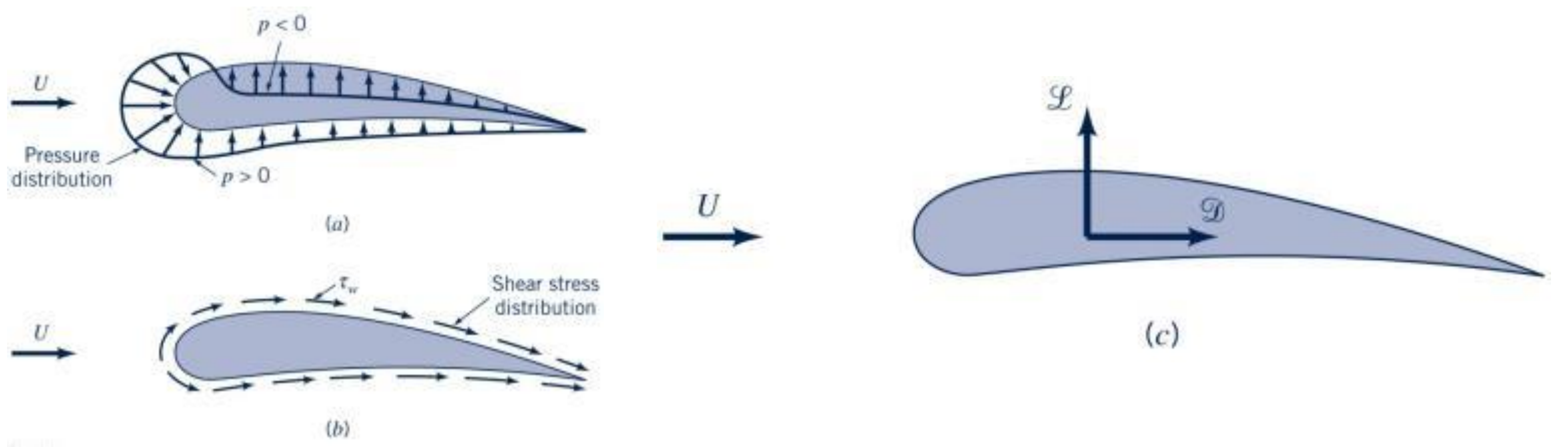
Lecture 14

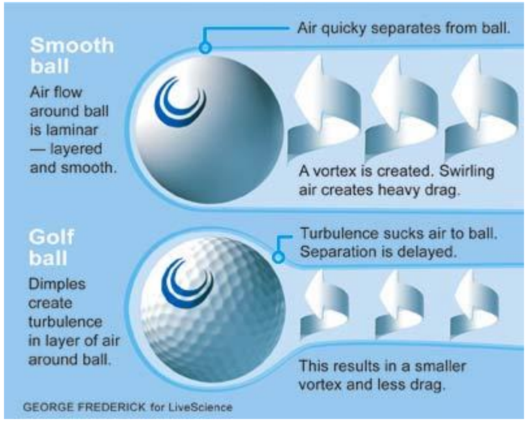
Lift and Drag in Incompressible Flow



Contents

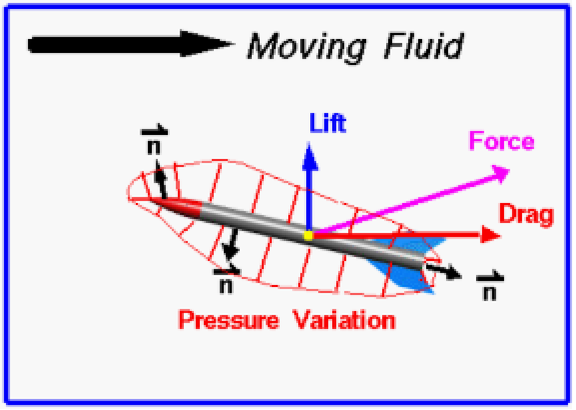
Objectives

- A knowledge of fluid resistance and lift would be needed for the effective and safe design of buildings, bridges, automobiles, and trains as well as ships and airplanes.
- Outline the fundamental and elementary aspects of external flow about immersed objects for incompressible flow
- Learn the key fundamentals and essential to describe the lift and drag



14.1 Definitions

- When flow occurs about an object that is either asymmetrical or whose axis is not aligned with the flow, the flow field will be asymmetrical, the local velocities and pressures on either side of the object will be different, and a **force normal to the oncoming flow** will be exerted.
- Along with this force, the action of the **frictional stress** due to the boundary layers on the surface of the body will produce a net force along the direction of the on coming flow and opposing the motion of the body.
- These pressure and friction forces produce a pair of net forces, which are perpendicular to each other and are called ***lift*** (양력) and ***drag*** (항력).



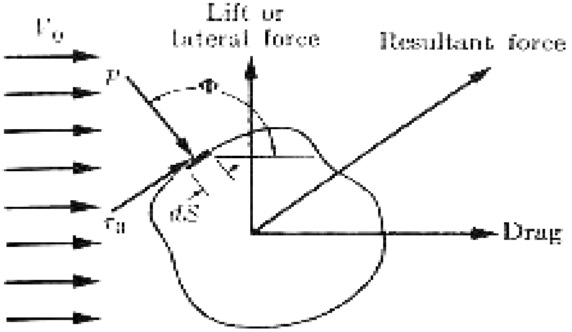


FIG. 8-7. Definition diagram for flow-induced forces.

Definitions

- Consider the element *dA* on the right figure

$$dD = p\,dA\sin\theta + \tau_0\,dA\cos\theta$$

$$dL = -p\,dA\cos\theta + \tau_0\,dA\sin\theta$$

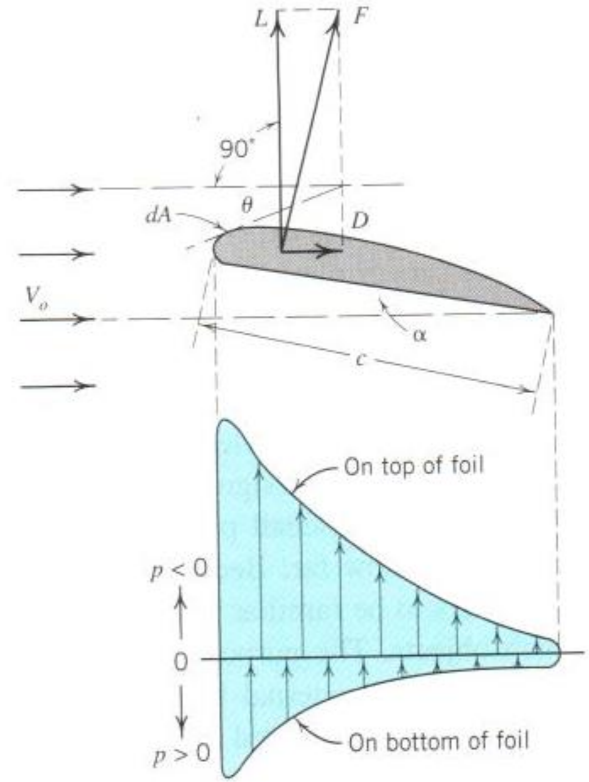
- Integrating yield

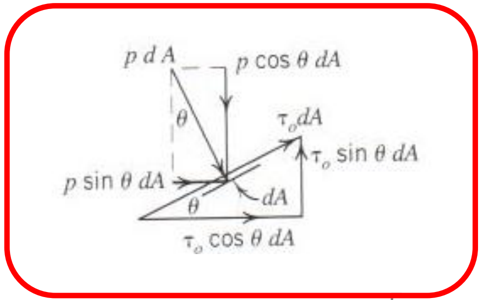
$$D = \iint_s p\,dA\sin\theta + \iint_s \tau_0\,dA\cos\theta \qquad (14.1)$$

$$L = -\iint_s p\,dA\cos\theta + \iint_s \tau_0\,dA\sin\theta \qquad (14.2)$$

- In the lift force, the contribution of the shear stress is negligible (use of ideal fluid theory)

$$L \approx -\iint_s p\,dA\cos\theta$$





Definitions

- The drag force: Both pressure and frictional forces must be considered.
- Combination of hydrodynamics and boundary-layer theory can be used

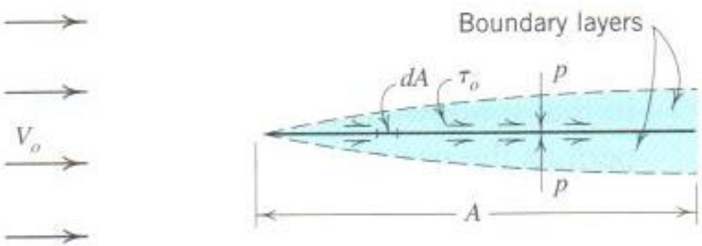
$$D = \iint_S p\,dA \sin\theta + \iint_S \tau_0\,dA \cos\theta$$

$$= D_p + D_s \qquad (14.3)$$

D_p : pressure drag (form drag)

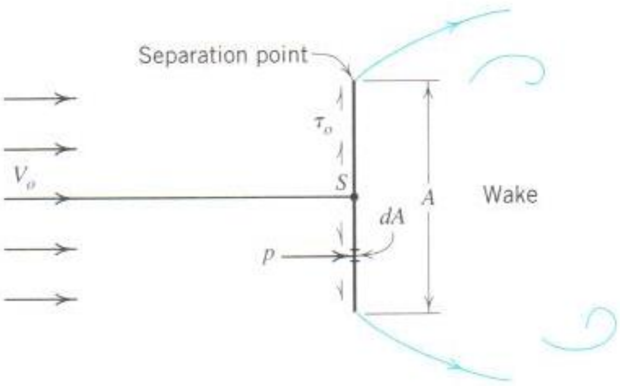
D_s : frictional drag (skin drag)

- When the pressure drag depends on the form of the object and flow separation, the frictional drag is on the extent and character of the <u>boundary layer.</u>



(a) Thin plate parallel to the flow

Skin drag only



(b) Thin plate normal to the flow

Form drag only

Definitions

- Disk, sphere and streamlined body have the same cross-sectional area normal to the flow.
- Disk:
  - Separation occurs at the edges, with increasing velocity, decreases of pressure (Bernoulli's).
  - No skin friction therefore, wholly pressure cause a large drag force.
- Sphere:
  - The wake is smaller than disk, frictional force is not zero but still small
- Streamlined body:
  - Wake may be small, pressure drag is small, and <u>friction drag becomes large</u>.
  - Total drag is <u>1/40 of disk</u>.

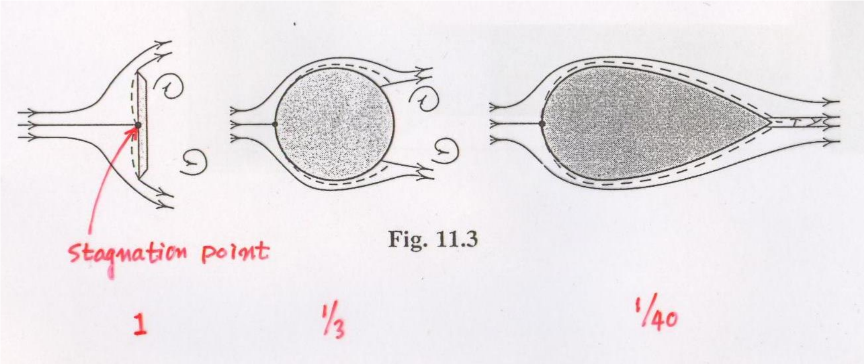
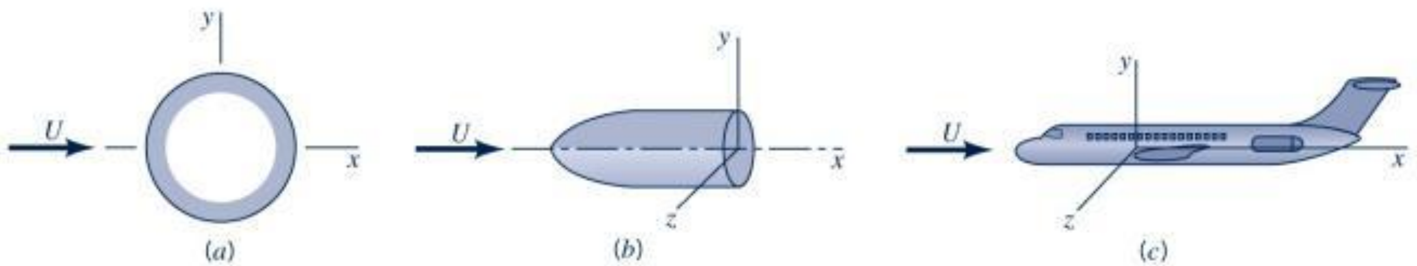


Fig. 11.3

For airfoil, hydrofoil, and slim ships: surface drag > form drag

For bluff objects like spheres, bridge piers: surface drag < form drag



Drag (항력, 저항력):

- 흐르는 유체가 물체에 작용하는 힘(건물, 교량에 미치는 풍하중)

- 정지된 유체속을 움직이는 물체에 작용하는 힘 (야구공, 골프공)

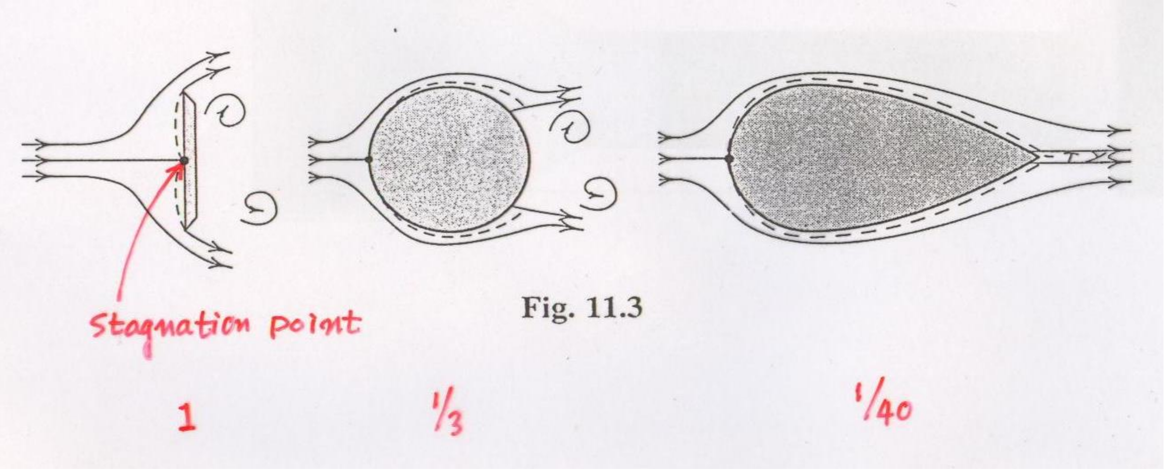


Fig. 11.3

14.2 Dimensional Analysis

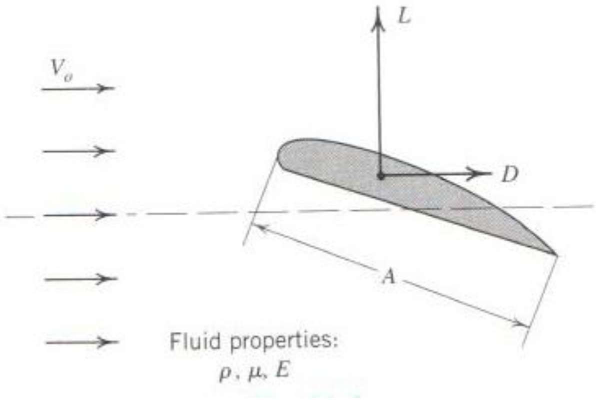
- A: area, ρ: density of fluid,

  μ: viscosity, E: modulus of elasticity

  D: drag force, V_0: flow velocity

  $$D = f_1\left(A, \rho, \mu, V_0, E\right)$$

  $$L = f_2\left(A, \rho, \mu, V_0, E\right)$$



- With dimensional analysis,

$$\Pi_1 = \frac{\rho\sqrt{A}V_0}{\mu} = Re$$

$$\Pi_2 = \frac{\rho V_0^2}{E} = \frac{V_0^2}{a^2} = Ma^2 \qquad \left(a = \sqrt{E/\rho}\right)$$

$$\Pi_3 = \frac{D}{A\rho V_0^2}; \qquad \Pi_4 = \frac{L}{A\rho V_0^2}$$

Dimensional Analysis

- If drag and lift coefficients, C_D and C_L respectively are defined by

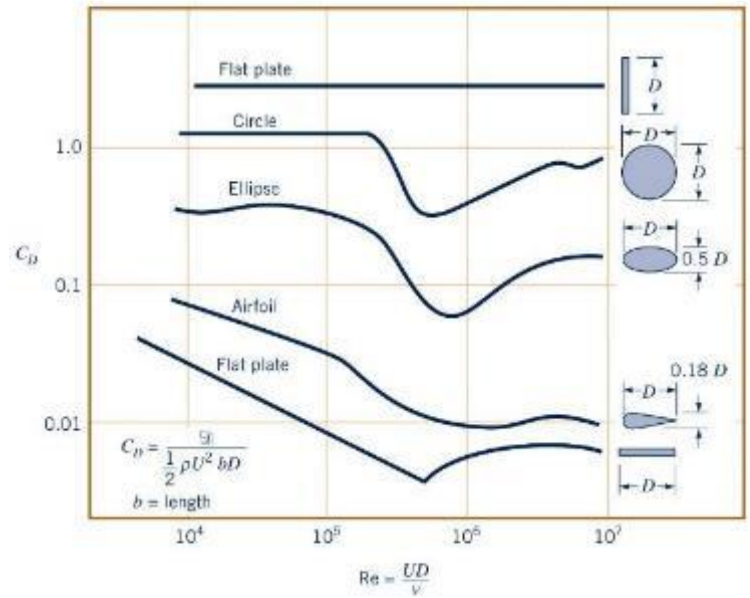
$$D = \frac{f_3(Re, Ma) A \rho V_0^2}{2}, \qquad L = \frac{f_4(Re, Ma) A \rho V_0^2}{2}$$

$$C_D = f_3(Re, Ma), \quad C_L = f_4(Re, Ma)$$

- These equations indicate
  - Bodies having the same shape and the same alignment with the flow posses the same drag and lift coefficients if their Reynolds numbers and <u>Mach numbers</u> are the same
  - The <u>drag and lift coefficients of bodies may depend on their Reynolds and Mach number only.</u>
  - Mach number is insignificant in <u>incompressible fluid.</u>

$$D = \frac{C_D A \rho V_0^2}{2} \qquad (14.4)$$

$$L = \frac{C_L A \rho V_0^2}{2} \qquad (14.5)$$



14.3 Drag and Lift

- In the lower Reynolds number (1 < Re < 10)
  - We can estimate the drag force of sphere from potential flow theory

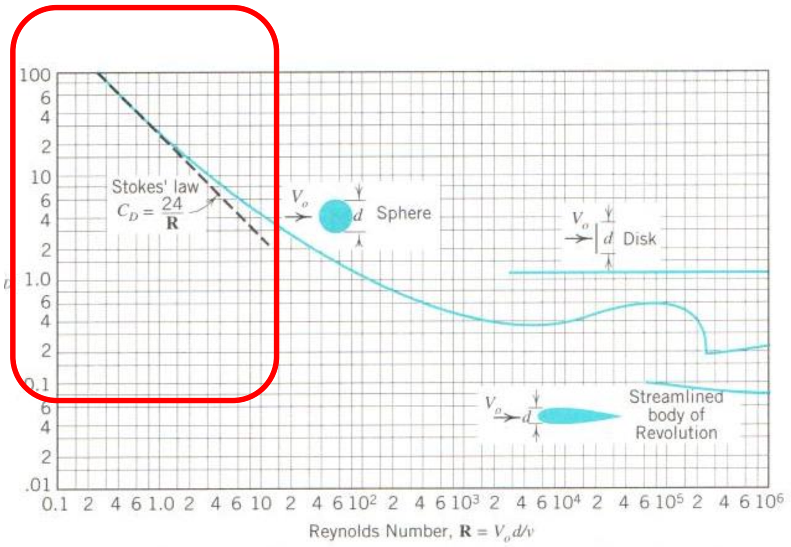
$$D = 3\pi\mu V_0 d \qquad (14.6)$$

  - Since we can ignore the inertial effects.

$$\frac{C_D A \rho V_0^2}{2} = 3\pi\mu V_0 d$$

$$\boxed{C_D = \frac{24\mu}{V_0 d \rho} = \frac{24}{Re}} \quad \left(A = \frac{\pi d^2}{4} \right)$$

Stokes' law (14.7)



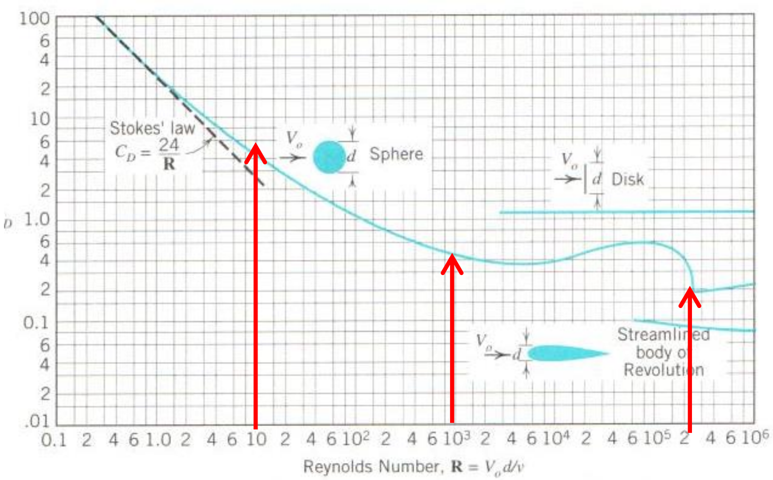
- 1,000 < Re < 250,000

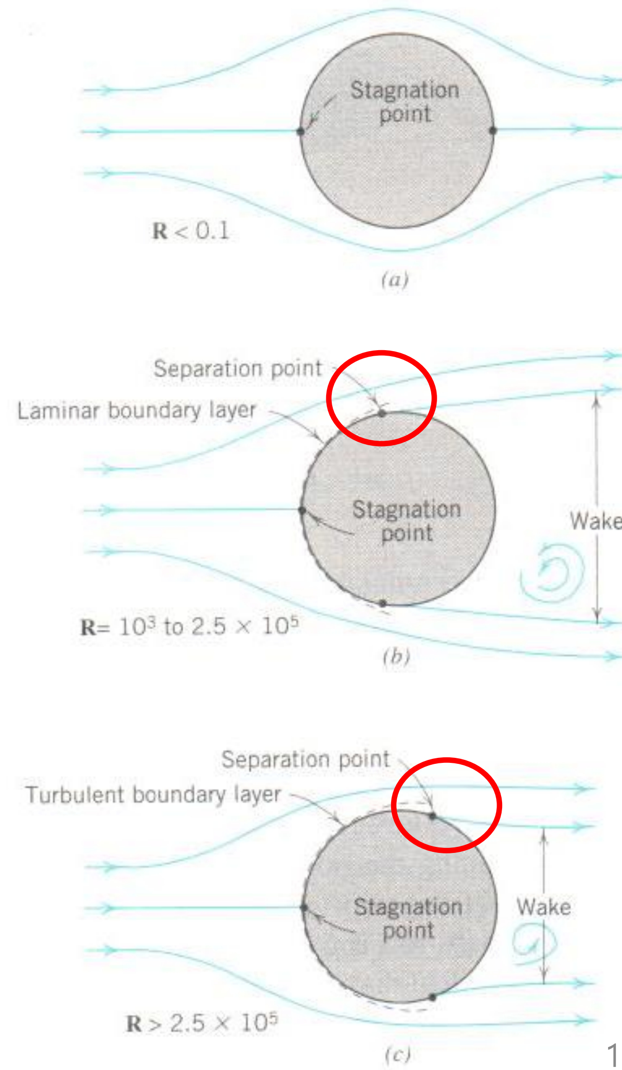
<u>- Separation and weak eddies begin to form</u>, enlarging into a fully developed wake near a Reynolds number of 1,000.

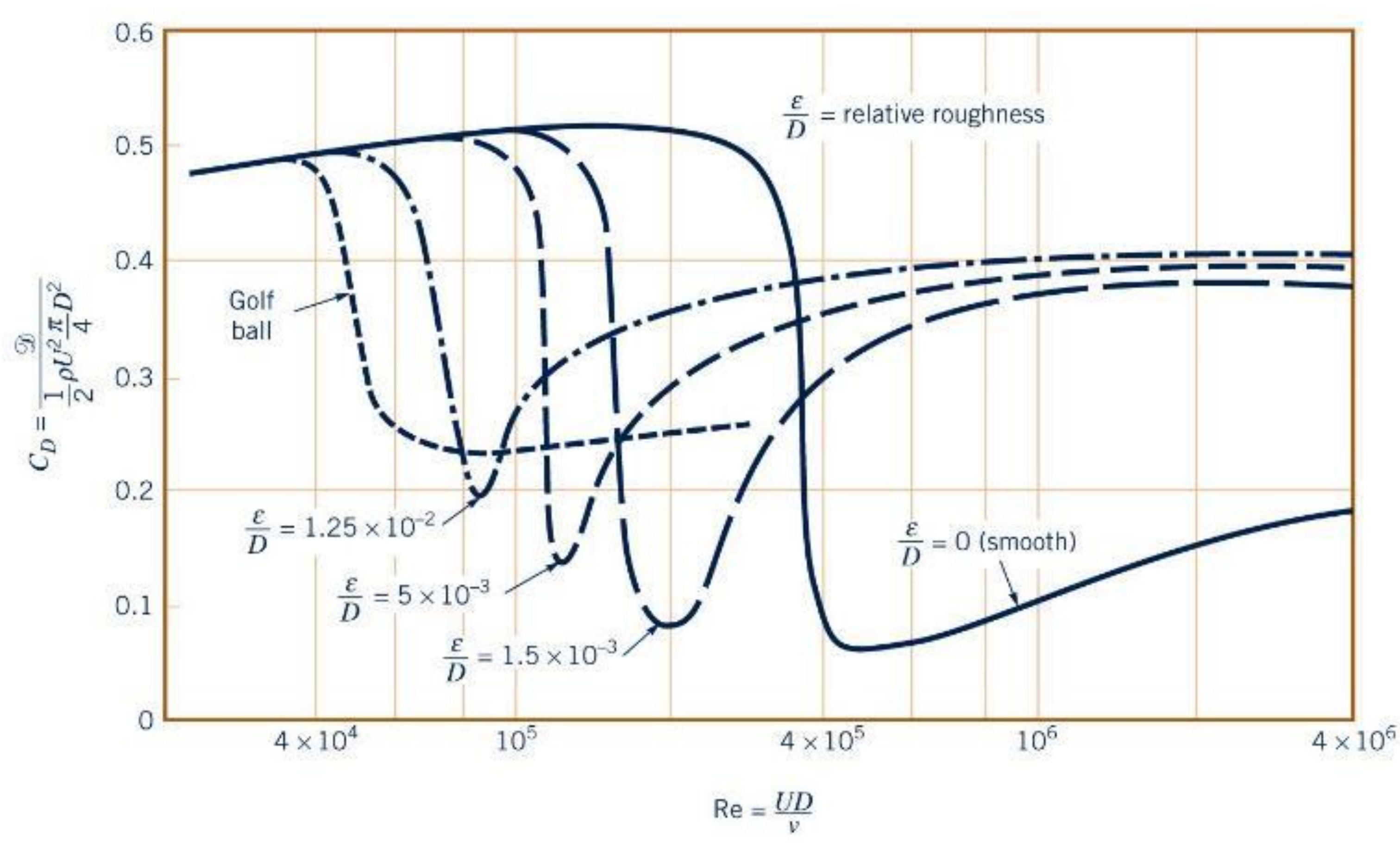
- pressure drag dominates

- 250,000 < Re

- drag coefficient suddenly drops







0.6
0.5
0.4
0.3
0.2
0.1
0
$C_D = \dfrac{\mathscr{D}}{\frac{1}{2}\rho U^2 \frac{\pi}{4} D^2}$
$\dfrac{\varepsilon}{D}$ = relative roughness
Golf ball
$\dfrac{\varepsilon}{D} = 1.25 \times 10^{-2}$
$\dfrac{\varepsilon}{D} = 5 \times 10^{-3}$
$\dfrac{\varepsilon}{D} = 1.5 \times 10^{-3}$
$\dfrac{\varepsilon}{D} = 0$ (smooth)
4×10^4
10^5
4×10^5
10^6
4×10^6
$\mathrm{Re} = \dfrac{UD}{\nu}$

Flow behind a cylinder

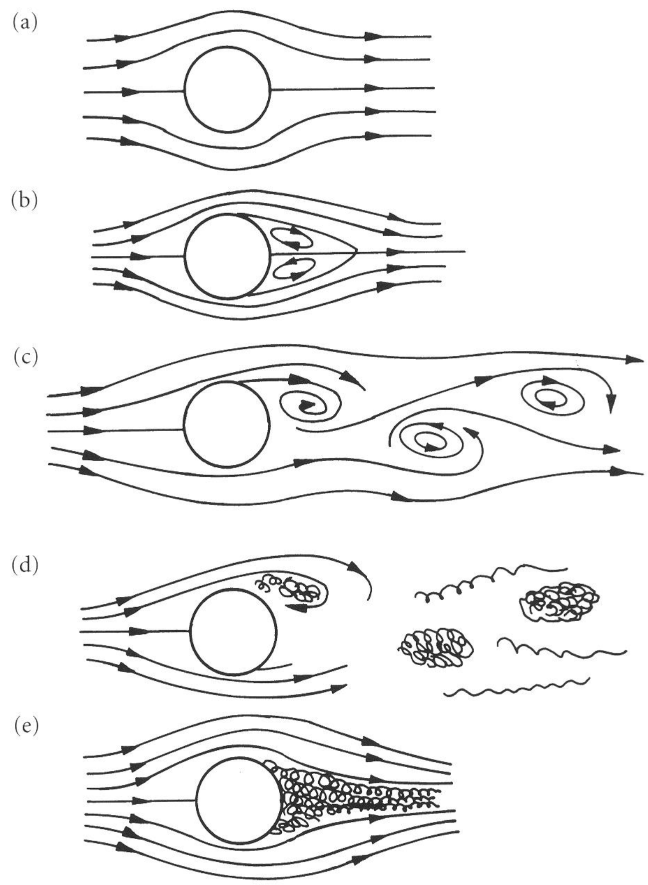
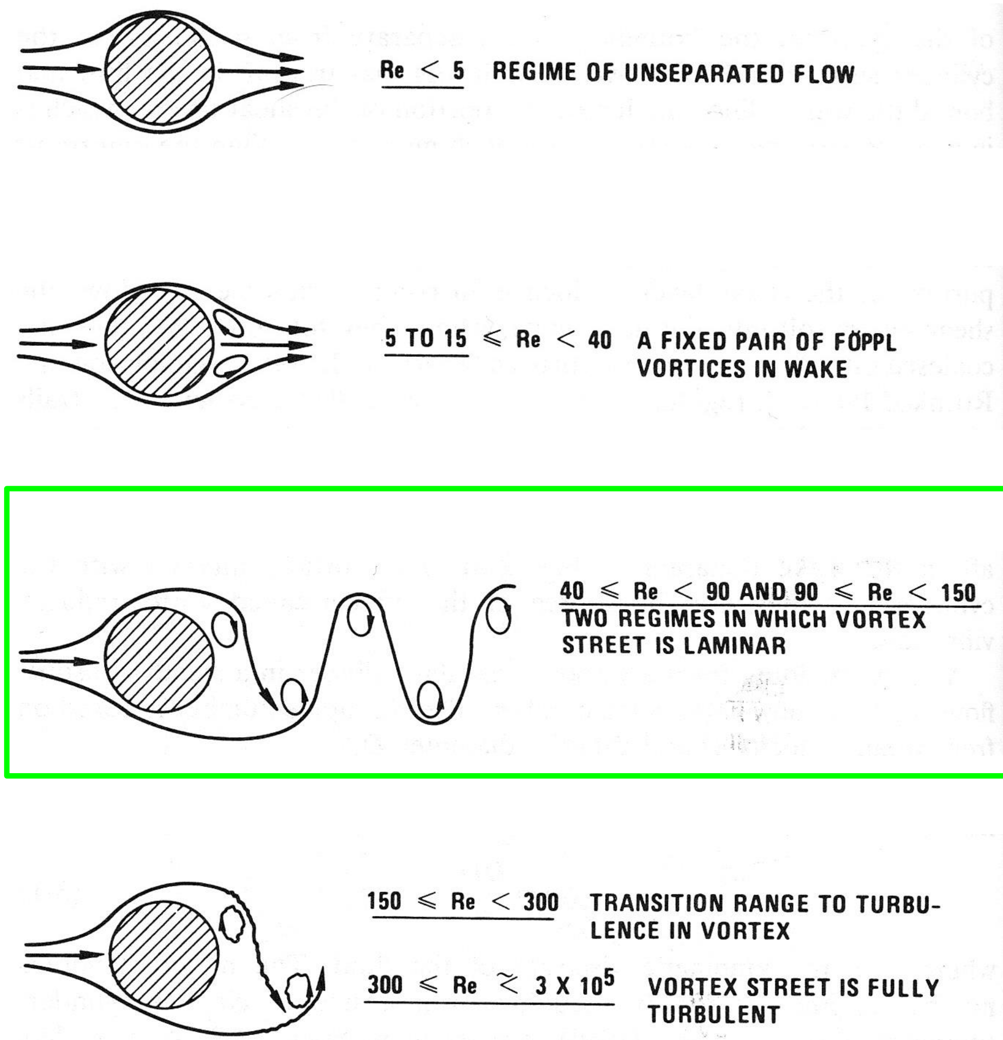


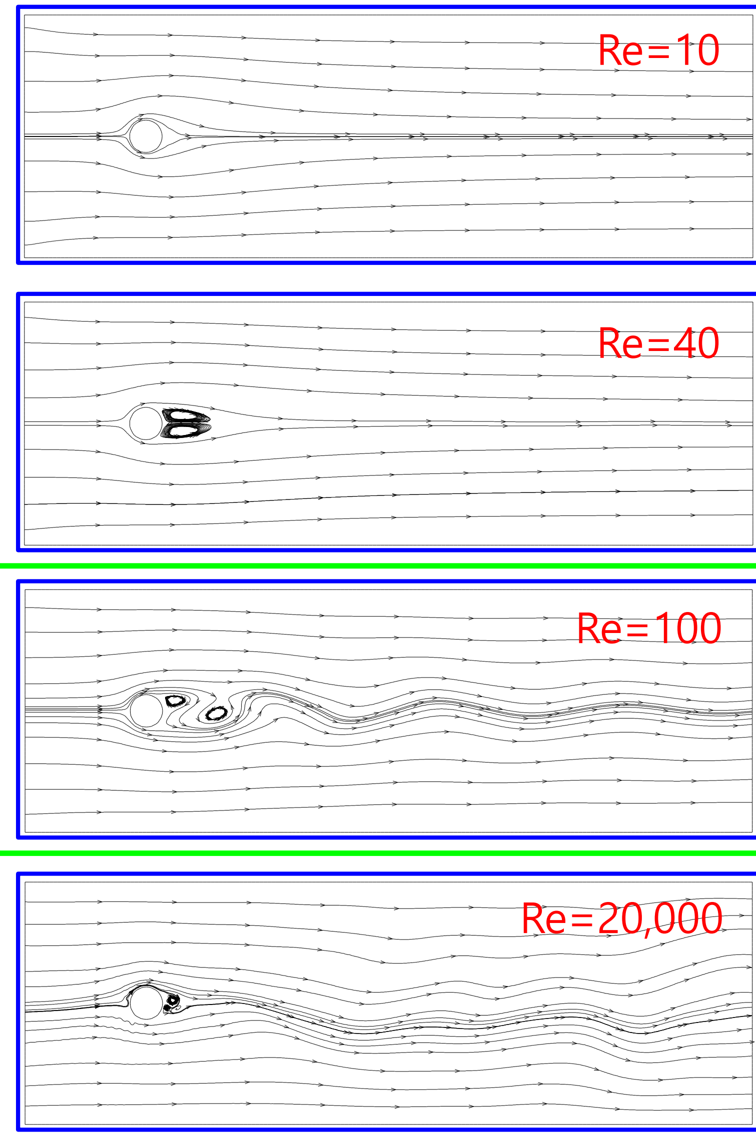
Figure 1.4 Flow behind a cylinder:
(a) $Re < 1$; (b) $5 < Re < 40$;
(c) $100 < Re < 200$; (d) $Re \sim 10^4$;
and (e) $Re \sim 10^6$.

Flow across smooth circular cylinder

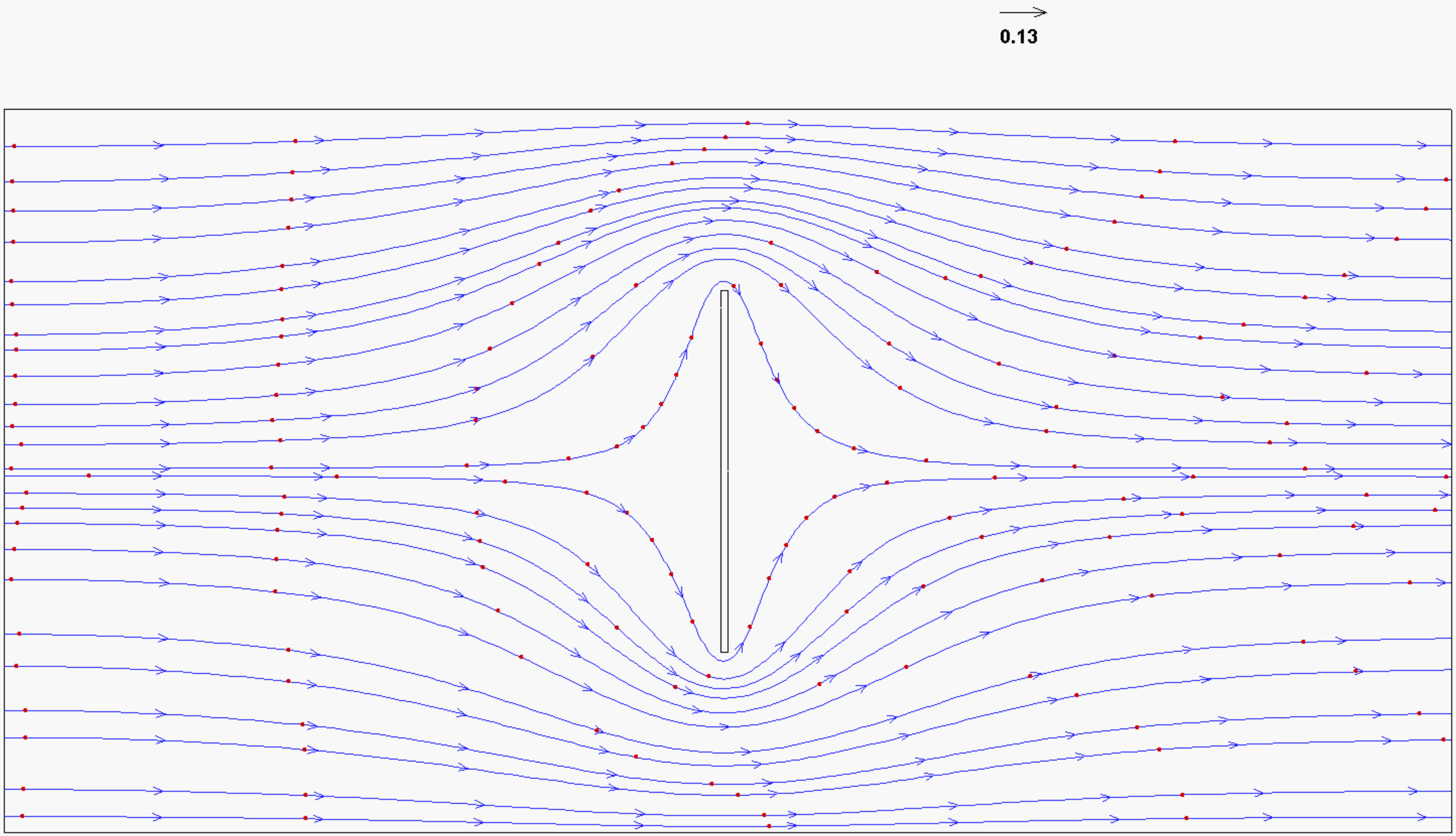
Lienhard (1966)



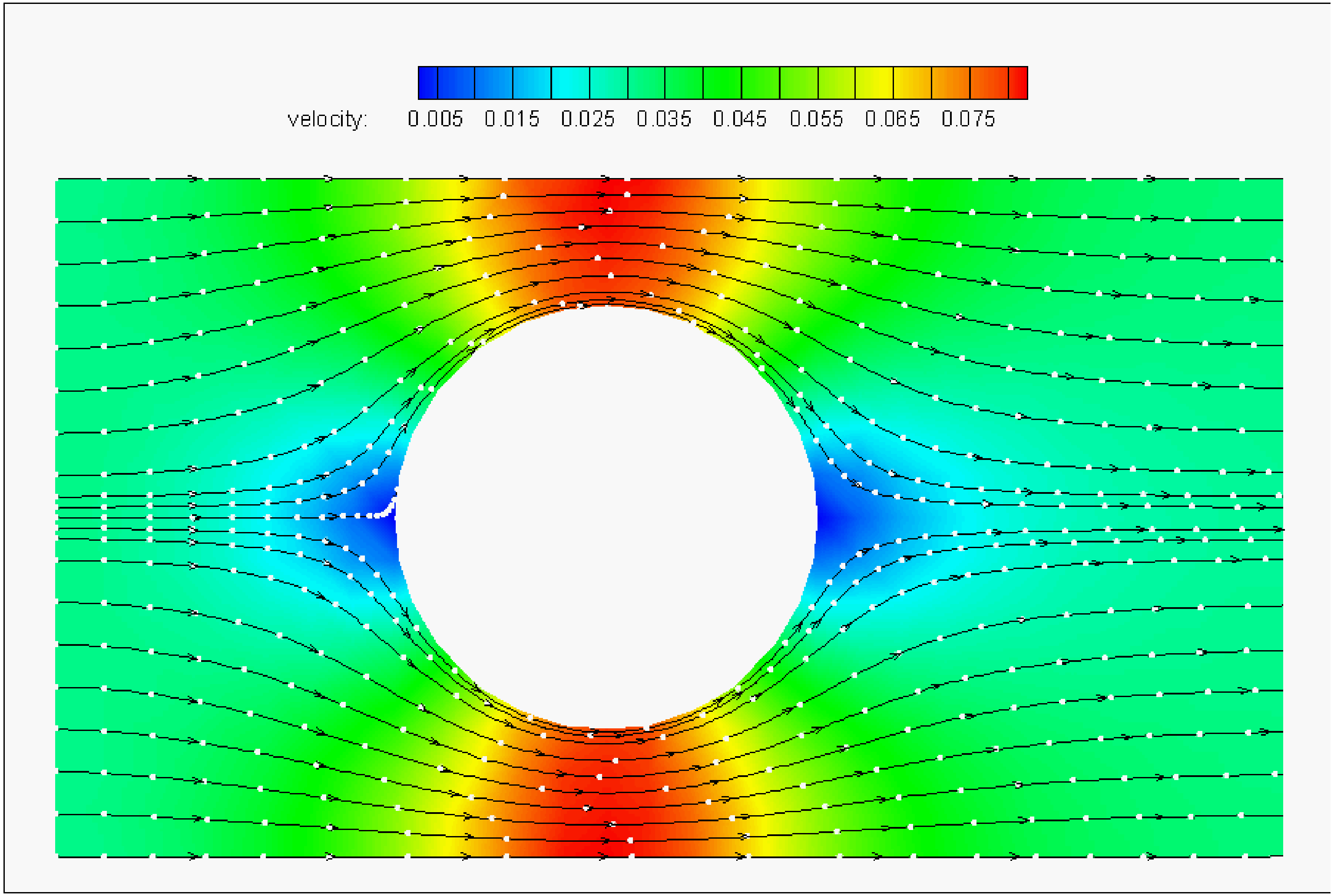
Simulation by HDM2D

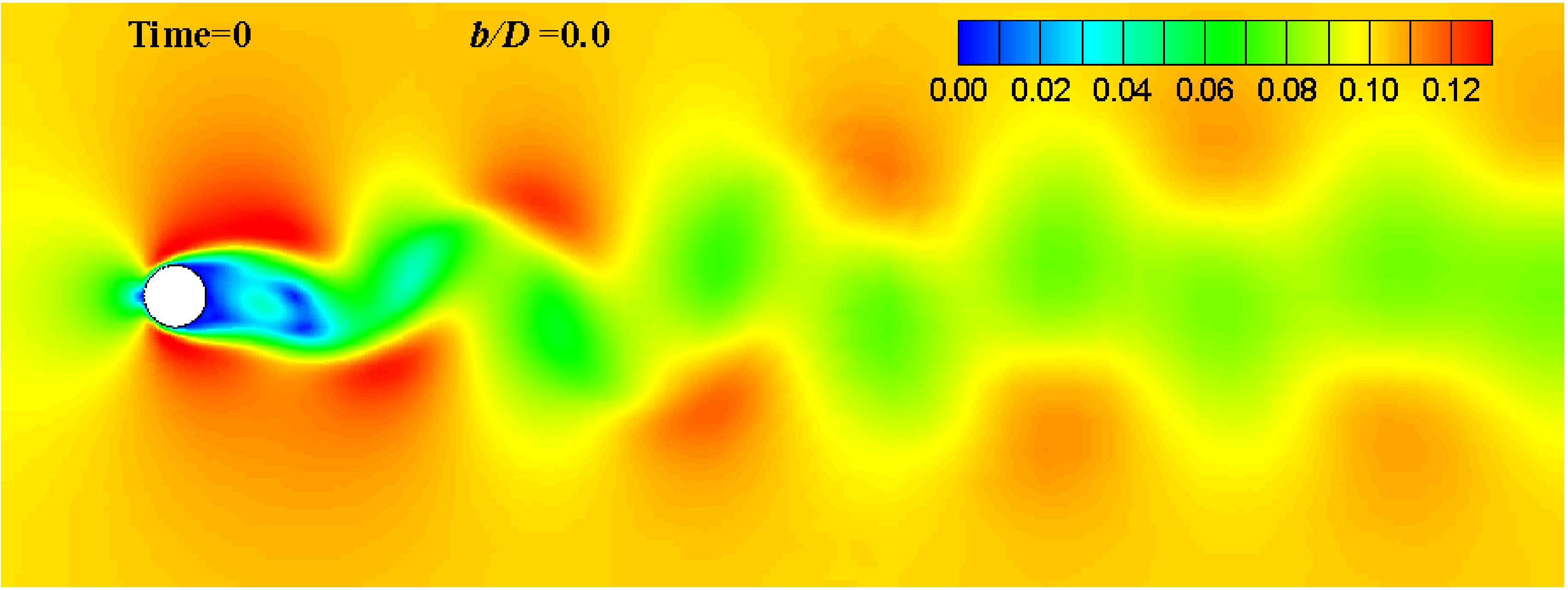


At low Reynolds number



At low velocity

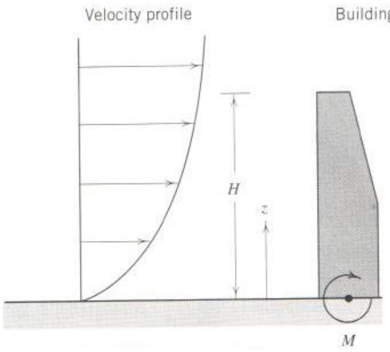



Time=0
b/D =0.0
0.00 0.02 0.04 0.06 0.08 0.10 0.12

Tacoma bridge

14.4 Forces on Structures

- The range of structure is large and there are many ways to analyze the forces acting on these structures.
- We focus on a few key issues,
- the drag forces on the structure as a whole
- integration of the drag force per unit section of structure (moment of forces acting about the base)
- the potential for lift forces to act on a structure
- the concept of decomposing the structure into subunits to calculate the forces
- effect of unsteady flow on the structural forces.

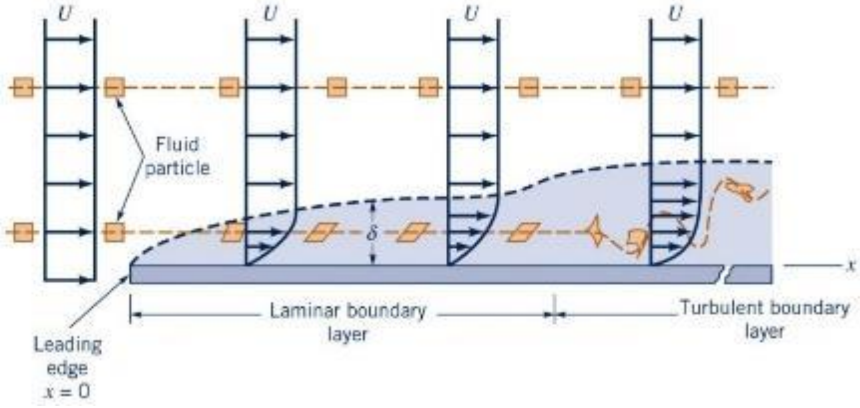


14.4 Forces on Structures

- Drag coefficients for buildings are provided from Hoerner (1965).

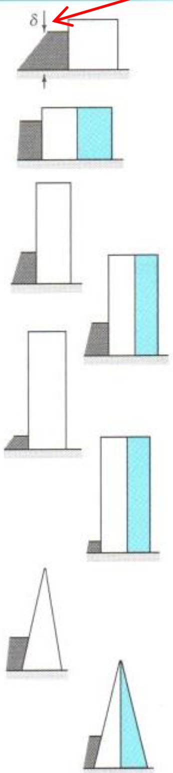
- These are experimental results set on the ground in a boundary layer, where generally the building is significantly taller than the boundary layer thickness.

- Wind speed is typical of atmospheric values (14 m/s) far from the surface.



Drag coefficients for simple buildings

Configuration	Height relative to boundary layer thickness	Orientation of building relative to wind stream	Drag coefficient C_D
Cube	> 1	face on	1.05
Cube	> 1	diagonal	0.80
Rectangle - square base	about 2 times larger	face on	1.30
Rectangle - square base	about 2 times larger	diagonal	0.95
Rectangle - square base	≫ 1	face on	1.50
Rectangle - square base	≫ 1	diagonal	1.05
Pyramid - square base	about 2-3 times larger	face on	1.14
Pyramid - square base	about 2-3 times larger	diagonal	0.83
Large sphere (275 ft)	unknown	without ground effect	0.19 [no lift]
		with ground effect	0.30 [C_L= 0.03]



boundary layer thickness

Forces on Structures

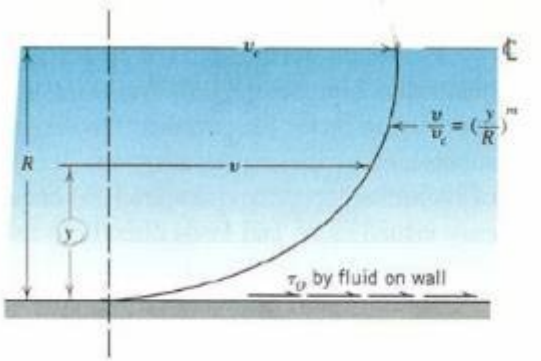
- In general, the velocity of wind varies with height in the atmospheric boundary layer.
- Thus the drag force caused by the mean wind field should be calculated the variation of velocity (Cermak, 1976)

$$\frac{V}{V_0} = \left(\frac{z}{d}\right)^{\frac{1}{n}} \qquad (14.8)$$

- V_0 is the velocity at the edge of the boundary layer whose thickness is δ.
- The coefficient 1/n ~ 0.4 in the building complexes

[Re]

Blasius smooth turbulent flow profile: 1/n =1/7



Forces on Structures

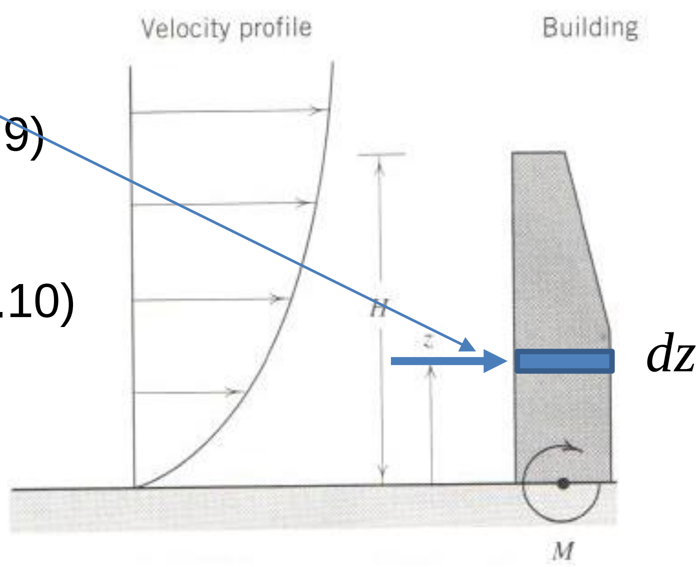
- The force on any element of the building will be known if the drag coefficient is known,

$$dD = \frac{1}{2} dA \rho V^2 C_D = \frac{1}{2} \rho V^2 C_D W dz$$

- Here, *Wdz* is <u>the projected area </u>of the structure at any elevation *z* above the ground.

$$D = \frac{1}{2} \rho \int_0^H \left(V^2 C_D W \right) dz \qquad (14.9)$$

$$M = \frac{1}{2} \rho \int_0^H \left(V^2 C_D W \right) z dz \qquad (14.10)$$

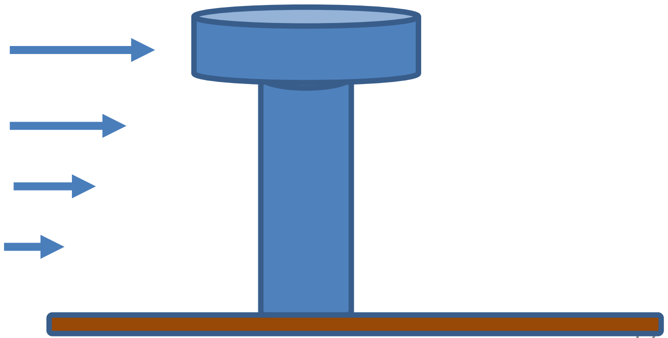


IP 11.6; pp. 521-522

- An architect decides to build a theme building for a fair and selects cylindrical shapes for the structure. The building is to have a restaurant at the top. The geometry follows:
  - Restaurant segment: 50 ft diameter and 20 ft high flat roof
  - Building stem : 25 ft diameter and 200 ft high.
- Compute the drag force on the total structure and the moment of that force about the center of the stem at the ground. The maximum load is expected to occur under gale force winds (say about 42 mph), the boundary layer thickness under theses conditions is about 1,500 ft and the profile coefficient appears to be about 0.3 for the conditions at the fairgrounds by Cermank's standards.

$$V_0 = 42\, mph = 42 \times 1.465 = 61.5\ ft/s$$

$$\delta = 1{,}500\ ft$$



- Decompose two parts, using the wind velocity equations, Eq. 14.8

$$V = 62\left(z/1,500\right)^{0.3} = 6.91z^{0.3}$$

- The Reynolds number is around

$$Re = 6.91\left(220\right)^{0.3} \times 50 \times 0.00238 / 1.93 \times 10^{-5} = 2.1 \times 10^{5}$$

- The Reynolds number of stem based on the velocity at 200 ft and its 25 ft diameter <u>will be less.</u> Based on Fig. 11.10, C_D = 1.0.

$$D = \frac{1}{2}\rho\int_0^H \left(V^2 C_D W\right) dz =$$

$$D_{stem} + D_{restaurant} = 4,264 + 498 = 4,762\,lb$$

$$M = \frac{1}{2}\rho\int_0^H \left(V^2 C_D W\right) z dz$$

$$= N_{stem} + M_{restaurant} = 524,800 + 295,160 = 819,960\ ft - lb$$