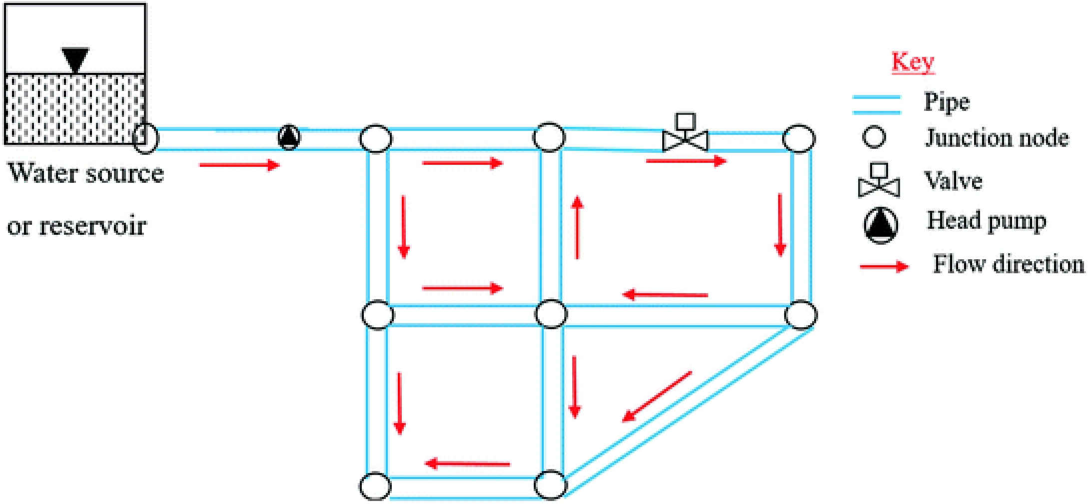
Lecture 8
Pipe Problems (2)



Contents

Objectives

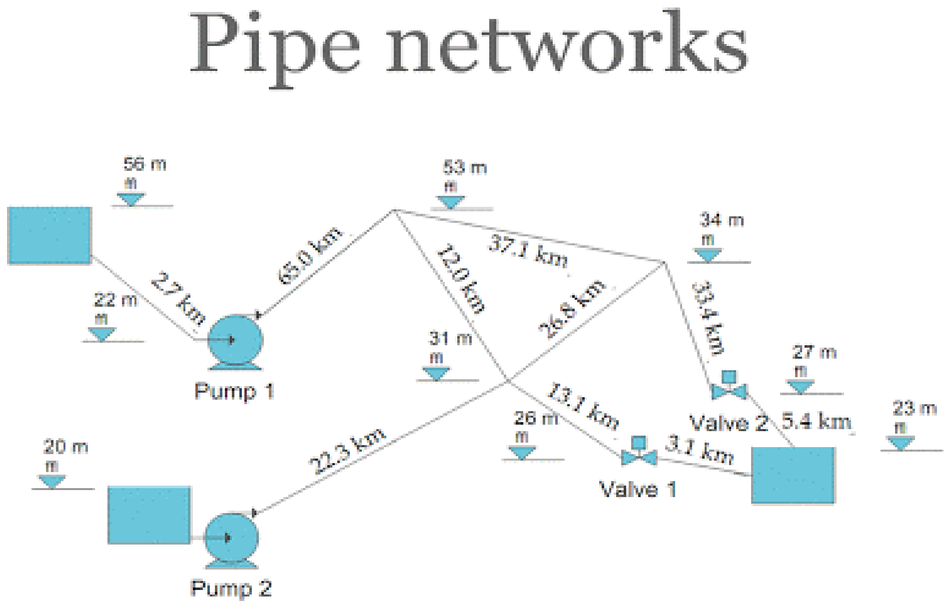
- Solve problems of multiple pipes
- Learn methods to solve pipe network problems

8.1 Multiple Pipes

- In real world, pipe is not single but connected with others.

- Sometimes, there are connections among hundred pipes.

- Even though there are many pipes, basic principles are the same.



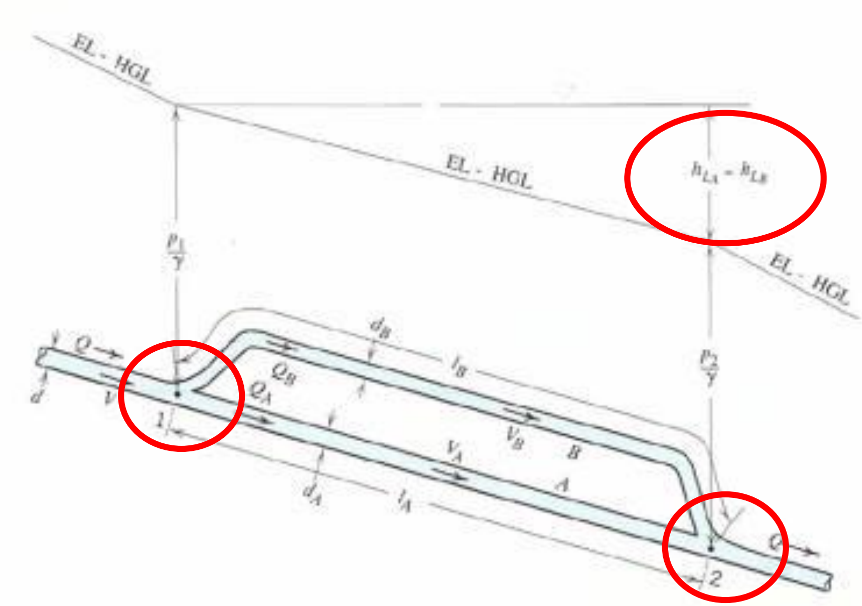
- **Two diverged pipes**

- Make problem simple, local loss and velocity head are neglected in the Bernoulli eq. with the EL-HGL considered coincident.

- As a consequence, the EL-HGLs of the pipes form a continuous network above the pipes, joining at the pipe junctions.

→ The head loss through both branches of the loop must be the same.

$$h_{L_A} = h_{L_B} \qquad (8.1)$$



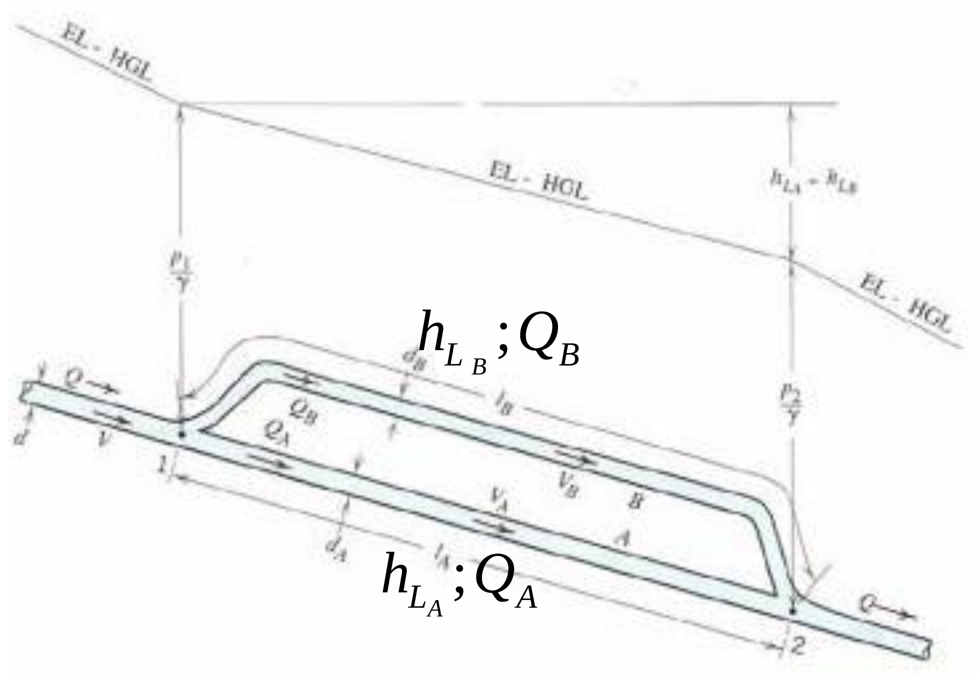
- The flowrate in the main pipe is equal to the sum of the flowrates in the branches.

→ Continuity equation

$$Q = Q_A + Q_B \qquad (8.2)$$

$$\rightarrow \; 2\,equations - 4\,unknowns$$

$$h_{L_A}, Q_A, h_{L_B}, Q_B$$



- To reduce the unknowns, <u>head loss is expressed in terms of flowrate</u> through the Darcy-Weisbach equation.

$$h_L = f\frac{l}{d}\frac{V^2}{2g_n} = \frac{fl}{2g_n d}\frac{16Q^2}{\pi^2 d^4} = \left(\frac{16\,fl}{2\pi^2 g_n d^5}\right)Q^2 \qquad (8.3)$$

- This equation may be generalized as

$$\boxed{h_L = KQ^n} \qquad (8.4)$$

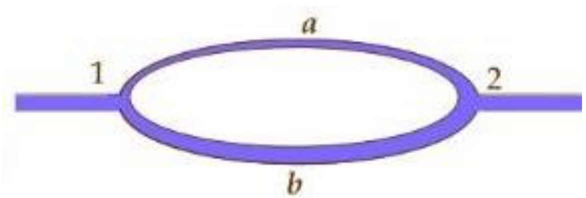
- For *K*, we need to know *f, l, d*.

- *n* = 2 for Darcy-Weisbach equation

- *n* = 1.85 for Hazen-Williams equation $V = 0.849 C_{hw} R_h^{0.63} S_f^{\ 0.54}$

- Substituting Eq. (4) into (1) gives the general description of head loss as

$$K_A Q_A^n = K_B Q_B^n \qquad (8.5)$$



$$Q = Q_A + Q_B \qquad (8.6)$$

$\rightarrow \quad 2\,equations - 2\,unknowns$

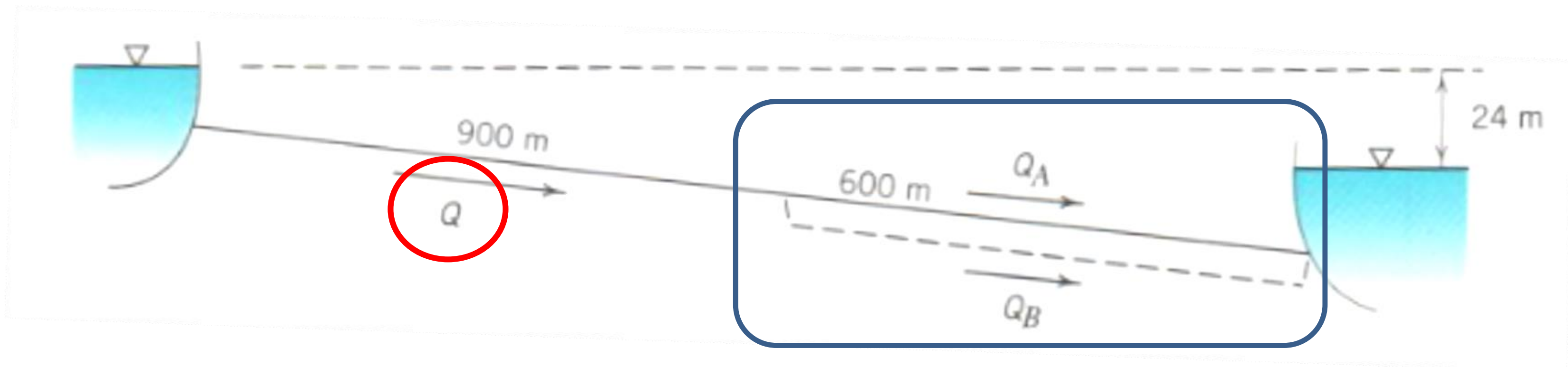
- Solution of these simultaneous equations allows prediction of the division of a flowrate *Q* into flowrates of two diverged pipes.

- Application of these equations also allows prediction of the increased flowrate obtainable by looping an existing pipeline.

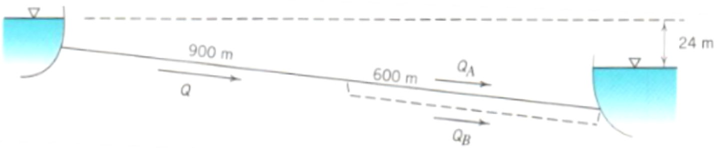
I.P. 9.20 (p. 382)

A 300 mm pipe 1,500 m long is laid between two reservoirs having a difference in surface elevation of 24 m. The maximum flowrate obtainable through this line (with all valves wide open) is 0.15 m³/s.

When this pipe is looped with a 600 m pipe of the same size and material laid parallel and connected to it, what percent increase in maximum flowrate may be expected?



Solution:



(I) Before looped; $h_L = KQ^n$

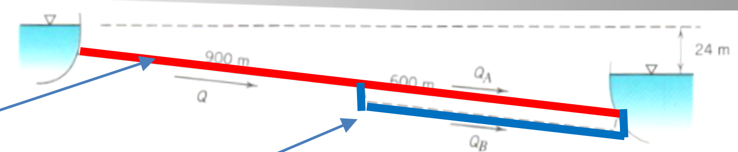
For original 1,500 m line, $K_{1500} = \dfrac{h_L}{Q_{old}^{\ 2}} = \dfrac{24m}{\left(0.15m^3/s\right)^2} = 1{,}067$

- In equation of $K = \dfrac{16fl}{2p^2 g_n d^5}$

We know that <u>*K* is linear with length</u> if the size (diameter) and material of the pipe are the same, then

$$K_{600} = K_{1500}\frac{600}{1500} = 427 \text{ for looped section}$$

$$K_{900} = K_{1500}\frac{900}{1500} = 640 \text{ for unlooped portion}$$



(II) After looped: Q is increased

1) For the original pipeline (red pipe), the head loss in the unlooped (900 m) plus pipe A (600 m)gives

$$h_{L,ori} = 24m = K_{900}Q_{new}{}^2 + K_{600}Q_A^2 = 640Q_{new}{}^2 + 427Q_A^2 \qquad (1)$$

2) For looped pipe(red+blue), the head loss in the unlooped (900 m) plus the head loss in the pipe B in the looped portion is

$$h_{L,new} = 24m = K_{900}Q_{new}{}^2 + K_{600}Q_B^2 = 640Q_{new}{}^2 + 427Q_B^2 \qquad (2)$$

- Eliminating Q_{new} shows that $\boxed{Q_A = Q_B.}$ Then $Q_A = Q_{new}/2$. From (1)

$$Q_{new} = 0.18m^3/s$$

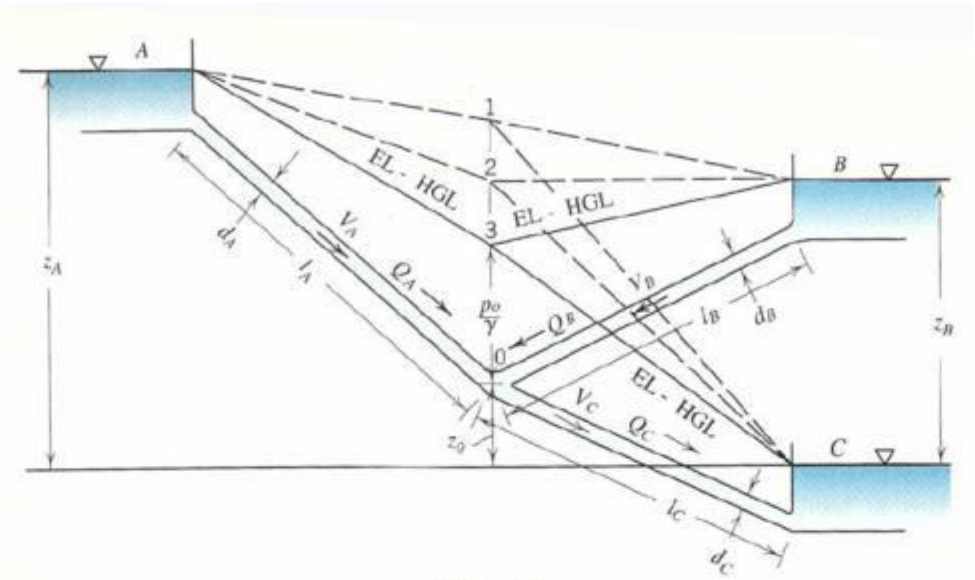
- Thus, the gain in capacity is $0.03 m^3/s$.

$$\frac{(0.18-0.15)}{0.15} \times 100 = 20\%$$

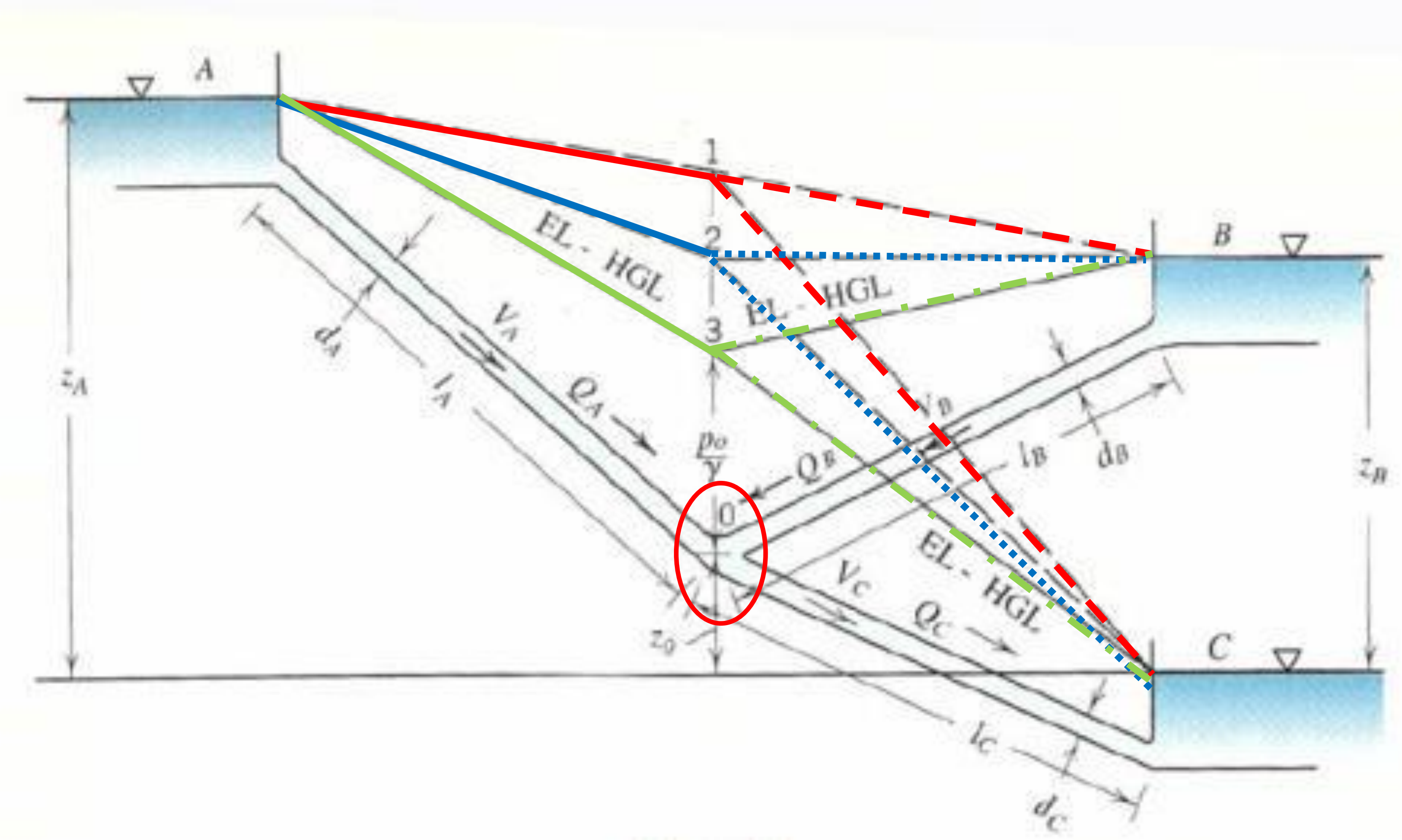
8.2 Three Reservoir Problems

- Three-reservoir problem: Flow may take place
  1) From reservoir A to reservoirs B and C
  2) From reservoir A to C without inflow or outflow from reservoir B
  3) From reservoirs A and B into reservoir C

- Solve this problem using the energy line



Three reservoir problems



Three reservoir problems

1) Situation 1: flow may take place from reservoir *A* to *B* and *C*

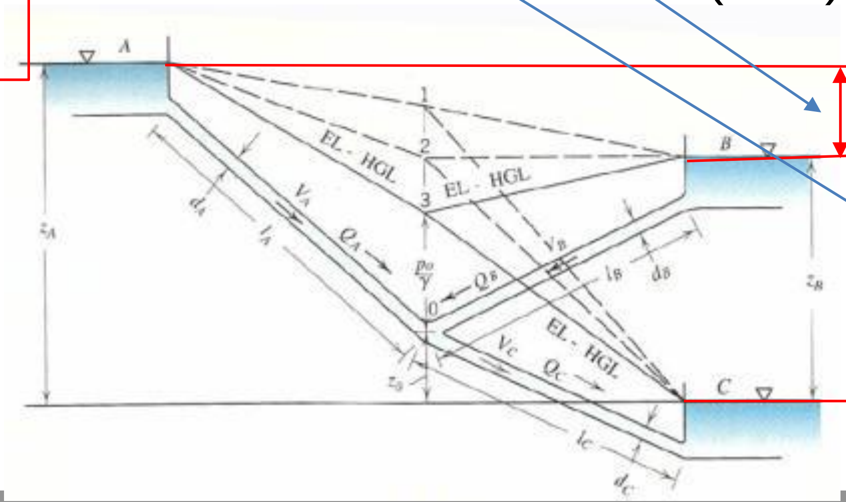
$$h_{L_{A-B}} = K_A Q_A{}^2 + K_B Q_B^2 \qquad (8.7)$$

$$h_{L_{A-C}} = K_A Q_A{}^2 + K_C Q_C^2 \qquad (8.8)$$

$$z_A - K_A Q_A^n - K_B Q_B^n = z_B$$ (since flow is to B) (8.9a)

$$z_A - K_A Q_A^n - K_C Q_C^n = 0$$ (since $z_C = 0$) (8.9b)

$$Q_A = Q_B + Q_C$$ (8.9c)



$3 equations - 3 unknowns$

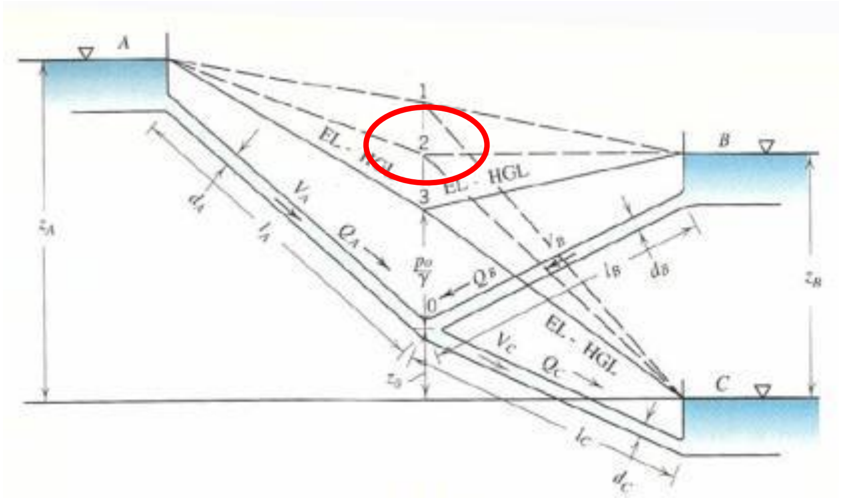
Three reservoir problems

2) Situation 2: flow may take place from reservoir *A* to *C* without flowing to *B* (Q_B=0)

$$z_A - K_A Q_A^n - K_C Q_C^n = 0 \qquad (8.10a)$$

$$z_A - K_A Q_A^n = z_B \qquad (8.10b)$$

$$\boxed{Q_A = Q_C} \qquad (8.10c)$$



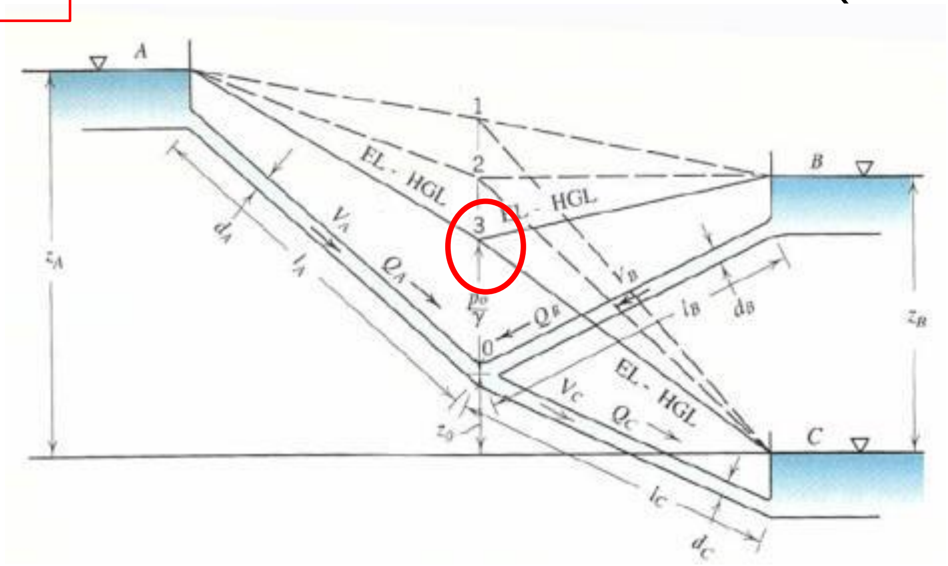
Three reservoir problems

3) Situation 3: Flow may take place from reservoir A and B to C,

$$z_A - K_A Q_A^n - K_C Q_C^n = 0 \quad (\text{since} \;\; z_C = 0) \qquad (8.11a)$$

$$z_A - K_A Q_A^n = z_0 = z_B - K_B Q_B^n \qquad (8.11b)$$

$$\boxed{Q_A + Q_B = Q_C} \qquad (8.11c)$$

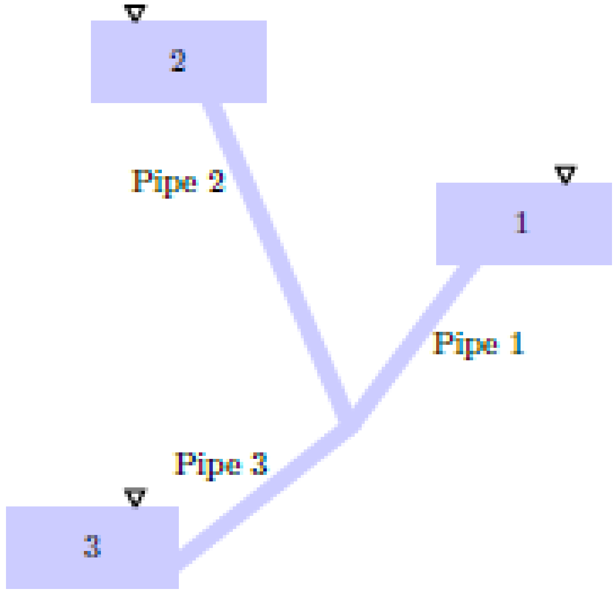


- Three reservoir problems

Assumptions: neglect velocity head and minor losses

Given: pipe length, pipe diameter, *f*, water surface level

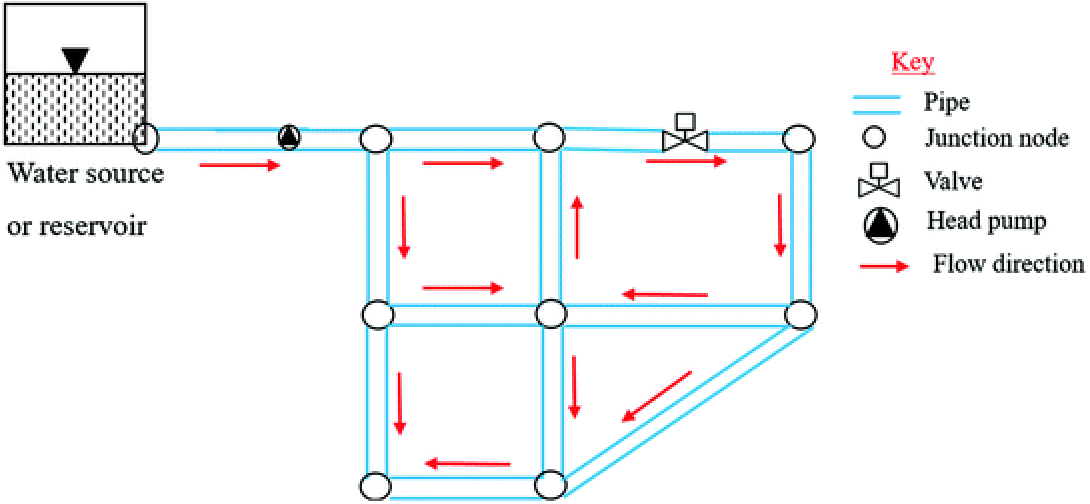
Find: flowrate



Pipe	Length m	Diameter cm	Z m
1	95	28	85
2	125	42	115
3	160	14	25

8.3 Pipe Networks

- A pipe network of a water system is the aggregation of connected pipes used to distribute water to users in a specific area, such as a city or subdivision.
- The network consists of pipes of various sizes, geometric orientations, and hydraulic characteristics plus pumps, valves, and fittings and so forth.



Networks of pipes

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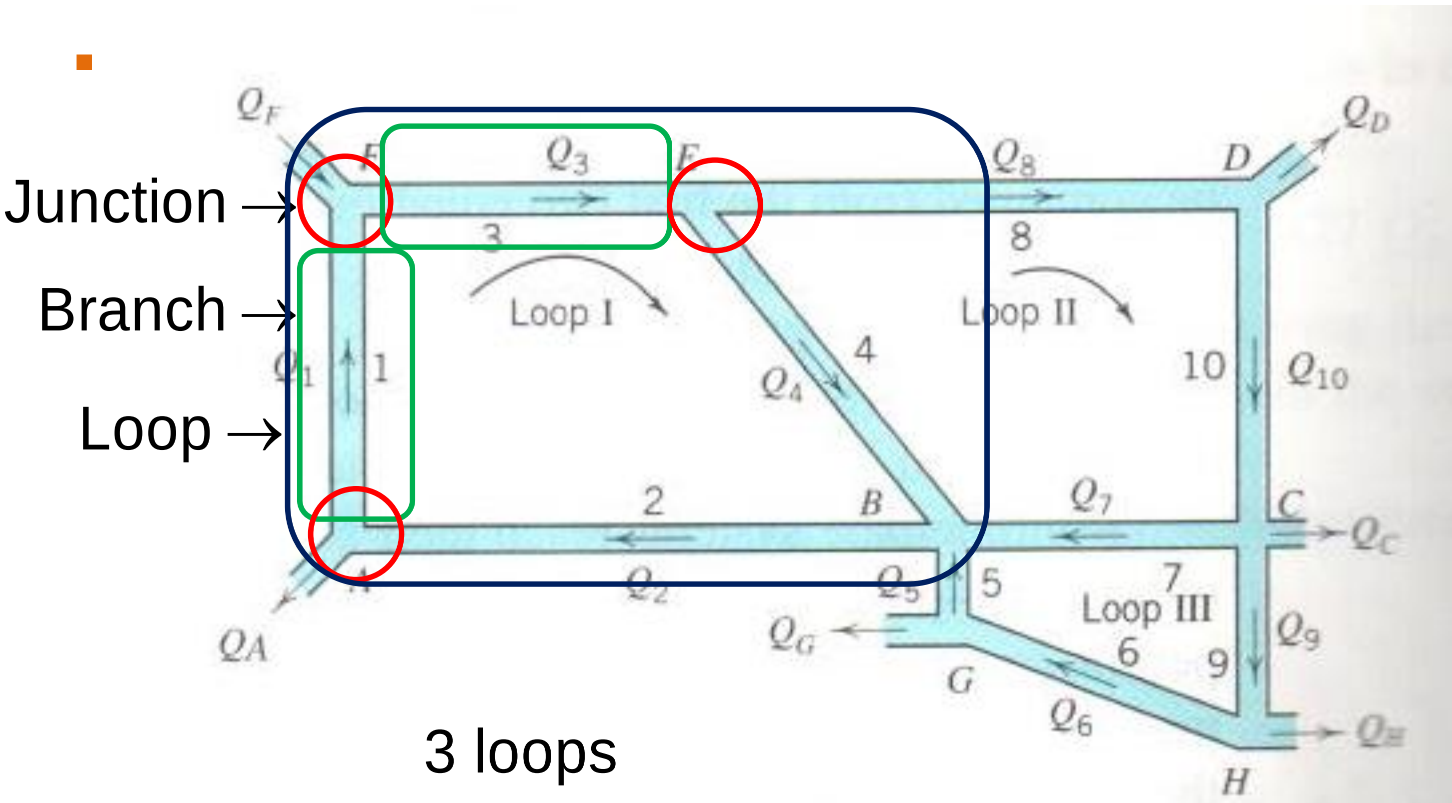


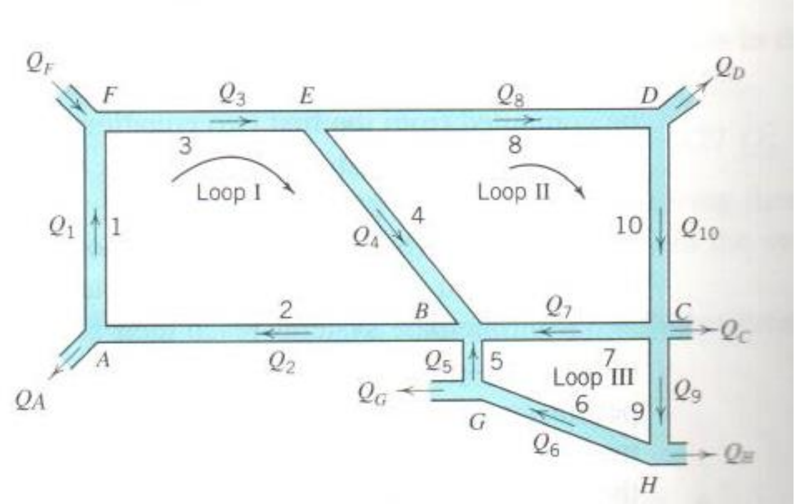
3 loops

10 branches

8 junctions

Networks

- Pipe junction: A~H
- Branch pipe: 1~10
- Loop (closed circuit of pipes): I~III



1) The continuity principle states that the net flowrate into any pipe junction must be zero.

2) The work-energy principle requires that at any junction there be only one position of the EL-HGL.

→ Net head loss around any single loop of the network must be zero.

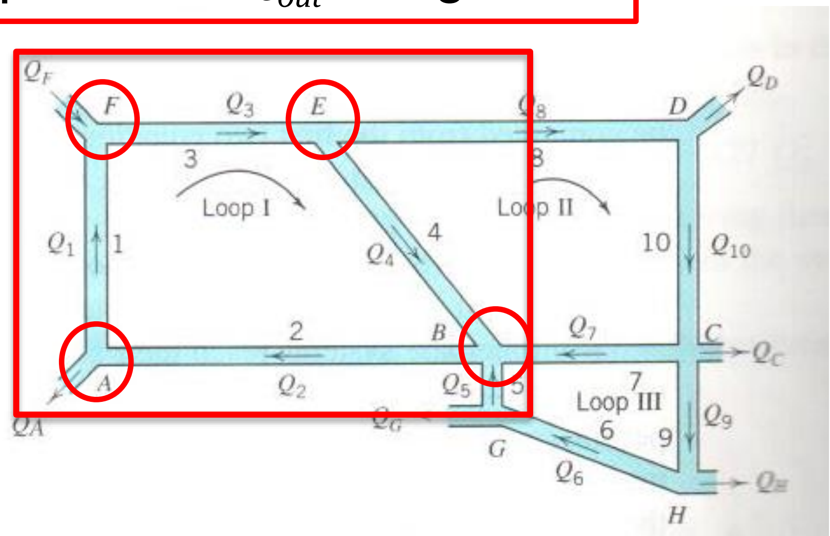
3) Clockwise is positive, and opposite is negative.

- For example, pipe 4 for Loop II is negative and positive for Loop I.

- Equations of flowrate and head loss

Q_{in} - positive; Q_{out} - negative

- The equations for Loop I,



$$\sum_A Q = -Q_A + Q_2 - Q_1 = 0$$

$$\sum_F Q = Q_1 + Q_F - Q_3 = 0$$

$$\sum_E Q = Q_3 - Q_4 - Q_8 = 0$$

$$\sum_B Q = -Q_2 + Q_4 + Q_7 + Q_5 = 0$$

$$\sum_I h_L = K_1 Q_1^n + K_3 Q_3^n + K_4 Q_4^n + K_2 Q_2^n = 0 \qquad (8.12)$$

$$K_1 Q_1^n + K_3 Q_3^n + K_4 Q_4^n = -K_2 Q_2^n$$

- To construct flownet, we need to <u>assume the flow direction of each pipe.</u>

- The equations for Loop II,

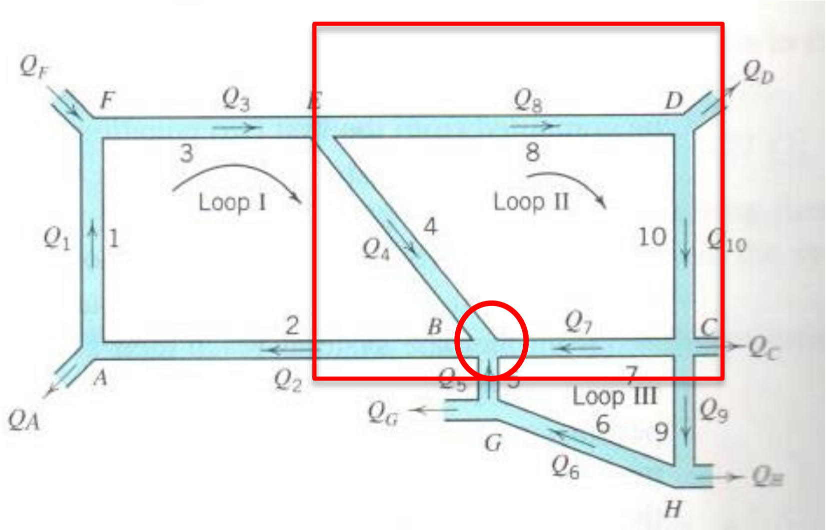
$$\sum_{B} Q = -Q_2 + Q_4 + Q_5 + Q_7 = 0$$

$$\sum_{E} Q = Q_3 - Q_4 - Q_8 = 0$$

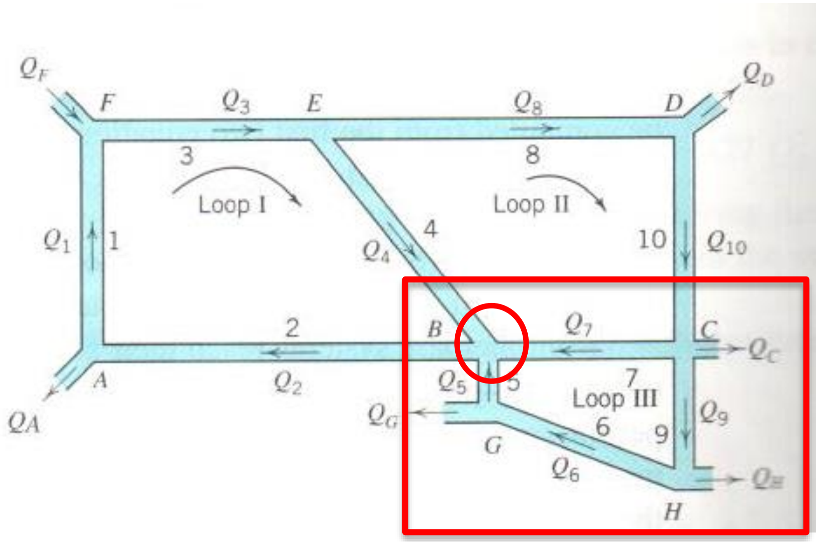
$$\sum_{D} Q = -Q_D + Q_8 - Q_{10} = 0$$

$$\sum_{C} Q = -Q_C - Q_7 - Q_9 + Q_{10} = 0$$

$$\sum_{II} h_L = -K_4 Q_4^n + K_8 Q_8^n + K_{10} Q_{10}^n + K_7 Q_7^n = 0 \qquad (8.13)$$



- The equations for Loop III,



$$\sum_{B} Q = -Q_2 + Q_4 + Q_5 + Q_7 = 0$$

$$\sum_{C} Q = -Q_7 + Q_{10} - Q_C - Q_9 = 0$$

$$\sum_{H} Q = -Q_6 + Q_9 - Q_H = 0$$

$$\sum_{G} Q = -Q_G - Q_5 + Q_6 = 0$$

$$\sum_{III} h_L = K_5 Q_5^n - K_7 Q_7^n + K_9 Q_9^n + K_6 Q_6^n = 0 \qquad (8.14)$$

Solution of pipe network

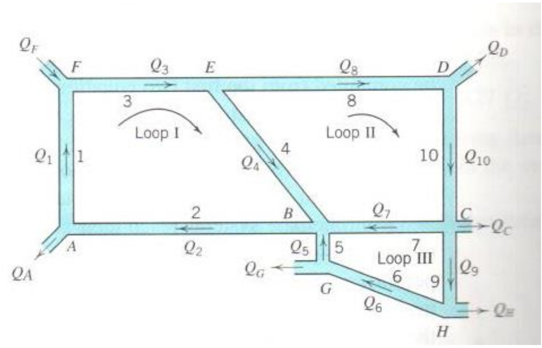
- Flow directions of each pipe have been assumed.
- The pipe size, length, and hydraulic characteristics are known as well as network inflows and outflows.
- Pump locations and pump characteristics, network layout and elevations are also given.
- Now we have 15 simultaneous equations: 12 equations for continuity and 3 equations for head loss.
- And we have the 10 unknown flowrates Q_i, for i=1~10 pipes, when Q_A, Q_F, Q_D, Q_C, Q_H, Q_G are given.
- The solution 15 simultaneous equations for the 10 unknown flowrates is obtained by a trial-and-correction method or iteration process.

- ## Hardy Cross Method (1936)

The most simple and easy one may be the **Hardy Cross method**.

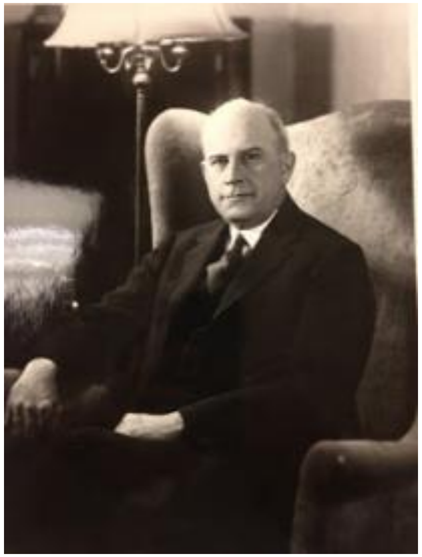
- The essence of the method is to start with a best estimate of a set of initial values, Q_{0i} that satisfy continuity at each junction.
- And then to systematically adjust these values keeping continuity satisfied until the head loss equations around each loop are satisfied to a desired level of accuracy.
- All the equations of continuity at the pipe junctions are automatically and continuously satisfied by this approach.
- Hence, only head loss equations remain and the number of simultaneous equations to be solved is reduced to the number of loops.

[Ex] 15 equations → 3 equations

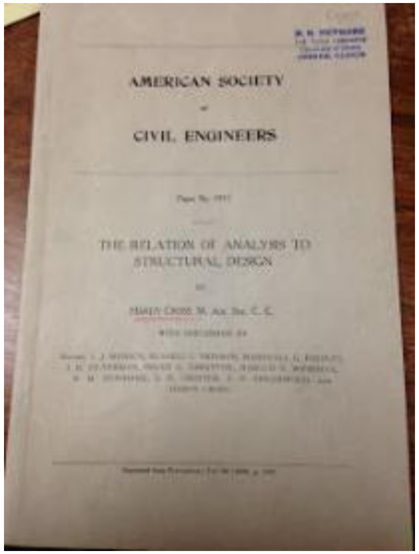


Hardy Cross (1885 – 1959)

Hardy Cross was an American structural engineer and the developer of the moment distribution method for structural analysis of statically indeterminate structures.



Another Hardy Cross method is also famous for modeling flows in complex water supply networks.



- First guess initial values, Q_{0i} that satisfy continuity at each junction

- If the first estimates are reasonably accurate, the true flowrate should only be a small increment different from the original (initial) flowrate in each loop.

$$Q_i = Q_{0i} \pm \Delta_L \quad (\Delta_L \text{ is loop correction}) \qquad (8.15)$$

$$Q_{0i} = \text{initial guess for pipe } i$$

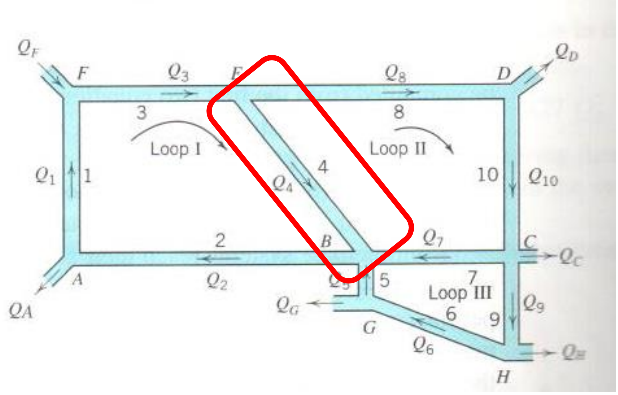
- For example

$$Q_3 = Q_{03} + \Delta_I \quad \text{(Loop I correction)}$$

$$Q_8 = Q_{08} + \Delta_{II} \quad \text{(Loop II correction)}$$

- But for pipe 4,

$$Q_4 = Q_{04} + \Delta_I - \Delta_{II}$$



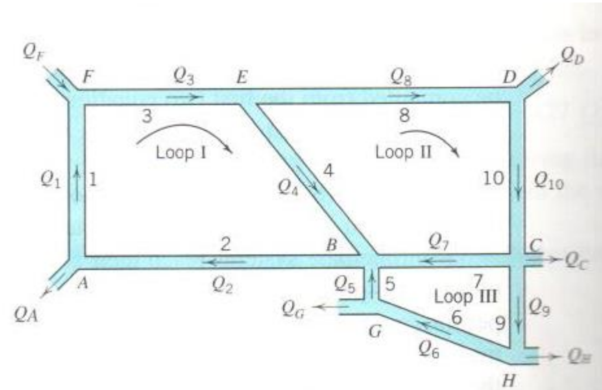
- For general head loss equations

$$\sum_L h_{Li} = \sum_L \pm K_i Q_i^n = 0 \qquad (8.16)$$

$$\sum_{I} h_L = K_1Q_1^n + K_3Q_3^n + K_4Q_4^n + K_2Q_2^n = 0$$

$$\sum_{II} h_L = -K_4Q_4^n + K_8Q_8^n + K_{10}Q_{10}^n + K_7Q_7^n = 0$$

$$\sum_{III} h_L = K_5Q_5^n - K_7Q_7^n + K_9Q_9^n + K_6Q_6^n = 0$$



- Substitute (18.15) into (18.16)

$$\boxed{\sum_{L} h_{Li} = \sum_{L} \pm K_i \left(Q_{0i} \pm \Delta_L\right)^n = 0} \qquad (8.17)$$

- Now, we need to find Δ_L to obtain Q_i in Eq. (8.15)

- Expanding Eq. (8.17) by the binomial theorem and neglecting all terms higher order of containing small increments, $\Delta_L^2, \Delta_L^3,$

$$\sum_L h_{Li} = \sum_L \pm K_i \left(Q_{0i} \pm \Delta_L\right)^n = \sum_L \pm K_i \left(Q_{0i}^n \pm n Q_{0i}^{n-1} \Delta_L + \cancel{O(\Delta_L^2)}\right) = 0$$

- Solving for Δ_L, the first correction for loop *L* becomes

$$\boxed{\Delta_L = -\frac{\sum_L \pm K_i Q_{0i}^n}{\sum_L \left| n K_i Q_{0i}^{n-1} \right|} \quad (n = 2)} \qquad (8.18)$$

- The absolute value must be used in the denominator to insure the proper sign for Δ_L

- Several pipes share loop and we neglect the higher order flow increment. Therefore, Eq. (8.18) does not produce Δ_L that precisely corrects all the Q_{0i} to the final Q_i which satisfy the head loss equations.
- Therefore, we need to iterate to find final values.
- So, iteration equation is

$$\boxed{\Delta_L^{(j+1)} = -\frac{\sum_L \pm K_i \left(Q_{0i}^{(j)}\right)^n}{\sum_L \left| nK_i \left(Q_{0i}^{(j)}\right)^{n-1} \right|}} \tag{8.19}$$

j : *j*th trial-and-correction for loop *L*

- The iterative process will be stopped when all the Δ_L have dropped below an accuracy limit.

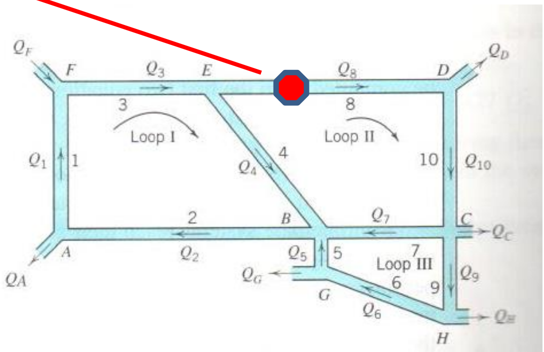
- Pump in pipe network

- To add a **pump** to a pipe in the network, an expression representing the head increase versus the capacity curve is required.
- One method to accomplish this is to a fit a polynomial curve to the pump characteristics to form an equation of the form

$$E_{pi} = a_0 + a_1 Q_i + a_2 Q_i^2 + a_3 Q_i^3 + \cdots \qquad (8.20)$$

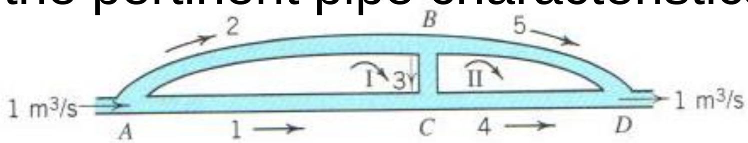
- If a pump (head increase) is added to the line 8 in loop II, the head loss equation for loop II becomes

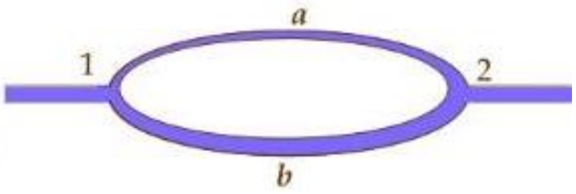
$$-K_4 Q_4^n + K_8 Q_8^n - \left(a_0 + a_1 Q_8 + a_2 Q_8^2 + \cdots\right) + K_{10} Q_{10}^n + K_7 Q_7^n = 0 \qquad (8.21)$$



I.P. 9.21 (p. 387~388)

- A parallel commercial steel pipe network was build in two parts. As shown below, section ACD is the original line the parallel section ABC was then added; then section BD was added to complete the job. By accident, a valve is left open in the short pipe BC. What are the resulting flowrates in all the pipes, neglecting local losses and assuming the flows are wholly rough.
- The pipe table below constructed using the Darcy-Weisbach equation gives all the pertinent pipe characteristics.





Pipe No.	Length (m)	Diameter (m)	e/d	f	K_i (Eq. 9.50)
1	1 000	0.5	9×10^{-5}	0.012	31.7
2	1 000	0.4	1×10^{-4}	0.012	96.8
3	100	0.4	1×10^{-4}	0.012	9.7
4	1 000	0.5	9×10^{-5}	0.012	31.7
5	1 000	0.3	1.4×10^{-4}	0.013	442.0

Solution:

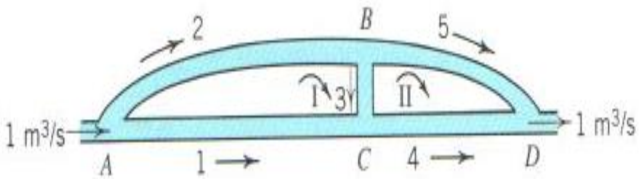
- We begin the analysis by writing out the equations for increment for each loop.

$$\Delta_L = -\frac{\sum_L \pm K_i Q_{0i}^n}{\sum_L \left| n K_i Q_{0i}^{n-1} \right|} \quad (n = 2)$$

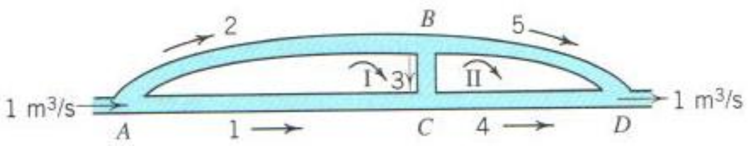
zero subscript for first iteration

$$\Delta_I = -\frac{K_2 Q_{02}^2 + K_3 Q_{03}^2 - K_1 Q_{01}^2}{2\left(\left|K_2 Q_{02}\right| + \left|K_3 Q_{03}\right| + \left|K_1 Q_{01}\right|\right)}$$

$$\Delta_{II} = -\frac{-K_3 Q_{03}^2 + K_5 Q_{05}^2 - K_4 Q_{04}^2}{2\left(\left|K_3 Q_{03}\right| + \left|K_5 Q_{05}\right| + \left|K_4 Q_{04}\right|\right)}$$



Solution:



Initial calculation	Subsequent calculations	
$Q_1 = Q_{01} + \Delta_I$	$Q_1^{(j+1)} = Q_1^{(j)} + \Delta_I^{(j)}$	
$Q_2 = Q_{02} + \Delta_I$	$Q_2^{(j+1)} = Q_2^{(j)} + \Delta_I^{(j)}$	
$Q_3 = Q_{03} + \Delta_I - \Delta_{II}$	$Q_3^{(j+1)} = Q_1^{(j)} + \Delta_I^{(j)} - \Delta_{II}^{(j)}$	(Loop I)
$Q_3 = Q_{03} - \Delta_I + \Delta_{II}$	$Q_3^{(j+1)} = Q_1^{(j)} - \Delta_I^{(j)} + \Delta_{II}^{(j)}$	(Loop II)
$Q_4 = Q_{04} + \Delta_{II}$	$Q_4^{(j+1)} = Q_4^{(j)} + \Delta_{II}^{(j)}$	
$Q_5 = Q_{05} + \Delta_{II}$	$Q_5^{(j+1)} = Q_5^{(j)} + \Delta_{II}^{(j)}$	

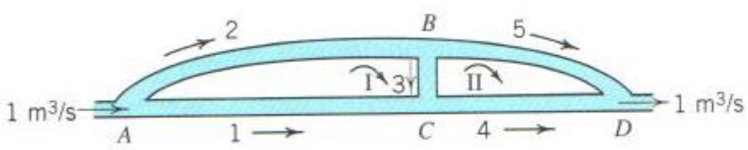
- The iteration is carried out by setting up a table for systematically calculating the increment and correcting the flowrates in all pipes. The following table illustrates for the first two iterations.

Loop I			First iteration			*Second iteration*		
pipe	*K*	Q_{0i}	KQ_{0i}^2	KQ_{0i}	$Q^{(1)}$	KQ^2	KQ	$Q^{(2)}$
1	31.7	-0.5	-7.93	15.89	-0.63	-12.58	19.97	-0.64
2	96.8	0.5	24.20	48.40	0.37	13.25	35.82	0.36
3	9.7	0.4	1.55	3.88	0.12	0.14	1.16	0.15
		∑=	17.82	68.13		0.81	56.95	
			$\Delta_I^{(1)}$	=-0.13		$\Delta_I^{(2)}$	=-0.01	

$$\Delta_I = -\frac{-K_1Q_{01}^2 + K_2Q_{02}^2 + K_3Q_{03}^2}{2\left(\left|K_1Q_{01}\right| + \left|K_2Q_{02}\right| + \left|K_3Q_{03}\right|\right)}$$

=0.4+(−0.13)−0.15

$$Q_3 = Q_{03} + \Delta_I - \Delta_{II} \quad \text{(Loop I)}$$

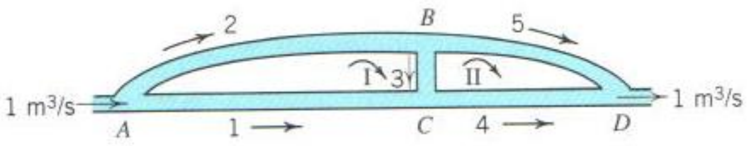


$$Q_3 = Q_{03} - \Delta_I + \Delta_{II} \quad \text{(Loop II)}$$

=−0.4−(−0.13)+0.15

Loop II			First iteration			Second iteration		
pipe	K	Q_0	KQ_0^2	KQ_0	$Q^{(1)}$	KQ^2	KQ	$Q^{(2)}$
3	9.7	-0.4	-1.55	3.88	-0.12	-0.14	1.16	-0.15
4	31.7	-0.9	-25.68	28.53	-0.75	-17.83	23.78	-0.79
5	442.0	0.1	4.42	44.20	0.25	27.63	110.50	0.21
			-22.81	76.61		9.66	135.44	
			$\Delta_{II}^{(1)}$	0.15		$\Delta_{II}^{(2)}$	=-0.04	

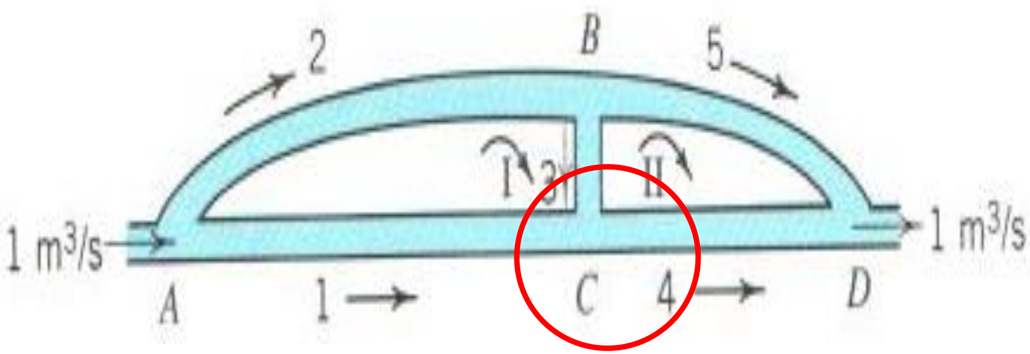
$$\Delta_{II} = -\frac{-K_3 Q_{03}^2 + K_5 Q_{05}^2 - K_4 Q_{04}^2}{2\left(\left|K_3 Q_{03}\right| + \left|K_5 Q_{05}\right| + \left|K_4 Q_{04}\right|\right)}$$



- We assign an algebraic sign to each flowrate in the network, giving a positive sign to those flows which move in a clockwise direction around the loop and a negative sign to those that flow counterclockwise.

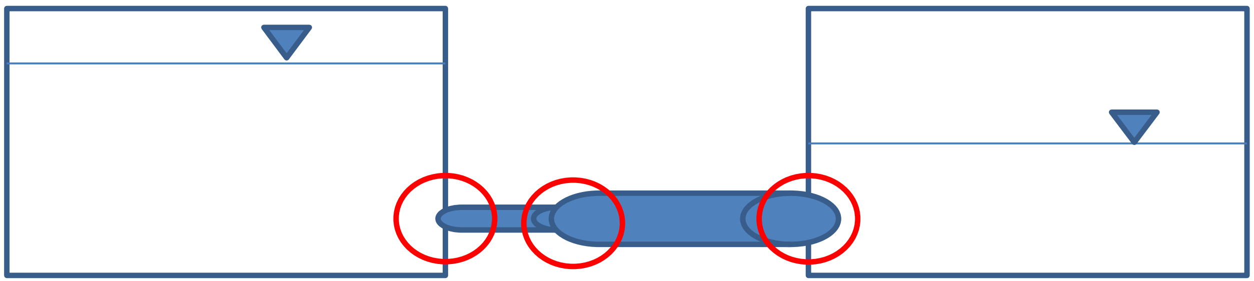
$$Q_1 = -0.64$$
$$Q_2 = +0.36$$
$$Q_3 = +0.15$$
$$Q_3 = -0.15$$
$$Q_4 = -0.79$$
$$Q_5 = +0.21$$

$$\sum_C Q = 0.64 + 0.15 - 0.79 = 0$$

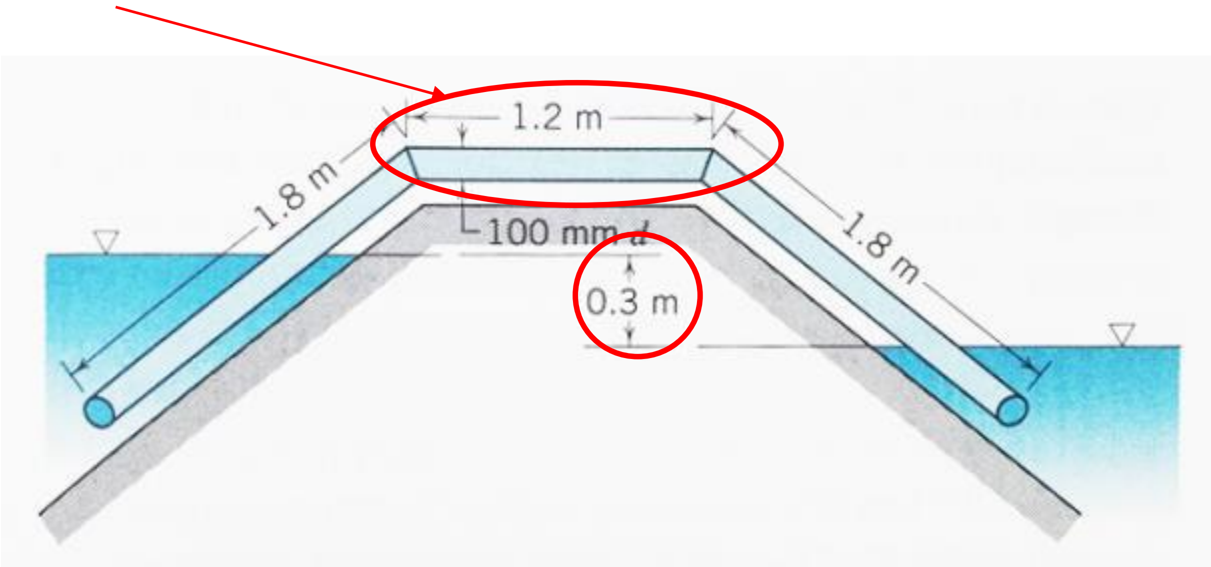


Homework Assignment No. 4
Due: 2 weeks from today
Answer questions in Korean or English

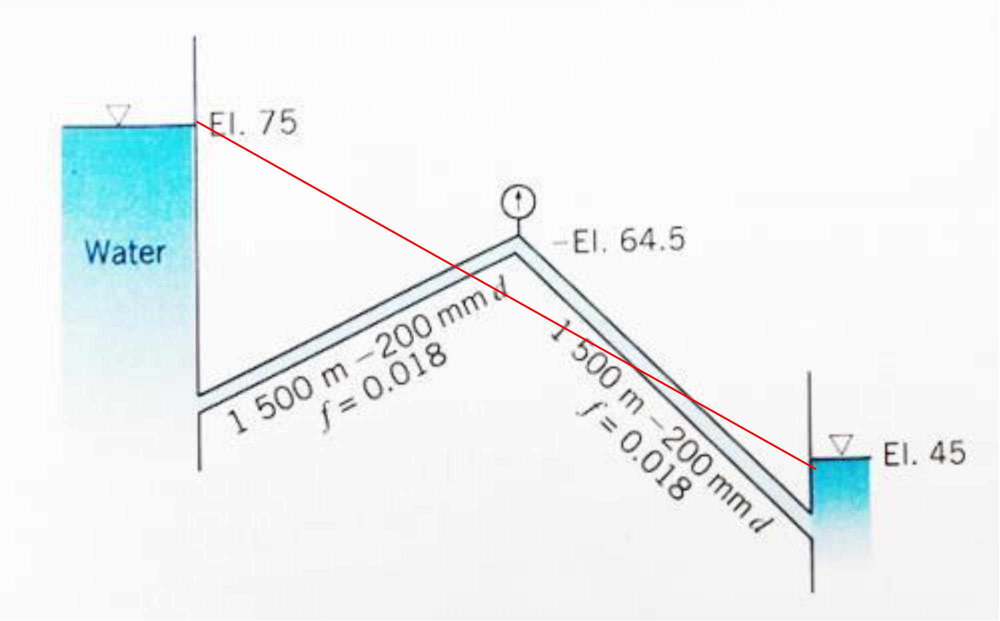
1.(9-111) A horizontal 50 *mm* PVC (smooth) pipeline leaves (square-edged entrance) a water tank 3 *m* below its free surface. At 15 *m* from the tank, it enlarges abruptly to a 100 *mm* pipe which runs 30 *m* horizontally to another tank, entering it 0.6 *m* below its surface. Calculate the flowrate through the line (water temperature 20 ℃), including all head losses.→ Type 2



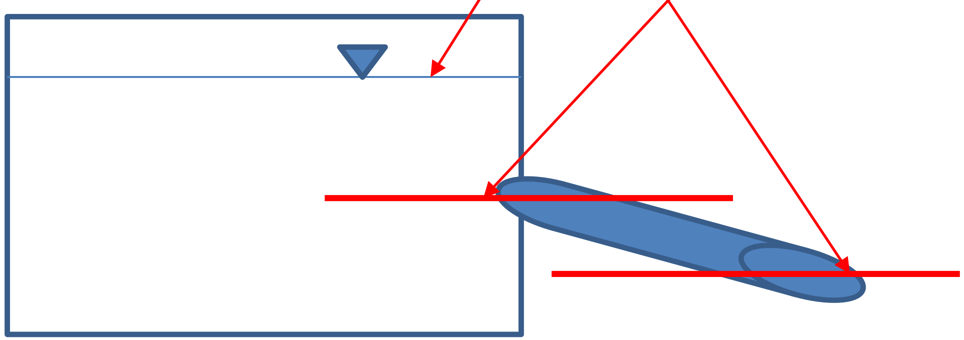
2. (9–123) An irrigation siphon has the dimensions shown and is placed over a dike. Estimate the flowrate to be expected under a head of 0.3 *m*. Assume a re–entrant entrance, a friction factor of 0.020, and bend loss coefficients of 0.20.



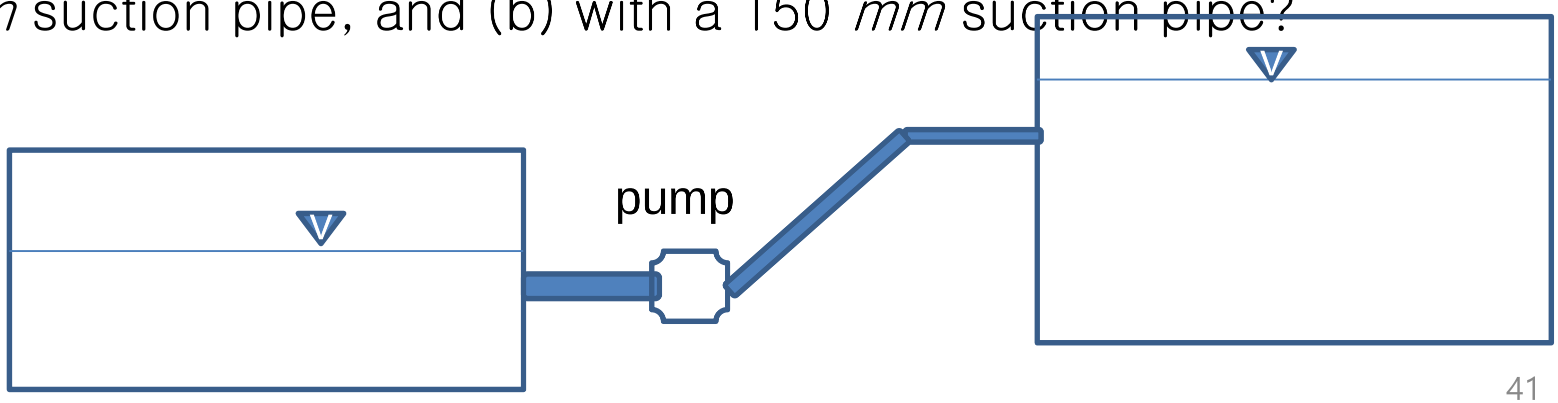
3. (9-124) Calculate the flowrate and the gage reading, neglecting local losses and velocity heads.



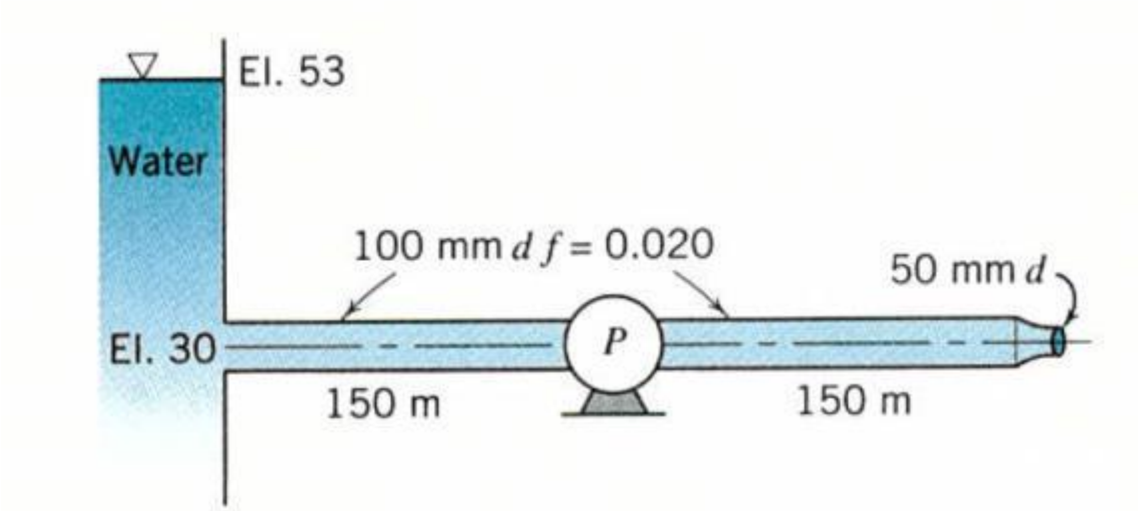
4. (9–132) A 0.3 *m* pipeline 450 *m* long leaves (square-edged entrance) a reservoir of surface elevation 150 at elevation 138 and runs to elevation 117, where it discharges into the atmosphere. Calculate the flowrate and sketch the energy and hydraulic grade lines (assuming that f = 0.022) (a) for these conditions, and (b) when a 75 *mm* nozzle is attached to the end of the line, assuming the lost head caused by the nozzle to be 1.5 *m*. How much power is available in the jet?



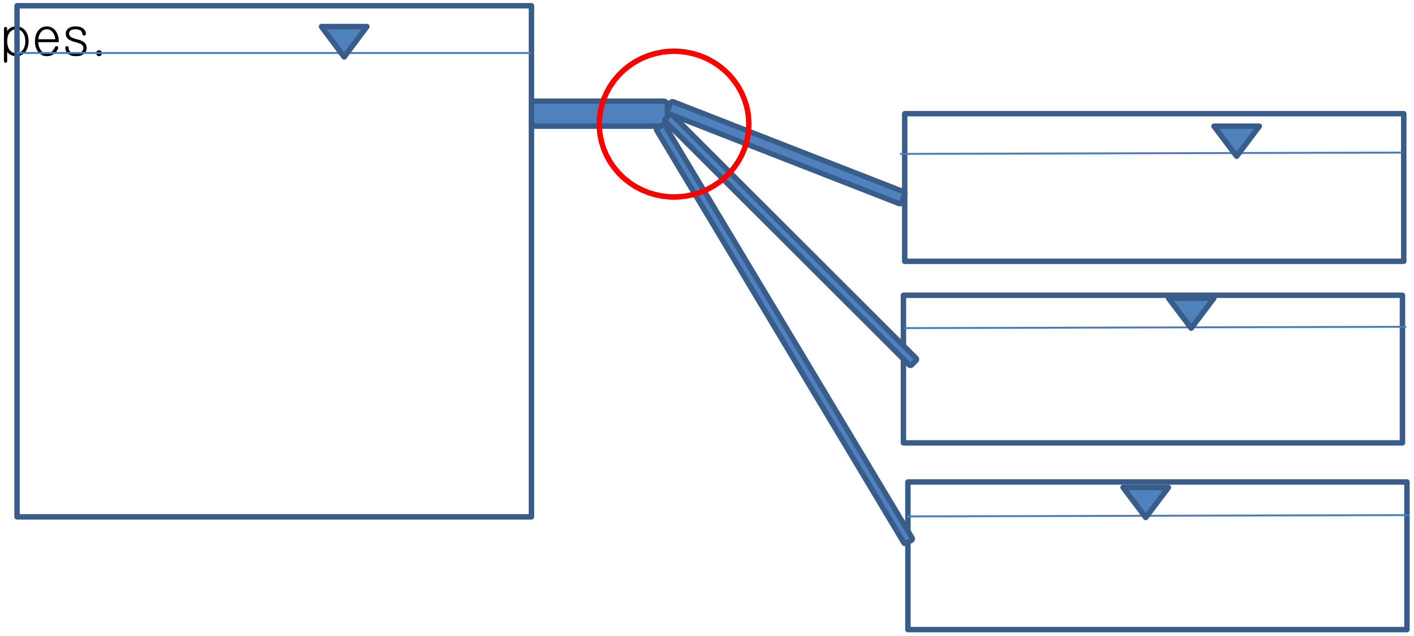
5. (9–138) The horizontal 200 *mm* suction pipe of a pump is 150 *m* long and is connected to a reservoir of surface elevation 90 *m*, 3 *m* below the water surface. From the pump, the 150 *mm* discharge pipe runs 600 *m* to a reservoir of surface elevation 126, which it enters 10 *m* below the water surface. Taking *f* to be 0.020 for both pipes, calculate the power required to pump 0.085 m^3/s from the lower reservoir. What is the maximum dependable flowrate that may be pumped through this system (a) with the 200 *mm* suction pipe, and (b) with a 150 *mm* suction pipe?



6. (9-143) The pump is required to maintain the flowrate which would have occurred without any friction. What power pump is needed? Neglect local losses.



7. (9–166) A 0.9 *m* pipe divides into three 0.45 *m* pipes at elevation 120. The 0.45 *m* pipes run to reservoirs which have surface elevations 90, 60, and 30, these pipes having respective lengths of 3.2, 4.8, and 6.8 kilometers. When 1.4 m^3/s flows in the 0.9 *m* line, how will the flow divide? Assume that f = 0.017 for all pipes.



8. (9-175) Calculate the flowrates on the pipes of this loop if all friction factors are 0.020.