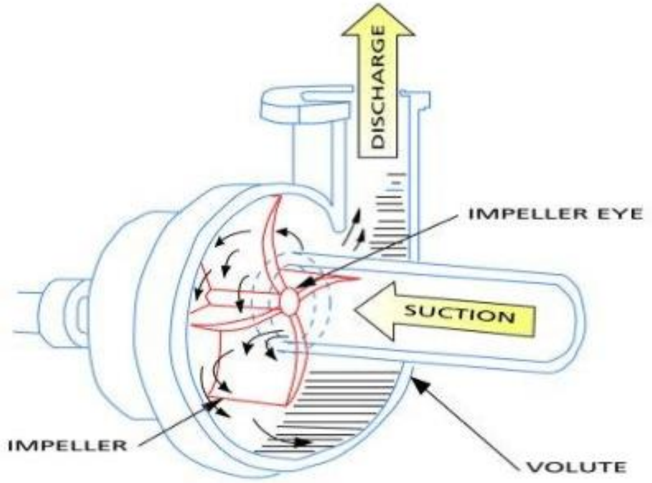
$$z_1 + \frac{p_1}{g} + \frac{V_1^2}{2g_n} + E_p = z_2 + \frac{p_2}{g} + \frac{V_2^2}{2g_n} + h_L \qquad (7.8)$$

- E_p is work per unit weight added to the fluid by the pump,
- Power equation is

$$P(KW) = \frac{Q\gamma E_P}{1{,}000} \qquad (7.9)$$

- Water pump





Centrifugal pump

- Waste water treatment plant: gravity-driven flows + booster pump

▪ Designing pump

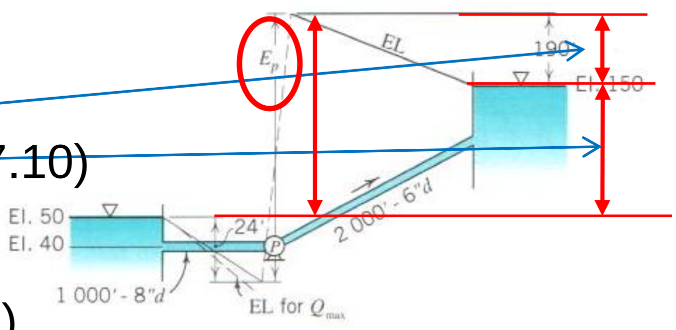
- To determine <u>power requirements</u> for the motor driven pump, the <u>performance of the pump</u> should be known.
- A pump is a <u>reactive element in a pipe system</u>.

1) Pump supply curve: The flowrate (*Q*) through the pump and head increase (*H*) across the pump depend on the pipeline system in which the pump is installed. The pump has own supply curve for given flowrates.

2) System demand curve: The demand (E_p) the system makes on the pump depends on the friction losses (h_L) in the system as functions of flowrate (*Q*) and the vertical lift (static lift; head increase, Δz) required between the two ends of the pipeline.

$$E_p = H = h_L(Q) + \Delta z \qquad (7.10)$$

$$h_L = f\,\frac{l}{d}\,\frac{V^2}{2g_n} \qquad (7.11)$$

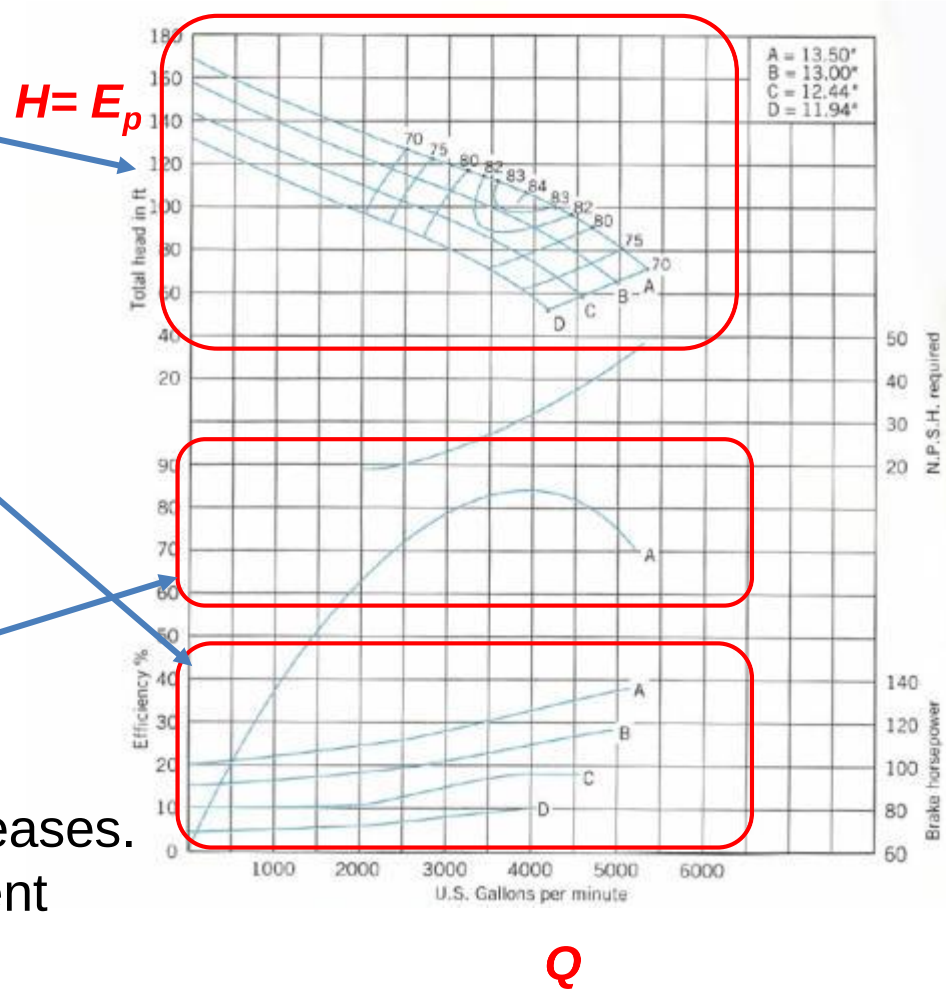


- **Pump characteristic diagram**

- **Pump supply curves:** total head increase (*H*) the pump will supply for a given flowrate for the four different impeller sizes (curves A, B, C, and D).

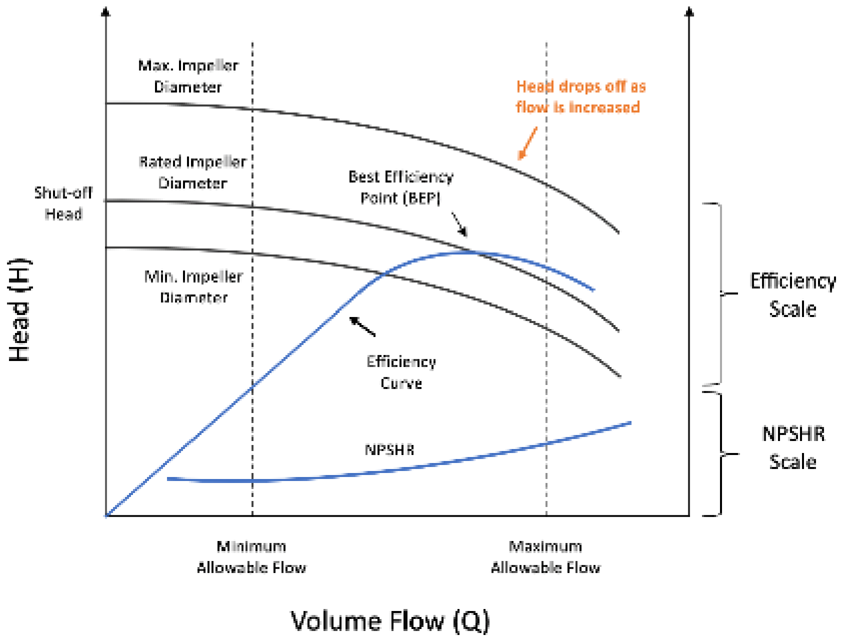
- **Power requirement curves:** power necessary to drive the various-sized impellers

- **Pump efficiency curves**

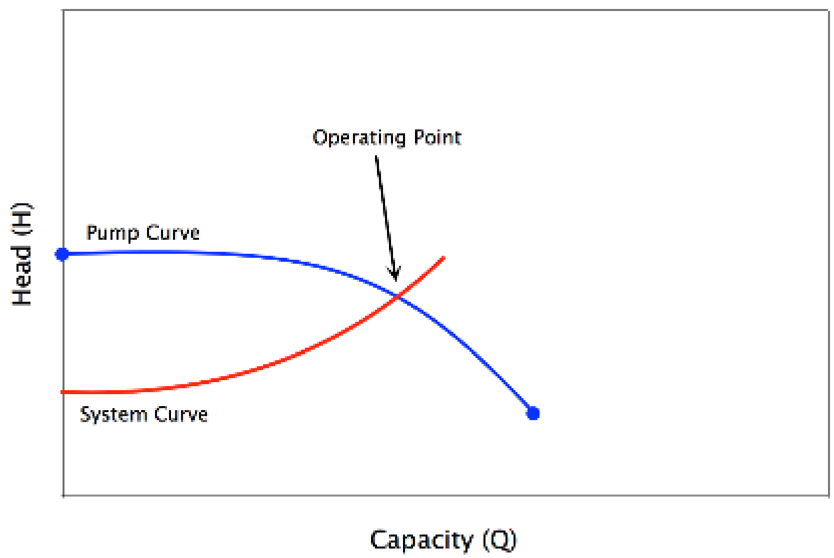


- As Q increases, H by pump decreases.
- As Q increases, power requirement from pump decreases.

▪ Pump curve



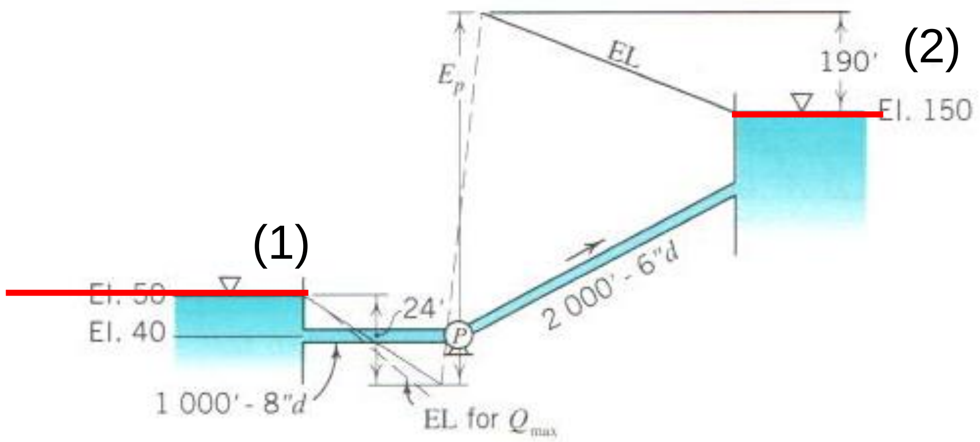
Pump characteristic curves



Pump supply curve vs System demand curve

IP 9.18 (p. 375~6) – Booster pump

1) Calculate the horsepower that the pump must supply to the water (50°F) in order to pump 2.5 ft^3/s through a clean cast iron pipe from the lower reservoir to the upper reservoir. Neglect local losses and velocity heads. → For given *Q*, find head loss
2) Using the criteria for minimum allowable pressure to prevent the cavitation, compute the maximum dependable flowrate which can be pumped through this system. → For given *H*, find flowrate



Solution:

- To get the pump power, apply work-energy eq. between 1 & 2

$$z_1 + \cancel{\frac{p_1}{\gamma}} + \cancel{\frac{V_1^2}{2g_n}} + E_p = z_2 + \cancel{\frac{p_2}{\gamma}} + \cancel{\frac{V_2^2}{2g_n}} + h_L \rightarrow E_p = H + h_L$$

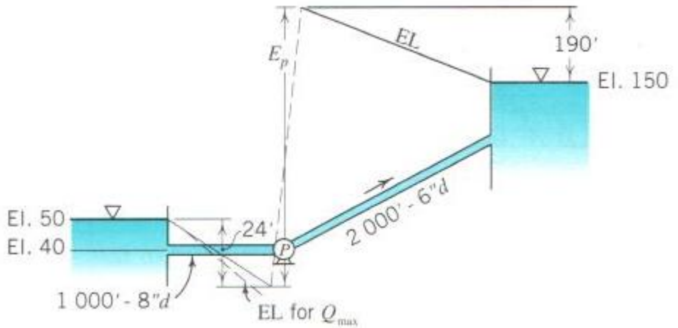
1) Calculate friction loss, h_L

$$V = \frac{Q}{A}; \quad V_8 = 7.16\ ft/s \quad and \quad V_6 = 12.72\ ft/s$$

$$\frac{V_8^{\ 2}}{2g} = 0.796\ ft; \quad \frac{V_6^{\ 2}}{2g} = 2.51\ ft$$

- Reynolds numbers for each pipe are

$$\mathrm{Re}_8 = 338,500 \qquad \mathrm{Re}_6 = 451,000$$



- Since we already know this pipe is cast iron pipe then we can use Fig. 9.11 for determining the roughness height, then we can find the friction factor.

$$\left(\frac{e}{d}\right)_8 = 0.00128 \qquad f = 0.021$$

$$\left(\frac{e}{d}\right)_6 = 0.00171 \qquad f = 0.022$$

- The head loss in each of the pipes can now be calculated.

$$h_{L_8} = f\frac{l}{d}\frac{V^2}{2g_n} = 0.021\frac{1{,}000\,ft}{8in./12}\frac{(7.16\,ft/s)^2}{2\times 32.2} = 25\,ft$$

$$h_{L_6} = f\frac{l}{d}\frac{V^2}{2g_n} = 0.022\frac{2{,}000\,ft}{6in./12}\frac{(12.72\,ft/s)^2}{2\times 32.2} = 221\,ft$$

- Consider the total losses from the both pipes

$$50 + 0 + 0 + E_P = 150 + 0 + 0 + 25 + 221$$

$$E_P = 346\,ft$$

$$WHP = \frac{Q\gamma E_P}{550} = \frac{2.5\,ft^3/s \times 62.4\,lb/ft^3 \times 346\,ft}{550} = 98\,hp$$

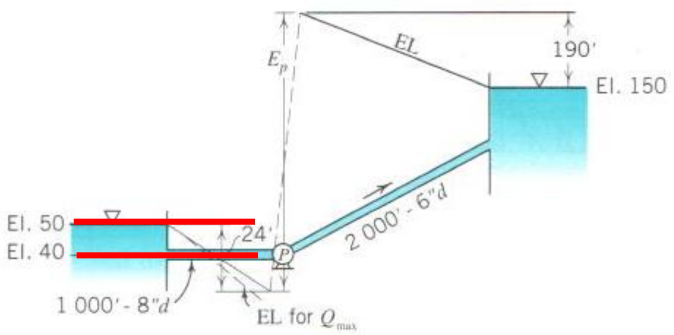
2) We need to check the **pressure drop**, which should <u>not be lower than 20 ft (cavitation occurs).</u>

Apply work-energy eq. <u>between 1 and suction side</u> (neglecting the local loss)

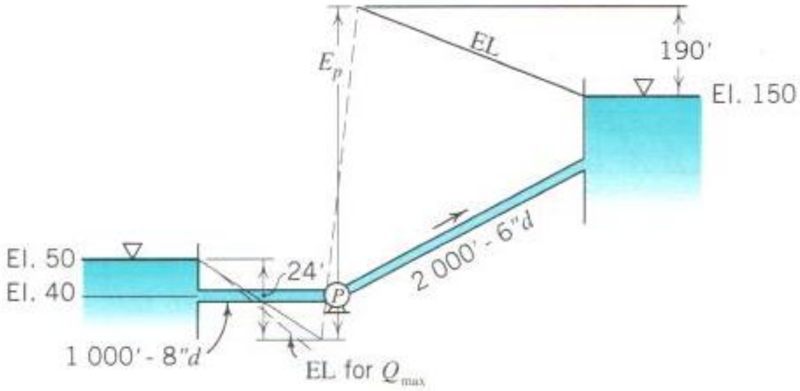
$$z_1 + \frac{p_1}{g} + \frac{V_1^2}{2g_n} = z_s + \frac{p_s}{g} + \frac{V_s^2}{2g_n} + h_{L_8}$$

$$50 + 0 + 0 = 40 + \boxed{(-20)} + 0 + h_{L_8}$$

Maximum allowable pressure drop

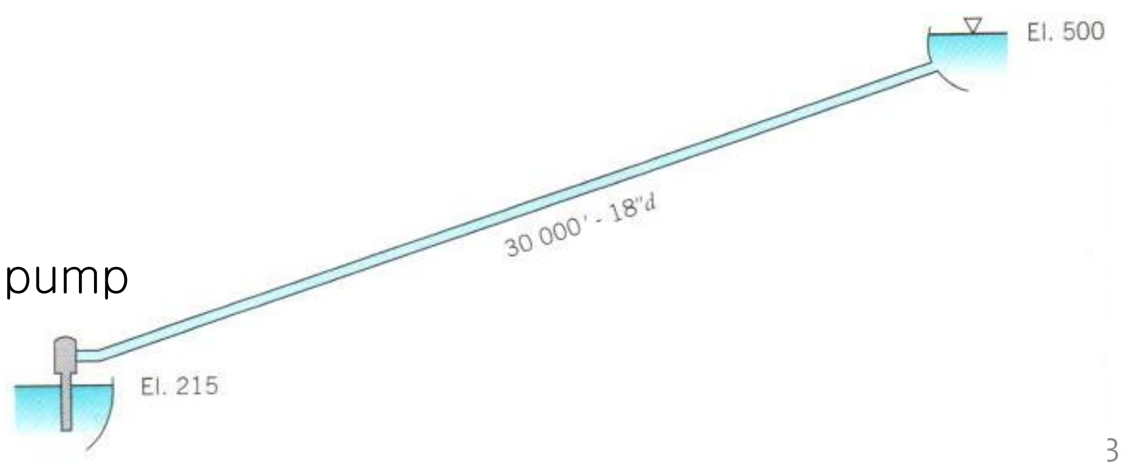


- Therefore, $$h_{L_8} = 30\,ft = f\,\frac{l}{d}\frac{V_8^2}{2g_n}$$
$$V_8 = 7.8\,ft/s \qquad > 7.16\,ft/s$$
$$Q_{\max} = V_8 A = 2.7\ ft^2/s$$
- So this is the maximum flow rate that can be pumped through this pipe system preventing the cavitation in the pipe and pump.



IP 9.19 (p. 378~9) – Source pump

The pump whose characteristics are given in Fig. 9.25 (p. 374) is proposed for use in the pipe system shown below. In order to provide the total head (*H*) necessary to deliver the required flowrate (*Q*) to the upper reservoir, a four-stage pump is planned. Using the graphical technique, determine the flowrate (*Q*) produced by the proposed pumping configuration and estimate the efficiency of the pump. Neglect local losses and use a Darcy-Weisbach *f*-value of 0.018. → For given *H* (Δz), find flowrate



1) System demand curve: Apply work-energy eq. between 1 and 2

$$z_1 + \frac{p_1}{g} + \frac{V_1^2}{2g_n} + E_p = z_2 + \frac{p_2}{g} + \frac{V_2^2}{2g_n} + f\frac{L}{d}\frac{V^2}{2g_n}$$

$$E_p = H = \Delta z + h_L$$

Δz

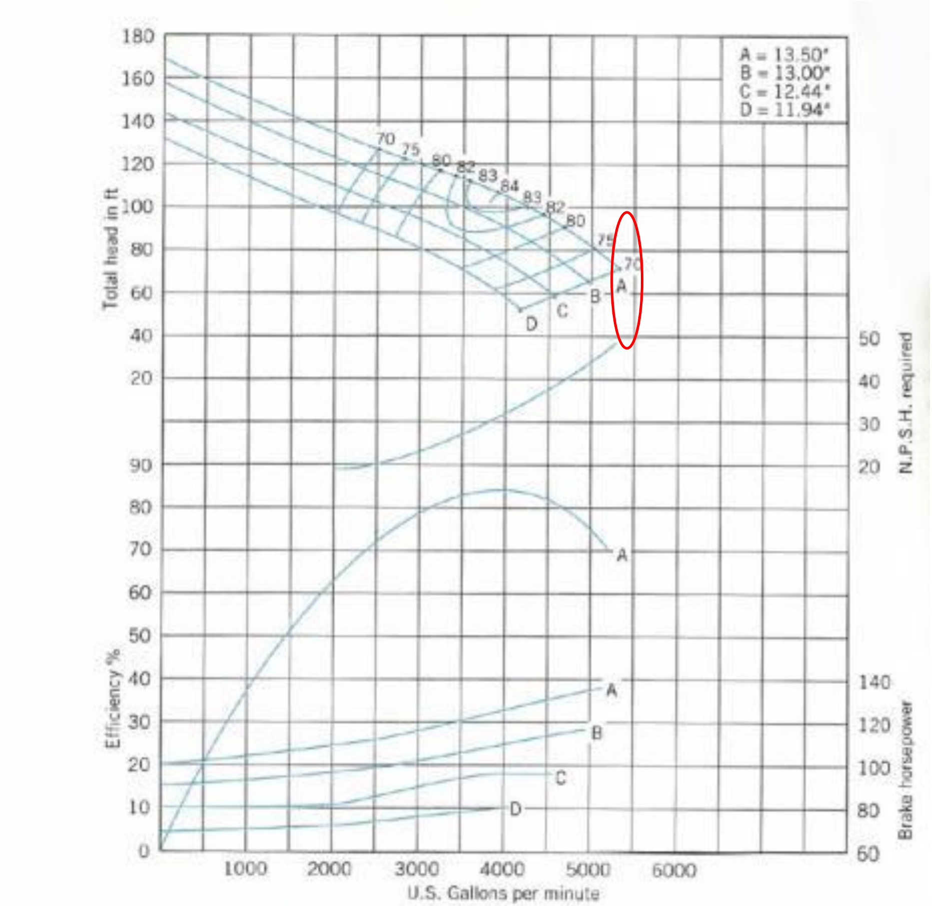
$$215 + 0 + 0 + E_p = 500 + 0 + 0 + 0.018\frac{3000\,ft}{18in/12}\frac{V^2}{2 \times 32.2}$$

$$\boxed{E_p = 285 + 5.59V^2}$$

Q (ft³/s)	V	$5.59V^2$	$E_p=285+5.59V^2$
0	0	0	285
2	1.13	7.1	292
4	2.26	28.6	314
6	3.40	64.6	350
8	4.53	114.7	400
10	5.66	179.1	464

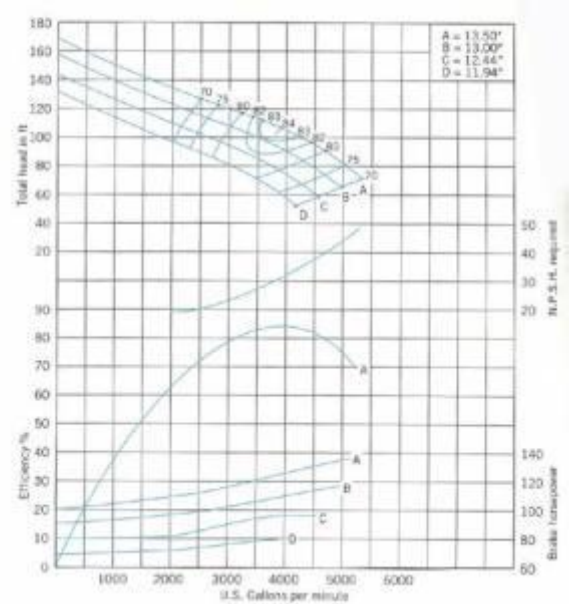
2) Pump supply curve

- The flowrate through the pump and the head increase across the pump <u>depend on the pipeline system.</u>
- Fig. 9.25 depicts the head increase the pump will supply for a given flowrate for the four different impeller sizes (curves A, B, C, and D).
- <u>Use curve A</u>



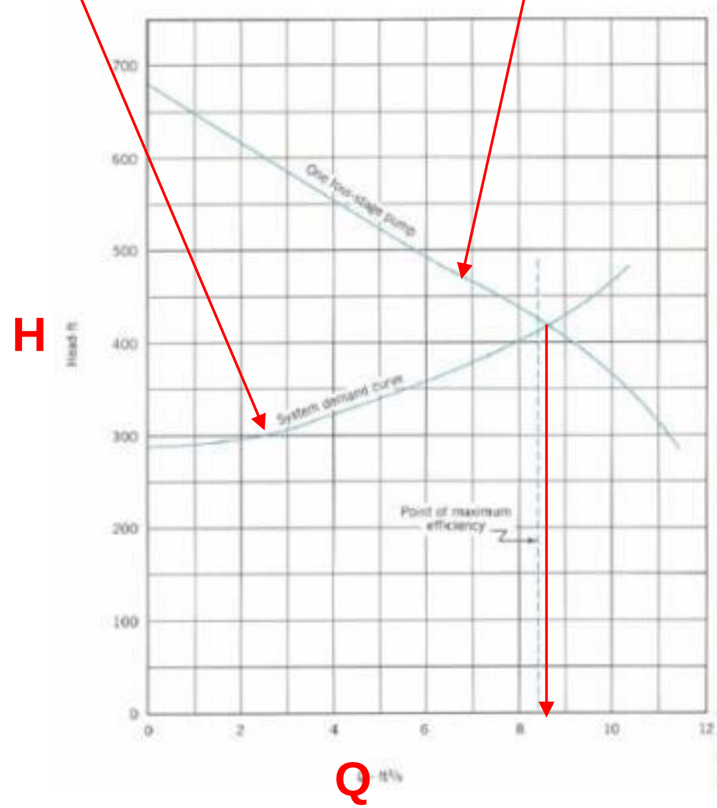
- Pump Supply Curve

- Use curve A for 1 stage pump
- For 4 stage pump, multiply 4 times

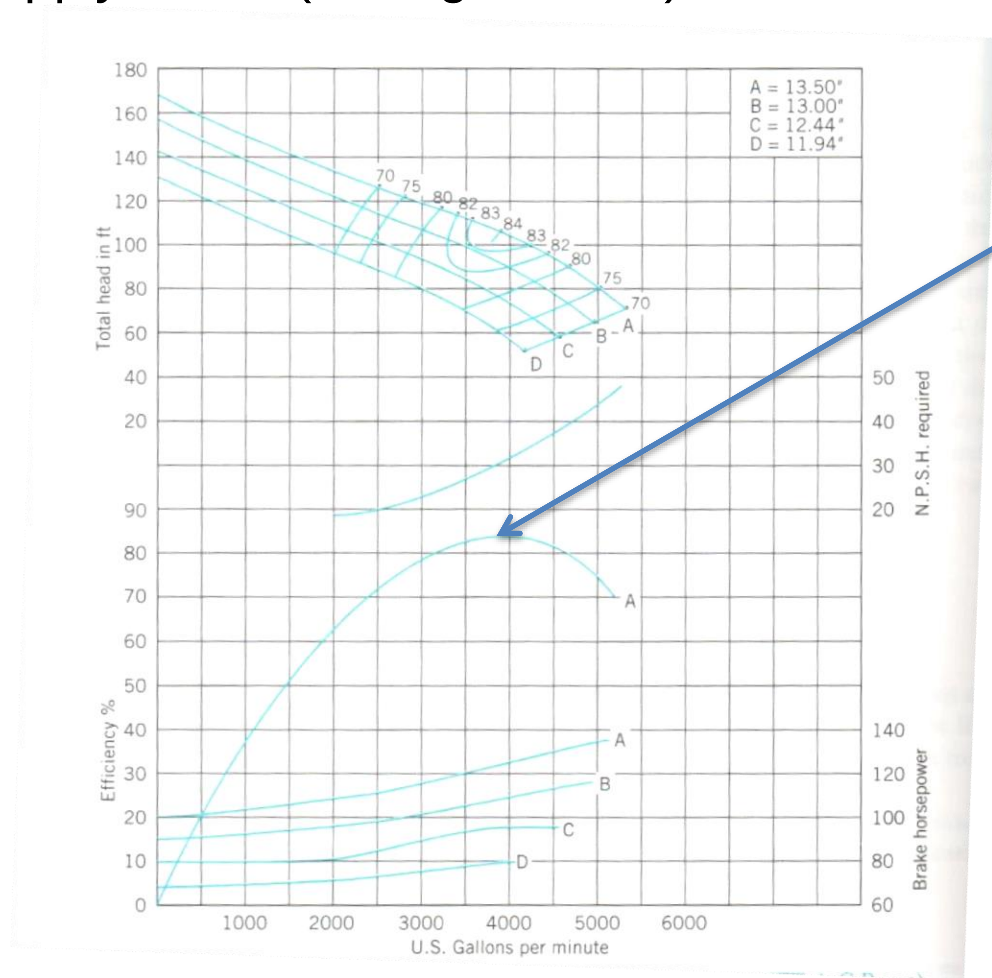


Q (gal/min)	Q(ft^3/sec)	1 stage head, H	4 stage head, H
0	0	168	672
1,000	2.23	150	600
2,000	4.45	133	532
3,000	6.68	119	476
4,000	8.91	103	412
5,000	11.14	78	312

- Plot the system demand curve and pump supply curve and find the flowrate from their intersection, which is 8.7 ft^3/s (3,900 gal/min).
- Pump's maximum efficiency is found at the similar value to the cross point of two curves.



- Pump Supply Curve (use figure 9.25)



Maximum Efficiency 84% at Q=3900