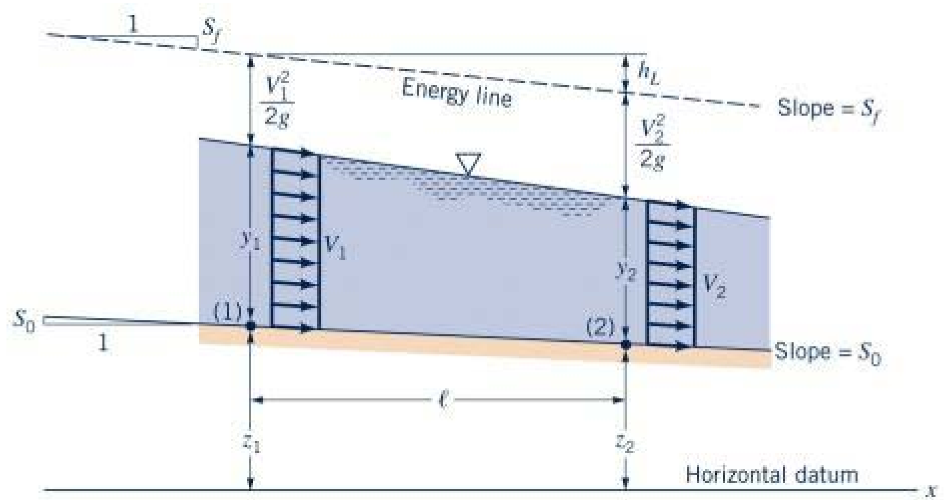
Ch. 8 Varied Flow in Open Channels

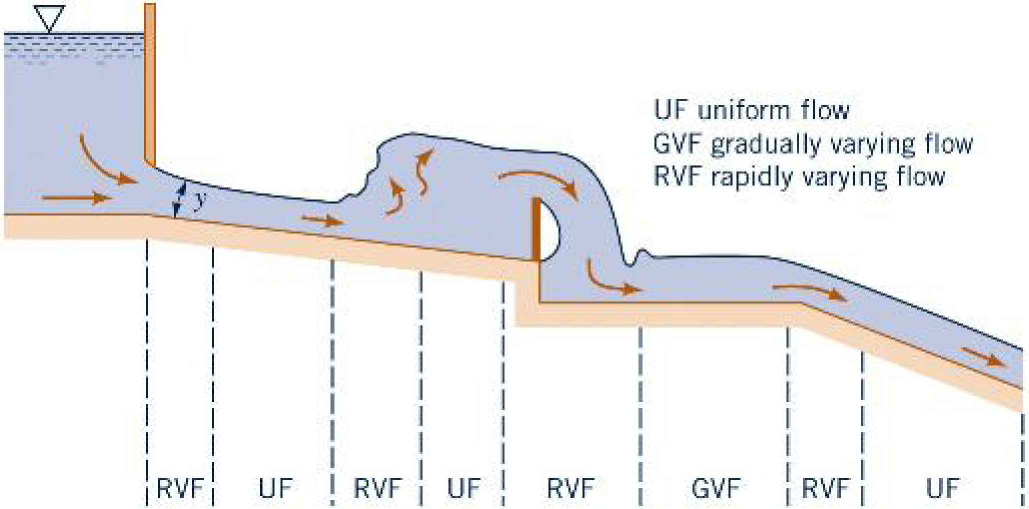
8-2 Gradually Varied Flows



Contents

Objectives

- Derive differential equation of varied flow using energy equation
- Classify water surface profiles of GVF

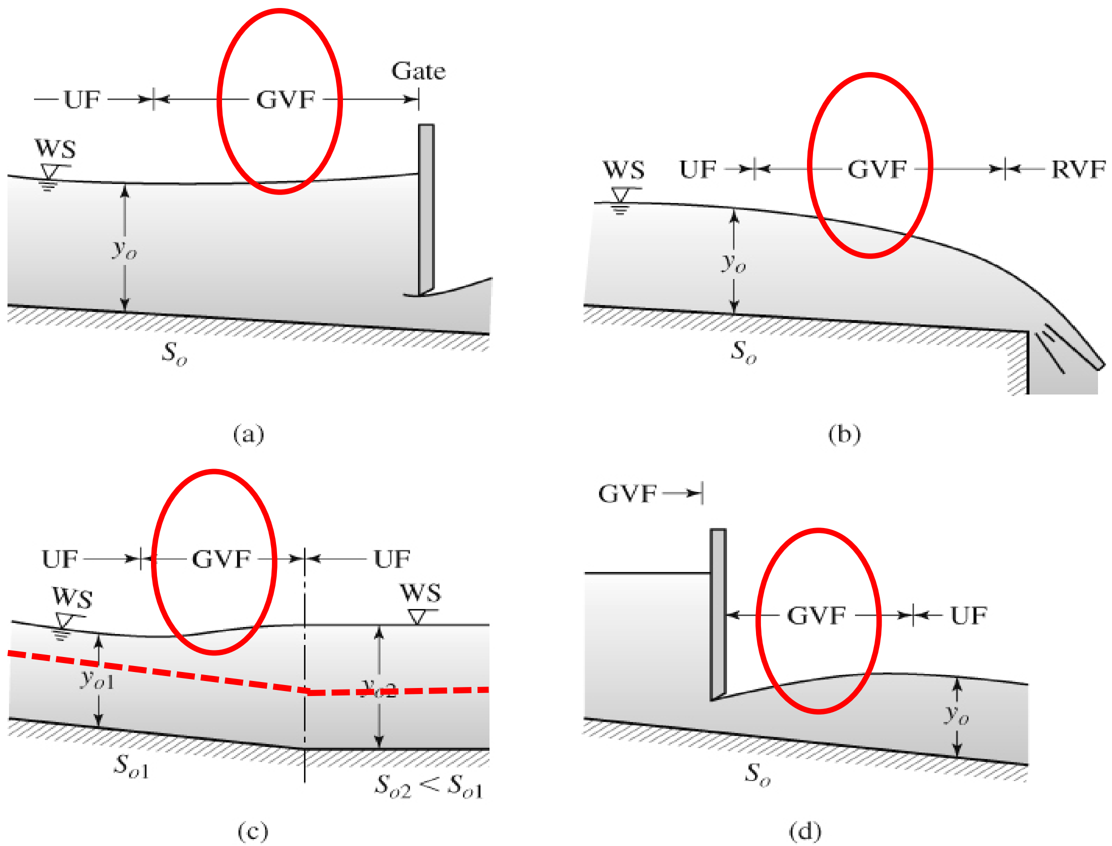


8.4 Gradually Varied Flow Equation

- In real situation, open channel's depth is not constant
- In the short distance, flow changes fast, then we call it as Rapid Varied Flow (RVF) such as hydraulic jump
- In gradually varied flow (GVF) case, energy loss mainly comes from <u>bottom friction in the long distance.</u>
- For the design of open channels and the analysis of their performance, the engineer must be able to <u>predict the forms</u> and <u>calculate the various types of water surface profiles</u>.

Gradually varied flow equation

- In uniform flow $S_0 = S_w = S_f$
  - Slope of the water surface = slope of bed = slope of total energy
- Typical gradually varied flow is found upstream of gates, and controls.



Differential equation for gradually varied flow

- Assumption

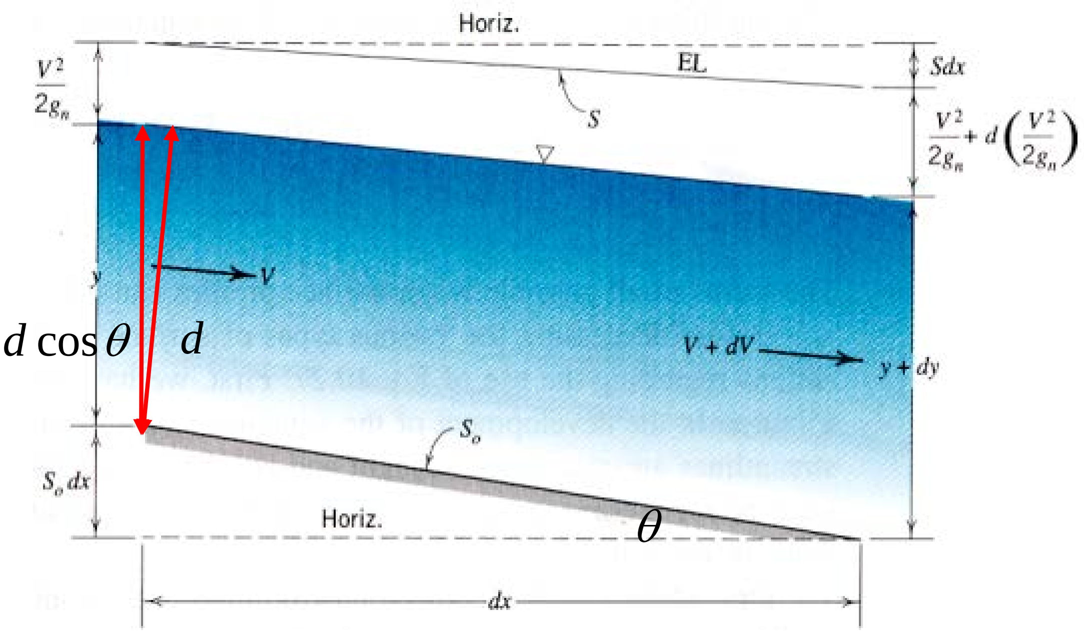
1. Channel is prismatic and the flow is steady (depth is constant with time)
2. The bed slope (S_0) is relatively small. Therefore, the vertical depth of flow is almost equal to the depth measured normal to the channel bed.
3. The velocity distribution in the vertical section is uniform and the kinetic energy correction factor is close to unity.
4. Streamlines are straight and parallel and the pressure distribution is hydrostatic.
5. The channel roughness (Manning's *n*) is constant along its length and does not depend on the depth of flow

Gradually varied flow equation

Derivation of gradually varied flow equation

dx = length of a channel segment
y ~ *d* = depth of flow (varies by *dy*);
V = average velocity (varies by *dV*)
S_0 = bed slope
S = energy (friction) slope

$$H = z + d\cos\theta + \frac{V^2}{2g} \approx z + y + \frac{V^2}{2g}$$

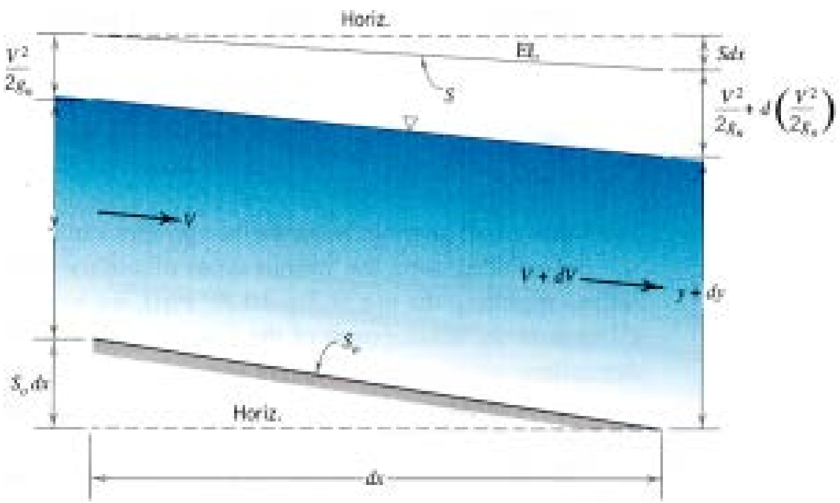


Gradually Varied Flow

One-dimensional longitudinal water surface profile can be described by a simple ordinary differential equation.

The total energy per unit weight of water measured from a horizontal datum, or the total head, is given as

$$H = z + y + \frac{V^2}{2g} = z + E$$



Apply to Sec.1 and 2,

$$S_0 dx + y + \frac{V^2}{2g} = y + dy + \frac{V^2}{2g} + d\left(\frac{V^2}{2g}\right) + Sdx$$

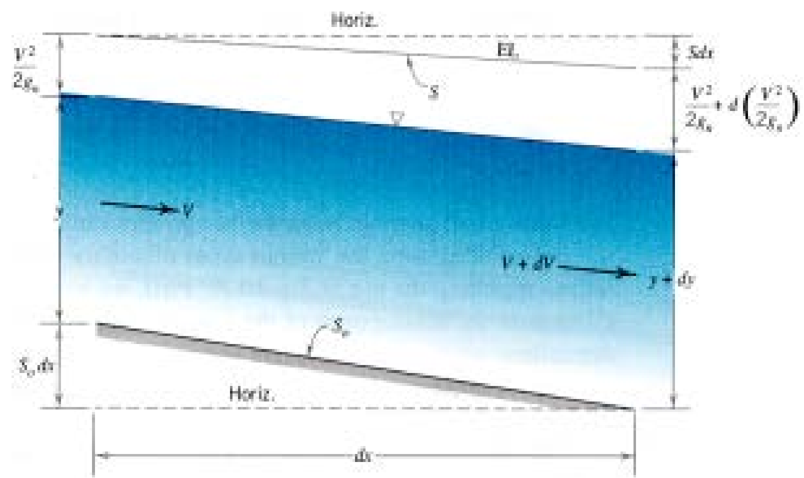
Gradually Varied Flow

Dividing this equation by *dx* and canceling equal terms produces

$$\frac{dy}{dx}+\frac{d}{dx}\left(\frac{V^2}{2g}\right)=S_0-S$$

Multiplying the second term by *dy/dy*, factoring out *dy/dx*, and solving for *dy/dx* yields

$$\frac{dy}{dx}=\frac{S_0-S}{1+\dfrac{d}{dy}\left(\dfrac{V^2}{2g}\right)}$$



Gradually Varied Flow

The second term in the denominator can be written as

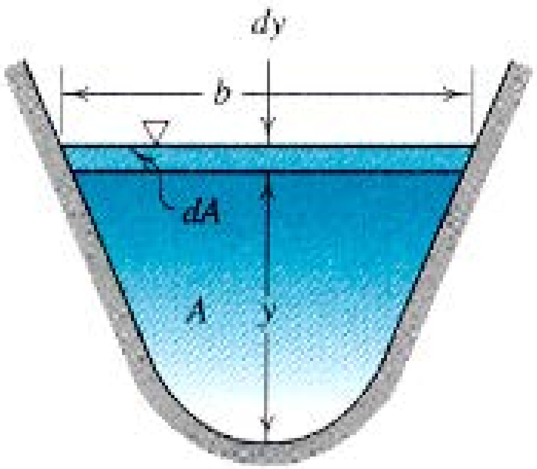
$$\frac{d}{dy}\left(\frac{V^2}{2g}\right)=\frac{1}{2g}\frac{d}{dy}\left(\frac{Q^2}{A^2}\right)=-\frac{Q^2 b}{gA^3}=-Fr^2$$

$$\frac{d}{dy}\left(\frac{Q^2}{A^2}\right)=Q^2\frac{dA^{-2}}{dy}$$

$$=Q^2\left(-2A^{-3}\frac{dA}{dy}\right)=Q^2\left(-2A^{-3}b\right)$$

Substitute into the varied flow equation

$$\boxed{\frac{dy}{dx}=\frac{S_0-S}{1-F_r^2}} \qquad (10.27)$$



Gradually Varied Flow

- Approximation: Even though Chezy-Manning equation is generally considered a uniform flow equation, it can be used to evaluate the slope of energy line, *S* for varied flow as long as the streamlines are straight and parallel.

Substitute Chezy-Manning equation into (10.27)

$$\frac{dy}{dx}=\frac{S_0-\frac{V^2n^2}{R_h^{4/3}}}{1-F_r^2}=\frac{S_0-\frac{Q^2n^2}{A^2R_h^{4/3}}}{1-F_r^2} \qquad (10.28)$$

$$V=\frac{1}{n}R_h^{2/3}S^{1/2}$$

$$S=\frac{n^2V^2}{R_h^{4/3}}$$

$$Q=\frac{1}{n}AR_h^{2/3}S^{1/2}$$

$$S=\frac{Q^2n^2}{A^2R_h^{4/3}}$$

Review of Critical Slope

- The critical slope (S_c) is the bed slope of the channel that produces critical uniform flow for a given discharge.
- The Manning equation is applicable to the case of uniform assuming that the normal depth is equal to the critical depth,
- For wide rectangular channel, $R_h \sim y$

$$q = \sqrt{g y_c^{\ 3}} = \frac{1}{n} y_c^{\ 5/3} S_c^{\ 1/2}$$

$$y_c = \sqrt[3]{\frac{q^2}{g}}$$

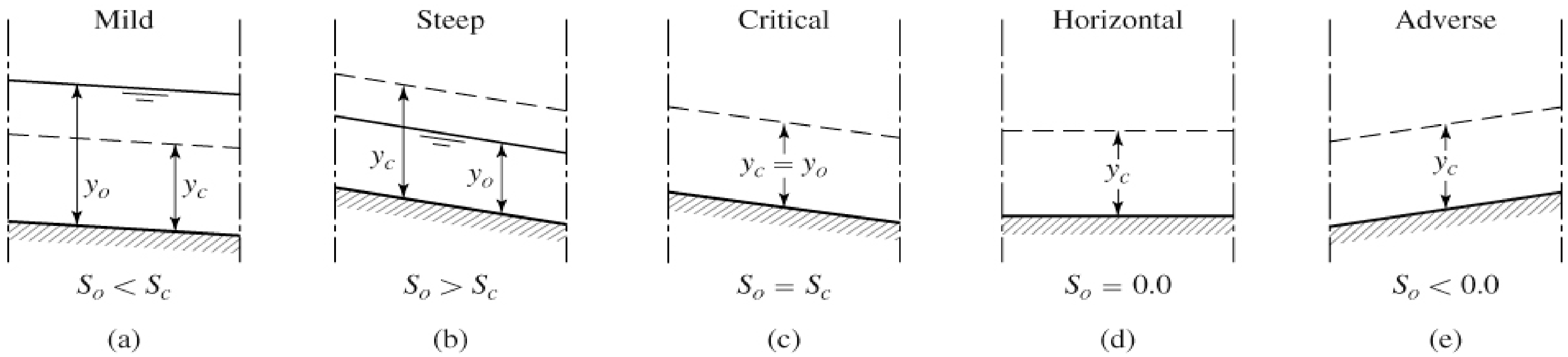
$$\boxed{S_c = \frac{g n^2}{y_c^{\ 1/3}}} \qquad (10.17)$$

8.5 Gradually Varied Flows

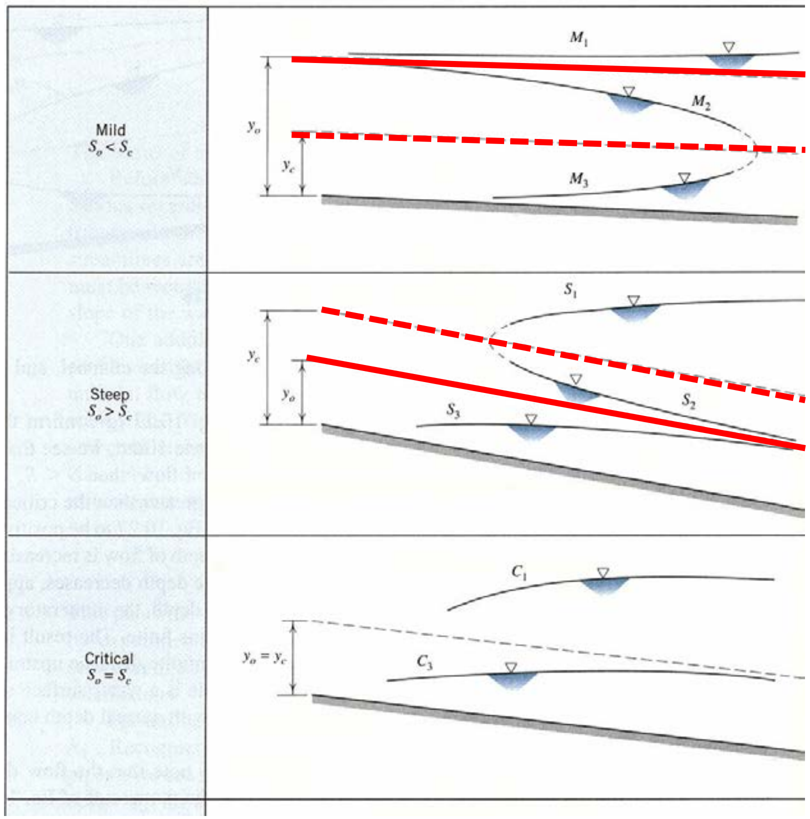
1. Classification of channels according to bed slope

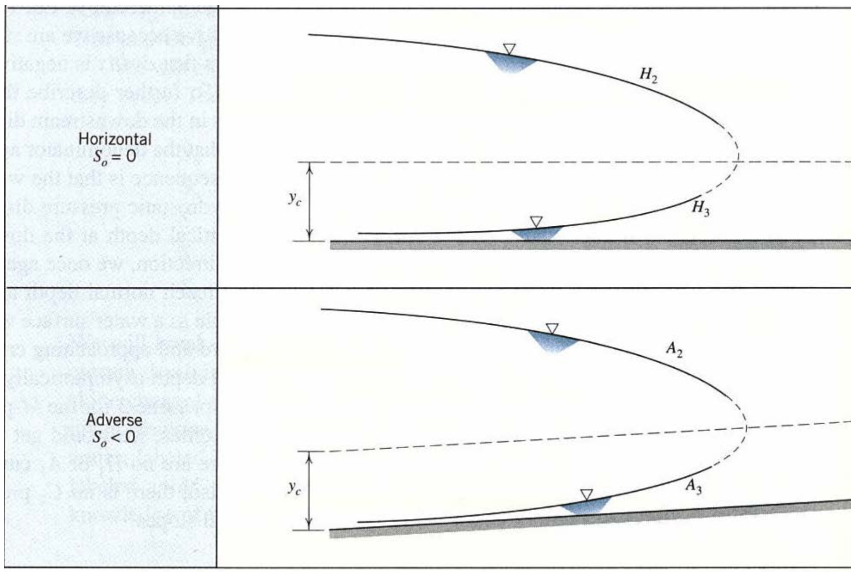
- The information of normal depth, critical depth and current depth can be used for classification of flow type.

- First, classify the type of channel slope



Classification of channels according to bed slope





Classification of channels according to bed slope

1. **The mild slope (*M-profile*)**: The bed slope S_0 is less than the critical slope S_c. In this category, the normal depth y_0, is located above the critical depth y_c and the Froude number, F_r will be less than unity under normal flow condition

$$S_0 < S_c \quad y_0 > y_c \quad V_0 < V_c$$

2. **The steep slope (*S-profile*)**: The bed slope S_0 is steeper than the critical slope. The normal depth is less than the critical depth y_c and the Froude number, F_r will be greater than unity under normal flow condition

$$S_0 > S_c \quad y_0 < y_c \quad V_0 > V_c$$

3. **The critical slope (*C-profile*)**: The bed slope is exactly equal to the critical slope. The Froude number will be exactly equal to unity under normal flow conditions.

$$S_0 = S_c \quad y_0 = y_c \quad V_0 = V_c$$

$$S_c = \frac{gn^2}{y_c^{1/3}}$$

Classification of channels according to bed slope

4. Horizontal slope (*H-profile)*: The bed slope is equal to zero. The critical depth can be evaluated while the normal depth will be equal to infinity. Substituting S_0 as zero in the Chezy or Manning equation will give infinite value for the normal depth.

5. Adverse slope (*A-profile)*: The bed slope is less than zero. Normal depth is not defined.

2. Stage zone according to normal and critical depths

Classify three zones that are defined according to the current depth, y with respect to normal and critical depths for a given discharge.

Zone 1: y is located above both y_0 and y_c

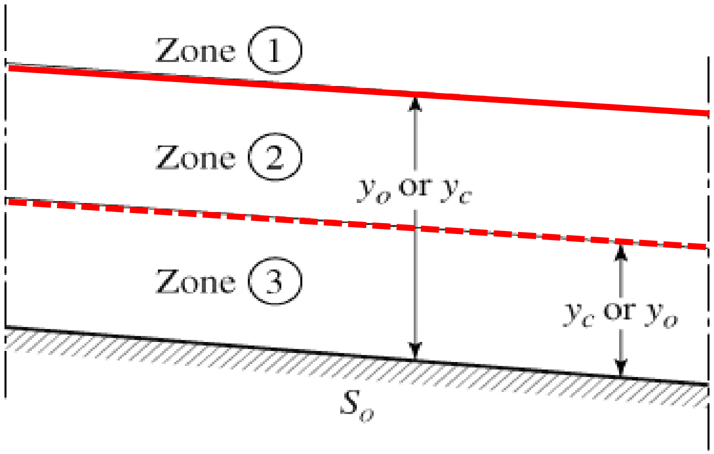
i.e. $y > y_0 > y_c$ or $y > y_c > y_0$

Zone 2: y is located between y_0 and y_c

i.e. $y_0 > y > y_c$ or $y_c > y > y_0$

Zone 3: y is located below both y_0 and y_c

i.e. $y_0 > y_c > y$ or $y_c > y_0 > y$

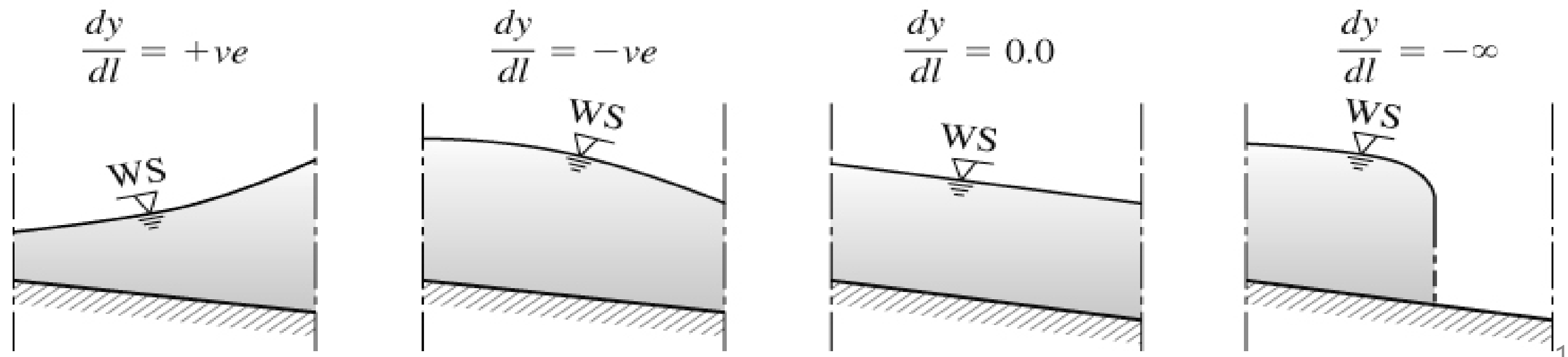


3. Classification of profiles according to (dy/dx)

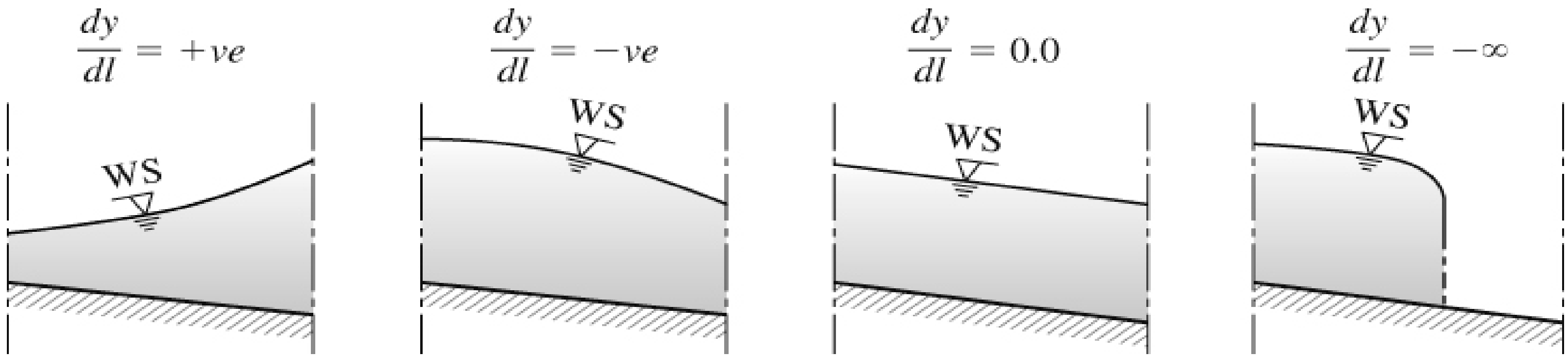
$$\frac{dy}{dx} = \frac{S_0 - S}{1 - F_r^2}$$

According to the rate of change of the flow depth with distance, *dy/dx* the trend of the water surface in gradually varied flow is divided into regimes:

1. *dy/dx* >0: This means that the depth of flow is increasing with distance. The water surface forms a rising curve.
2. *dy/dx*<0: This means that the depth of flow is decreasing with distance. The water surface forms a falling curve.
3. *dy/dx* =0: This means that the flow is uniform and the water surface is parallel to the bottom of the channel.



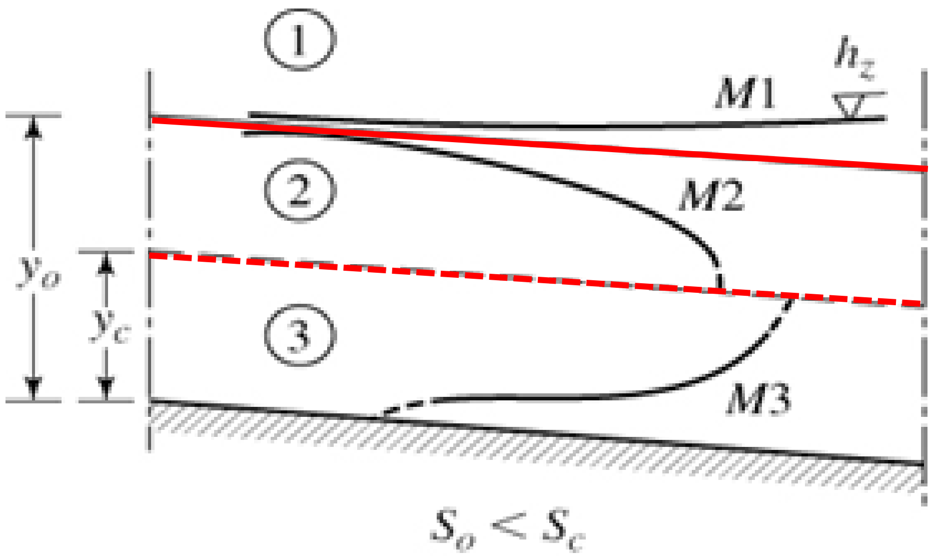
4. dy/dx is equal to negative infinity. This means that the water surface forms a right angle with the channel bed. This condition can never be exactly encountered in nature. This type of profile may occur at the free fall of a mild channel or at the transition between a mild reach to a steep or critical reach



4. Graphical representation of the gradually varied flow equation

(1) Mild slope curves

Zone 1. y > y_0 > y_c *M1 → rising*
Zone 2. y_0 > y > y_c *M2 → falling*
Zone 3. y_0 > y_c > y *M3 → rising*



(a) Mild slope M

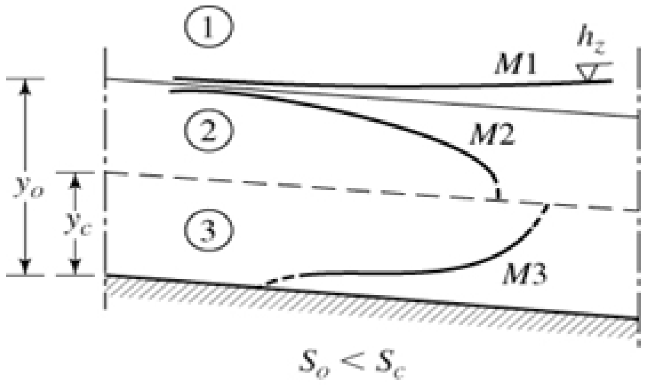
M1 curve: $y > y_0 > y_c$

$y > y_0$, then $S < S_0$
$y > y_c$, then $F_r < 1$

$$\frac{dy}{dx} = \frac{S_0 - S}{\left(1 - F_r^2\right)}$$

$$\frac{dy}{dx} = \frac{S_0 - S}{1 - F_r^2} = \frac{+}{+} = +$$

$$S = \frac{Q^2 n^2}{A^2 R_h^{4/3}}$$



(a) Mild slope *M*

Thus, the depth is increasing in the positive direction. Indicating that this is case 1 of the mild slope situation, we denote this case as the *M1* profile. "backwater profile (배수곡선)" occurs when dam or hump exist at the downstream.

As we move upstream the depth decreases, approaching the normal depth asymptotically. Thus, the *M1* curve tends to be parallel to the bottom at infinity.

Uniform flow equation

$$S = \frac{Q^2 n^2}{A^2 R_h^{4/3}}$$

For uniform flow of a given discharge and bed slope, we have a depth y_0 which satisfy the equation given below

$$S_0 = \frac{Q^2 n^2}{A_U^{\,2} R_{h_U}^{\,4/3}}$$

If the depth increases over the normal depth y_0, then $A^2 R_h^{4/3}$ also increases causing S to decrease below S_o

Therefore, from varied flow equation $\dfrac{dy}{dx} = \dfrac{S_0 - S}{\left(1 - F_r^{\,2}\right)}$

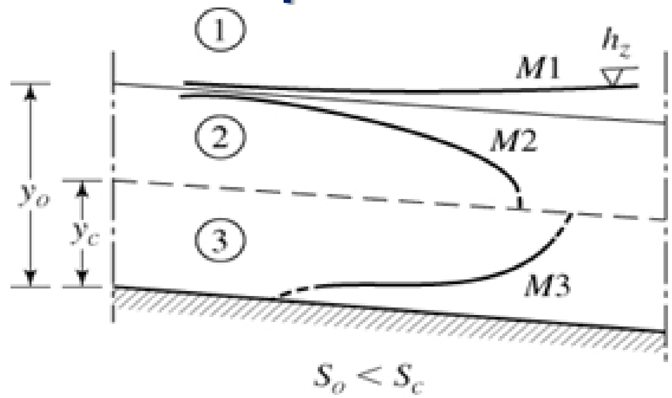
$y > y_0$, then $S < S_0$

Graphical representation of the Gradually varied flow equation

M2 curve: $y_0 > y > y_c$

$y < y_0$, then $S > S_0$
$y > y_c$, then $F_r < 1$

$$\frac{dy}{dx} = \frac{S_0 - S}{\left(1 - F_r^2\right)}$$



(a) Mild slope *M*

$$\frac{dy}{dx} = \frac{S_0 - S}{1 - F_r^2} = \frac{-}{+} = -$$

- The *M2* curve is thus falling curve in which the depth decreases in the flow (downstream) direction.
- At the critical depth *Froude number* will be equal to 1. Therefore as the *M2* approaches y_c, *dy/dx* tends to infinite. Thus the *M2* profile tends to be perpendicular to the critical depth line. It approaches the normal depth asymptotically in the upstream direction.
- M2 profile: drawdown curve (저하곡선).

Graphical representation of the Gradually varied flow equation

M3 curve: $y_0 > y_c > y$

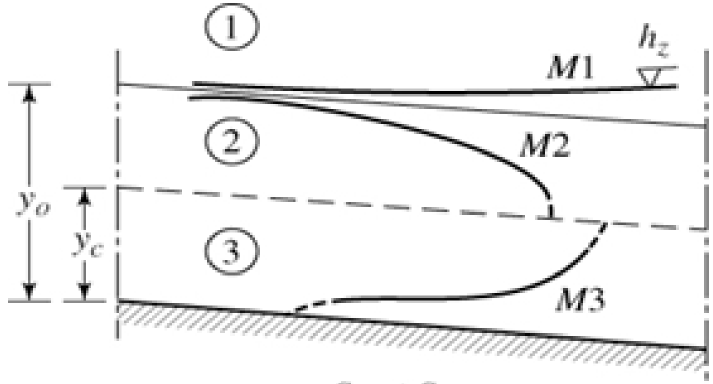
$y < y_0$, then $S > S_0$

$y < y_c$, then $F_r > 1$

$$\frac{dy}{dx} = \frac{S_0 - S}{\left(1 - F_r^2\right)}$$

$$\frac{dy}{dx} = \frac{S_0 - S}{1 - F_r^2} = \frac{-}{-} = +$$

- The curve is rising.
- Approaches the critical depth with a right angle and approaches the channel bed with an acute angle

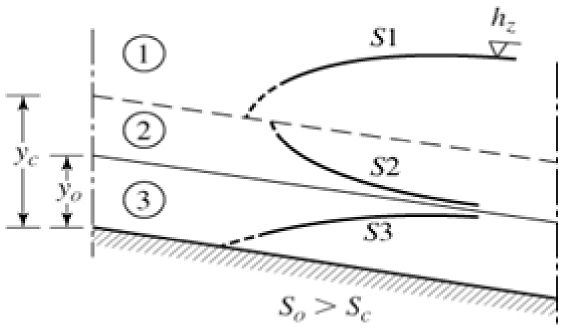


(a) Mild slope M

- M3 profile cannot end in either the normal depth or critical depth down stream. The profile is rising toward y_c since there must be a transition (hydraulic jump) between the supercritical flow of the *M3* profile and the subcritical flow downstream.

Graphical representation of the Gradually varied flow equation

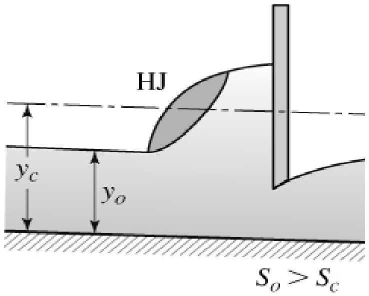
(2) Steep slope curves
S1 profile ($y>y_c>y_o$)



(b) Steep slope S

$y > y_{0,}$ then $S < S_0$
$y > y_c$, then $F_r < 1$

$$\frac{dy}{dx} = \frac{S_0 - S}{1 - F_r^2} = \frac{+}{+} = + \quad \rightarrow \text{rising curve}$$



- Downstream control must be a choke by dam, while, at the upstream boundary, it approaches the critical depth with an acute angle.
- Transition between the subcritical flow downstream and the supercritical normal depth upstream must be present. Thus *S1* profile must be led by a hydraulic jump

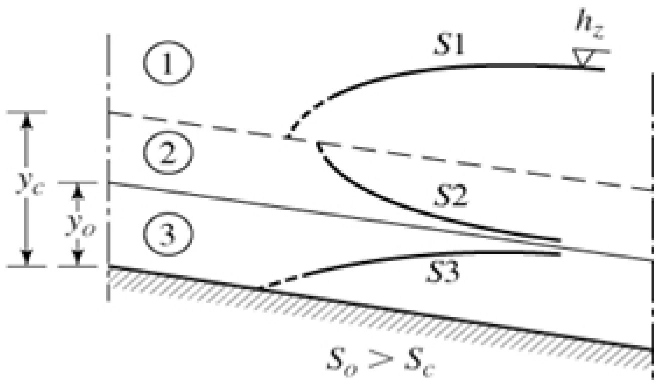
Graphical representation of the Gradually varied flow equation

S2 profile ($y_c > y > y_o$)

$y > y_{0,}$ then $S < S_0$

$y < y_c$, then $F_r > 1$

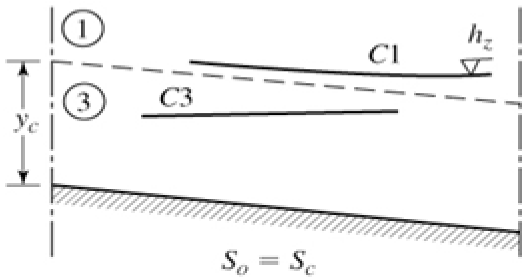
$$\frac{dy}{dx} = \frac{S_0 - S}{1 - F_r^2} = \frac{+}{-} = -$$



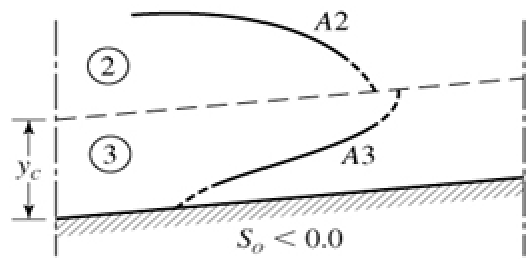
(b) Steep slope S

- Drawdown curve since it provides the transition between a critical section upstream and the supercritical normal depth associated with the steep downstream slope.

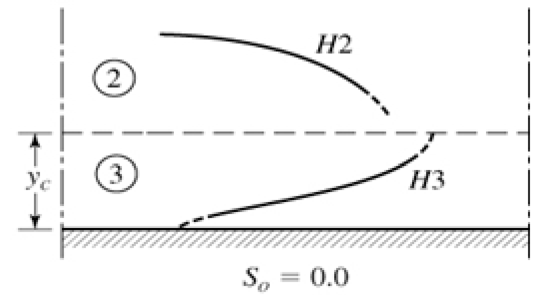
Graphical representation of the Gradually varied flow equation



(c) Critical slope C



(e) Adverse slope A



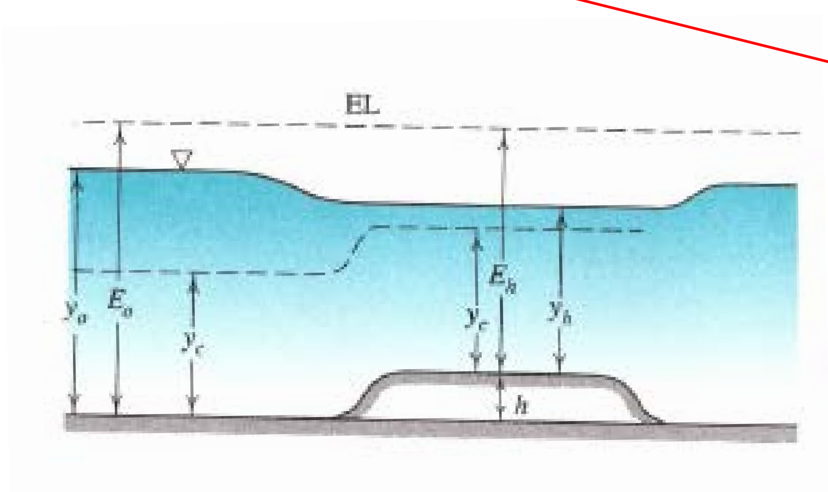
(d) Horizontal slope H

- In the case of the horizontal and adverse slopes, the normal depth does not exist and thus there can be no type 1 ($y>y_o$). In these cases the type 2 profile must exist approach the horizontal boundary for y is infinite upstream.

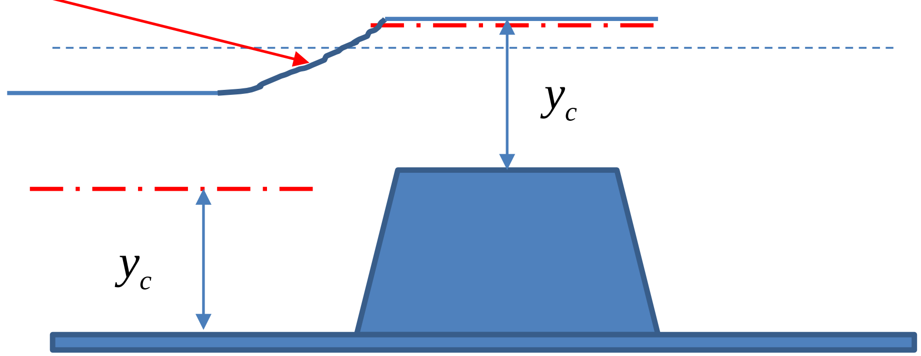
- In the critical slope case $y_o=y_c$:and thus there can be no type 2 profile.

IP 10-8 (p. 467): Identify the water surface profiles

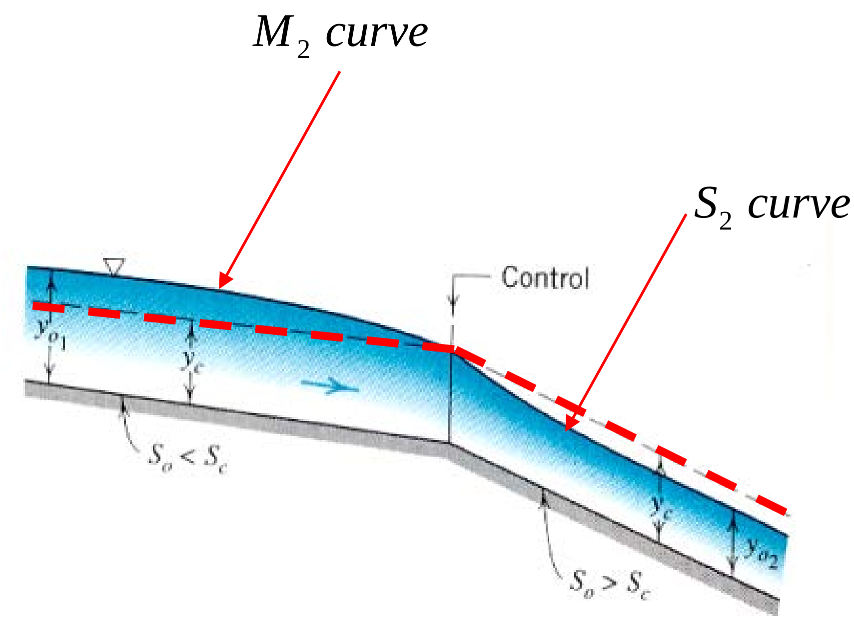
a) I.P. 10.5: If the hump were made higher, the flow would not have enough specific energy to flow over the hump and the hump would become a dam. This will cause the depth of the flow upstream of the hump to increase. Then, water surface profile on a mild slope at a depth greater than the normal is M_1 curve.



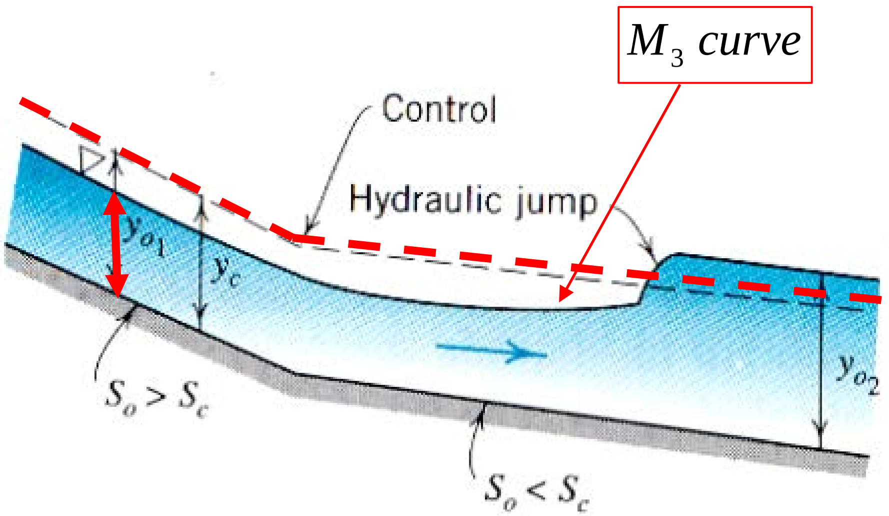
Subcritical flow upstream



b) Upstream and downstream of the brink



c) Just upstream of the hydraulic jump on the mild slope channel



d) Upstream of the free outfall