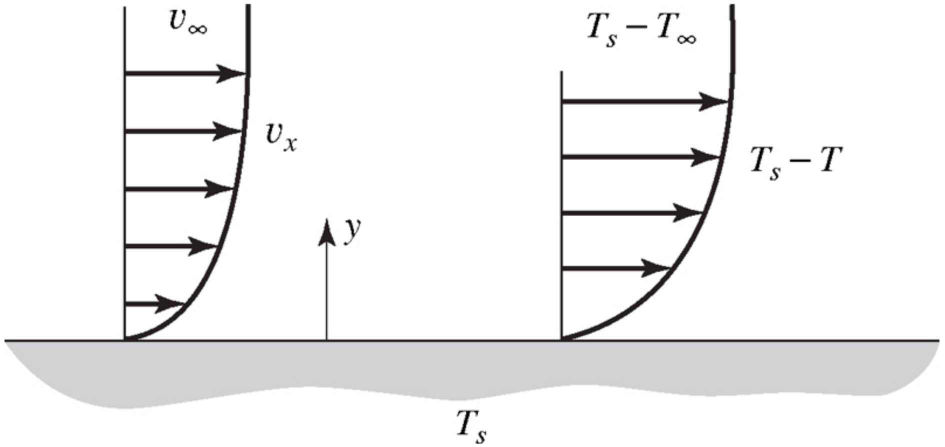
# convective heat transfer




$$\rho \frac{\mathrm{D}\mathbf{v}}{\mathrm{Dt}} = -\nabla \mathrm{P} + \mu \nabla^2 \mathbf{v} + \rho \mathbf{g}$$

$$\rho c_p \frac{DT}{Dt} = -k\nabla^2 T + \dot{q} + \boldsymbol{\tau} : \nabla \mathbf{v}$$

momentum diffusivity : $\nu \equiv \dfrac{\mu}{\rho}$

themal diffusivity : $\alpha \equiv \dfrac{k}{\rho c_p}$

$$\mathrm{Pr} \equiv \frac{\nu}{\alpha} = \frac{\mu c_p}{k}$$

$$h\left(T_s - T_\infty\right) = -k \frac{\partial}{\partial y}\left(T - T_s\right)\Big|_{y=0}$$

$$\frac{hL}{k} = \frac{\partial\left(T_s - T\right)/\partial y\big|_{y=0}}{\left(T_s - T_\infty\right)/L}$$

$$\mathrm{Nu} = \frac{hL}{k} \equiv \frac{\textit{conductive thermal resistance}}{\textit{convective thermal resistance}}$$

dimensional analysis

forced convection

in a closed conduit

| Variable | Symbol | Dimensions |
|---|---|---|
| Tube diameter | $D$ | $L$ |
| Fluid density | $\rho$ | $M/L^3$ |
| Fluid viscosity | $\mu$ | $M/Lt$ |
| Fluid heat capacity | $c_p$ | $Q/MT$ |
| Fluid thermal conductivity | $k$ | $Q/tLT$ |
| Velocity | $v$ | $L/t$ |
| Heat-transfer coefficient | $h$ | $Q/tL^2T$ |

\# of variables; 7
\# of dimensions; 5
\# of ranks; 4
\# of independent dimensionless groups; 3

$$\pi_1 = D^a k^b \mu^c v^d \rho$$

$$\pi_2 = D^e k^f \mu^g v^h c_p$$

$$\pi_3 = D^i k^j \mu^k v^l h$$

$$\pi_1 = \frac{Dv\rho}{\mu} = \text{Re} \qquad \pi_2 = \frac{\mu c_p}{k} = \text{Pr} \qquad \pi_3 = \frac{hD}{k} = \text{Nu}$$

$$\boxed{\text{Nu} = f_1(\text{Re},\ \text{Pr})}$$

## natural convection

| Variable | Symbol | Dimensions |
|---|---|---|
| Significant length | $L$ | $L$ |
| Fluid density | $\rho$ | $M/L^3$ |
| Fluid viscosity | $\mu$ | $M/Lt$ |
| Fluid heat capacity | $c_p$ | $Q/MT$ |
| Fluid thermal conductivity | $k$ | $Q/LtT$ |
| Fluid coefficient of thermal expansion | $\beta$ | $1/T$ |
| Gravitational acceleration | $g$ | $L/t^2$ |
| Temperature difference | $\Delta T$ | $T$ |
| Heat-transfer coefficient | $h$ | $Q/L^2tT$ |

$$\rho = \rho_0 \left(1 - \beta \Delta T\right)$$

$$F_{\text{buoyant}} = \left(\rho_0 - \rho\right) g$$

$$F_{\text{buoyant}} = \beta g \rho_0 \Delta T$$

# of variables; 9
# of dimensions; 5
# of ranks; 5
# of independent dimensionless groups; 4

$$\pi_1 = L^a \mu^b k^c \beta^d c_p$$

$$\pi_2 = L^f \mu^g k^h g^j \rho$$

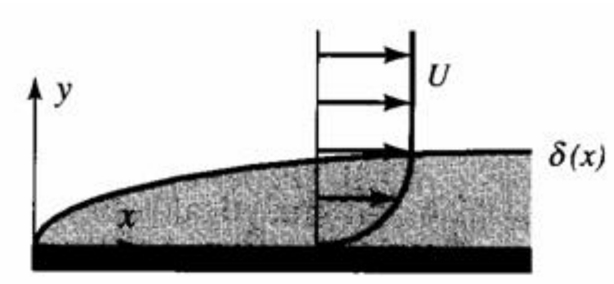
$$\pi_3 = L^k \mu^l k^m \beta^n g^o \Delta T$$

$$\pi_4 = L^p \mu^q k^r \beta^s g^t h$$

$$\pi_1 = \frac{\mu c_p}{k} = \text{Pr} \qquad \pi_2 = \frac{L^3 g \rho^2}{\mu^2} \qquad \pi_3 = \beta \Delta T \qquad \pi_4 = \frac{hL}{k} = \text{Nu}$$

$$\text{Gr} \equiv \frac{\beta g \rho^2 L^3 \Delta T}{\mu^2}$$

$$\boxed{\text{Nu} = f_3\left(\text{Gr}, \text{Pr}\right)}$$




# Laminar boundary layer

No matter how turbulent the flow is far from the surface

Negligible viscous effect outside the boundary layer

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0$$

$$\rho\left(u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2}\right)$$

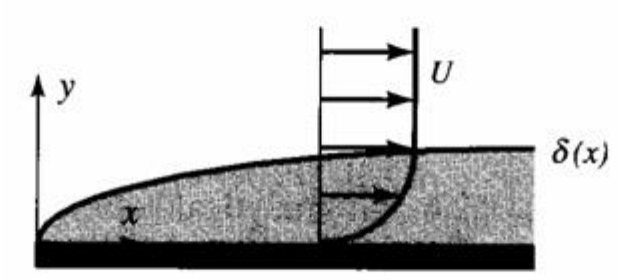
$$u_y = 0; \quad \frac{\partial p}{\partial y} = 0 \quad \text{within the boundary layer}$$

$$\rho\left(u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y}\right) = -\cancel{\frac{\partial p}{\partial x}} + \mu\left(\cancel{\frac{\partial^2 u_x}{\partial x^2}} + \frac{\partial^2 u_x}{\partial y^2}\right)$$

$$\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}=0 \qquad \rho\left(u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}\right)=\mu\frac{\partial^2 u_x}{\partial y^2}$$

$$\boxed{\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=\nu\frac{\partial^3\psi}{\partial y^3}}$$

## Boundary conditions




$$u_x=u_y=0 \quad \text{on} \quad y=0, \text{ for } x>0$$

$$u_x=U \quad \text{for} \quad x<0$$

$$u_x\to U \quad \text{for} \quad y\to\infty$$

$$\frac{\partial\psi}{\partial y}=-\frac{\partial\psi}{\partial x}=0 \quad \text{on} \quad y=0, \text{ for } x>0$$

$$\frac{\partial\psi}{\partial y}\to U \quad \text{for} \quad x<0, \text{ and } y\to\infty$$

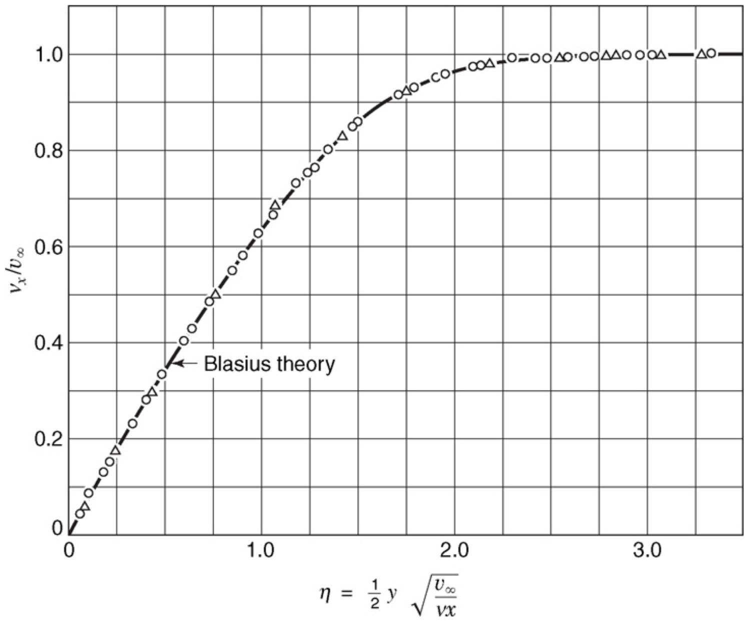
## Blasius' similarity transformation

$$f(\eta)\equiv\frac{\psi}{(U\nu x)^{1/2}} \qquad \eta\equiv\frac{y}{2}\left(\frac{U}{\nu x}\right)^{1/2}$$

$$\boxed{f'''+ff''=0}$$

$$f=f'=0 \quad \text{at} \quad \eta=0$$

$$f'\to 2 \quad \text{as} \quad \eta\to\infty$$




$U_x$ becomes nearly $U$
along the boundary layer $y = \delta(x)$

$$\delta\left(\frac{U}{\nu x}\right)^{1/2} = 5$$

Boundary layer thickness

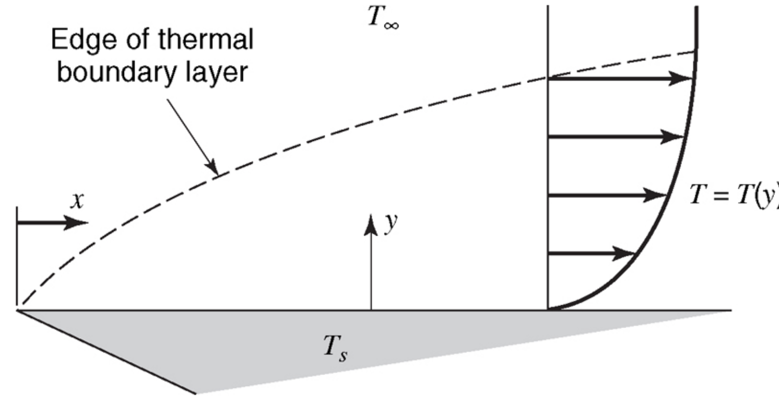
Friction coeff. $$C_f \equiv \frac{-\mu(\partial u_x/\partial y)_{y=0}}{\frac{1}{2}\rho U^2} \equiv 0.664\left(\frac{\nu}{xU}\right)^{1/2} = \frac{0.664}{(\mathrm{Re}_x)^{1/2}}$$

Shear force $$F_s = W\int_0^L -\mu\left(\frac{\partial u_x}{\partial y}\right)_{y=0} dx \qquad \frac{F_s/LW}{\frac{1}{2}\rho U^2} \equiv \bar{C}_f = \frac{1.328}{(UL/\nu)^{1/2}} = \frac{1.328}{(\mathrm{Re}_L)^{1/2}}$$

Local Reynolds number $$\mathrm{Re}_x \equiv \frac{xU\rho}{\mu}$$

$$\left.\frac{\partial v_x}{\partial y}\right|_{y=0} = v_\infty\left[\frac{0.332}{x}\mathrm{Re}_x^{1/2}\right]$$

laminar boundary layer; Blasius




$$\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \nu \frac{\partial^2 v_x}{\partial y^2}$$

$$\frac{v_x}{v_\infty} = \frac{v_y}{v_\infty} = 0 \quad \text{at } y = 0$$

$$\frac{v_x}{v_\infty} = 1 \quad \text{at } y = \infty$$

$$\left. \frac{\partial v_x}{\partial y} \right|_{y=0} = v_\infty \left[ \frac{0.332}{x} \mathrm{Re}_x^{1/2} \right]$$

$$v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$$

for Pr=1

$$\frac{T - T_s}{T_\infty - T_s} = 0 \quad \text{at } y = 0$$

$$\frac{T - T_s}{T_\infty - T_s} = 1 \quad \text{at } y = \infty$$

$$\left. \frac{\partial T}{\partial y} \right|_{y=0} = \left( T_\infty - T_s \right) \left[ \frac{0.332}{x} \mathrm{Re}_x^{1/2} \right]$$

$$\frac{q_y}{A} = h_x \left( T_s - T_\infty \right) = -k \left. \frac{\partial T}{\partial y} \right|_{y=0}$$

$$\frac{h_x x}{k} = \mathrm{Nu}_x = 0.332 \, \mathrm{Re}_x^{1/2}$$

effect of Pr; Pohlhausen $$\frac{\delta}{\delta_t} = \mathrm{Pr}^{1/3}$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = \left(T_\infty - T_s\right)\left[\frac{0.332}{x}\mathrm{Re}_x^{1/2}\right]$$

$$\frac{h_x x}{k} = \mathrm{Nu}_x = 0.332\,\mathrm{Re}_x^{1/2}$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = \left(T_\infty - T_s\right)\left[\frac{0.332}{x}\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}\right]$$

$$\frac{h_x x}{k} = \mathrm{Nu}_x = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$$

$$\frac{hL}{k} = \mathrm{Nu}_L = 0.664\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3}$$

fluid properties are evaluated at the film temperature

$$T_f = \frac{T_s + T_\infty}{2}$$

; laminar boundary layer over a flat plate

(von Karman) $$\mathrm{Nu}_x = 0.36\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$$

energy and momentum transfer analogy

Reynolds analogy ; for Pr=1

$$h\left(T_s - T_\infty\right) = -k\frac{\partial}{\partial y}\left(T - T_s\right)\Big|_{y=0}$$

$$\frac{d}{dy}\frac{v_x}{v_\infty}\Big|_{y=0} = \frac{d}{dy}\left(\frac{T - T_s}{T_\infty - T_s}\right)\Big|_{y=0}$$

$$h = \frac{\mu c_p}{v_\infty}\frac{dv_x}{dy}\Big|_{y=0}$$

coefficient of skin friction

$$C_f \cong \frac{\tau_0}{\rho v_\infty^2 / 2} = \frac{2\mu}{\rho v_\infty^2}\frac{dv_x}{dy}\Big|_{y=0}$$

$$h = \frac{C_f}{2}\left(\rho v_\infty c_p\right)$$

$$\boxed{\frac{h}{\rho v_\infty c_p} \equiv \mathrm{St} = \frac{C_f}{2}}$$

Colburn analogy ; 0.5<Pr<50, no form drag

$$\boxed{\mathrm{St}\,\mathrm{Pr}^{2/3} = \frac{C_f}{2}}$$

$$j_H = \frac{C_f}{2} \qquad j_H = \mathrm{St}\,\mathrm{Pr}^{2/3}$$

Colburn j-factor

## turbulent flow

$$\bar{v}_x\Big|_{y\pm L} - \bar{v}_x\Big|_y = \pm L\frac{d\bar{v}_x}{dy} \qquad v'_x = \pm L\frac{d\bar{v}_x}{dy} \qquad \tau_{yx}^{turb} = -\overline{\rho v'_x v'_y}$$

$$T\Big|_{y\pm L} - T\Big|_y = \pm L\left.\frac{dt}{dy}\right|_y \qquad T' = \pm L\frac{d\bar{T}}{dy} \qquad \left.\frac{q_y}{A}\right|_{\mathrm{turb}} = \rho c_p\overline{\left(v'_y T'\right)}$$

Prandtl analogy $\quad \mathrm{St} = \dfrac{C_f/2}{1+5\sqrt{C_f/2}\left(\mathrm{Pr}-1\right)}$

von Karman analogy $\quad \mathrm{St} = \dfrac{C_f/2}{1+5\sqrt{C_f/2}\left\{\mathrm{Pr}-1+\ln\left[1+\frac{5}{6}\left(\mathrm{Pr}-1\right)\right]\right\}}$

## convective heat transfer correlations

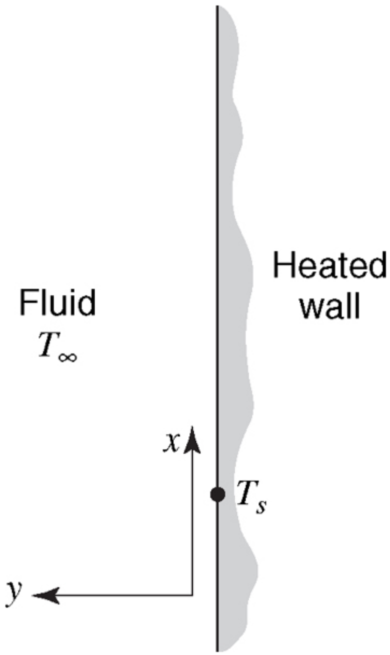
natural convection;
- vertical plates, vertical cylinders,
- horizontal plates, horizontal cylinders,
- spheres, rectangular enclosures

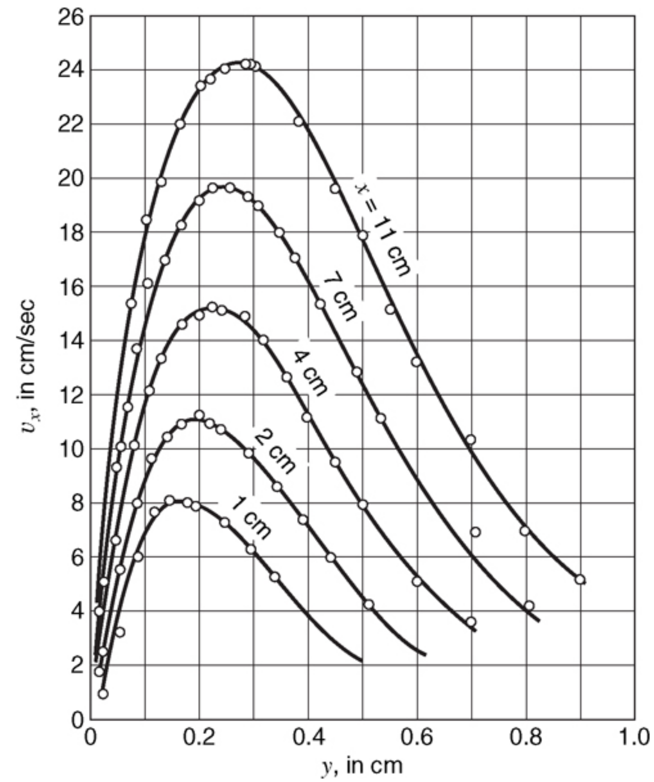
forced convection for internal flow;
- laminar flow, turbulent flow

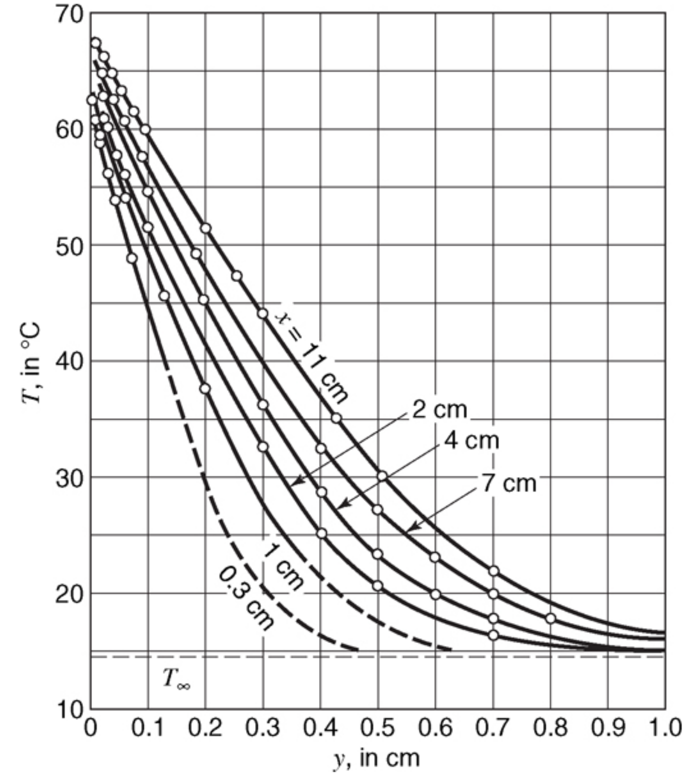
forced convection for external flow;
- flow parallel to plane surfaces, cylinders in crossflow,
- single spheres, tube banks in crossflow,

special considerations;
- whether to evaluate fluid properties at bulk or film temperature
- what significant length is used
- what is the allowable Pr, Re range for a given set of data

# natural convection; vertical plates




12.5cm high
Ts=65C
Tinf=15C

$$\mathrm{Nu}=\left\{0.825+\frac{0.387\mathrm{Ra}_L^{1/6}}{\left[1+\left(0.492/\mathrm{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2$$

$$Ra=\mathrm{Pr}\cdot Gr \qquad \mathrm{Pr}=\frac{\mu c_p}{k} \qquad \mathrm{Gr}\equiv\frac{\beta g\rho^2L^3\Delta T}{\mu^2}$$

forced convection for internal flow

laminar flow; Graetz solution

$$v_x = 2v_{\text{avg}}\left[1-\left(\frac{r}{R}\right)^2\right] \qquad v_x \frac{\partial T}{\partial x} = \alpha\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right]$$

$$2v_{\text{avg}}\left[1-\left(\frac{r}{R}\right)^2\right]\frac{\partial T}{\partial x} = \alpha\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right]$$

$$T = T_e \quad \text{at} \;\; x=0 \quad \text{for} \quad 0 \le r \ge R$$

$$T = T_s \quad \text{at} \;\; x>0, \quad r=R$$

$$\frac{\partial T}{\partial r} = 0 \quad \text{at} \quad x>0, \quad r=0$$

$$\frac{T-T_e}{T_s-T_e} = \sum_{n=0}^{\infty} c_n f\left(\frac{r}{R}\right)\exp\left[-\beta_n^2 \frac{\alpha}{Rv_{\text{avg}}}\frac{x}{R}\right]$$

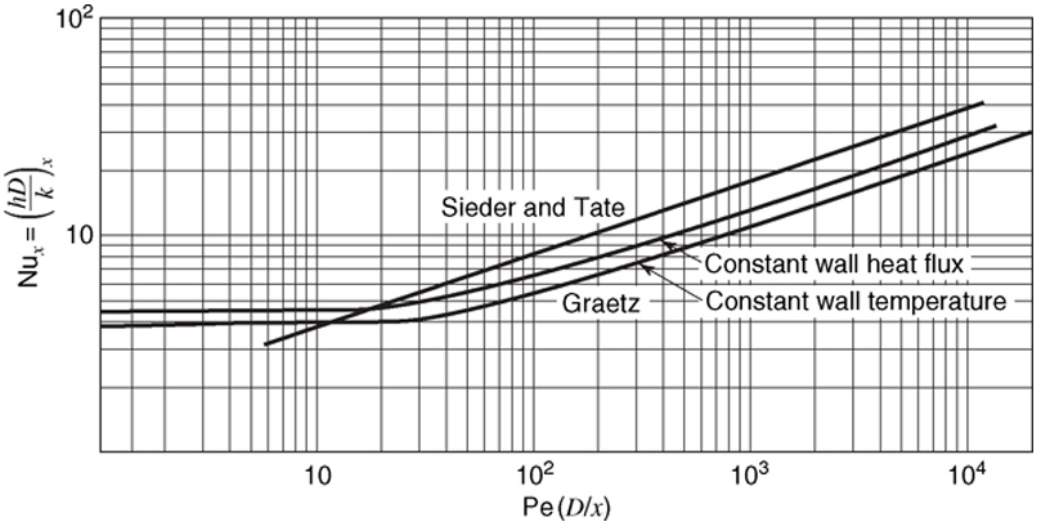
$$\frac{4}{\text{Re}\,\text{Pr}\,D/x} = \frac{4x/D}{\text{Pe}}$$

$$\boxed{\text{Gz} \equiv \frac{\pi}{4}\frac{D}{x}\text{Pe}}$$

$$\frac{T - T_e}{T_s - T_e} = \sum_{n=0}^{\infty} c_n f\left(\frac{r}{R}\right) \exp\left[-\beta_n^2 \frac{\alpha}{R v_{\text{avg}}} \frac{x}{R}\right]$$

$$\text{Nu}_x = 4.364 \quad \text{for } q/A_{\text{wall}} = \text{constant}$$

$$\text{Nu}_x = 3.658 \quad \text{for } T_{\text{wall}} = \text{constant}$$




Sieder and Tate

$$\text{Nu}_D = 1.86\left(\text{Pe}\frac{D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

turbulent flow; Dittus and Boelter

$$\mathrm{Nu}_D = 0.023\mathrm{Re}_D^{0.8}\,\mathrm{Pr}^n$$

1. n=0.4 if the fluid is being heated, n=3 if the fluid is being cooled
2. all fluid properties are evaluated at the arithmetic-mean bulk temperature
3. Re>$10^4$
4. 0.7<Pr<100
5. L/D>60

Colburn; $\mathrm{St}=0.023\mathrm{Re}_D^{-0.2}\,\mathrm{Pr}^{-2/3}$

Sieder and Tate ; $\mathrm{St}=0.023\mathrm{Re}_D^{-0.2}\,\mathrm{Pr}^{-2/3}\left(\dfrac{\mu_b}{\mu_w}\right)^{0.14}$

## forced convection for external flow

## flow parallel to plane surfaces

laminar;

$$\mathrm{Nu}_x = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$$

$$\mathrm{Nu}_L = 0.664\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3}$$

turbulent;

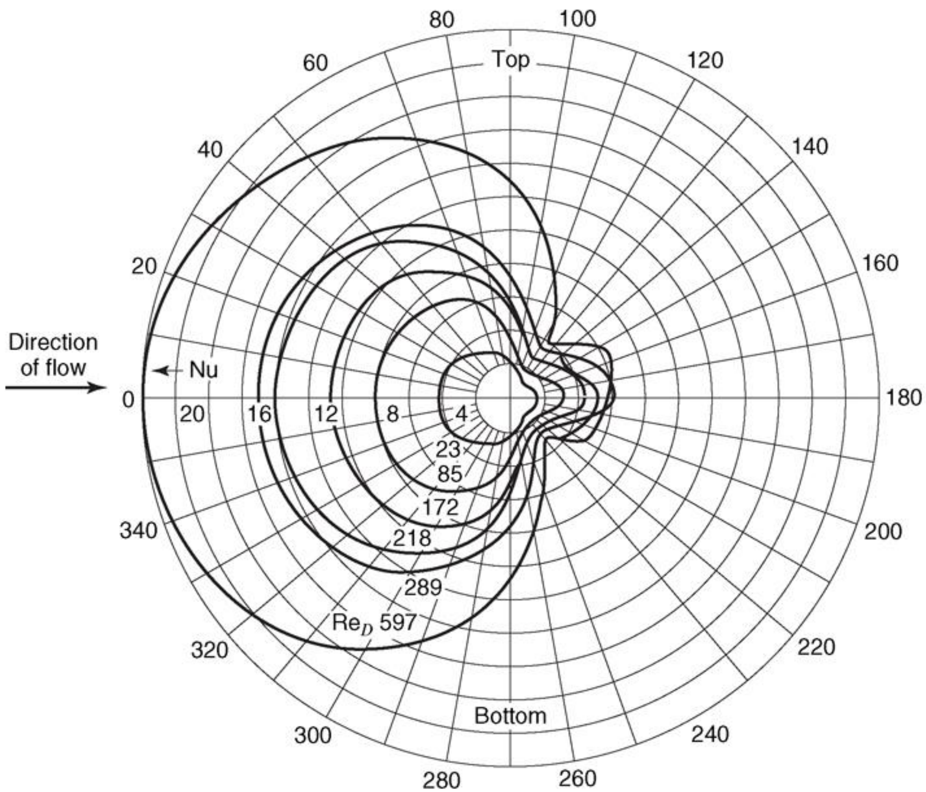
$$\mathrm{St}_x\,\mathrm{Pr}^{2/3} = \frac{C_{fx}}{2}$$ (Colburn analogy)

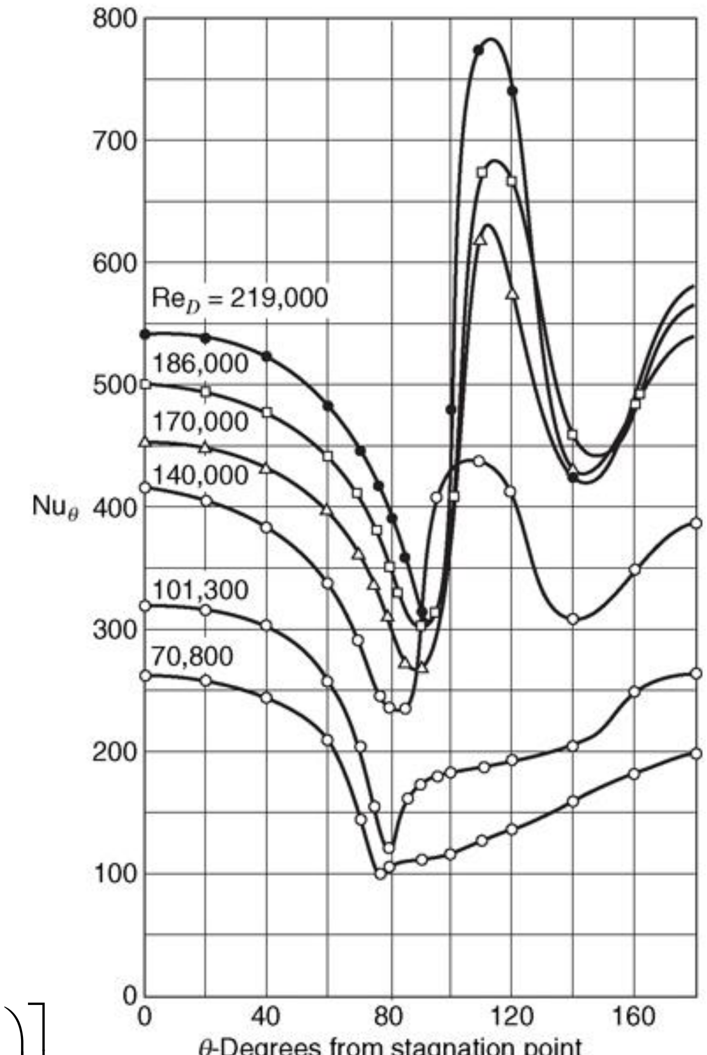
$$C_{fx} = \frac{0.0576}{\mathrm{Re}_x^{\;1/5}}$$ (von Karman)

$$\mathrm{Nu}_x = 0.0288\,\mathrm{Re}_x^{4/5}\,\mathrm{Pr}^{1/3}$$

$$\mathrm{Nu}_L = 0.036\,\mathrm{Re}_L^{4/5}\,\mathrm{Pr}^{1/3}$$

# cylinders in crossflow




$$Nu_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+\left(0.4/Pr\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282{,}000}\right)\right]$$