Ch. 6 Unsteady Flow in Pipes

Contents

Objectives of class

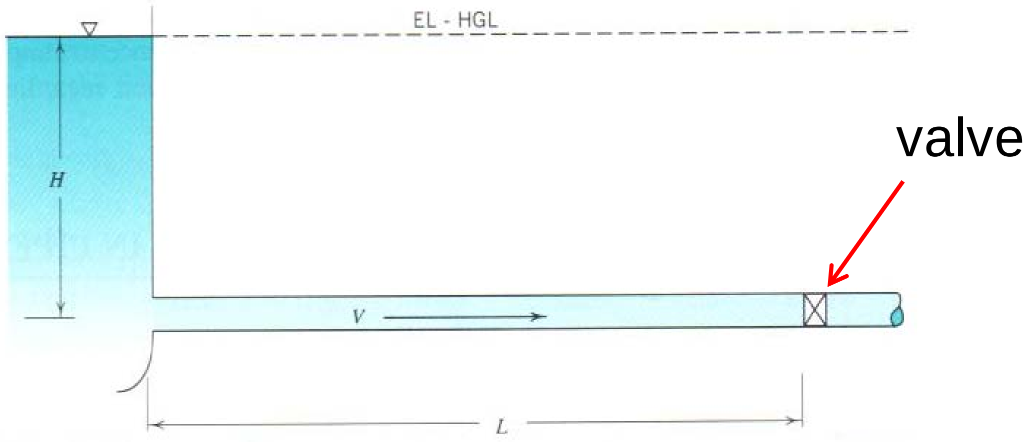
- Learn how to account for the unsteadiness of flow through pipe.
- Learn rigid water column theory and water hammer theory

6.1 Unsteady Flow in Pipes

- Steady flow is so rare in real situation.
- But still in many case, the unsteadiness is small. So, a few cases are interested.
- Once unsteady flow occurs, that is very important since it can cause excessive pressure, vibration, cavitation and noise etc.
- In fact, the problems created by hydraulic transients may be so severe as to cause physical or performance failure of a system.

The analysis of unsteady flow

- Then which theory should be applied to description of flows?
- Therefore we will examine the action of water hammer in a simple pipeline situation.
- In unsteady flow, the friction is important, therefore, we remember the friction factor which we learned in the last classes.
- **Simple water hammer example**
  - A valve is placed a distance *L* from the reservoir. Let's first ignore friction for simplification and velocity head is assumed to be very small.



The Valve

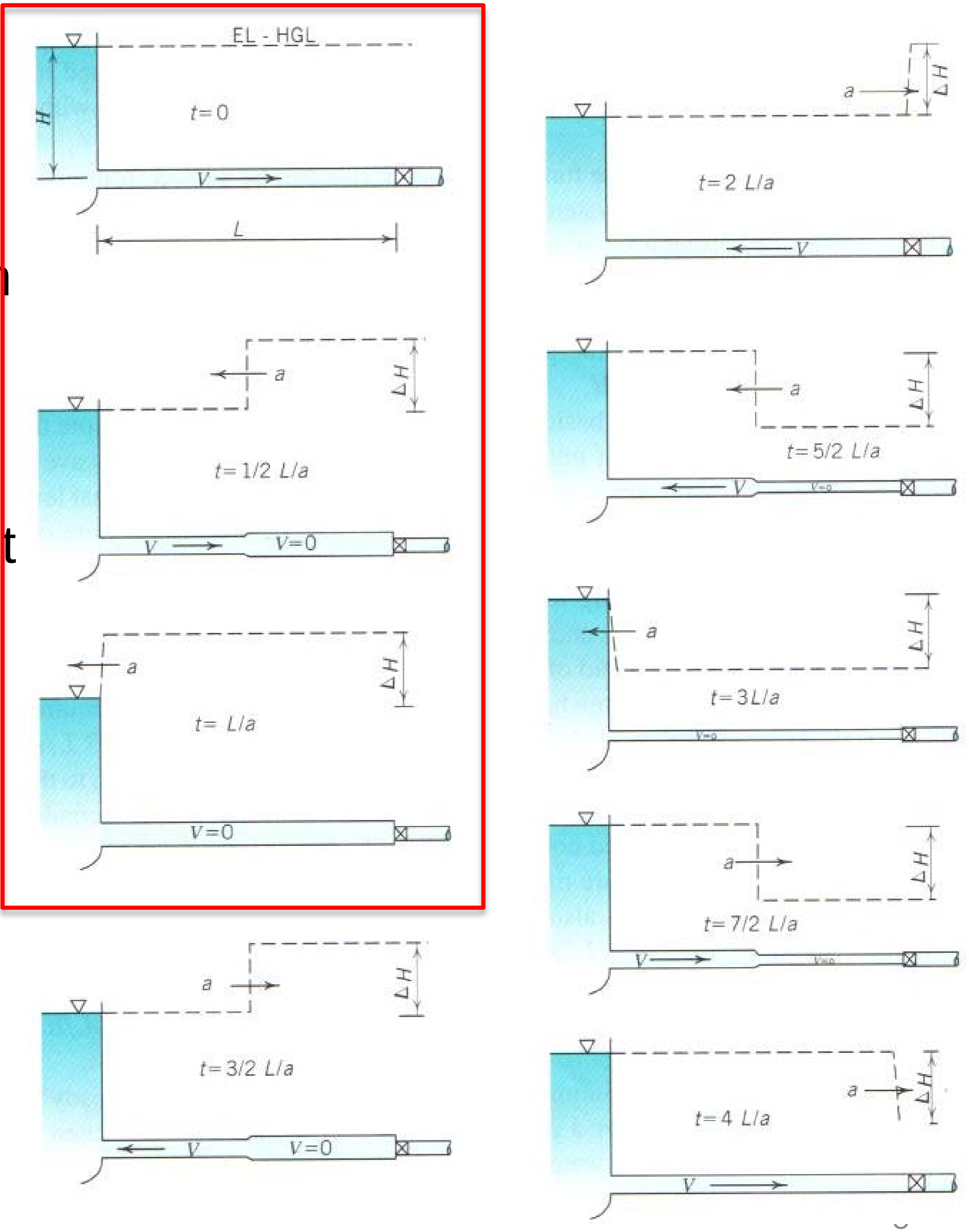
- A **valve** is a device attached to a pipe or a tube which controls the flow of air or liquid through the pipe or tube.
  - Globe valve
  - Needle valve
  - Sluice valve
  - Butterfly valve
  - Check valve

The analysis of unsteady flow

- Water hammer will be introduced into the system by suddenly closing the valve.
- Activity will occurs down and upstream, but we care only upstream.
- Sudden closure of the valve leads the velocity of water to be zero suddenly.
- As a consequence, the pressure head at the valve increases suddenly by an amount ΔH.
- The amount of ΔH is just the amount of pressure head necessary to change the momentum of the liquid initially flowing at velocity V at the valve to zero.
- The increase in pressure at the valve results in a stretching of the pipe and an increase in the density of the liquid.
- Those are dependent of the pipe materials and size of pipe and the liquid elasticity (compressibility).
- Generally for common pipe materials, change is less than 0.5%.

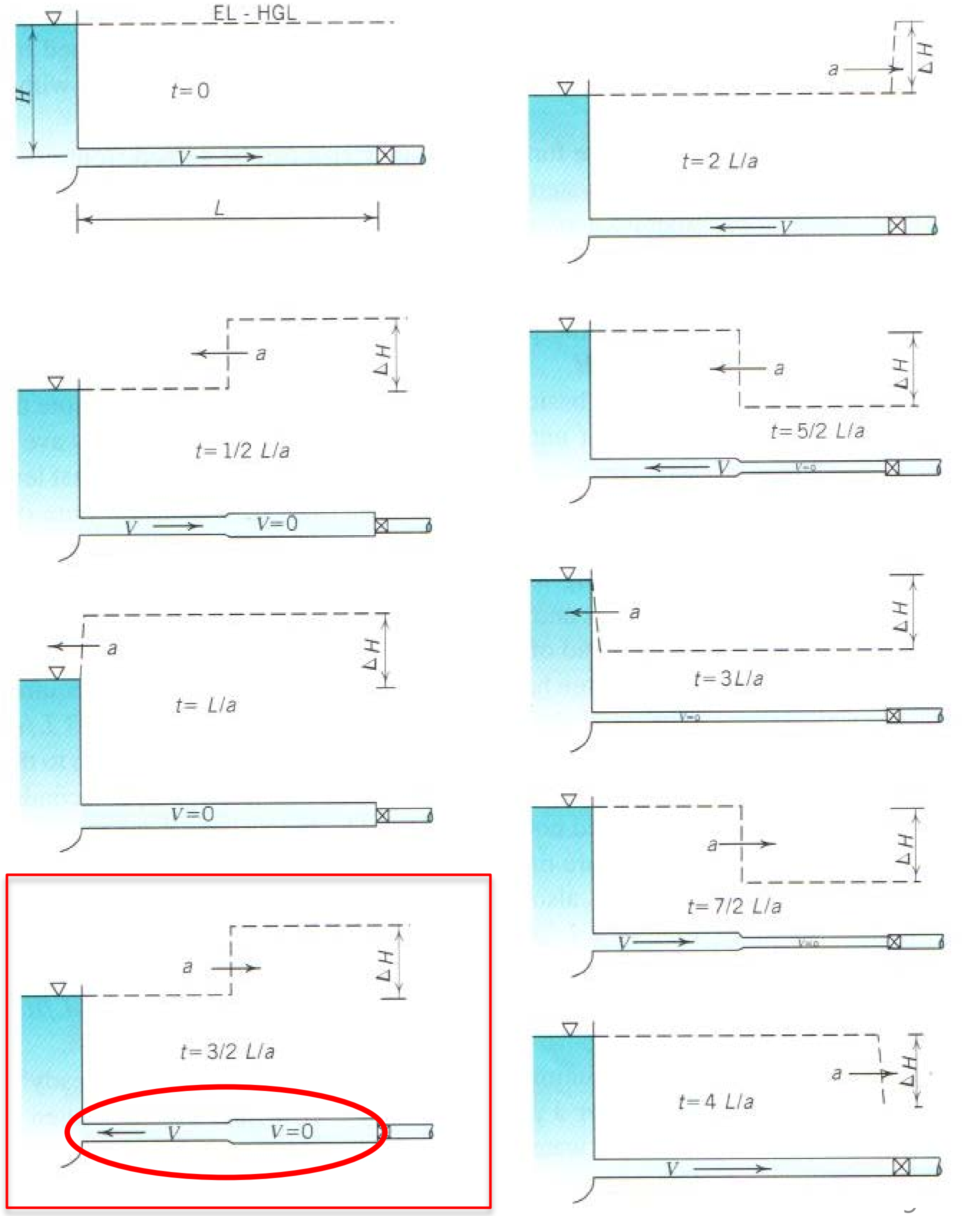
The water hammer
- *t=0~L/a*

- The pressure increase **propagates upstream at a wave speed *a***, which is determined by elastic properties of the system and the liquid and the system geometry.
- The wave speed will remain constant so long as they remain constant.
- Traveling at a speed ***a*,** the wave will <u>reach the reservoir in a time *L/a*</u>.
- The velocity in the pipe is everywhere zero.



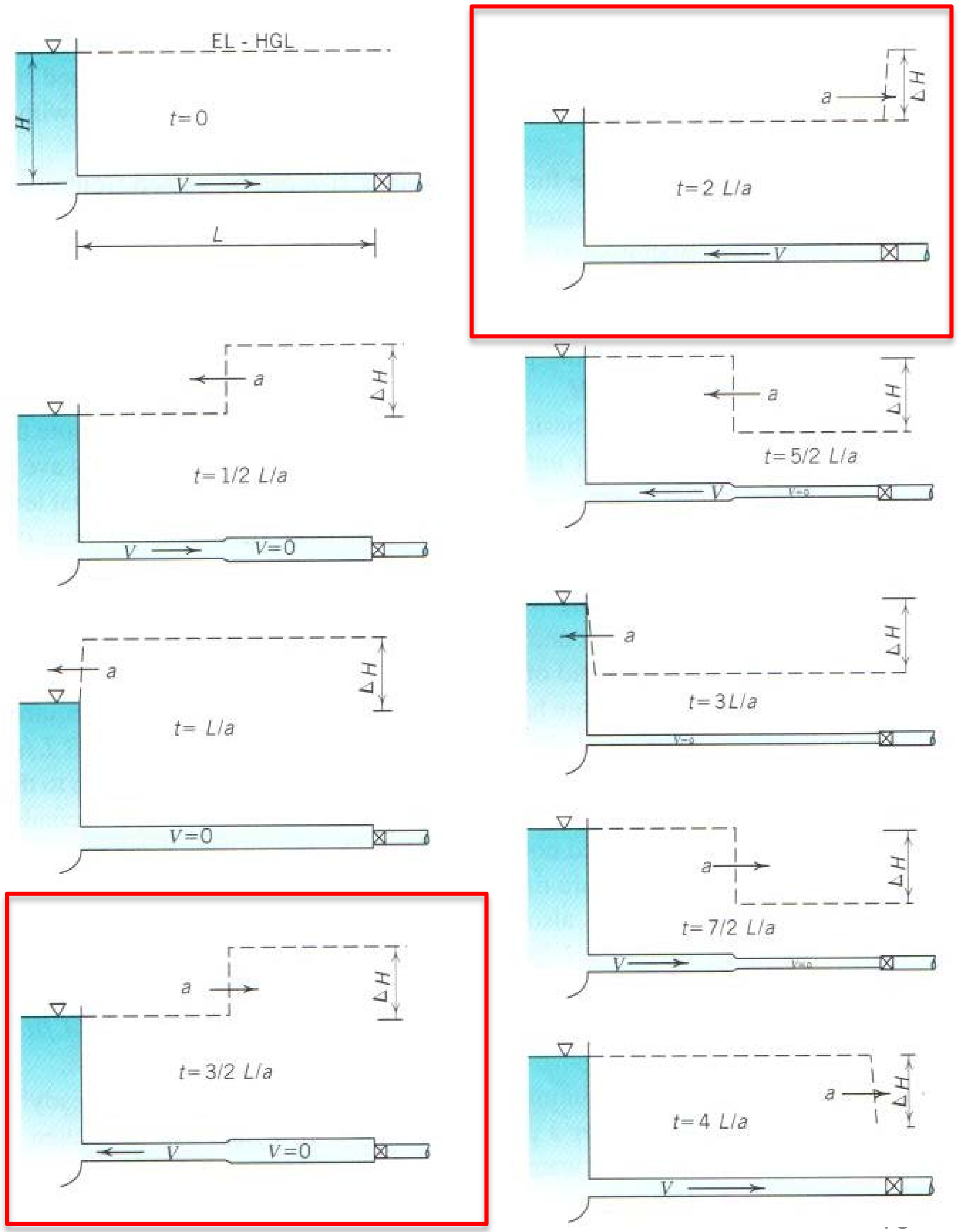
The water hammer
- *t=3/2 L/a*

- When velocity is zero everywhere, the pressure head is $H+\Delta H$, the <u>pipe is stretched</u>, and the <u>fluid is compressed</u>.
- Under these conditions the liquid in the pipe is not in equilibrium because the pressure head in the reservoir is only H. As a result, <u>flow begins to occur toward the reservoir</u> as the distended pipe ejects liquid in that direction.



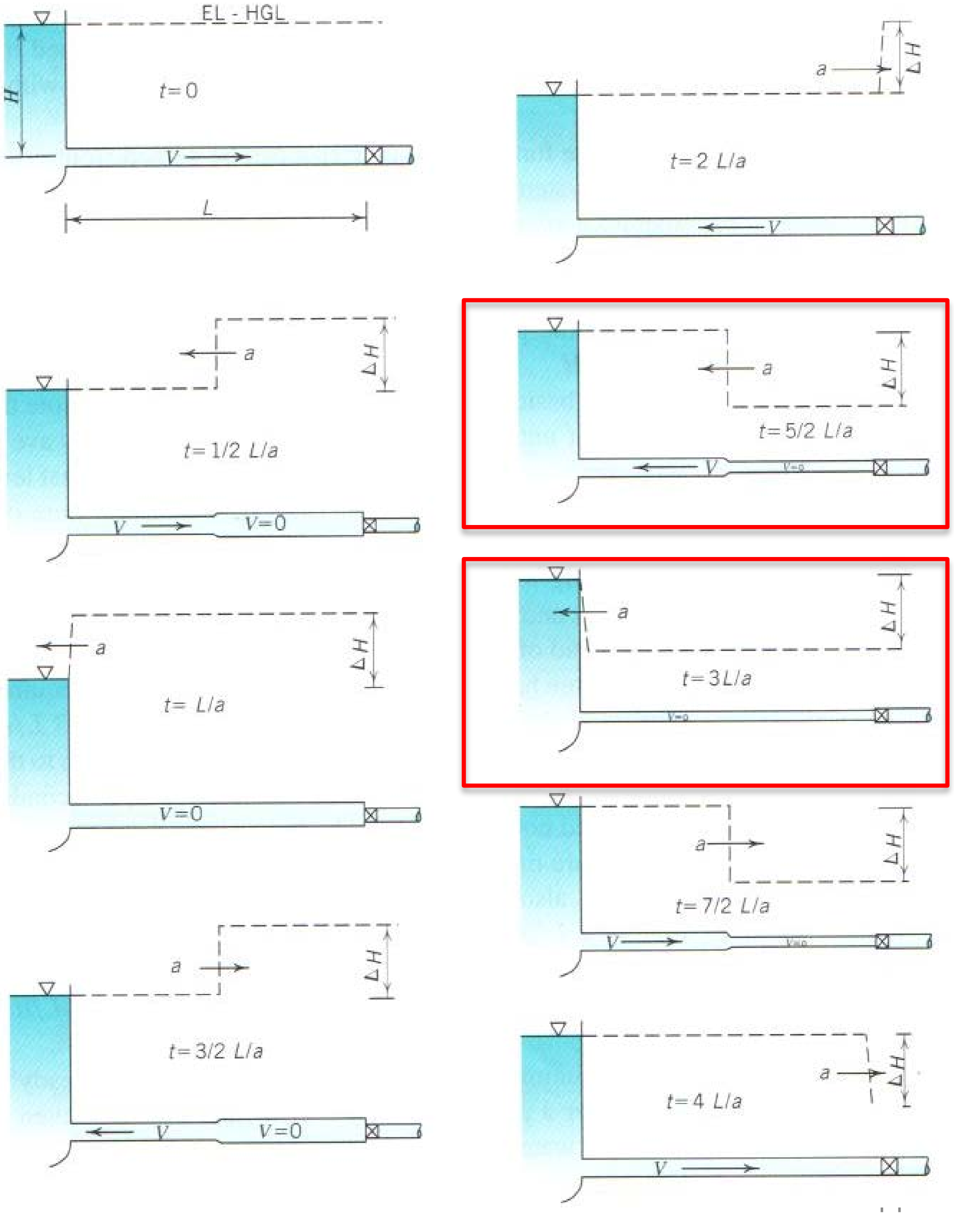
The water hammer
- *t=3/2 L/a~2 L/a*

- The **reverse velocity** is equal in magnitude to the initial steady velocity (as a result of neglecting friction) and the source of liquid for the reverse flow is the liquid previously stored in the stretched pipe walls as a compressed liquid.
- This process continues and a time 2 *L/a*, the pressure has returned to normal throughout the pipe.



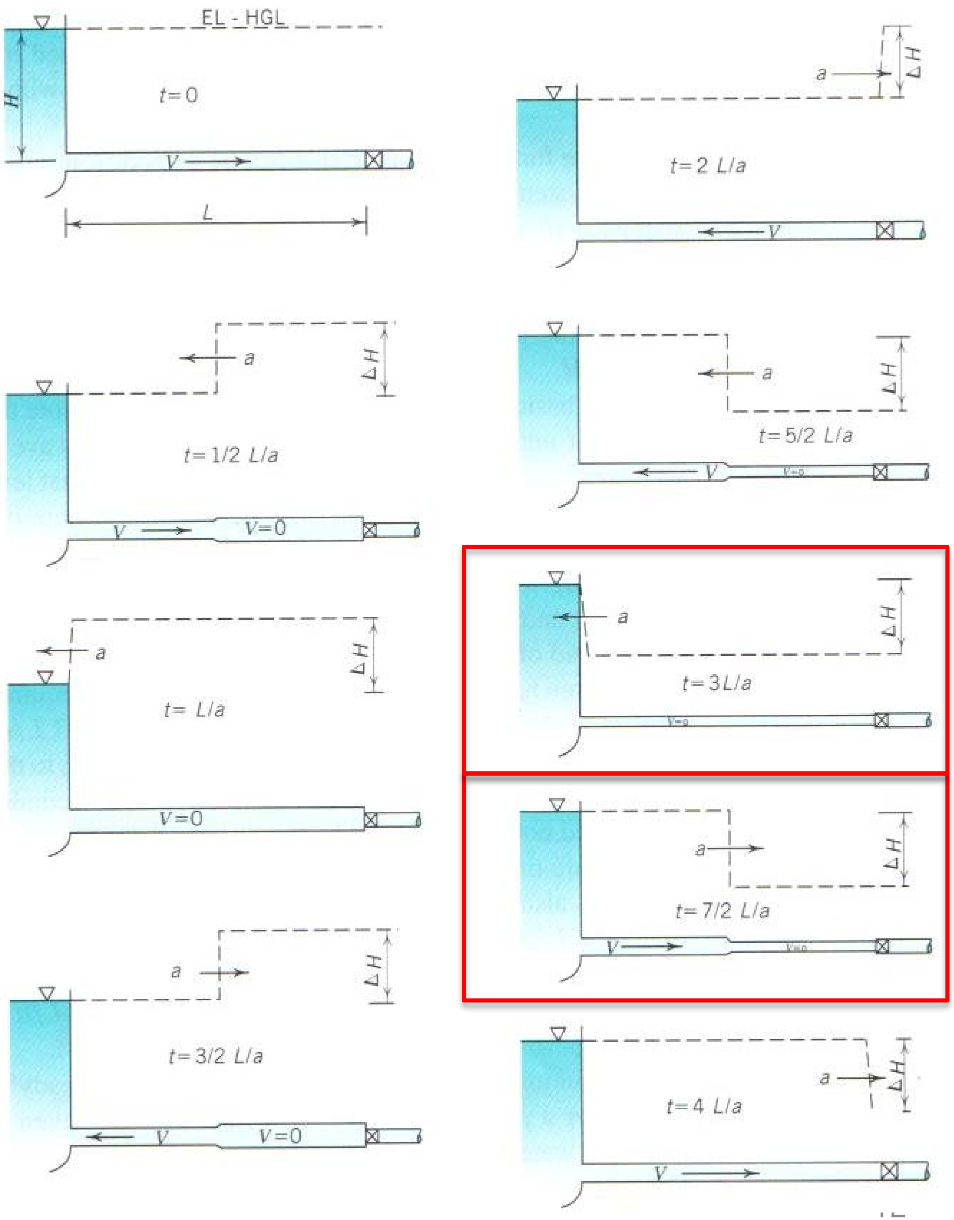
The water hammer
- *t=5/2 L/a~3 L/a*

- However, there is no source of liquid at the valve to supply the upstream flow hence the pressure had dropped an additional *ΔH* to force the reverse velocity to zero
- This drop in pressure causes the pipe to shrink and the liquid to expand.
- At time 3 *L/a* this effect has propagated to the reservoir and the velocity of flow is everywhere zero.



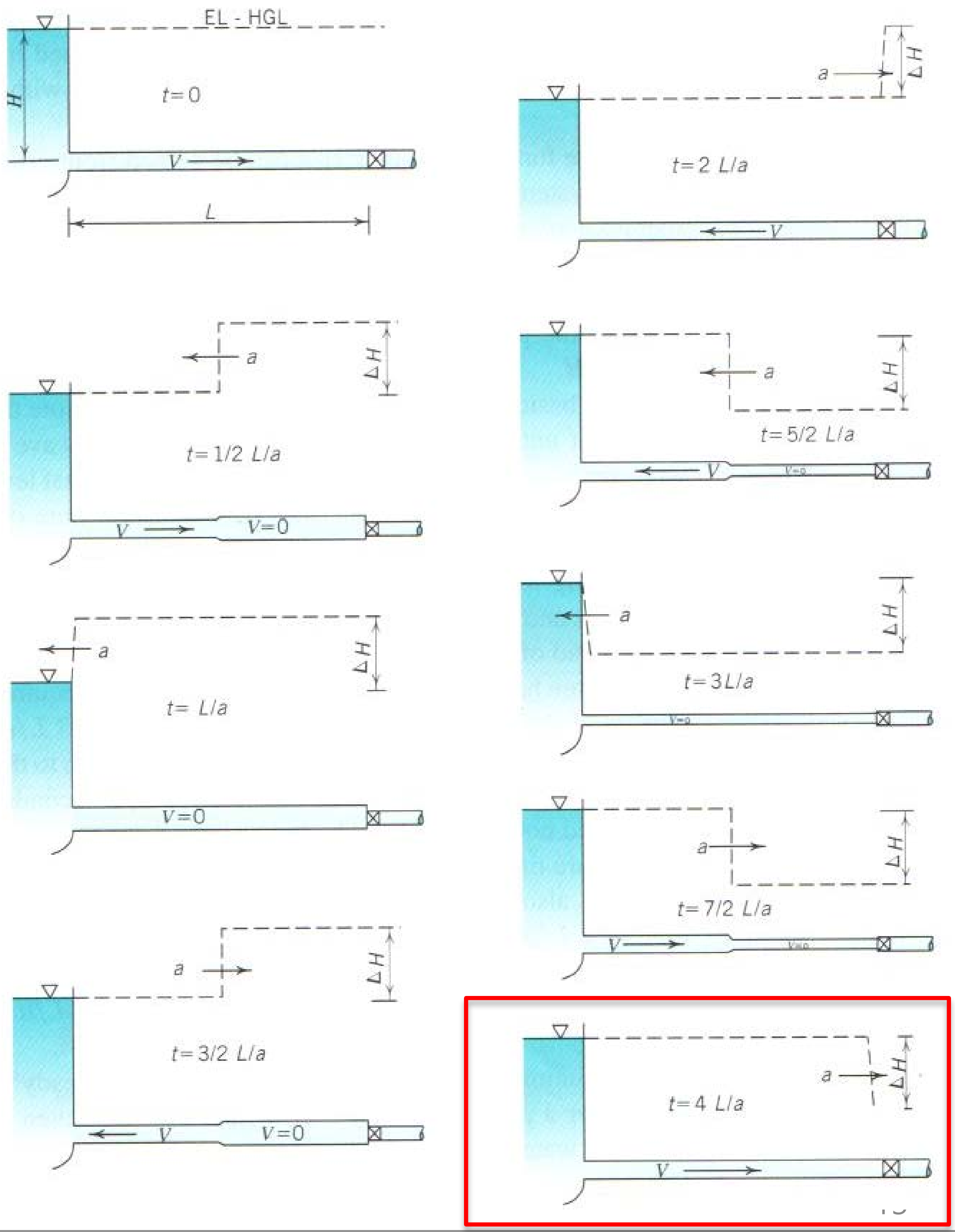
The water hammer
- *t=3 L/a~7/2 L/a*

- However, the pipe pressure head is *ΔH* below that of the reservoir. Consequently, the <u>pipe sucks in liquid from the reservoir</u> creating a velocity of flow equal to and in the same direction as the original steady flow.
- While this is occurring, the pressure in the pipe is also returning to its original value .



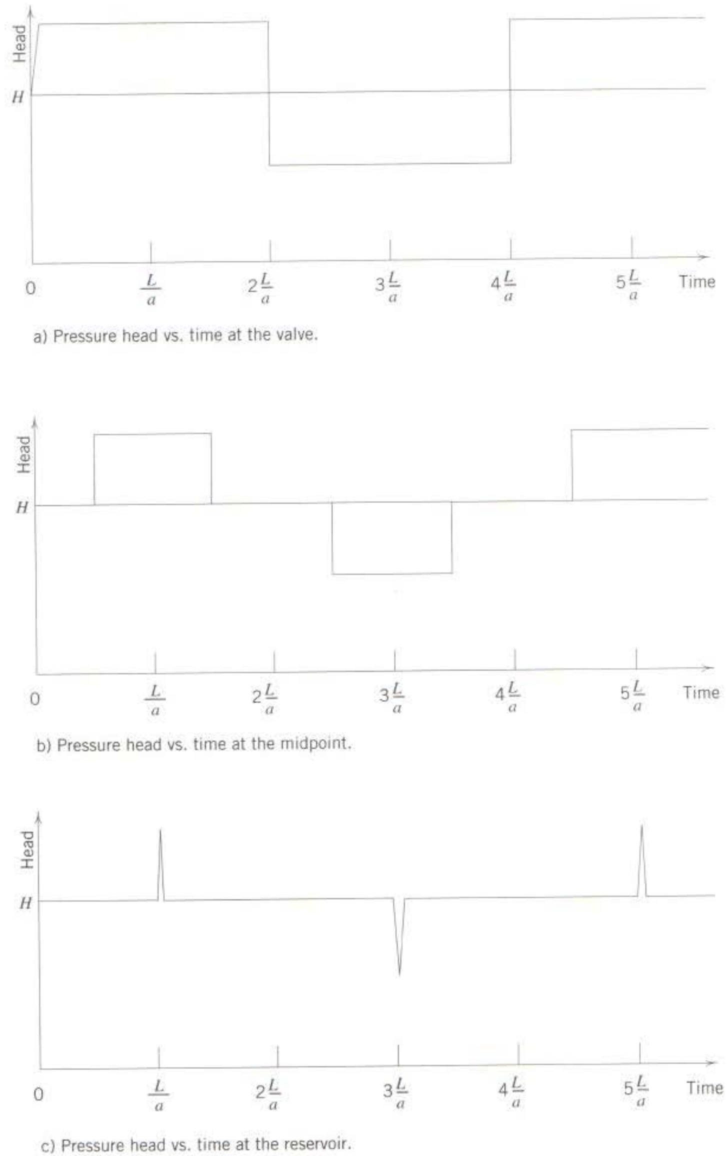
The water hammer
- *t=4 L/a*

- After time 4 *L/a*, this wave has reached the valve and at this instant the flow is identical to its original steady state configuration.
- This elapsed time constitutes **one wave period**.
- As time goes on, **this cycle** of events will continue without abatement.



The analysis of unsteady flow

- (Fig. b) Pressure does not increase at a point until enough time has occurred for the wave to travel from the closed valve.
- Once the pressure head has increased, it remains at that level only long enough for “**relief**” to arrive back from the reservoir.
- This is idea of “time of communication” or “message propagation time”.



a) Pressure head vs. time at the valve.

b) Pressure head vs. time at the midpoint.

c) Pressure head vs. time at the reservoir.

The analysis of unsteady flow

- Instead of closing the valve suddenly, we were to close to it in 10 steps, each increasing the pressure head at the valve by $\Delta H/10$.
- A further requirement would be that the complete closure of the valve would be accomplished before 2 L/a seconds had elapsed.
- It is clear that the pressure head at the valve would still build up to the full ΔH value because "relief" from the reservoir could arrive before 2 L/a seconds.
- The point to be made is that valve need not be closed suddenly to create the maximum water hammer pressure.
- Indeed, <u>any closure time less than the time necessary for relief</u> to return from a reservoir will <u>result in full water hammer pressures</u>.
- This time of 2 L/a is know as the "***Critical time of closure***".
- It may be necessary to close the valve in a time <u>much greater than 2 L/a to prevent high pressures</u> from occurring.

The analysis of unsteady flow

- What to use?

- Action causing unsteady flow takes place over a period of many 2 *L/a* time intervals, then it would be more appropriate to use rigid water column theory.
- On the other hand, the action was completed in only a few *L/a* time intervals or less, then elastic theory should be utilized.

6.2 Rigid Water Column Theory

- To analyze unsteady flow problems in pipe systems by rigid water column, deriving a equation.
- Please remind the Euler's equation

$$\frac{dp}{\rho} + VdV + g_n dz = 0 \quad \text{(one dimensional Euler's equation)}$$

$$\frac{dp}{\rho} + \frac{ds}{dt} dV + g_n dz = 0$$

$$\frac{1}{\gamma}\frac{dp}{ds} + \frac{1}{g_n}\frac{dV}{dt} + \frac{dz}{ds} = 0 \qquad \text{(modified Euler's equation)}$$

- Including shear stress and send to the infinitesimal control volume

$$-\frac{1}{\gamma}\frac{\partial p}{\partial s} - \frac{\partial z}{\partial s} - \frac{4}{\gamma}\frac{\tau}{d} = \frac{1}{g_n}\frac{dV}{dt}$$

Rigid water column theory

- When the system diameter is expanded to the size of the pipe diameter,

$$-\frac{1}{\gamma}\frac{\partial p}{\partial s}-\frac{\partial z}{\partial s}-\frac{4}{\gamma}\frac{\tau_0}{d}=\frac{1}{g_n}\frac{dV}{dt}$$

Wall shear stress

- Substitute the form of friction factor

$$-\frac{1}{\gamma}\frac{\partial p}{\partial s}-\frac{\partial z}{\partial s}-\frac{f}{d}\frac{V^2}{2g_n}=\frac{1}{g_n}\frac{dV}{dt}$$

- Since *s* is function of datum and distance,

$$-\frac{1}{\gamma}\frac{\partial p}{\partial s}-\frac{dz}{ds}-\frac{f}{d}\frac{V^2}{2g_n}=\frac{1}{g_n}\frac{dV}{dt}$$

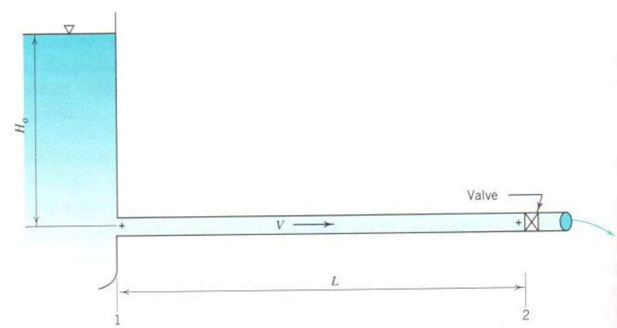
Rigid water column theory (Application to opening valve)

- If the discharge in the pipeline as in your figure, is controlled by the valve at the downstream end, the pressure in the pipe is everywhere equal to H_0 when the valve is closed.
- When the valve is suddenly opened, the pressure at the valve drops instantly to zero and the fluid begins to accelerate.

$$-\int \frac{1}{\gamma}\frac{\partial p}{\partial s}ds-\int \frac{dz}{ds}ds-\int \frac{f}{d}\frac{V^2}{2g_n}ds=\int \frac{1}{g_n}\frac{dV}{dt}ds$$

- In horizontal constant pipe ($\frac{dz}{ds}=0$) and V is a function of time only.
- Also, f in unsteady is the same to the steady flow at instantaneous speed.

$$\frac{p_1}{\gamma}-\frac{p_2}{\gamma}-\frac{fL}{2g_n d}V^2=\frac{L}{g_n}\frac{dV}{dt}$$



Rigid water column theory (Application to opening valve)

- Since the pressure head at reservoir is constant (H_0) and at position 2 ($\frac{p_2}{\gamma} = 0$), eq. becomes

$$H_0 \left(= \frac{p_1}{\gamma} \right) - \frac{fL}{2g_n d} V^2 = \frac{L}{g_n} \frac{dV}{dt}$$

- Integrating with separation variable methods

$$\int dt = \frac{L}{g_n} \int \frac{dV}{H_0 - (fL/2g_n d)V^2}$$

$$t = \sqrt{\frac{Ld}{2g_n f H_0}} \ln \left[\frac{\sqrt{(2g_n H_0 d / fL)} + V}{\sqrt{(2g_n H_0 d / fL)} - V} \right]$$

- When neglecting the local loss the total loss is same to the head

$$H_0 = h_L = f \frac{l}{d} \frac{V_0^2}{2g_n} \Rightarrow V_0 = \sqrt{\frac{2g_n H d}{fL}}$$ (steady state velocity)

Rigid water column theory (Application to opening valve)

- The equation for *t* becomes

$$t = \frac{LV_0}{2g_n H_0} \ln\left[\frac{V_0 + V}{V_0 - V}\right]$$

- It is important to note that as steady flow is approached, $V \rightarrow V_0$ and as a consequence $t \rightarrow \infty$. Of course, this answer is unacceptable so we propose that when $V=0.99V_0$, we have essentially steady flow. With this interpretation,

$$t_{99} = 2.65 \frac{LV_0}{g_n H_0}$$

- This is time to take for velocity to reach almost the same to the final velocity of flow (equilibrium)

Example problem #1: Application to opening valve

- A horizontal 24 inch pipe 10,000 ft long leaves a reservoir 100 ft below the surface and terminates in a valve. The steady-state friction factor for the pipe is 0.018 and it is assumed to remain constant during the acceleration processes. If the valve is opened suddenly, calculate how long it will take for the velocity to reach 99% of its final value. Neglect losses.
- First let's calculate the steady velocity

$$h_L = 100\,\text{ft} = f\frac{l}{d}\frac{V_0^2}{2g_n} = 0.018\frac{10{,}000\,\text{ft}}{2\,\text{in}/12}\frac{V_0^2}{2\times 32.2}$$

$$V_0 = 8.46\,\text{ft}/\text{sec}$$

$$t_{99} = 2.65\frac{LV_0}{g_n H_0} = \frac{2.65\times 10{,}000\,ft\times 8.46\,ft/\text{sec}}{32.2\times 100\,\text{ft}} = 70\,\text{sec}$$

Rigid water column theory (Application to closing valve)

- When valve is suddenly closed, more difficult, since the pressure just upstream of the valve is no longer zero.
- At t=0, the velocity is V_0. (we have non-zero p_2 here and this dependent on velocity)

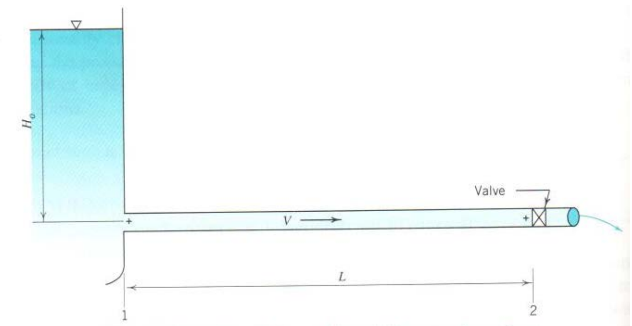
$$H_0 - \frac{p_2}{\gamma} - \frac{fL}{2g_n d}V^2 = \frac{L}{g_n}\frac{dV}{dt}$$

- Therefore, we need another equation for removing pressure term.

$$\frac{p_2}{\gamma} = K_L \frac{V^2}{2g_n}$$

- K_L is valve loss coefficient.

$$H_0 - \left(K_L + \frac{fL}{d}\right)\frac{V^2}{2g_n} = \frac{L}{g_n}\frac{dV}{dt}$$



Rigid water column theory (Application to closing valve)

- K_L is not usually constant and a function of the valve opening. Also, it is not directly related with time or velocity.
- The approach would be to write the equation in finite difference form. With a valve closing schedule specified, its value would be known at any time and would be averaged over each time increment interval.
- **Implicit** relationship

$$V(t+\Delta t)=V(t)+\frac{g_n\Delta t}{L}\left[H_0-\left(\bar{K}_L+f\frac{L}{d}\right)\frac{1}{2g_n}\left(\frac{V(t)+V(t+\Delta t)}{2}\right)^2\right]$$

$$\bar{K}_L=0.5\left[K_L(t)+K_L(t+\Delta t)\right]$$

$$\frac{dV}{dt}\approx\frac{V(t+\Delta t)-V(t)}{\Delta t}$$

Rigid water column theory (Application to closing valve)

- There is limitation of using of tis equation

$$H_0 - \frac{p_2}{\gamma} - \frac{fL}{2g_n d}V^2 = \frac{L}{g_n}\frac{dV}{dt}$$

- Since as faster as valve closure times are used, *dV/dt* becomes quite large (becomes infinity).
- In such case, we should move to the elastic theory (water hammer).

Rigid water column theory (Application to closing valve)

- When we have local loss, then there are two ways to solve.
  - One is break down into two parts
  - Or, just add the local loss to the total loss term

$$\frac{p_1}{\gamma} - \frac{p_2}{\gamma} - \frac{f' L}{2 g_n d} V^2 = \frac{L}{g_n} \frac{dV}{dt}$$

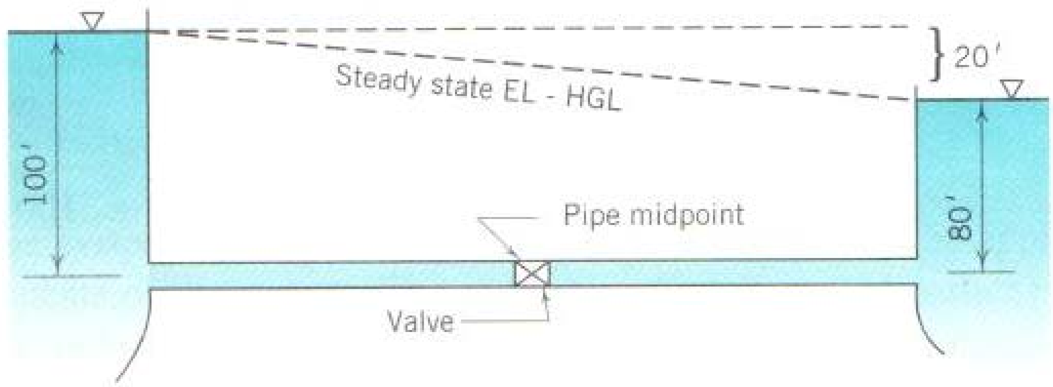
$$f' = f + K_L \frac{d}{L}$$

$$\frac{p_1}{\gamma} - \frac{p_2}{\gamma} - \left(f \frac{L}{d} + K_L \right) V^2 = \frac{L}{g_n} \frac{dV}{dt}$$

- It is important *not to use the traditional equivalent length method* to represent the local loss. This technique adds length to the pipe and the subsequent increases in liquid mass will distort the true dynamic behavior of the system.

Example problem #2

- Water flows from one reservoir to another through the horizontal pipe at a velocity of 10 ft/s. The shutdown plan calls for a valve closure schedule, which will cause the velocity to decrease linearly to zero in 100 sec. The valve is located at the center of the 6,440 ft long pipeline. Estimate the maximum and minimum pressures which will occur in the system, locate them and give the time at which they will occur.



Example problem #2

- The equation for unsteady flow,

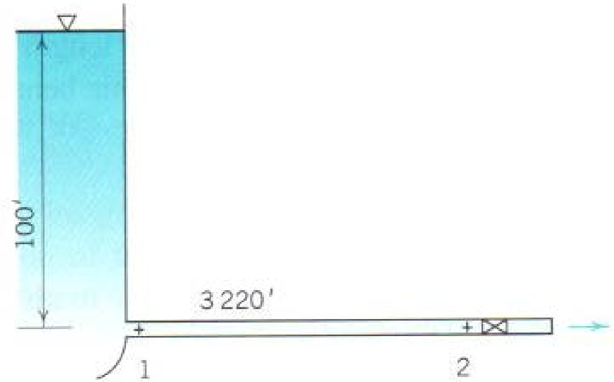
$$\frac{p_1}{\gamma} - \frac{p_2}{\gamma} - \frac{fl}{2g_n d}V^2 = \frac{l}{g_n}\frac{dV}{dt}$$

- Velocity will decrease linearly with time,

$$\frac{dV}{dt} = \frac{-10\,\text{ft/s}}{100\,\text{s}} = -0.10\,\text{ft/s}^2$$

- Now we break down into two parts,
- One is upstream section, another one is down stream section

Example problem #2



- Upstream section

$$\frac{p_1}{\gamma} - \frac{p_2}{\gamma} - \frac{fl}{2g_n d}V^2 = \frac{l}{g_n}\frac{dV}{dt}$$

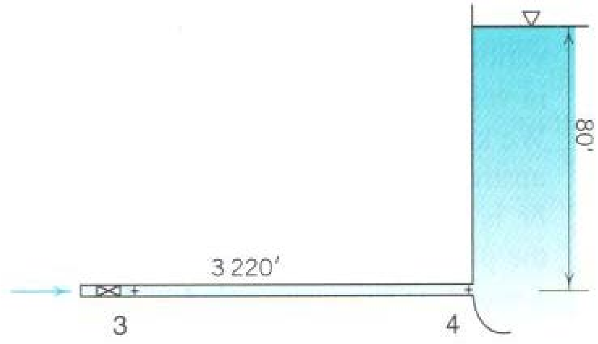
$$100\,ft - \frac{p_2}{\gamma} - f\frac{l}{d}\frac{V^2}{2g_n} = \frac{3220\,ft}{32.2}\left(-0.10\,ft/s^2\right)$$

$$\frac{p_2}{\gamma} = 110 - \frac{fl}{2g_n d}V^2$$

- Maximum pressure will occur when *V*=0, and minimum pressure occurs under steady flow just before the valve begins to close.

$$\left(\frac{p_2}{\gamma}\right)_{max} = 110\,\mathrm{ft} \quad at \quad t = 100\,\mathrm{sec}, \quad \left(\frac{p_2}{\gamma}\right)_{\min} = 90\,\mathrm{ft}\ ((100+80)/2)$$

Example problem #2



- Downstream section

$$\frac{p_3}{\gamma} - \frac{p_4}{\gamma} - \frac{fl}{2g_n d}V^2 = \frac{l}{g_n}\frac{dV}{dt}$$

$$\frac{p_3}{\gamma} - 80\,ft - f\frac{l}{d}\frac{V^2}{2g_n} = \frac{3220\,ft}{32.2}\left(-0.10\,ft/s^2\right)$$

$$\frac{p_3}{\gamma} = 70 + f\frac{l}{d}\frac{V^2}{2g_n}$$

- Under steady flow conditions, the pressure head just down stream of the valve is 90 ft. The instant the valve begins to move, it suddenly drops to 80ft. At the instance the valve reaches complete closure it has reduced to 70ft.

$$\left(\frac{p_3}{\gamma}\right)_{max} = 90\,\text{ft} \quad , \quad \left(\frac{p_3}{\gamma}\right)_{\min} = 70\,\text{ft} \quad at \;\; t = 100\,\text{sec}$$

6.3 Elastic Theory (Water Hammer)

- Velocity changes suddenly and the pipeline is relatively long.
- We'll employ the impulse-momentum equation and the conservation of mass principle
- For the description of wave, moving reference often used.
- Here, we will use this concept to analysis (then it looks similar to the steady flow)
- One-dimensional impulse-momentum equation (Force is the rate of change of the impulse)

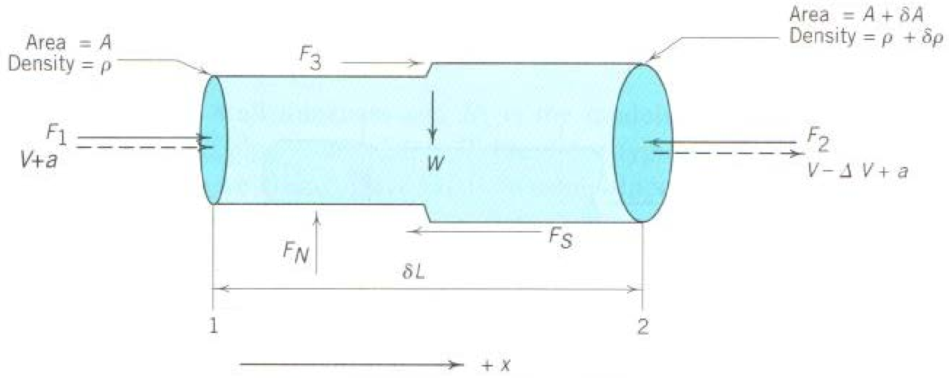
$$\sum \mathbf{F}_{ext} = \left(\sum Q\rho\mathbf{V}\right)_{out} - \left(\sum Q\rho\mathbf{V}\right)_{in}$$

- Momentum correction factor is 1.

Elastic Theory (Water Hammer)

- Considering only the component of this vector equation parallel to the pipe and noting that momentum enters and leaves the section of pipe δL long at only one section each, we can write

$$\sum \mathbf{F}_{ext} = \dot{m}\left(V_{out} - V_{in}\right)$$



- Choose control volume
- Rigid pipe, small bulge.
- Then

$$F_1 - F_2 = \dot{m}\left[(V - \Delta V + a) - \left(V + a\right)\right] = \dot{m}\left(-\Delta V\right)$$

$where\ \ \dot{m} = \left(V + a\right)A\rho;\ \ \Delta V$ is reduction in velocity

- Force by pressure should be equivalent to momentum change

$$pA - \left(p + \Delta p\right)\left(A + \delta A\right) = \left(V + a\right)A\rho\left(-\Delta V\right)$$

Elastic Theory (Water Hammer)

- Expanding and with

$$\Delta p = \gamma \Delta H \quad (\delta A \text{ is very small})$$

$$-A\gamma\Delta H = (V + a) A\rho(-\Delta V)$$

$$\Delta H = \frac{\rho}{\gamma} \Delta V (V + a)$$

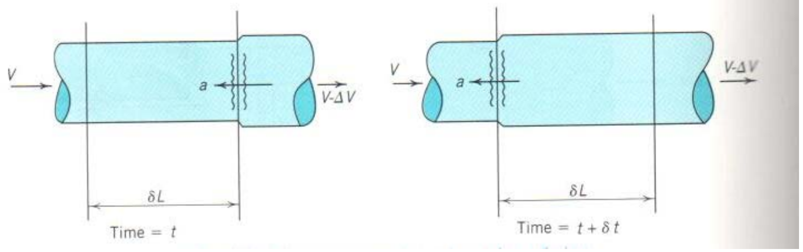
$$\Delta H = \frac{a\Delta V}{g_n}\left(1 + \frac{V}{a}\right)$$

- The value of *V/a* is less than 0.01 in most case since a wave speed even in PVC is 1,200 fps,

$$\Delta H = \frac{a}{g_n} \Delta V$$

- Therefore, we need to know a (wave speed) from continuity eq.

Elastic Theory (Water Hammer)



- Conservation of mass
- During the time period δt an amount of liquid has accumulated in the section of pipe given by the amount

$$\delta M = mass\ accumulated = VA\rho\delta t - (V - \Delta V)(\rho + \delta\rho)(A + \delta A)\delta t$$

$$\delta M = A\rho\Delta V\delta t \quad (neglecting\ higher\ order\ terms)$$

- With wave speed and δL.

$$\delta M = A\rho\Delta V\frac{\delta L}{a}$$

- This amount of extra liquid is accumulated in section δL by being <u>compressed slightly</u> and <u>by stretching the pipe slightly</u> to provide storage room.

Elastic Theory (Water Hammer)

- Because the pressure has increased during the passage of the wave, the volume of the liquid in the section will compress slightly to a higher density. The equation describing this relationship is that defining the bulk modulus of elasticity (see Ch. 1 of EFM)

$$E = -\frac{dp}{d\forall / \forall} \quad (E \text{ is the bulk modulus of elasticity})$$

- Then the volume of change with a little adjustment

$$\delta\forall = -\Delta p \frac{\delta LA}{E}$$

- Because the increased pressure stretched the pipe, there is more room made available to store the net mass inflow of liquid.
- Stretched circumferentially, lead to stretch longitudinally.

Elastic Theory (Water Hammer)

- Therefore

$$\delta \not{V} = \frac{\pi}{4} d^2 \delta L \left(\Delta\epsilon_1 + \Delta\epsilon_2 \right)$$

(ϵ_1 *and* ϵ_2 *are* unit strains in the logitudinal and radial direciton)

- If the pipe is restrained from longitudinal stretching, then

$$\delta \not{V} = \frac{\pi}{4} d^2 \delta L \left(\frac{1 - \mu_p^2}{E_p} \right) \left(\frac{\Delta p d}{e_p} \right)$$

- Where e_p is the pipe wall thickness and E_p is the modulus of elasticity and μ_p is the Poisson's ratio of the pipe material.

Elastic Theory (Water Hammer)

- We can write conservation of mass in the different form.

$$\delta M = (\rho + \delta\rho)(A\delta L + \delta V\!\!\!/) - \rho A \delta L$$

- With the previous mass change derivation, and volume changes we can simplify

$$a = \frac{\sqrt{E/\rho}}{\sqrt{1 + \frac{E}{E_p}\frac{d}{e_p}\left(1 - \mu_p^2\right)}}$$

- This is the wave speed.
- Small amount of free air suspended as bubbles can drastically reduce *E*.
- But in the design situation, the larger conservative value of E without free air is commonly used because it generally predicts the most severe water hammer pressures.

Equation for Hammering

- For rigid pipes, the velocity of the pressure wave is

$$c = \sqrt{\frac{\text{Bulk modulus of fluid}}{\text{density of fluid}}} = \sqrt{\frac{E}{\rho}}$$

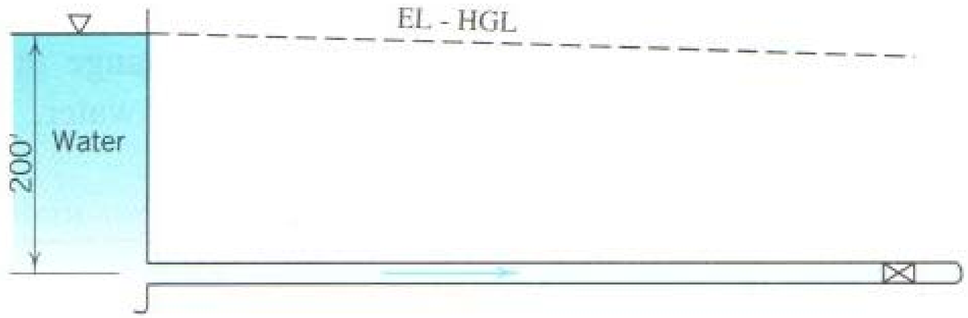
- For non-rigid pipes, the velocity is

$$c = \sqrt{\frac{E/\rho}{1 + \frac{E}{E_p}\frac{d}{e_p}\left(1 - \mu_p^2\right)}}$$

- Simply, this equation includes effects of the density change and volume change of the system.

Example problem #3

- Steady flow in a 24 inch pipe line 10,000ft long occurs at a velocity of 6 ft/s. The pipe fabricated of steel and has a wall thickness of 0.25 in. Calculate the wave speed in the pipe and the head increase resulting from sudden valve closure.
- What is the longest valve closure time that will produce the same maximum pressure at the valve?



Example problem #3

- Using the applicable properties from Table 3 and Appendix 2.

$$a = \frac{4860\ ft/s}{\sqrt{1 + \frac{E}{E_p}\frac{d}{e_p}\left(1-\mu_p^2\right)}} = \frac{4860\ ft/s}{\sqrt{1 + \frac{3.18\times10^5 lb/in^2 \times 24in}{3\times10^7 lb/in^2 \times 0.25in}\left(1-0.3^2\right)}} = 3{,}502\ \text{ft/s}$$

- To calculate the head increment,

$$\Delta H = \frac{a}{g_n}\Delta V = \frac{3.502\text{ft/s}}{32.2} \times 6\ \text{ft/sec} = 653\ \text{ft}$$

- To find the longest valve closure time that will produce the same high pressure, we recall that the critical valve closure time is 2 *L/a*.
- Any valve closure in a time less than 2 *L/a* will produce the same pressure as sudden valve closure. So the critical valve closure time is

$$2L/a = \frac{2\times10{,}000\ ft}{3{,}502\ ft/\sec} = 5.71\text{s}$$

Air Entrainment

- When **free air** occurs in a pipeline, the wave speed in the pipeline is decreased dramatically.
- Wave propagation and the pressure are substantially affected.
- The general speed equation can be used but determining *E* and density need to be taken care.
- Elasticity in **mixture**

$$E_{mix} = \frac{E}{1 + \alpha_{mix}\left(\frac{E}{E_{air}} - 1\right)}$$

- α is void fraction. For density

$$\rho_{mix} = \left(1 - \alpha_{mix}\right)\rho$$

The analysis of unsteady flow

- Surge or rigid water column theory
  - Fluid is treated as an inelastic substance
  - Pressure changes propagate instantaneously throughout the system
  - No wall's elasticity is considered
  - **Ordinary Differential Equations (ODE)**
- Elastic or water hammer theory
  - Elasticity of fluid and the pipe are taken into account in the calculations.
  - Pressure waves created by velocity changes depend on these elastic properties and they propagate throughout the pipeline system at speeds depending on these elastic properties.
  - **Partial Differential Equations (PDE)**

Air Entrainment

- The speed in pure water

$$a = \frac{\sqrt{E/\rho}}{\sqrt{1 + \frac{E}{E_p}\frac{d}{e_p}\left(1 - \mu_p^2\right)}}$$

- Put some mixture values

$$a = \frac{\sqrt{E/\rho_{mix}}}{\sqrt{1 + \frac{E}{E_p}\frac{d}{e_p}\left(1 - \mu_p^2\right) + \alpha_{mix}\frac{E}{E_{air}}}}$$

- It is clear that the wave speed depends on the pressure in the pipeline because the value of void fraction and Elasticity depend on the pressure.
- With wave speed, the **thermodynamic process occurs**…

6.4 Basic Differential Equations for Transient Flow

- Equation of Motion

$$g_n \frac{\partial H}{\partial x} + V \frac{\partial V}{\partial x} + \frac{\partial V}{\partial t} + \frac{f V \left| V \right|}{2D} = 0$$

By setting $\frac{\partial V}{\partial x} = \frac{\partial V}{\partial t} = 0$, it becomes Darcy-Weisbach equation:

$$\Delta H = \frac{f \Delta x V \left| V \right|}{2gD}$$

- Continuity Equation

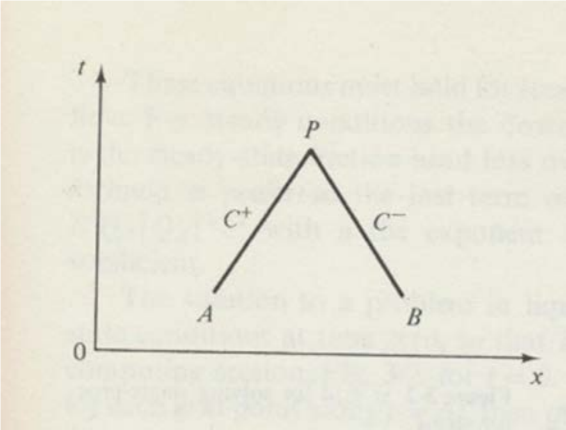
$$V \frac{\partial H}{\partial x} + \frac{\partial H}{\partial t} + \frac{a^2}{g_n} \frac{\partial V}{\partial x} = 0$$

Solution by Characteristics Method

- Characteristics equations

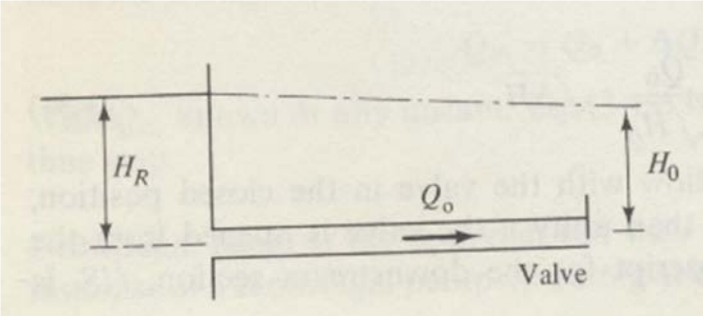
$$C^+ \begin{cases} \dfrac{g_n}{a}\dfrac{dH}{dt}+\dfrac{dV}{dt}+\dfrac{fV|V|}{2D}=0 \\ \dfrac{dx}{dt}=+a \end{cases}$$

$$C^- \begin{cases} -\dfrac{g_n}{a}\dfrac{dH}{dt}+\dfrac{dV}{dt}+\dfrac{fV|V|}{2D}=0 \\ \dfrac{dx}{dt}=-a \end{cases}$$

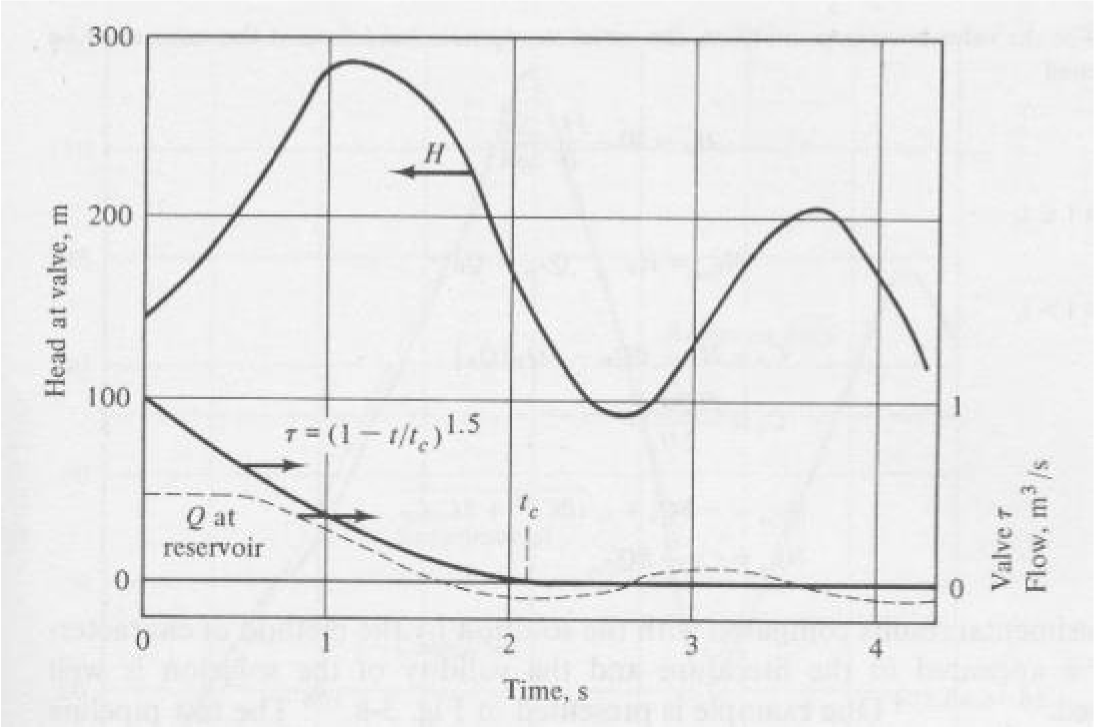


- 2 ODEs converted from 2 PDEs (motion and continuity)
- Valid only on the respective characteristic lines

Single Pipeline Application



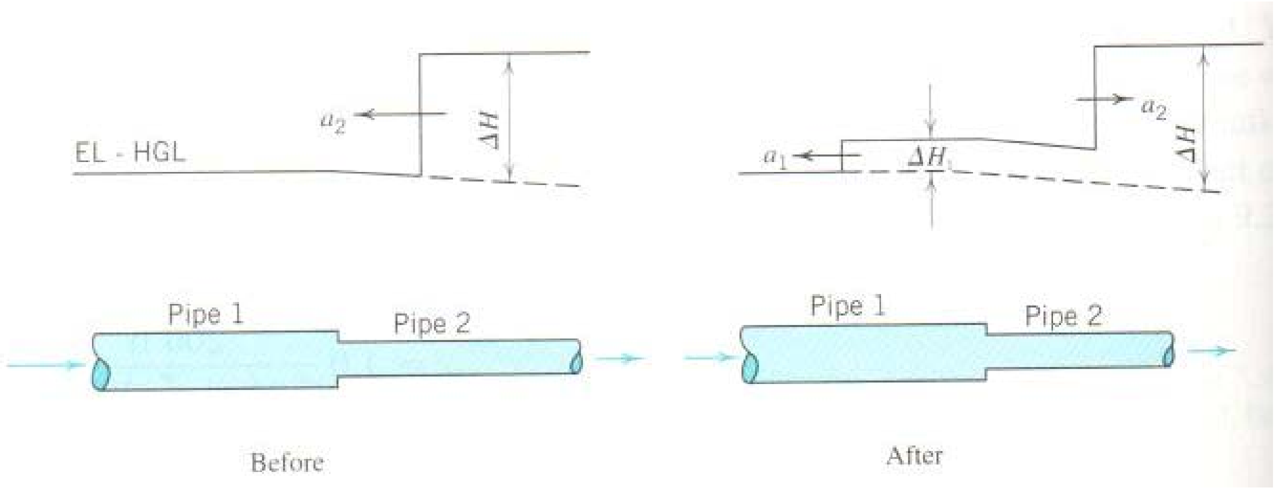
$$L = 600\,\text{m},\ a = 1{,}200\,\text{m},\ D = 0.5\,\text{m},\ f = 0.018,\ H_R = 150\,\text{m},\ t_c = 2.1\,\text{s}.$$



Series pipe junctions

- The equations of momentum and continuity are applied to a pressure head increase of ΔH approaching a junction. After the wave reaches the junction, ΔH_1 passes through (is transmitted) and $(\Delta H+\Delta H_1)$ is reflected.
- The results of analysis show that

$$\Delta H_1 = \frac{2a_1A_2}{a_1A_1 + a_2A_2}\Delta H, \quad \text{for equal } a,\ \Delta H_1 = \frac{2A_2}{A_1 + A_2}\Delta H$$



Tee junctions

- The situation for tee junctions is shown. Using the same analysis technique as before leads to the following equations

$$\Delta H_1 = \Delta H_2 = \frac{2a_1a_2A_3}{a_2a_3A_1 + a_1a_3A_2 + a_1a_2A_3}\Delta H,$$

for equal a, $\Delta H_1 = \Delta H_2 = \dfrac{2A_2}{A_1 + A_2 + A_3}\Delta H$