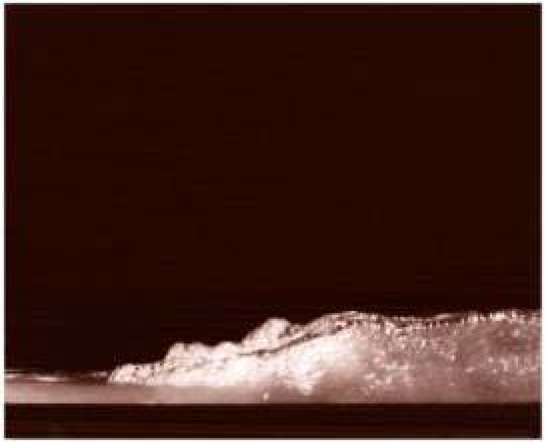
Ch. 8 Varied Flow in Open Channels

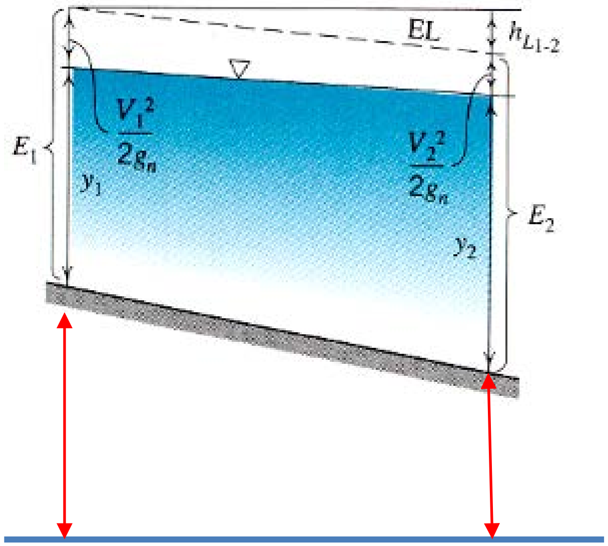
8-1 Concept of Non-uniform Flows



Contents

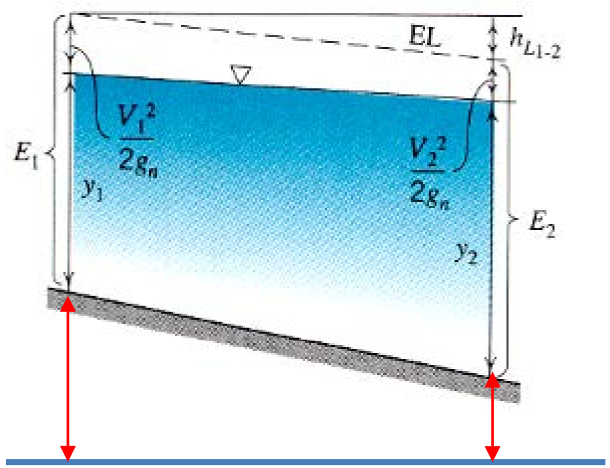
Class objectives

- Introduce basic concept of the steady non-uniform flows
- Apply the work-energy equation to the non-uniform open channel flow problems
- Derive equations of the hydraulic jump



Steady non uniform flow

- Steady non-uniform flow
  - The velocity, and depth are different at the different points
  - Important concepts are
    - Specific energy 비에너지
    - Specific force (or momentum) 비력
    - Rapidly varied flow 급변류
    - Gradually varied flow 점변류



8.1 Specific Energy

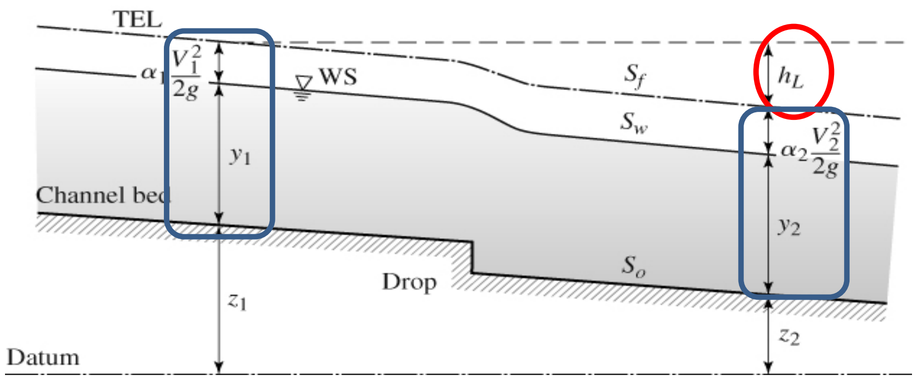
- The specific energy was first introduced by Bakhmeteff in 1912.

- The specific energy concepts permit determination of how water surface and velocity vary with change in flow cross-section.

- Specific energy (비에너지): total head above the channel bottom (not from datum) = distance between the channel bottom and the energy line

$$\boxed{E = y + \frac{V^2}{2g} = y + \frac{1}{2g}\left(\frac{Q}{A}\right)^2}$$

(10.13)



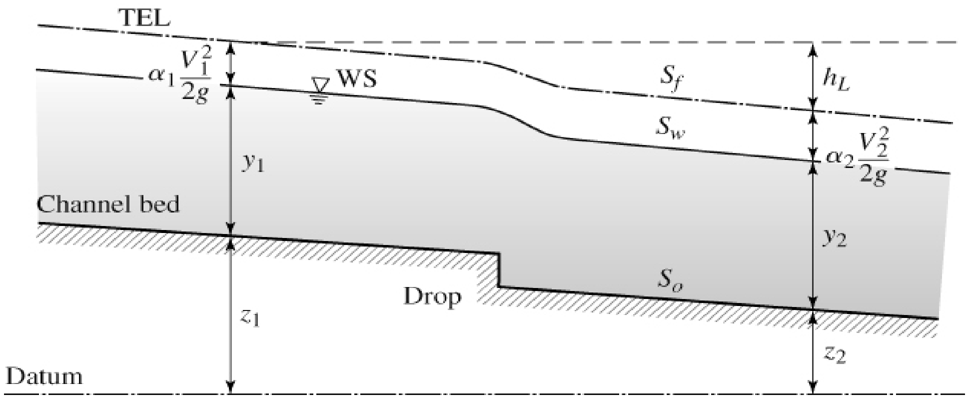
1. Specific energy

For a wide channel of rectangular cross section, specific energy is given by the two-dimensional flow rate, *q*

$$E = y + \frac{1}{2g}\left(\frac{q}{y}\right)^2$$

$$q = \sqrt{2g(y^2 E - y^3)}$$

(10.14)



Use diagram to solve Eq. (10.13)

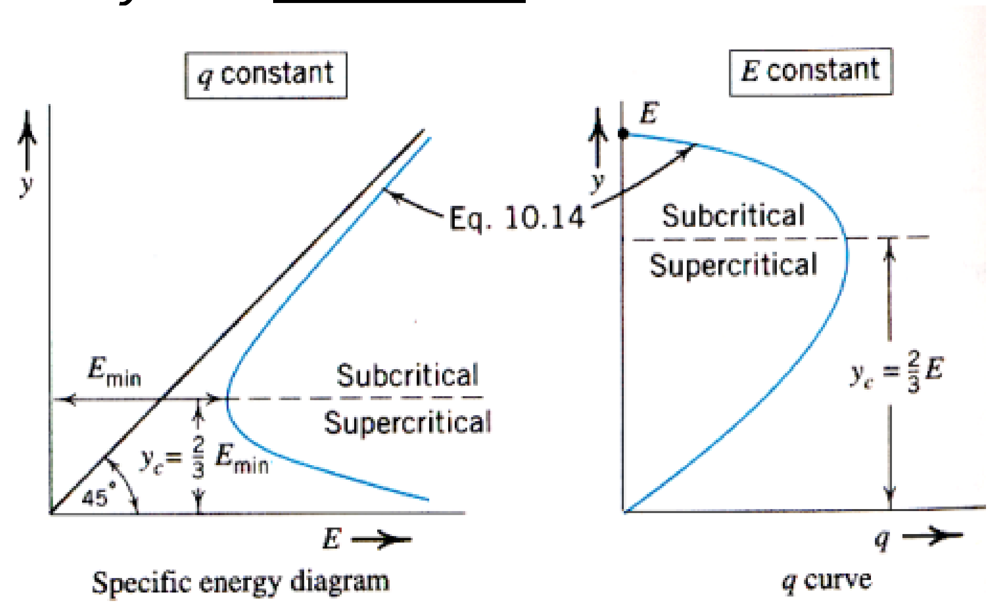
1) Specific energy diagram: plot of E versus y with q constant

- With a given flowrate, E is increasing as depth decreases or increases from the critical value, critical depth, y_c, in which E has the minimum value

***2) q*-curve**: plot of q versus y with E constant

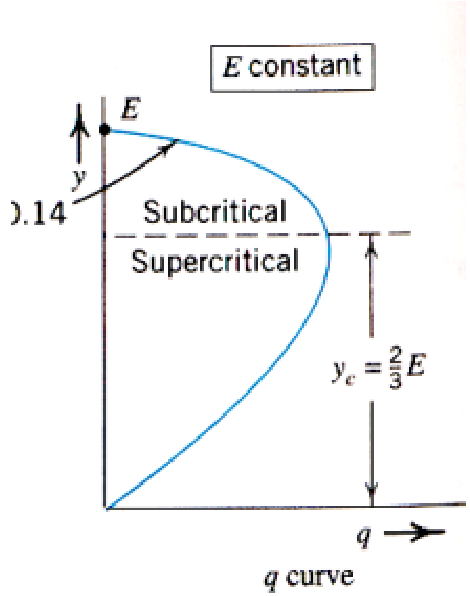
$$E = y + \frac{1}{2g}\left(\frac{q}{y}\right)^2$$

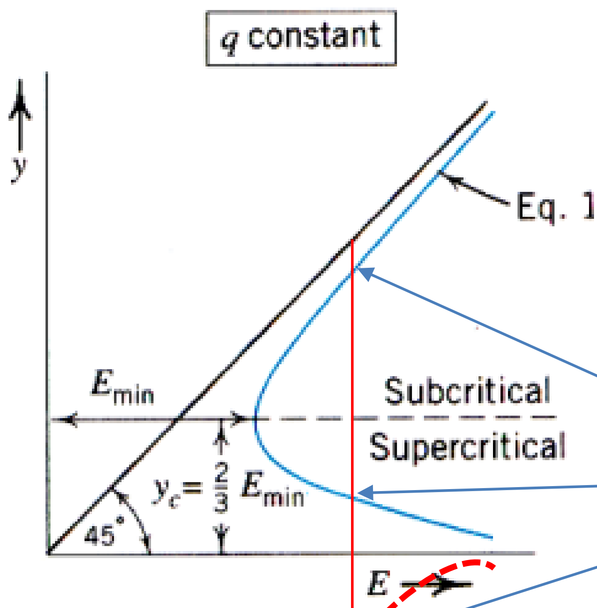
$$q = \sqrt{2g(y^2 E - y^3)}$$



2. E-diagram/q-diagram

$$\boxed{q = y\sqrt{2g(E-y)}}$$





Specific energy diagram

$$E = y + \frac{1}{2g}\left(\frac{q}{y}\right)^2$$

$$(E-y)\,y^2 = \frac{q^2}{2g}$$

$$\boxed{2\,\text{asymptotes} \rightarrow E - y = 0;\; y = 0}$$

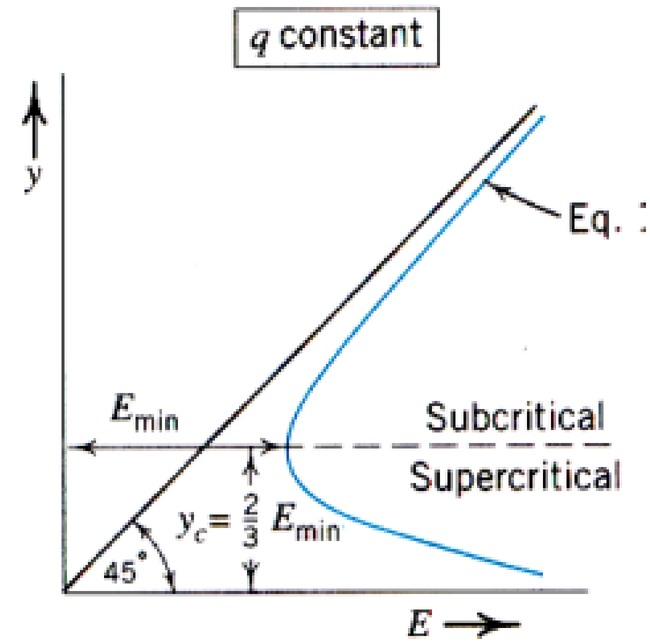
→ 3 alternate depths

Critical depth

$$\frac{dE}{dy} = 0 \;\; or \;\; \frac{dq}{dy} = 0$$

$$\boxed{\frac{dE}{dy} = 0 : q = \sqrt{g y_c^3}, \quad y_c = \sqrt[3]{\frac{q^2}{g}}} \qquad (10.15)$$

$$\boxed{\frac{dq}{dy} = 0 : y_c = \frac{2}{3} E_{\min} \;\; or \;\; E_{\min} = \frac{3 y_c}{2}} \qquad (10.16)$$

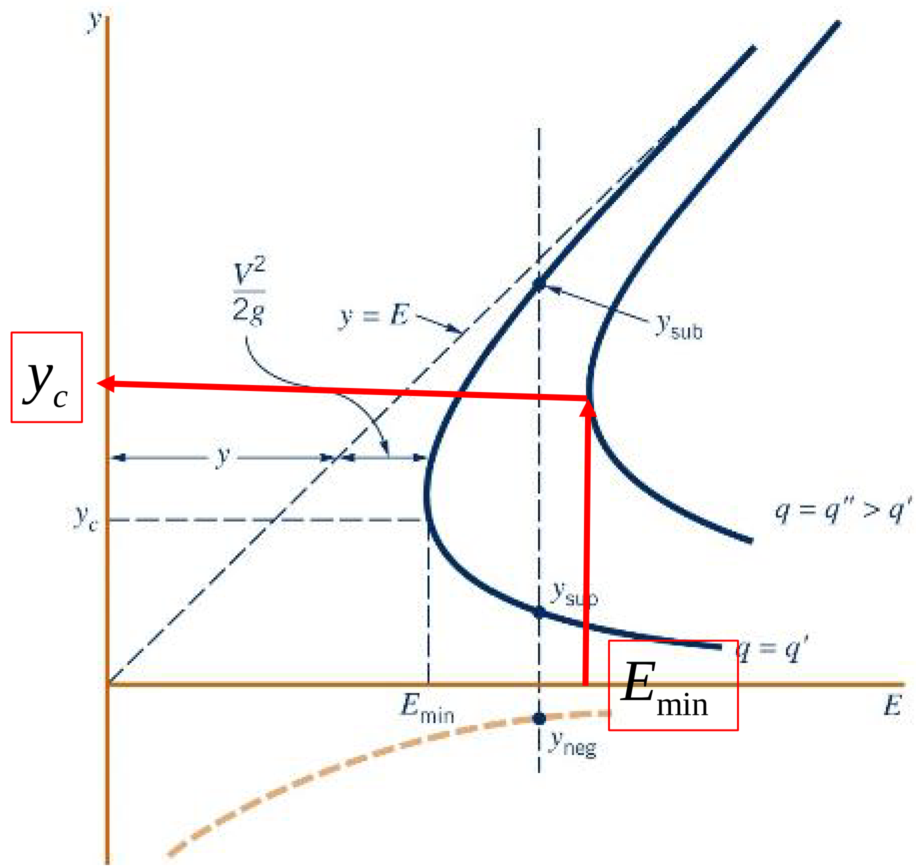


Specific energy diagram

- Eq. (10.15) shows that critical depth is dependent on flowrate only.
- Critical flow can be used as means of flowrate measurement.

  → critical-depth meters (broad-crested weir)

E-diagram

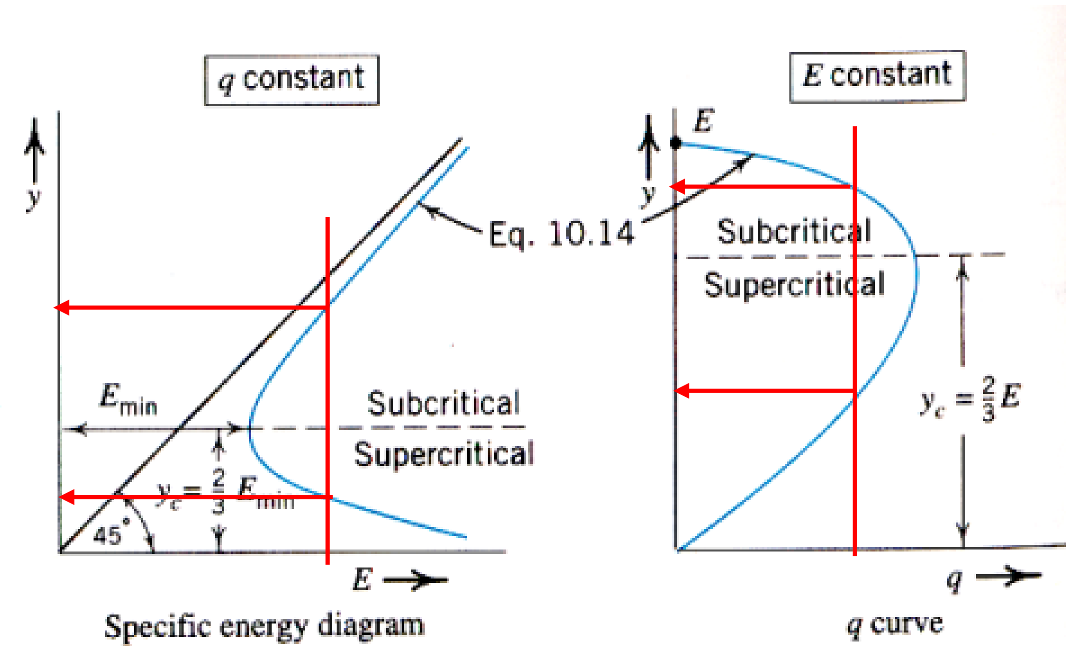


Alternate depths

In both specific energy diagram and *q*-curve, there exist two different depths <u>for a given specific energy</u> or unit discharge

→ Alternate depths: subcritical flow depth

대응수심 supercritical flow depth



3. Use of specific energy

Consider two types of local changes in flow cross section

- Channel hump (step) or broad-crested weir
- Channel constriction

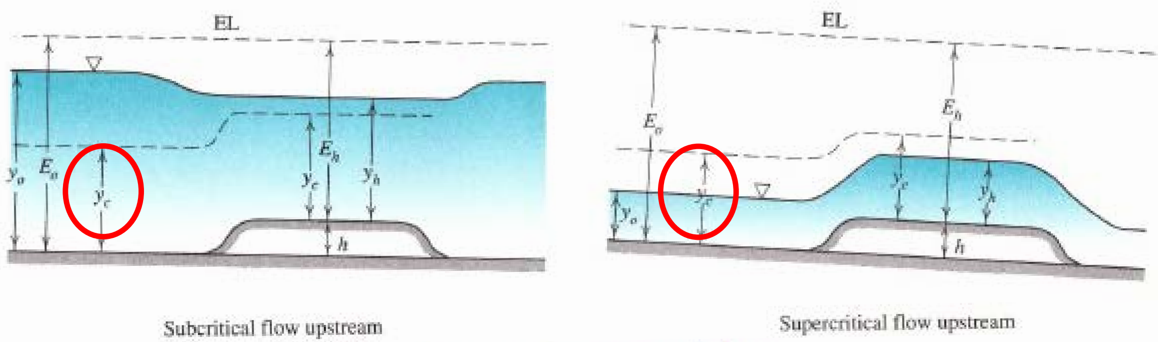


Fig. 10.12 Flow over a smooth hump.

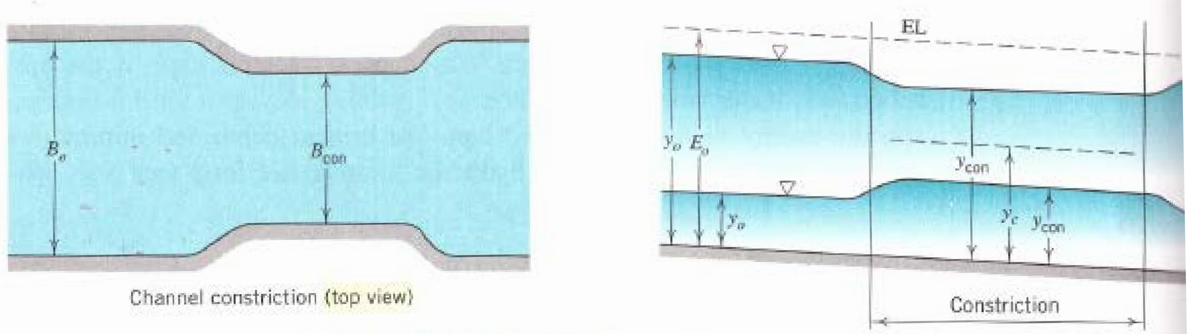


Fig. 10.14 Flow through a smooth constriction.

1) Channel hump (broad-crested weir)

Channel hump: The structure is sufficiently smooth and streamlined so that no additional friction losses are introduced.

→ Energy line remains parallel to the channel bottom.

$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + \cancel{h_L}$$

As flow moves from upstream channel over the hump, the distance from the channel bottom to the energy line, i.e. specific energy, *E*, decreases even though the total energy remains the same.

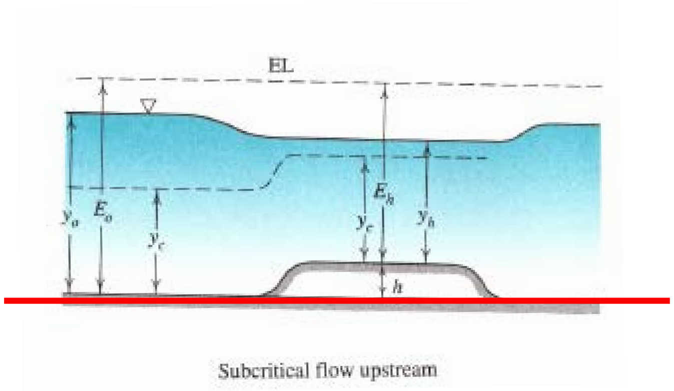
It appears that the depth of flow will either increase or decrease as flow moves onto the hump depending on whether the upstream flow is supercritical or subcritical.

1) Case I: upstream flow is subcritical

Work-energy eq. from channel bottom is

$$E_1 = y_1 + \frac{V_1^2}{2g} = \Delta z + y_2 + \frac{V_2^2}{2g}$$

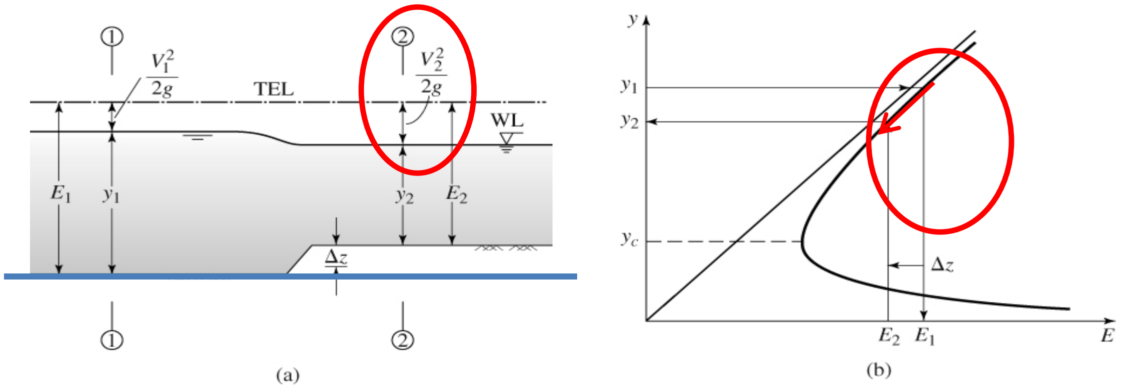
$$E_1 = \Delta z + E_2 \text{(specific energy)}$$



Subcritical flow upstream

So the new specific energy is just the initial specific energy minus the step dimensions, $E_2 = E_1 - \Delta z$

→ the specific energy decreases, even though total energy unchanged.



If the upstream flow is subcritical, then the decrease in E will cause a decrease in depth of flow.

→ This result in a drop in water surface over the hump

→ The flow velocity and the velocity head must increase.

In terms of the total energy given by the Bernoulli equation, the depth at the step is given by

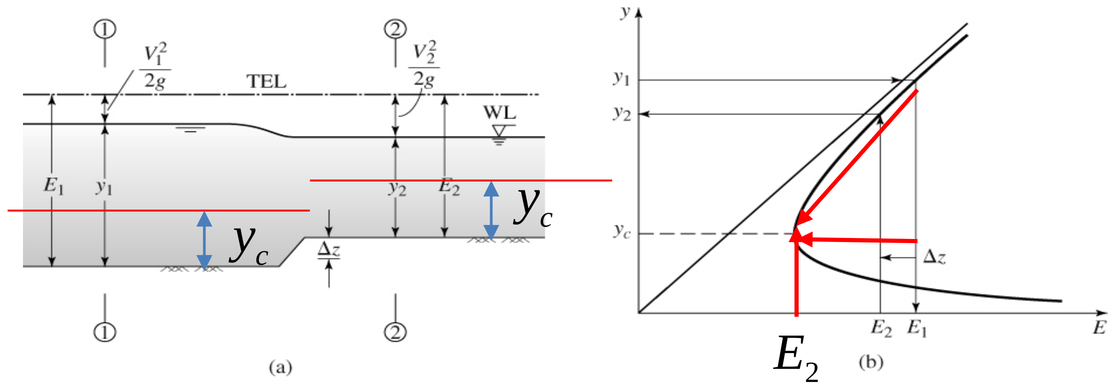
$$y_2 = E_1 - \left(\Delta z + \frac{V_2^2}{2g} \right) = E_1 - \left(\Delta z + \frac{Q^2}{2gA_2{}^2} \right)$$

For a rectangular channel section,

$$\boxed{y_2 = E_1 - \left(\Delta z + \frac{V_2^2}{2g} \right) = E_1 - \left(\Delta z + \frac{q^2}{2gy_2^2} \right)}$$

- If the hump were made high enough ($\Delta z = E_1 - E_{min}$), the critical depth would occur over the hump.

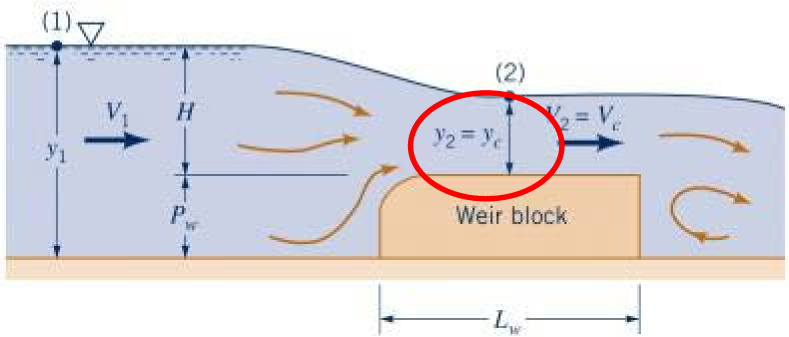
→ Broad-crested weir



- Further, if the hump were made even higher, the flow would not have enough specific energy to flow over the hump and the hump would become a dam. The depth at the hump will still remain the critical depth.

→ Flow would back up, gaining enough elevation to flow over the hump at minimum energy and critical depth.

Broad-crested weirs



- Broad-crested weirs:

$$0.08 < \frac{H}{L_w} < 0.5$$

For real weir flow the relation between flowrate and head is

$$q = \sqrt{g y_c^3} = \sqrt{g\left(\frac{2}{3}E_{\min}\right)^3} = \left(\frac{2}{3}\right)^{3/2}\sqrt{g}\,E^{3/2} = 0.577 \times \boxed{\frac{2}{3}\sqrt{2g}\,E^{3/2}} \qquad (10.15)$$

$$q = C_w \frac{2}{3}\sqrt{2g}\,H^{3/2} \qquad (14.23)$$

Combine (10.15) and (14.23)

$$C_w = f\left(\mathrm{Re}, \mathrm{We}, \frac{P}{H}\right)$$

$$\boxed{C_w = \frac{1}{\sqrt{3}}\left(\frac{E}{H}\right)^{3/2}}$$

$$E = \frac{3}{2}\left(E_1 - \left(P + \frac{V_c^{\,2}}{2g}\right)\right)$$

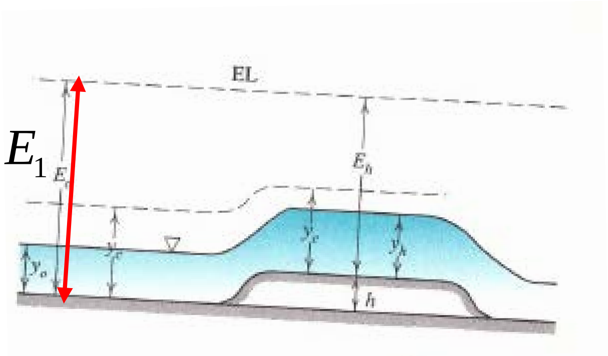
2) Case II: upstream flow is supercritical

If the upstream flow is supercritical, the decrease in *E* will cause a increase in depth over the hump.

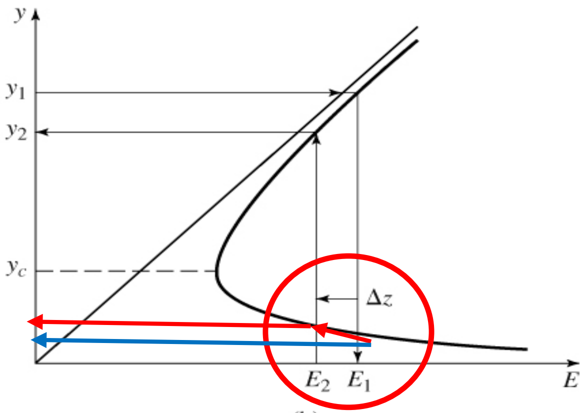
$$E_1 = y_1 + \frac{V_1^2}{2g} = \Delta z + y_2 + \frac{V_2^2}{2g}$$

$$E_1 = \Delta z + E_2 \text{(specific energy)}$$

$$y_2 = E_1 - \left(\Delta z + \frac{V_2^2}{2g} \right) = E_1 - \left(\Delta z + \frac{Q^2}{2gA_2^{\;2}} \right)$$

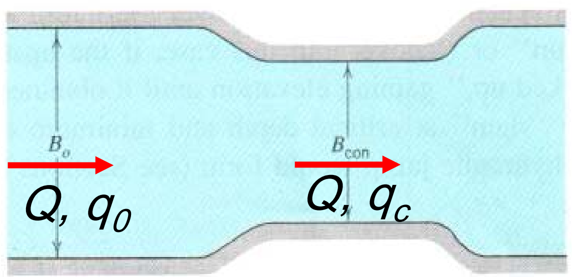


Supercritical flow upstream

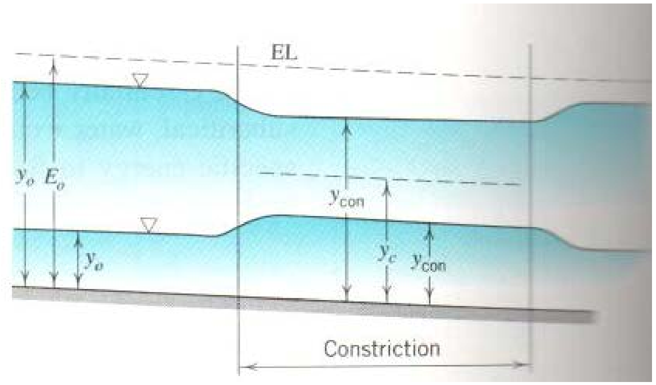


(b)

2) Channel constriction



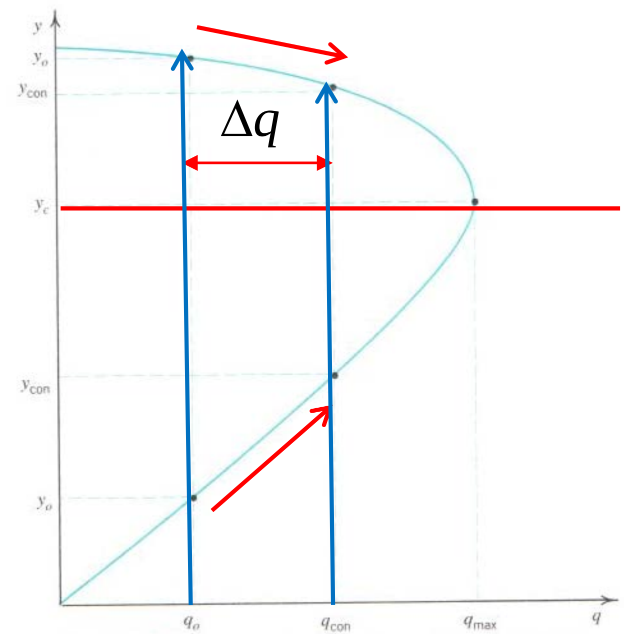
Channel constriction (top view)



- When contraction occurs, specific energy, <u>*E* is constant, but unit discharge (*q*) increases</u>.

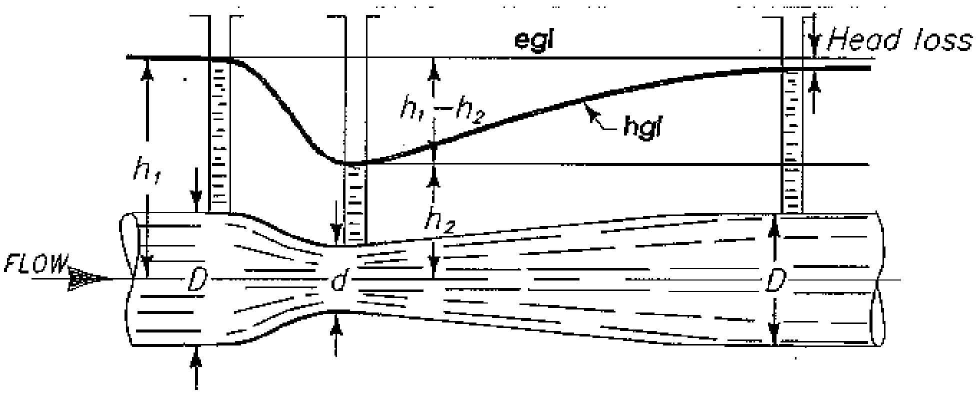
$$q_c = \frac{Q}{B_c} > q_0 = \frac{Q}{B_0}$$

- If the upstream flow is <u>subcritical</u>, a <u>decrease in flow depth</u> occurs in the constriction.
- If the upstream flow is <u>supercritical</u>, a <u>depth increase</u> occurs in the constriction.



- If the channel constriction is made sufficiently narrow, the critical depth would occur in the constriction.

→ Venturi flume, Parshall flume



- If the constriction is narrowed further, the flow will be unable to pass through with the specific energy available and the constriction becomes a choke. For subcritical flow upstream, flow will then back up much as occurred with the hump, creating an unconventional dam, until sufficient specific energy is built up to pass through the constriction at the critical depth. If the supercritical flow exists upstream, a hydraulic jump will form and the flow will back up, gaining enough elevation to flow through the constriction at minimum energy condition.

8.2 Critical Flow

1. Critical velocity

- Critical velocity may be obtained from combining the two equations.

$$q = V_c y_c, \quad q = \sqrt{g y_c^3}$$

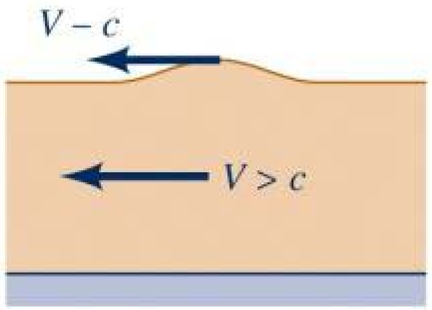
$$\boxed{V_c = \sqrt{g y_c}} \qquad \text{(a)}$$

- This equation is similar to another equation of open channel flow.

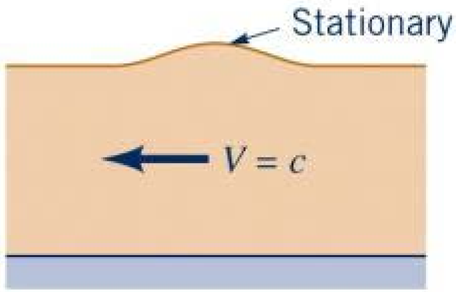
→ speed of propagation of a small gravity wave on the surface of a liquid of depth *y* → *Ch. 6 (pp. 200-203)*

$$\boxed{c = \sqrt{g y}} \qquad \text{(b)}$$

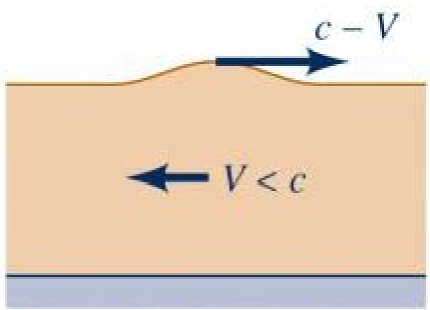
- In supercritical flow, $V > V_c$, small gravity waves will not propagate upstream but rather will be swept downstream.
- In subcritical flow, $V < V_c$, small gravity waves will propagate upstream.



Supercritical flow



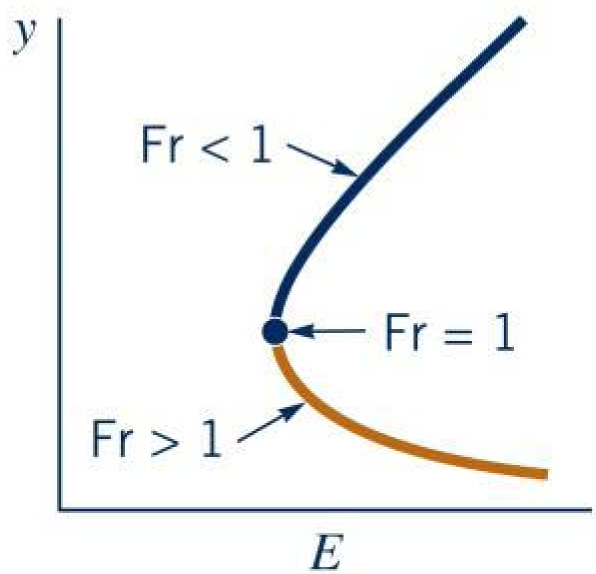
Critical flow

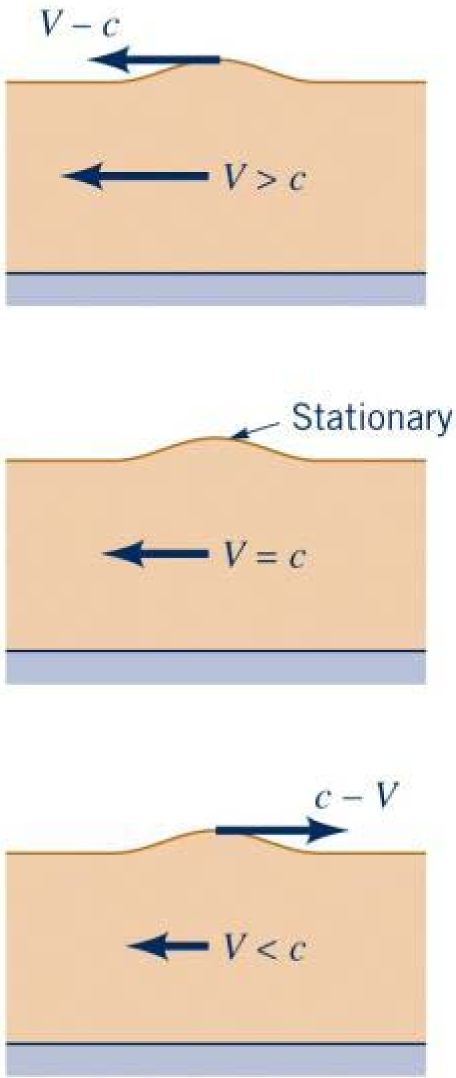


Subcritical flow

- In <u>subcritical flow</u>, $Fr < 1$
- In <u>supercritical flow</u>, $Fr > 1$

$$Fr = \frac{V}{\sqrt{gy}}$$





Supercritical flow

$$Fr = \frac{V}{\sqrt{gy}} > 1$$

Critical flow

Subcritical flow

$$Fr = \frac{V}{\sqrt{gy}} < 1$$

[Example 1]

Water flows in a rectangular channel of width 5 ft. 1) When discharge is 50 ft³/s, find the critical depth and minimum specific energy. 2) When depth of flow is 0.5 ft, find the specific energy and the alternate depth. 3) If discharge increases to 75 ft³/s, what is the critical depth?

$$1)\ y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{10^2}{32.2}} = 1.46\,ft$$

$$E_{\min} = \frac{3}{2} y_c = 2.19\,ft.$$

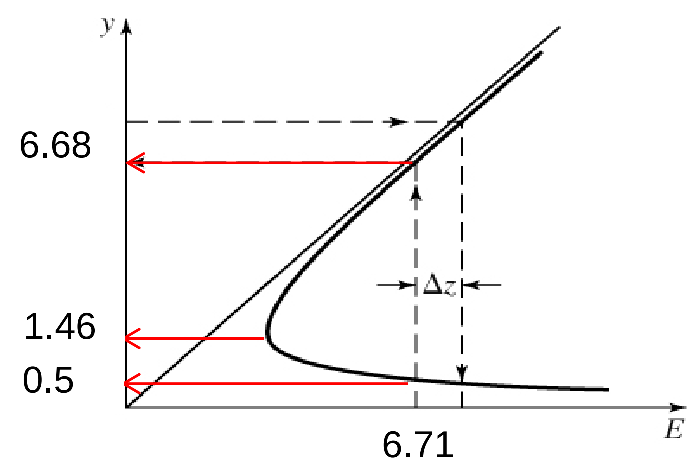
2) At the dpeth of 0.5ft (supercritical flow)

$$E = 0.5 + \frac{10^2}{2g(0.5)^2} = 6.71\,ft$$

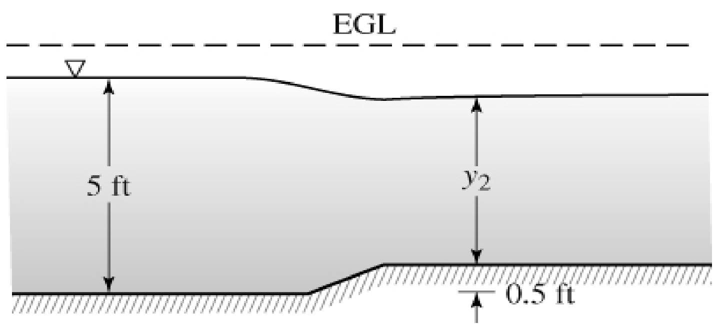
$$6.71 = y_2 + \frac{10^2}{2g(y_2)^2} = y_2 + \frac{1.55}{(y_2)^2}$$

$$y_2 = 6.68$$

$$3)\ y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{15^2}{32.2}} = 1.91\,ft$$

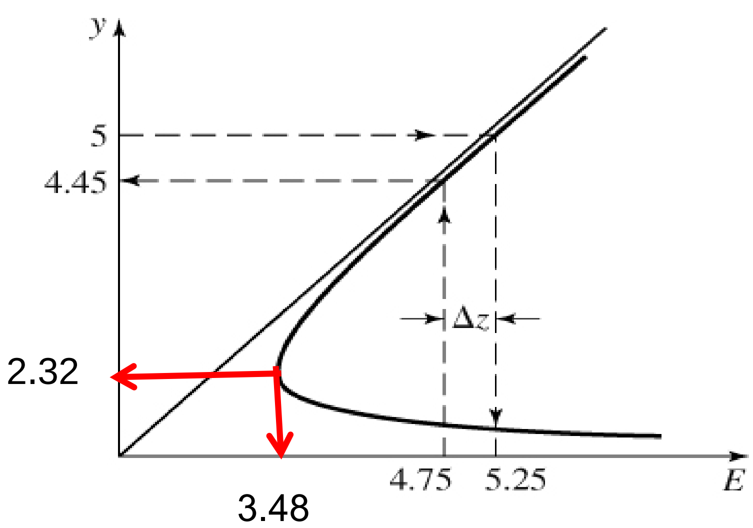


[Example 2] A uniform flow of 100 ft³/s occurs in a rectangular channel of 5 ft width and 5 ft depth. A smooth rise of 0.5 ft is formed in the channel bed. 1) Determine the depth of flow above step. 2) What is the height of the step that will produce a critical flow?



The discharge per unit width is

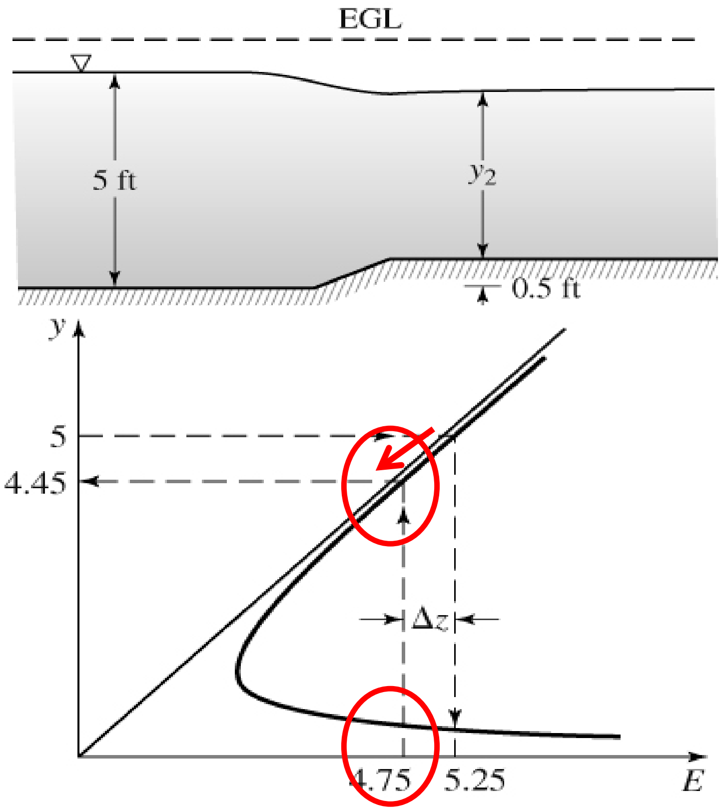
$$q = \frac{Q}{b} = \frac{100}{5} = 20\ ft^2/s$$



The critical depth is

$$y_c = \sqrt[3]{\frac{q^2}{g}} = 2.32\,ft,\quad E_{\min} = 1.5(2.32) = 3.48\,ft$$

Initial flow depth is larger than critical depth, so the flow is subcritical.



1) The initial energy available for flow,

$$E_1 = y + \frac{q^2}{2gy^2} = 5 + \frac{(20)^2}{2g(5)^2} = 5.25\,ft$$

Applying the Bernoulli equation across the step and ignoring losses,

$$y_1 + \frac{q^2}{2gy_1^2} = \Delta z + y_2 + \frac{q^2}{2gy_2^2}$$

$$5.25 = 0.5 + y_2 + \frac{6.21}{y_2^2} \rightarrow E_2 = y_2 + \frac{6.21}{y_2^2} = 4.75$$

$$y_2 = 1.35\, or\, 4.45\ ft \rightarrow \text{choose}\ 4.45\ ft$$

$$E_2 = 4.75\ ft$$

2) Required Δz to cause critical depth at the step: Critical depth will occur at the step when the available energy E_1 is equal to E_{min}.

$$E_1 = E_{\min} + \Delta z, \quad 5.25 = 3.48 + \Delta z, \quad \Delta z = 1.77\ ft$$

2. Critical slope

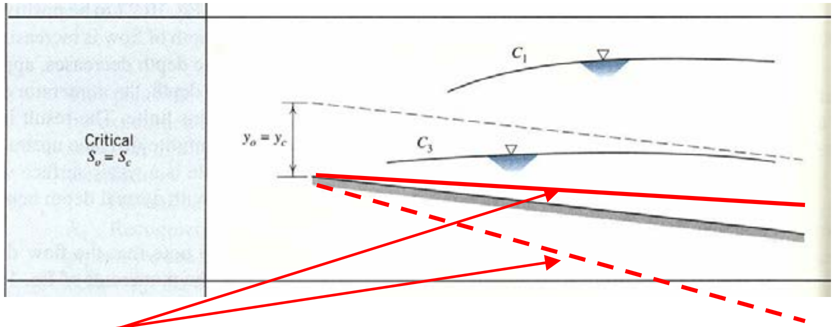
- If uniform flow were to occur at critical depth, the slope of the channel bottom would be called the critical slope S_c.
- For a rectangular channel of great width,

$$R_h = y_c$$

- S_c is obtained by equating the flowrate of (10.5) and (10.15)

$$q = \sqrt{g y_c^3} = \frac{u}{n} y_c^{5/3} S_c^{1/2}$$

$$\boxed{S_c = \frac{g n^2}{u^2 y_c^{1/3}}}$$



- $S > S_c$ → steep slope → will produce supercritical uniform flow
- $S < S_c$ → mild slope → will produce subcritical uniform flow

IP 10.6 (p. 455)

For a flowrate of 500 cfs in a rectangular channel 40 ft wide, the water depth is 4 ft. 1) Is this flow subcritical or supercritical? 2) If n = 0.017, what is the <u>critical slope</u> of this channel <u>for this flowrate</u>? 3) What channel slope would be required to produce <u>uniform flow at a depth of 4 ft</u>?

$$1)\ y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{12.5^2}{32.2}} = 1.69\ ft \leftarrow q = \frac{Q}{B} = \frac{50}{4} = 12.5$$

$$y = 4\ ft > y_c \rightarrow \text{subcritical flow}$$ ← uniform flow ?

2) For wide rectangular channel, critical uniform flow (y = 1.69 ft) occurs at the slope

$$S_c = \frac{gn^2}{u^2 y_c^{\ 1/3}} = \frac{32.2 \times (0.017)^2}{1.49^2 \times 1.69^2} = 0.0035$$

3) To compute uniform flow, use Manning Eq.

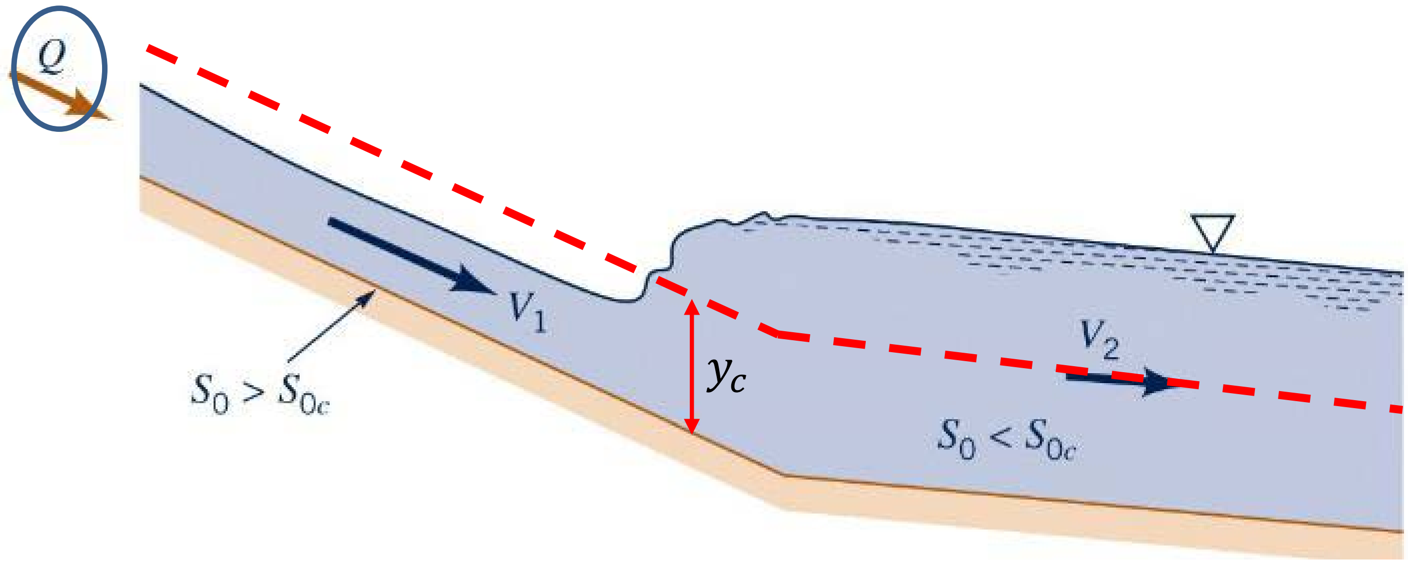
$$Q = \left(\frac{u}{n}\right) A R_h^{2/3} S_0^{1/2}$$

$$S_0^{\ 1/2} = Q\left(\frac{n}{u}\right)\frac{1}{AR_h^{2/3}} = 500\left(\frac{0.017}{1.49}\right)\frac{1}{(4\times 40)\left(\dfrac{4\times 40}{40+2\times 4}\right)^{2/3}} = 0.01598$$

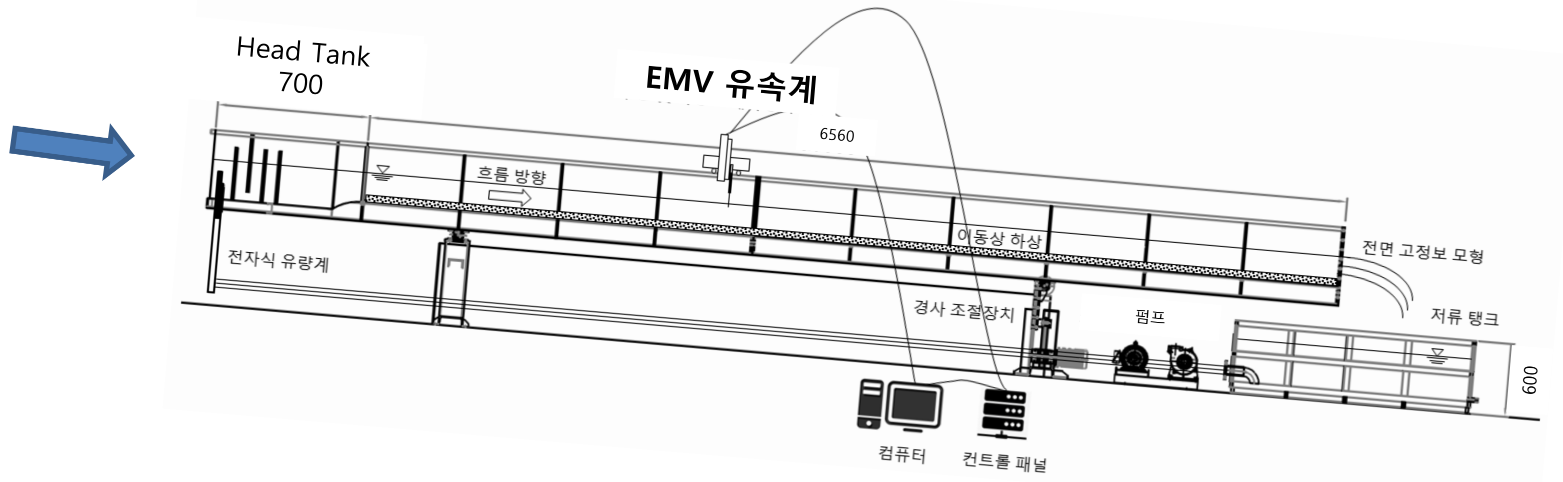
$$S_0 = 0.000255 < S_c \rightarrow \text{mild slope} \rightarrow \text{subcritcal unifrom flow}$$

Critical slope

- For upstream channel, flow is supercritical uniform flow
- For downstream channel, flow is subcritical uniform flow



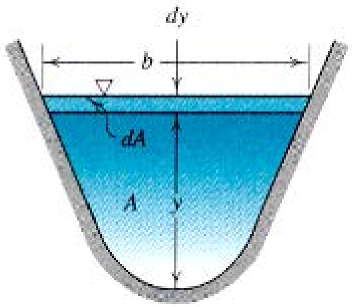
For given Q, adjust the channel slope to create the critical uniform flow

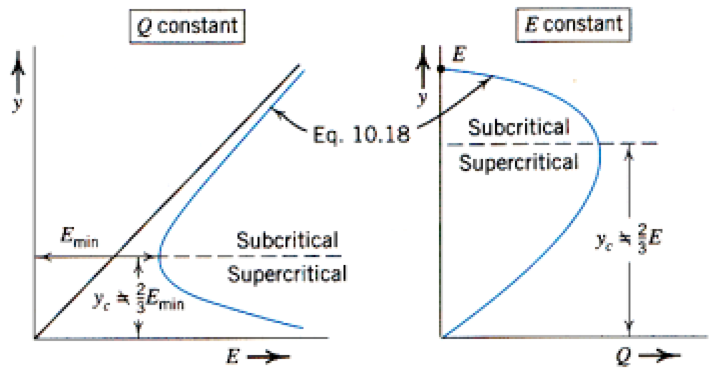


3. Critical depth in non-rectangular channels

- For non-rectangular channels, specific energy equation is given as

$$E = y + \frac{1}{2g}\left(\frac{Q}{A}\right)^2 = y + \frac{1}{2g}\left(\frac{Q}{A(y)}\right)^2 = y + \frac{Q^2}{2g}A(y)^{-2} \qquad (10.18)$$



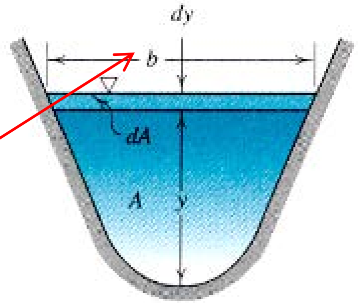


- The critical depth relationship can be found by

$$\frac{dE}{dy} = 0 \rightarrow 1 + \frac{Q^2}{2g}\left(-\frac{2}{A^3}\frac{dA}{dy}\right) = 0 \qquad \text{(A)} \qquad \rightarrow \frac{dA}{dy} = \frac{gA^3}{Q^2}$$

For non-rectangular channels,

$$dA = b\,dy \qquad \text{(B)}$$



Top width

Substituting this into (A) results in the critical depth relationship as

$$\boxed{\frac{Q^2 b}{gA^3} = 1} \qquad (10.19) \qquad \rightarrow Q = \sqrt{\frac{gA^3}{b}}$$

The critical flow can be defined by the Froude number as

$$\boxed{Fr_c = \sqrt{\frac{Q^2 b}{gA^3}} = \frac{V}{\sqrt{g(A/b)}} = 1} \qquad (10.20)$$

- The critical slope (for critical uniform flow) can be derived by following the same procedure as for the wide rectangular channel

$$Q = \sqrt{\frac{gA^3}{b}} = \frac{u}{n} A R_h^{2/3} S_c^{1/2}$$

$$\boxed{S_c = \frac{gn^2}{u^2}\left(\frac{A}{bR_h^{4/3}}\right)} \qquad (10.20)$$

A, b, and R_h are all functions of depth and dependent on the section shape. Critical slope is thus a function of section shape.

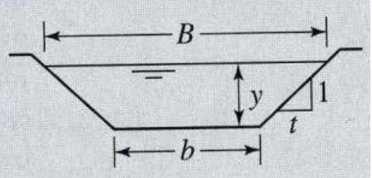
IP 10.7 (pp. 457-8)

A flow of 28 m^3/s occurs in an earth-lined canal having a base width of 3 m, side slopes *z* = 2 and Manning *n*-value of 0.022. Calculate the critical depth and critical slope.

- Solution

$$\frac{Q^2 b}{gA^3} = 1 = \frac{(28)^2 \times (3 + 4y_c)}{9.81 \times (3y_c + 2{y_c}^2)^3}$$

$$y_c = 1.50\, m$$

Trapezoidal	$(b + ty)y$	$b + 2yw$, $w = (1 + t^2)^{0.5}$	A/P	$b + 2ty$	A/B	

- Area and hydraulic radius corresponding to the critical flow conditions are

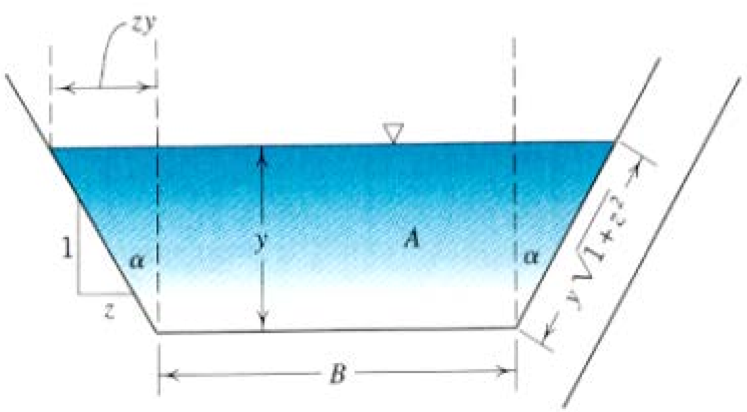
$$A = 3y_c + 2y_c{}^2 = 3\times1.50 + 2\times1.50^2 = 9.0\,m^2$$

$$R_h = \frac{A}{P} = \frac{9.0}{3+2\sqrt{5}\times1.50} = 0.93\,m$$

$$b = B + 2zy = 3 + 2\times2\times1.50 = 9.0\,m$$

$$S_c = \frac{gn^2}{u^2}\left(\frac{A}{bR_h{}^{4/3}}\right)$$

$$= \frac{9.81\times0.022^2}{1^2}\left(\frac{9.0}{9.0\times0.93^{4/3}}\right) = 0.00523$$

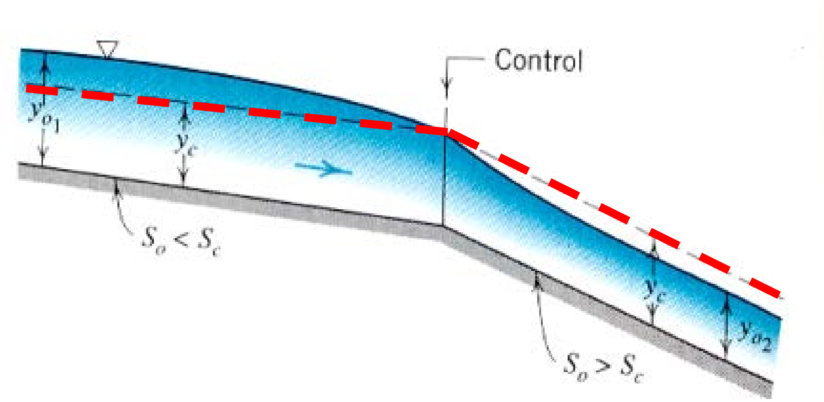


4. Occurrence of critical depth

(1) Mild slope to steep slope channel

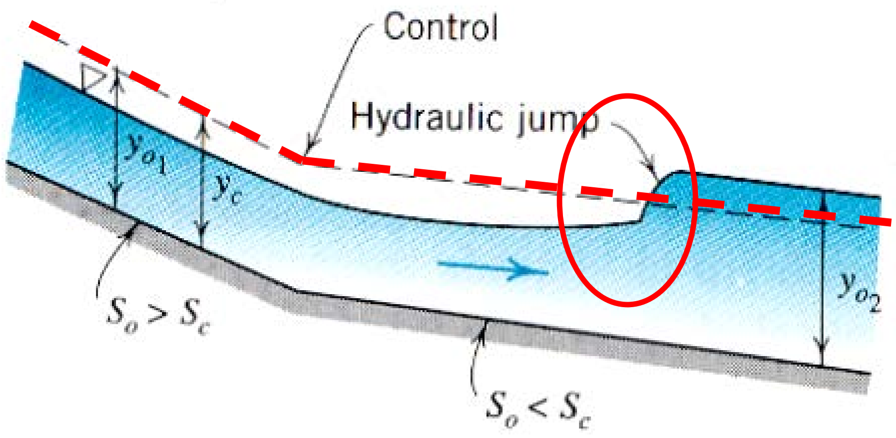
The critical depth can be expected in the situation where a long channel of mild slope is connected to a long channel of steep channel.

- Far up the former channel, uniform subcritical flow at normal depth, y_{01}, will occur, and far down the latter a uniform supercritical flow at a smaller normal depth, y_{02}, will occur.
- In a reach of varied flow, depth must pass through the critical.



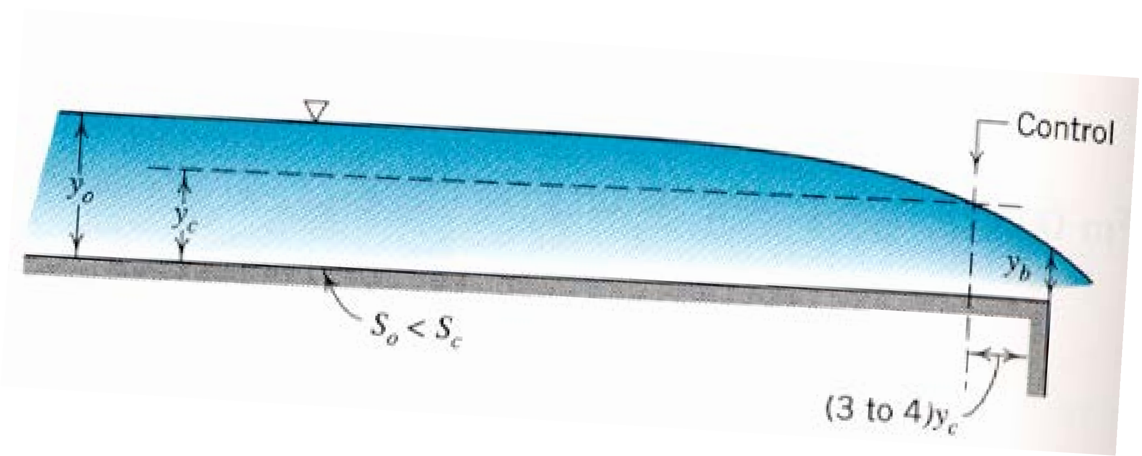
(2) Steep slope to mild slope channel

- When a long channel of steep slope discharges into to a long channel of mild channel, normal depth will occur upstream and downstream from the point of slope change.
- However, critical depth will not be found near this point.
- The hydraulic jump will form whose location will be dictated by the details of slopes, roughness, channel shapes, and so forth.
- The critical depth will be found within the hydraulic jump.



(3) Free outfall

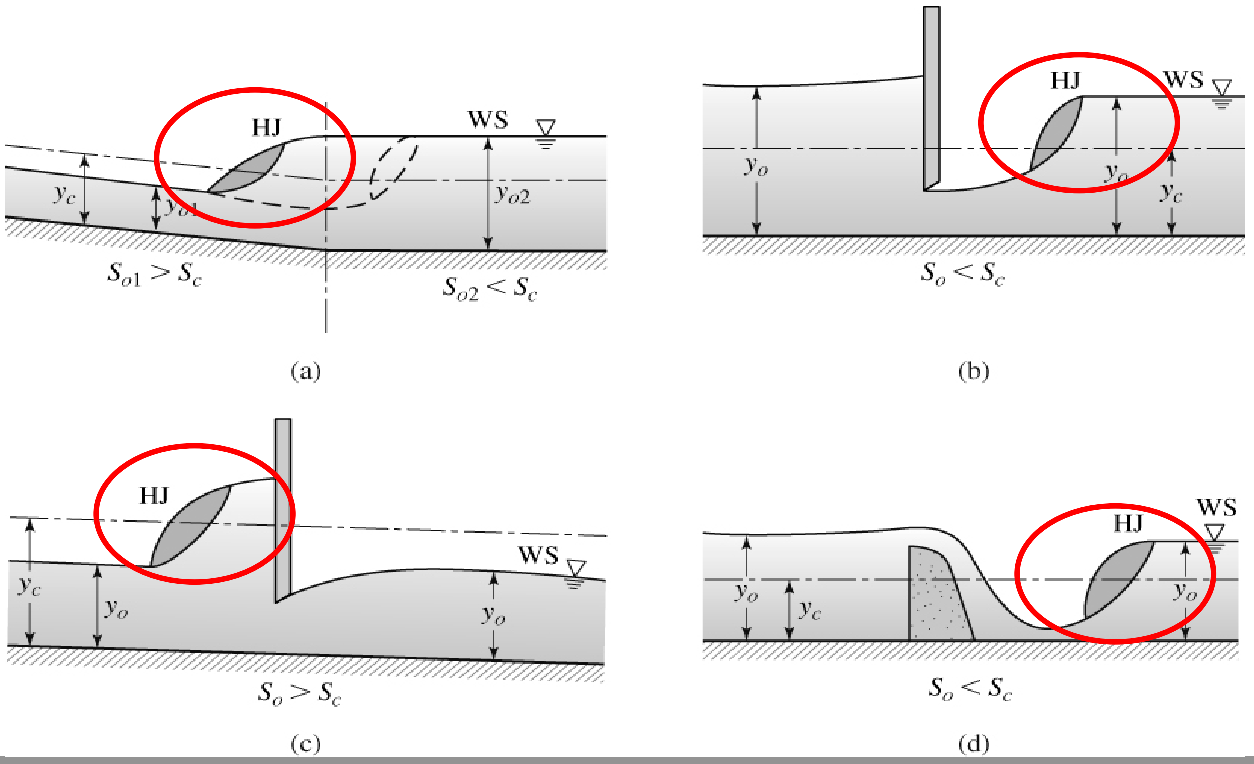
- A long channel of mild slope is concluded to the brink
- The critical depth occurs a <u>short distance (3~4 y_c) upstream</u> from the brink
- The brink depth (y_b) is 71.5% of the critical depth (Rouse, 1970)



8.3 Hydraulic Jump

1. Hydraulic jump

Hydraulic jump occurs, when the flow moves from the depth lower than critical depth (y_1) to the depth which is deeper than critical depth (y_2). That is, flow change from super-critical to sub critical



When channel is wide rectangular, there are two conjugate depths (공액수심) exist, and from this conjugate set of depths we can estimate the height of the jump.
Ignore the friction (short distance), then momentum equation becomes.

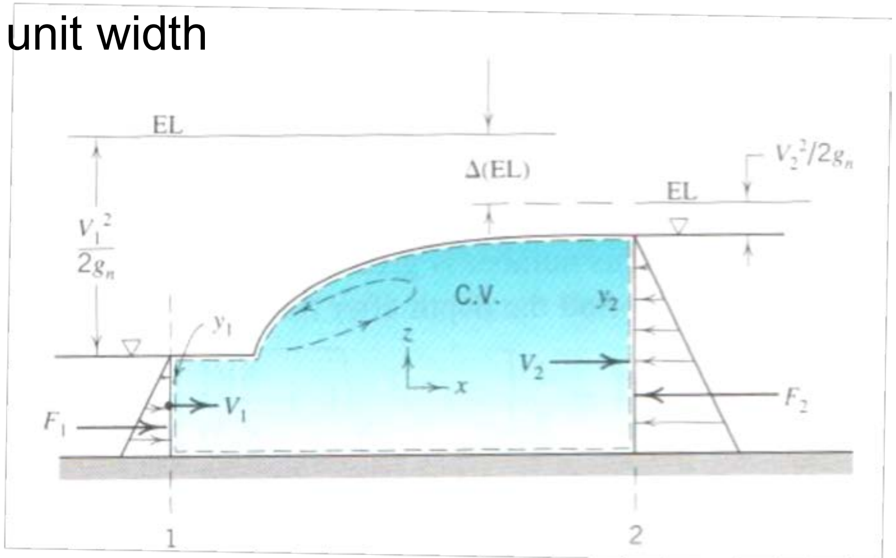
$$F_1 - F_2 = q\rho(V_2 - V_1)$$

Force per unit width

$$\frac{\gamma y_1^{\ 2}}{2} - \frac{\gamma y_2^{\ 2}}{2} = q\rho\left(\frac{q}{y_2} - \frac{q}{y_1}\right)$$

$$\boxed{\frac{q^2}{g y_1} + \frac{y_1^{\ 2}}{2} = \frac{q^2}{g y_2} + \frac{y_2^{\ 2}}{2}} \qquad \text{(A)}$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + \frac{8V_1^{\ 2}}{g y_1}}\right]$$



Specific force = force per unit width per specific weight

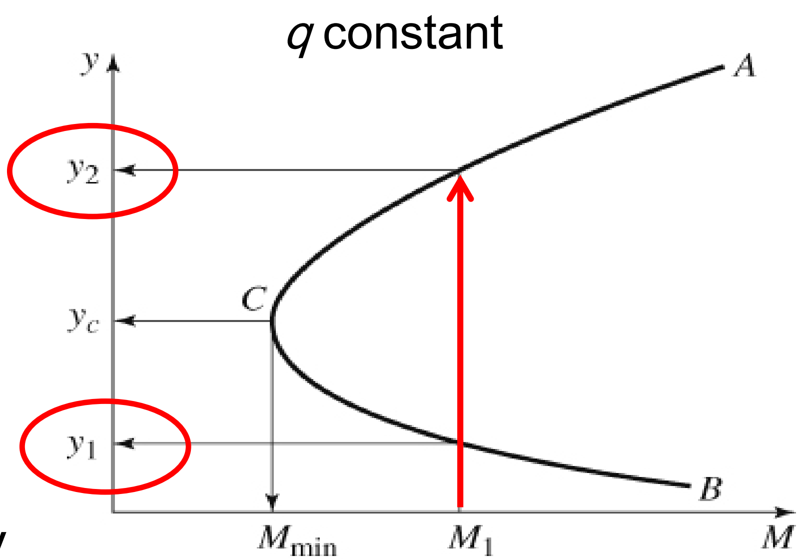
2. Specific force

- From (A), the specific force (비력) is defined as a function of depth

  → *M-curve*

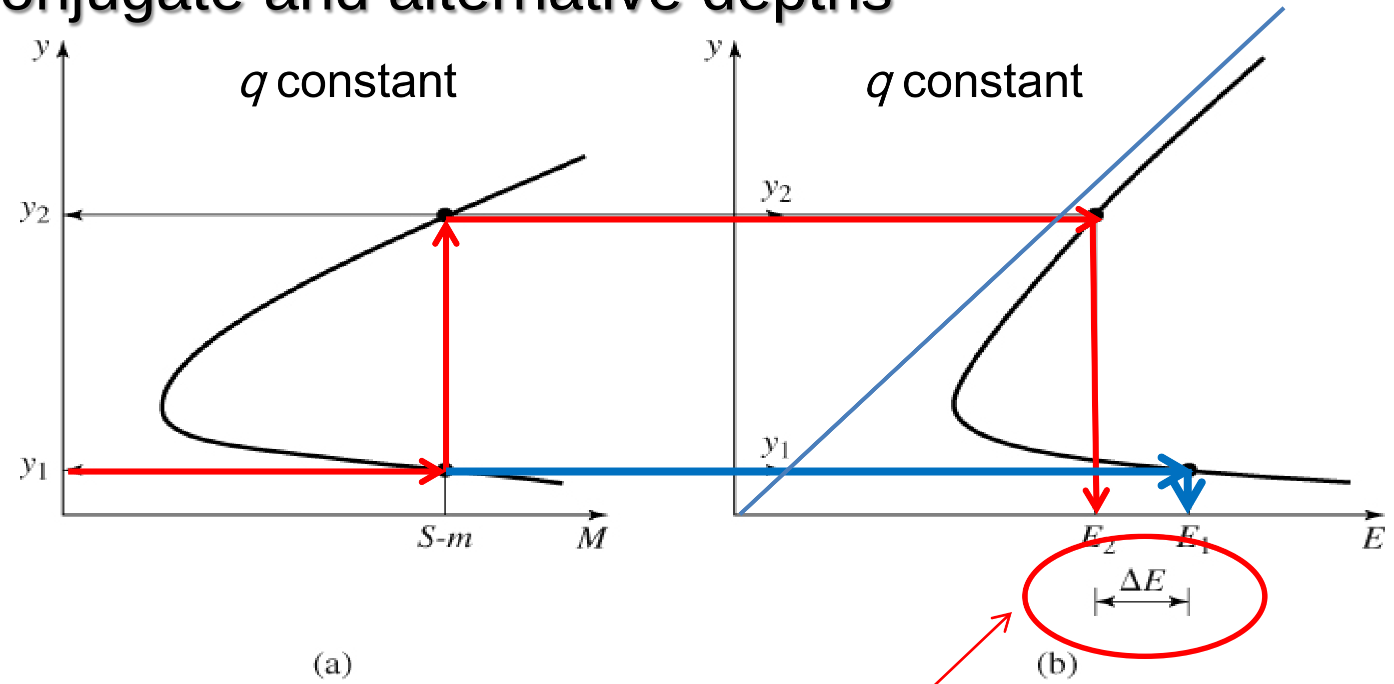
$$\boxed{M = \frac{q^2}{gy} + \frac{y^2}{2}}$$

$$\frac{dM}{dy} = 0 \rightarrow \frac{q^2}{g{y_c}^3} = 1$$



- A set of conjugate depths (공액수심) exists.
- Conjugate depth can be found by passing a vertical line since a vertical line is a line of constant *M*.
- The hydraulic jump can take place only across the critical depth.

3. Conjugate and alternative depths



- Conjugate depth (공액수심): same pressure and momentum relation.
- Alternate depth(대응수심): same specific energy
- Left figure is specific force, right figure is specific energy
- From the left hand side, find conjugate depth and then plug into the right hand side, then we can figure out how much energy is changed

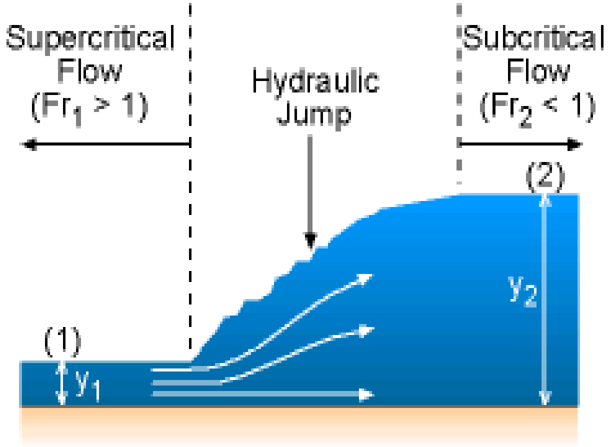
4. Energy loss by hydraulic jump

- Because of eddies (rollers), air entrainment, and flow decelerations in the hydraulic jump, large head losses are to be expected. In the horizontal channel, the loss of energy is given as

$$\Delta E = \left(y_1 + \frac{V_1^2}{2g} \right) - \left(y_2 + \frac{V_2^2}{2g} \right)$$

- When the channel is very wide or rectangular, the unit discharge expression is (q / y),

$$\Delta E = (y_1 - y_2) + \frac{q^2}{2g} \left(\frac{1}{y_1^2} - \frac{1}{y_2^2} \right)$$



$$\Delta E = (y_1 - y_2) + \frac{q^2}{2g}\left(\frac{1}{y_1^2} - \frac{1}{y_2^2}\right)$$

$$= (y_1 - y_2) + \frac{y_1 y_2 (y_1 + y_2)}{4}\left(\frac{y_2^2 - y_1^2}{y_1^2 y_2^2}\right)$$

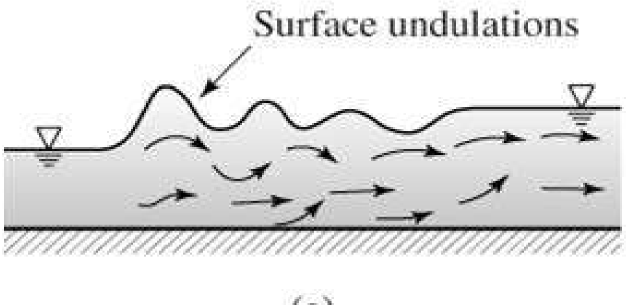
$$= (y_1 - y_2) + \frac{(y_1 + y_2)}{4}\left(\frac{y_2^2 - y_1^2}{y_1^2 y_2^2}\right)$$

$$= (y_2 - y_1)\left[-1 + \frac{(y_1 + y_2)^2}{4 y_1 y_2}\right]$$

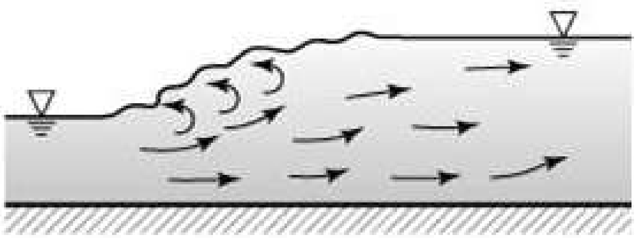
$$= (y_2 - y_1)\left[\frac{y_2^2 - 2 y_1 y_2 + y_1^2}{4 y_1 y_2}\right]$$

$$\boxed{\Delta E = \frac{(y_2 - y_1)^3}{4 y_1 y_2}}$$

5. Classification of Hydraulic jump

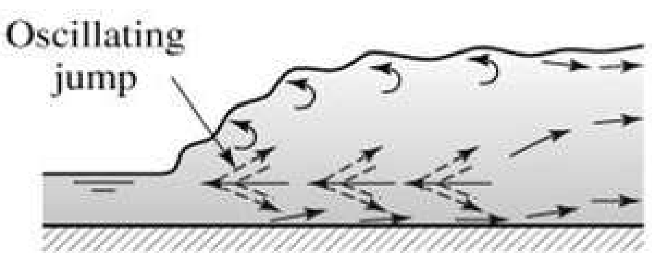


Undular jump (1<F_{r1}<1.7; 파상도수): A smooth standing wave is formed at the water surface. Transition from the supercritical to the subcritical.



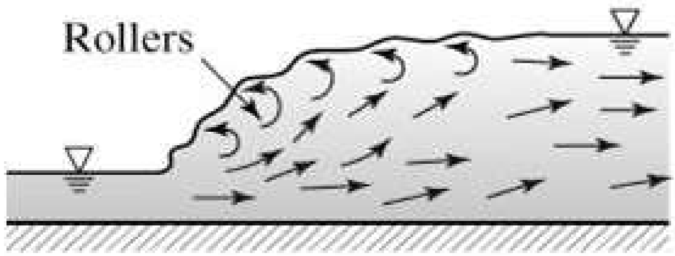
(b)

Weak jump (1.7<F_{r1}<2.5; 약도수): Small eddies and rollers are formed. Small loss of energy



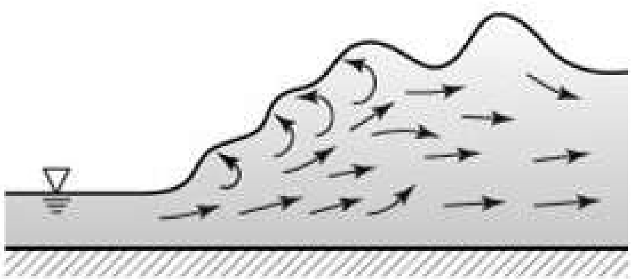
(c)

Oscillating jump (2.5<F_{r1}<4.5; 진동도수): Jet driven wave occurs from bottom to top. Destruction begins



(d)

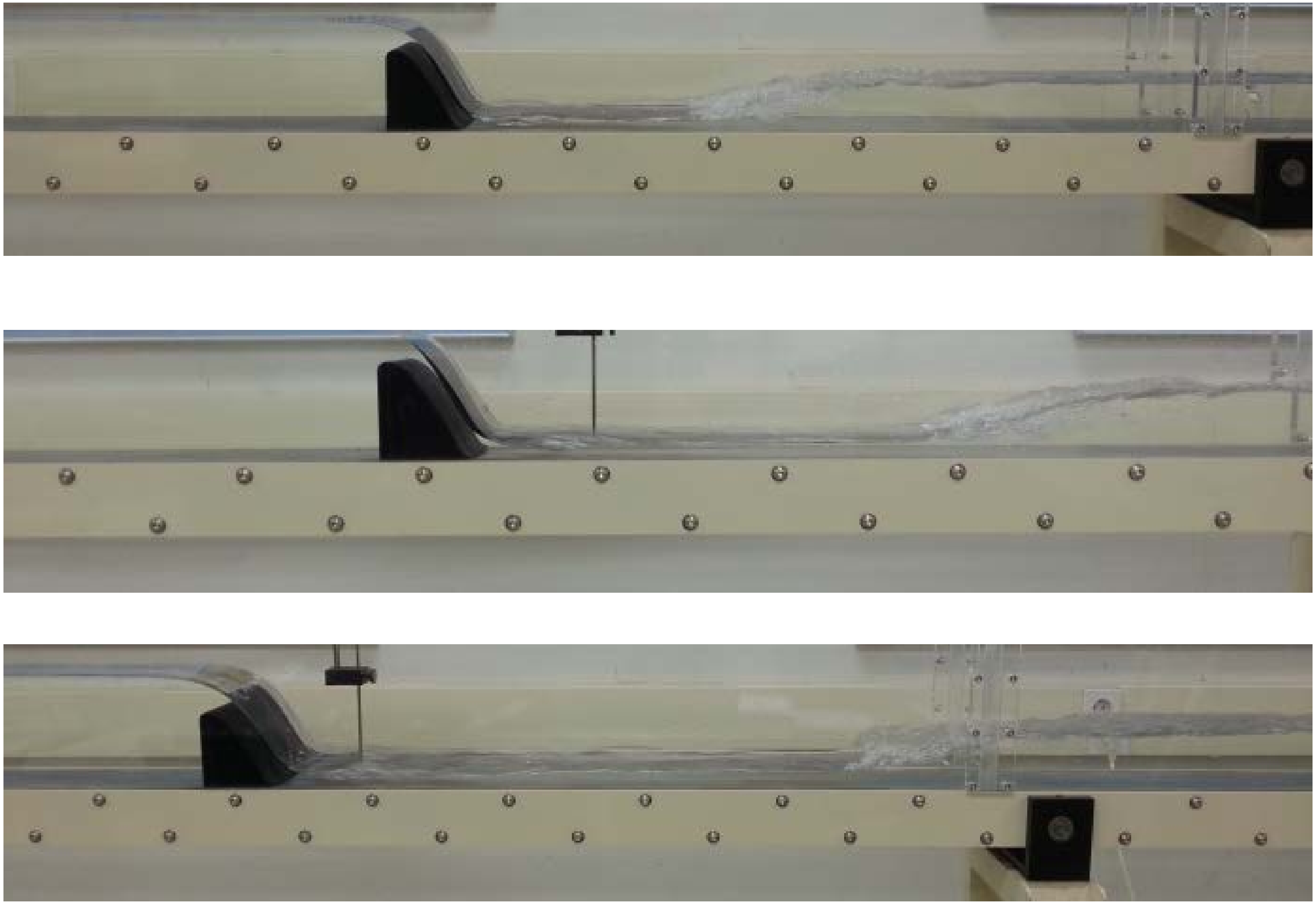
<u>Stable jump</u> (4.5<F_{r1}<9; 정상도수): Fixed position. Good <u>dissipation energy</u>



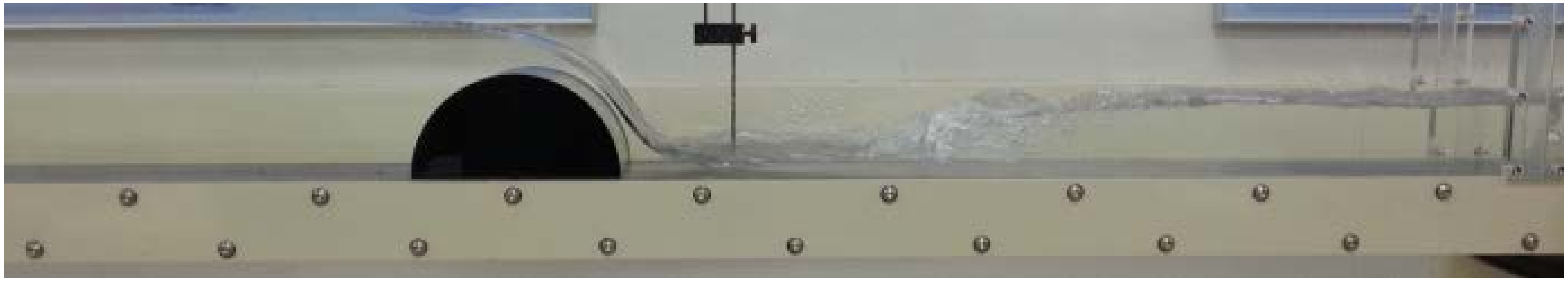
(e)

<u>Strong jump</u> (9<F_{n1}; 강도수): <u>Strong waves</u> are generator

Ogee-Crested Spillways (Q = 150, 200, 250 l/min)



Half-circle dam



Broad-crested weir

Trapezoidal dam