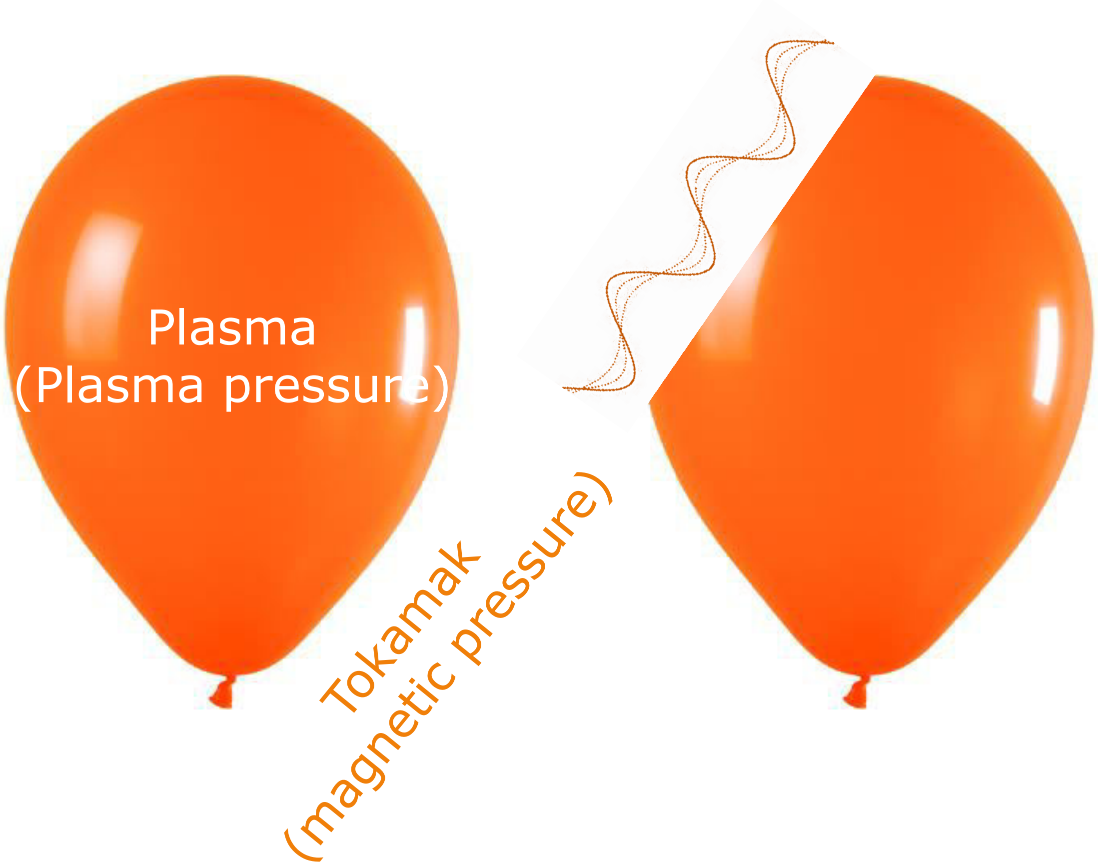
Introduction to Nuclear Fusion

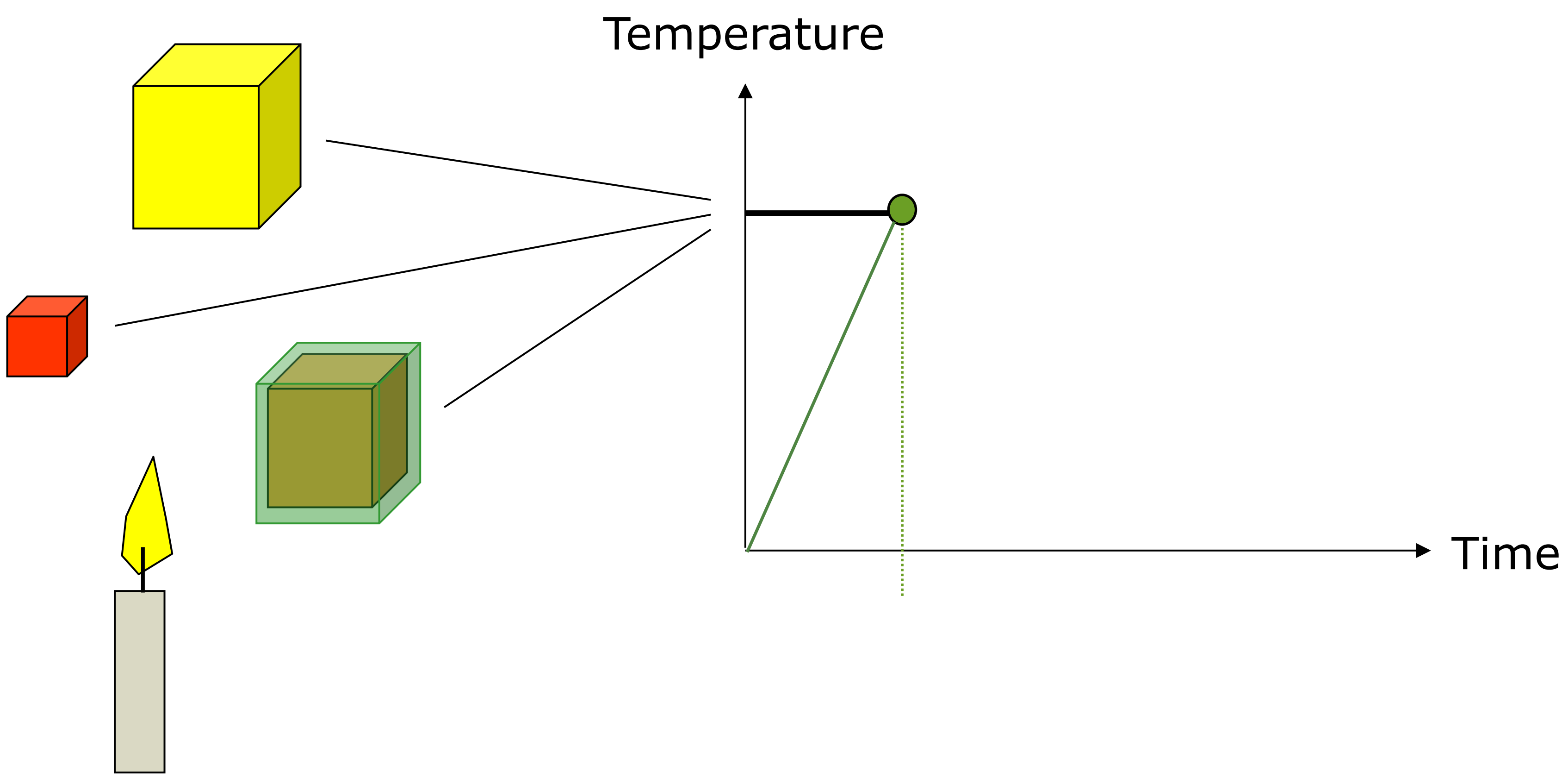
Prof. Dr. Yong-Su Na

Plasma Equilibrium, Stability and Transport

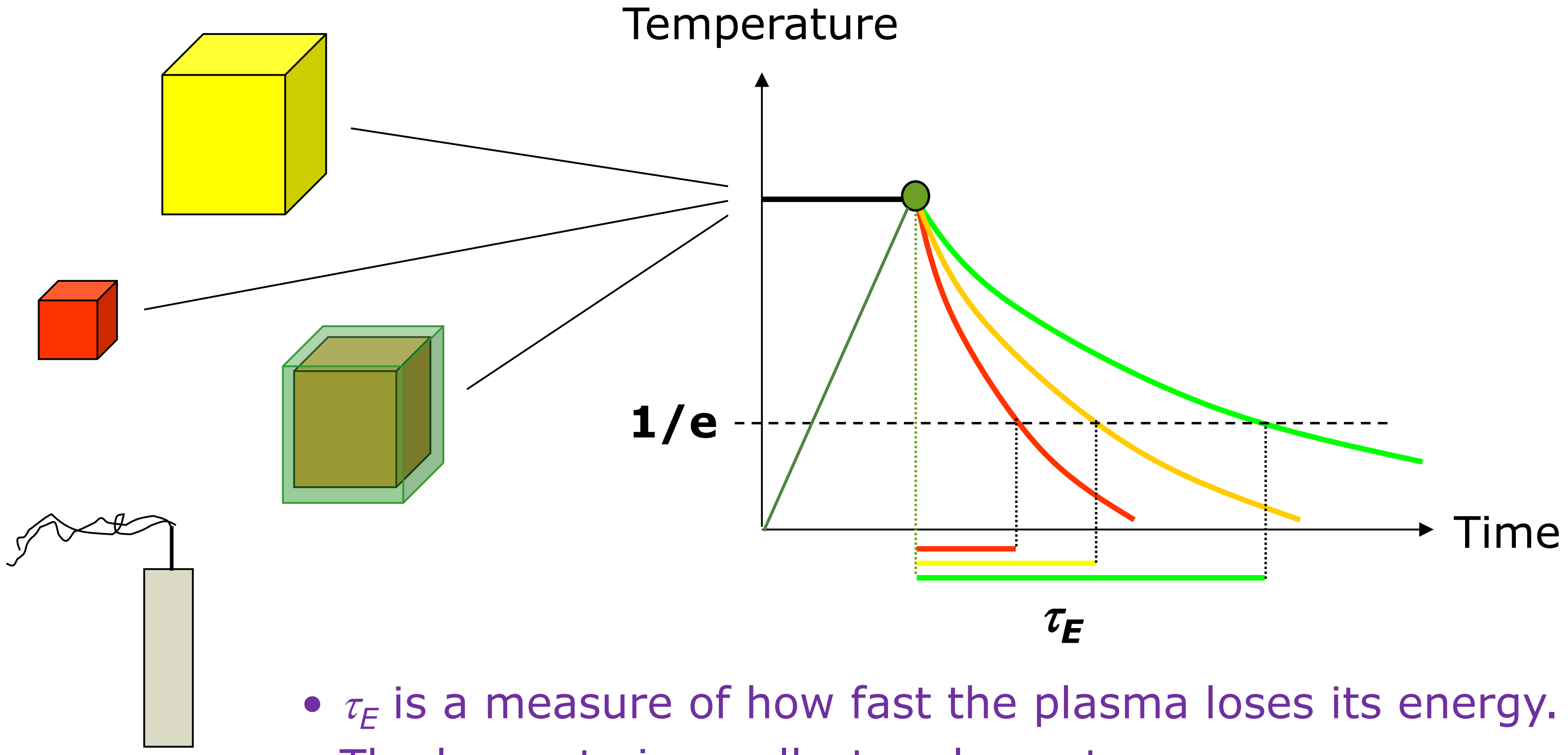


Plasma transport in a Tokamak

Energy Confinement Time



Energy Confinement Time



- τ_E is a measure of how fast the plasma loses its energy.
- The loss rate is smallest, τ_E largest
  if the fusion plasma is big and well insulated.

Tokamak Transport

- **Transport Coefficients**

$\Gamma = -D\nabla n$: Fick's law

$q = -\kappa\nabla T$: Fourier's law

$$D = \frac{(\Delta x)^2}{2\tau}$$: diffusion coefficient (m²/s)

$$D \sim v_{th}^2 \tau \sim \frac{\lambda_m^2}{\tau}$$

Thermal diffusivity

$$\chi \equiv \frac{\kappa}{n} \approx D \approx \frac{(\Delta x)^2}{\tau} \approx \frac{a^2}{\tau_E} \quad \rightarrow \quad \tau_E \approx \frac{a^2}{\chi}$$

- Particle transport in fully ionised plasmas with magnetic field

$$D_\perp = \frac{\eta_\perp n \sum kT}{B^2}$$

Tokamak Transport

- **Classical Transport**

- Classical thermal conductivity (expectation): $\chi_i \sim 40\chi_e$
- Typical numbers expected: ~10^{-4} m^2/s
- Experimentally found: ~1 m^2/s, $\chi_i \sim \chi_e$

Bohm diffusion (1946): $D_{\perp} = \frac{1}{16}\frac{kT_e}{eB}$

David Bohm (1917-1992)

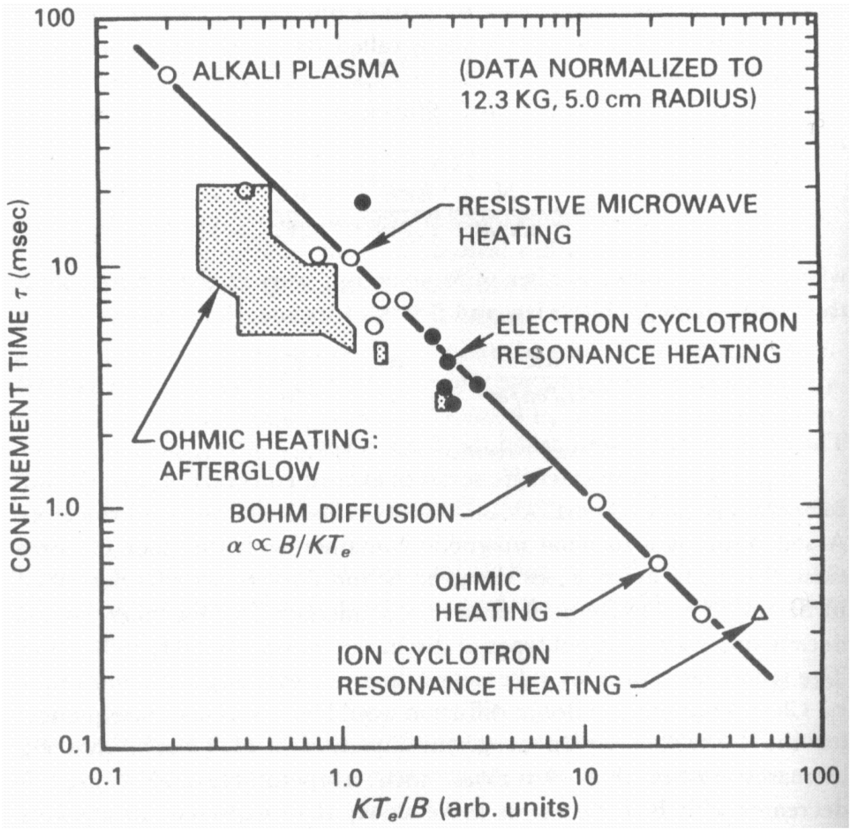
WIKIPEDIA
The Free Encyclopedia

Aharanov–Bohm effect

Tokamak Transport

- **Classical Transport**

Bohm diffusion: $$D_\perp = \frac{1}{16}\frac{kT_e}{eB}$$



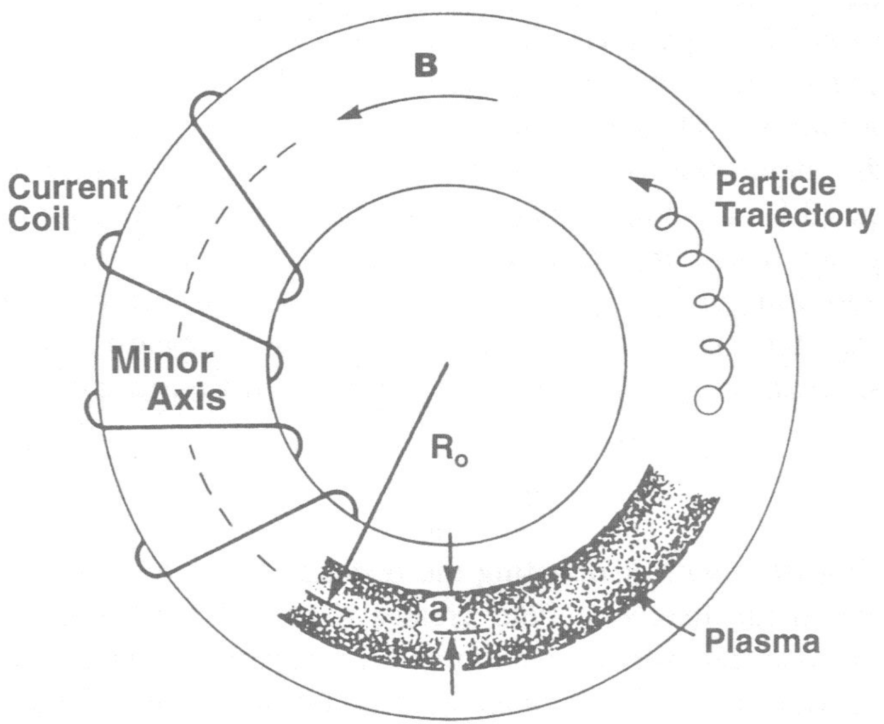
τ_E in various types of discharges in the Model C Stellarator

F. F. Chen, "Introduction to Plasma Physics and Controlled Fusion" (2006)

Tokamak Transport

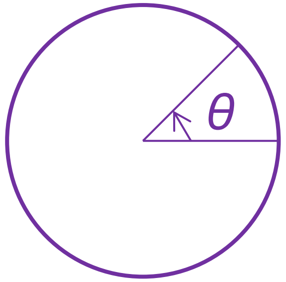
- **Neoclassical Transport**

  - Major changes arise from toroidal effects characterised by inverse aspect ratio, $\varepsilon = a/R_0$



Tokamak Transport

- **Particle Trapping**

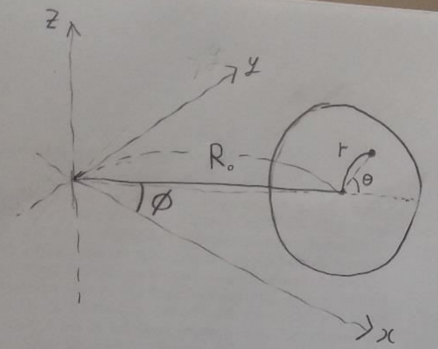


$$\nabla \cdot \vec{B} = 0$$

$$\Rightarrow \quad \frac{1}{1+\varepsilon\cos\theta}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r(1+\varepsilon\cos\theta)B_r\right]+\frac{1}{r}\frac{\partial}{\partial\theta}\left[(1+\varepsilon\cos\theta)B_\theta\right]+\frac{1}{rR_0}\frac{\partial(rB_\phi)}{\partial\phi}\right\}=0$$

$$\Rightarrow \quad B_\theta(r,\theta)=\frac{B_\theta^0(\theta=0)}{1+\varepsilon\cos\theta}$$

$$\left|B(r,\theta)\right|=\left|B_\theta(r,\theta)\hat{\theta}+B_\phi(r,\theta)\hat{\phi}\right|=\frac{B_0}{1+\varepsilon\cos\theta}$$



가정; $\frac{\partial}{\partial \phi} = 0$, $B_r = 0$, $J_r = 0$

$$\vec{r} = (R_o + r\cos\theta)\cos\phi\,\hat{x} + (R_o + r\cos\theta)\sin\phi\,\hat{y} + r\sin\theta\,\hat{z}$$

$$\frac{\partial \vec{r}}{\partial r} = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} + \sin\theta\,\hat{z}$$

$$\frac{\partial \vec{r}}{\partial \theta} = -r\sin\theta\cos\phi\,\hat{x} - r\sin\theta\sin\phi\,\hat{y} + r\cos\theta\,\hat{z}$$

$$\frac{\partial \vec{r}}{\partial \phi} = -(R_o + r\cos\theta)\sin\phi\,\hat{x} + (R_o + r\cos\theta)\cos\phi\,\hat{y}$$

$$\Rightarrow \hat{r} = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} + \sin\theta\,\hat{z}, \quad h_r = \left|\frac{\partial \vec{r}}{\partial r}\right| = 1$$

$$\hat{\theta} = -\sin\theta\cos\phi\,\hat{x} - \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}, \quad h_\theta = \left|\frac{\partial \vec{r}}{\partial \theta}\right| = r$$

$$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}, \quad h_\phi = \left|\frac{\partial \vec{r}}{\partial \phi}\right| = R + r\cos\theta$$

$$\Rightarrow \nabla\cdot\vec{A} = \frac{1}{h_r h_\theta h_\phi}\left[\frac{\partial}{\partial r}(h_\theta h_\phi A_r) + \frac{\partial}{\partial \theta}(h_r h_\phi A_\theta) + \frac{\partial}{\partial \phi}(h_r h_\theta A_\phi)\right]$$

$$\Rightarrow \nabla\cdot\vec{B} = \frac{1}{r(R_o + r\cos\theta)}\left[\cancel{\frac{\partial}{\partial r}(r(R_o + r\cos\theta)B_r)} + \frac{\partial}{\partial \theta}((R_o + r\cos\theta)B_\theta) + \cancel{\frac{\partial}{\partial \phi}(rB_\phi)}\right]$$

$$= 0$$

(Axisymmetric $\rightarrow \frac{\partial}{\partial \phi} = 0$

$\vec{B} = B_\theta\,\hat{\theta} + B_\phi\,\hat{\phi} \rightarrow B_r = 0$)

$$\Rightarrow \nabla\cdot\vec{B} = \frac{1}{r(R_o + r\cos\theta)}\left[\frac{\partial}{\partial \theta}(R_o + r\cos\theta)B_\theta\right] = 0$$

$$\Rightarrow B_\theta(r,\theta) = \frac{const(r)}{R_o + r\cos\theta} = \boxed{\frac{B_\theta(r,\frac{\pi}{2})}{1+\varepsilon\cos\theta}} \quad \left(\varepsilon \equiv \frac{r}{R_o}\right)$$

한편 Ampere's law $\nabla\times\vec{B} = \mu_o\vec{J}$

$$\nabla\times\vec{A} = \frac{1}{h_r h_\theta h_\phi}\begin{vmatrix} h_r\hat{r} & h_\theta\hat{\theta} & h_\phi\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ h_r A_r & h_\theta A_\theta & h_\phi A_\phi \end{vmatrix}$$

$$\vec{B} = B_\theta(r,\theta)\,\hat{\theta} + B_\phi(r,\theta)\,\hat{\phi}$$

$$\vec{J} = J_\theta(r,\theta)\,\hat{\theta} + J_\phi(r,\theta)\,\hat{\phi}$$ 에서

$$\Rightarrow \nabla\times\vec{B} = \frac{1}{r(R_o + r\cos\theta)}\begin{vmatrix} \hat{r} & r\hat{\theta} & (R_o + r\cos\theta)\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \cancel{\frac{\partial}{\partial \phi}} \\ 0 & rB_\theta & (R_o + r\cos\theta)B_\phi \end{vmatrix}$$ (∵ Axisymmetric)

$$= \mu_o J_\theta(r,\theta)\,\hat{\theta} + \mu_o J_\phi(r,\theta)\,\hat{\phi}$$

$\hat{r}$ 성분이 없으므로 $\frac{\partial}{\partial \theta}(R_o + r\cos\theta)B_\phi = 0$

$$\Rightarrow B_\phi(r,\theta) = \frac{Const(r)}{R + r\cos\theta} = \boxed{\frac{B_\phi(r,\frac{\pi}{2})}{1+\varepsilon\cos\theta}} \quad \left(\varepsilon = \frac{r}{R}\right)$$

$$\therefore \vec{B} = B_\theta(r,\theta)\,\hat{\theta} + B_\phi(r,\theta)\,\hat{\phi}$$

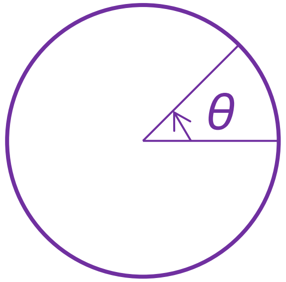
$$= \frac{1}{1+\varepsilon\cos\theta}\left[B_\theta(r,\tfrac{\pi}{2})\,\hat{\theta} + B_\phi(r,\tfrac{\pi}{2})\,\hat{\phi}\right]$$

$$\therefore \boxed{|\vec{B}| = \frac{B_o}{1+\varepsilon\cos\theta}} \quad \left(B_o = \sqrt{\left(B_\theta(r,\tfrac{\pi}{2})\right)^2 + \left(B_\phi(r,\tfrac{\pi}{2})\right)^2} = \left|\vec{B}(r,\tfrac{\pi}{2})\right|\right)$$

$\therefore$ Axisymmetric $\left(\frac{\partial}{\partial \phi} = 0\right)$ 이고, $\vec{B}$, $\vec{J}$ 의 r 성분이 없으면 $B = \frac{B_o(r)}{1+\varepsilon\cos\theta}$ 이다.

Tokamak Transport

- **Particle Trapping**



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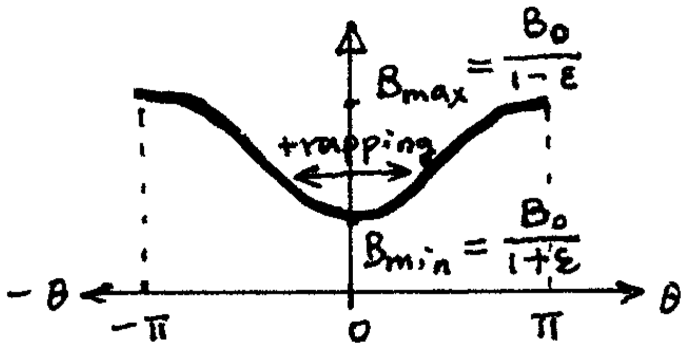


- Condition for trapping of particles

$$\frac{\left(v_\parallel^2\right)_{\max}}{\left(v_\perp^2\right)_{\min}}=\left(\frac{v_\parallel^2}{v_\perp^2}\right)_{mid-plane}\le\frac{B_{\max}}{B_{\min}}-1=\frac{\dfrac{B_0}{1-\varepsilon}}{\dfrac{B_0}{1+\varepsilon}}-1=\frac{2\varepsilon}{1-\varepsilon}\sim 2\varepsilon$$

$$\Rightarrow \quad v_\parallel^2\le 2\varepsilon v_\perp^2$$

Tokamak Transport

- **Particle Trapping**

  - Particle trapping by magnetic mirrors
    trapped particles with banana orbits
    untrapped particles with circular orbits

  - Trapped fraction: $$f_{trap} = \sqrt{1 - \frac{1}{R_m}} = \sqrt{1 - \frac{B_{\min}}{B_{\max}}} = \sqrt{1 - \frac{1-\varepsilon}{1+\varepsilon}} = \sqrt{\frac{2\varepsilon}{1+\varepsilon}}$$

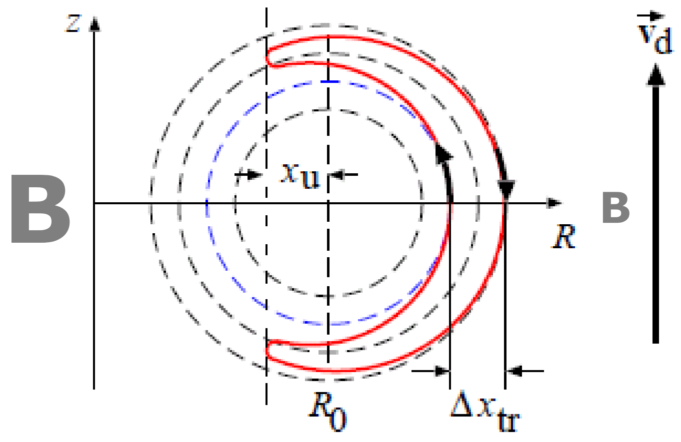
  for a typical tokamak, $\varepsilon \sim 1/3 \rightarrow f_{trap} \sim 70\%$

Tokamak Transport

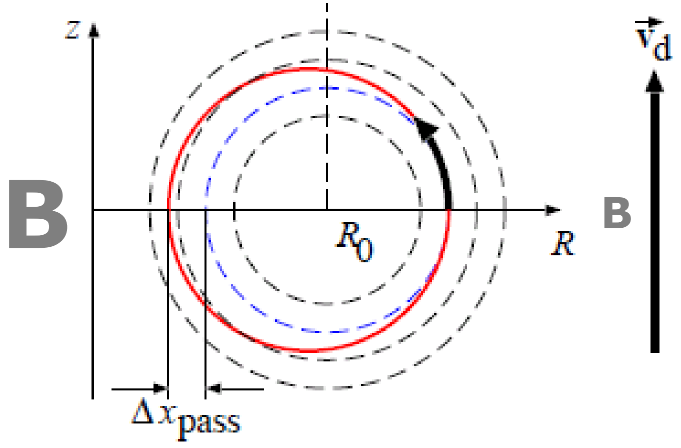
$$\mathbf{v}_{D,\nabla B} = \pm\frac{1}{2} v_\perp r_L \frac{\mathbf{B}\times\nabla B}{B^2}$$

$$\mathbf{v}_{D,R} = \frac{m v_\parallel^2}{q B_0^2}\frac{\mathbf{R}_0\times\mathbf{B}_0}{R^2}$$

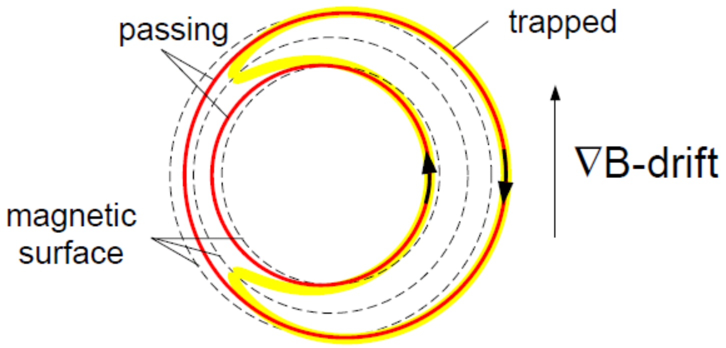
- **Particle Trapping**

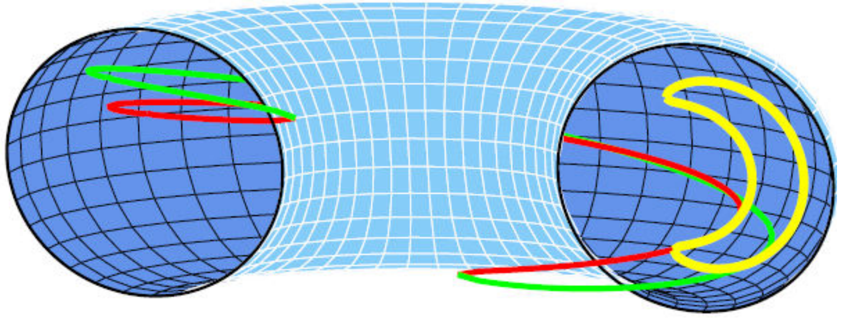


trapped particles



passing particles





Tokamak Transport

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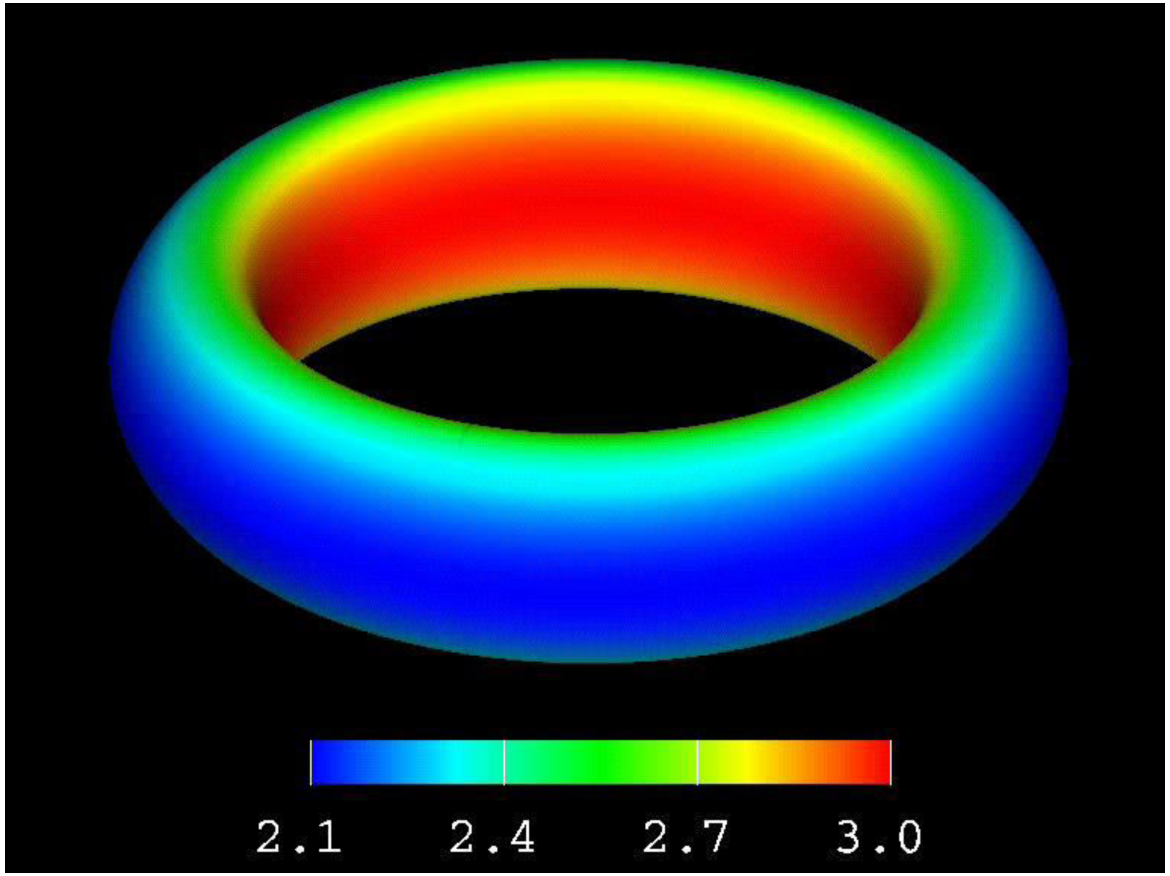
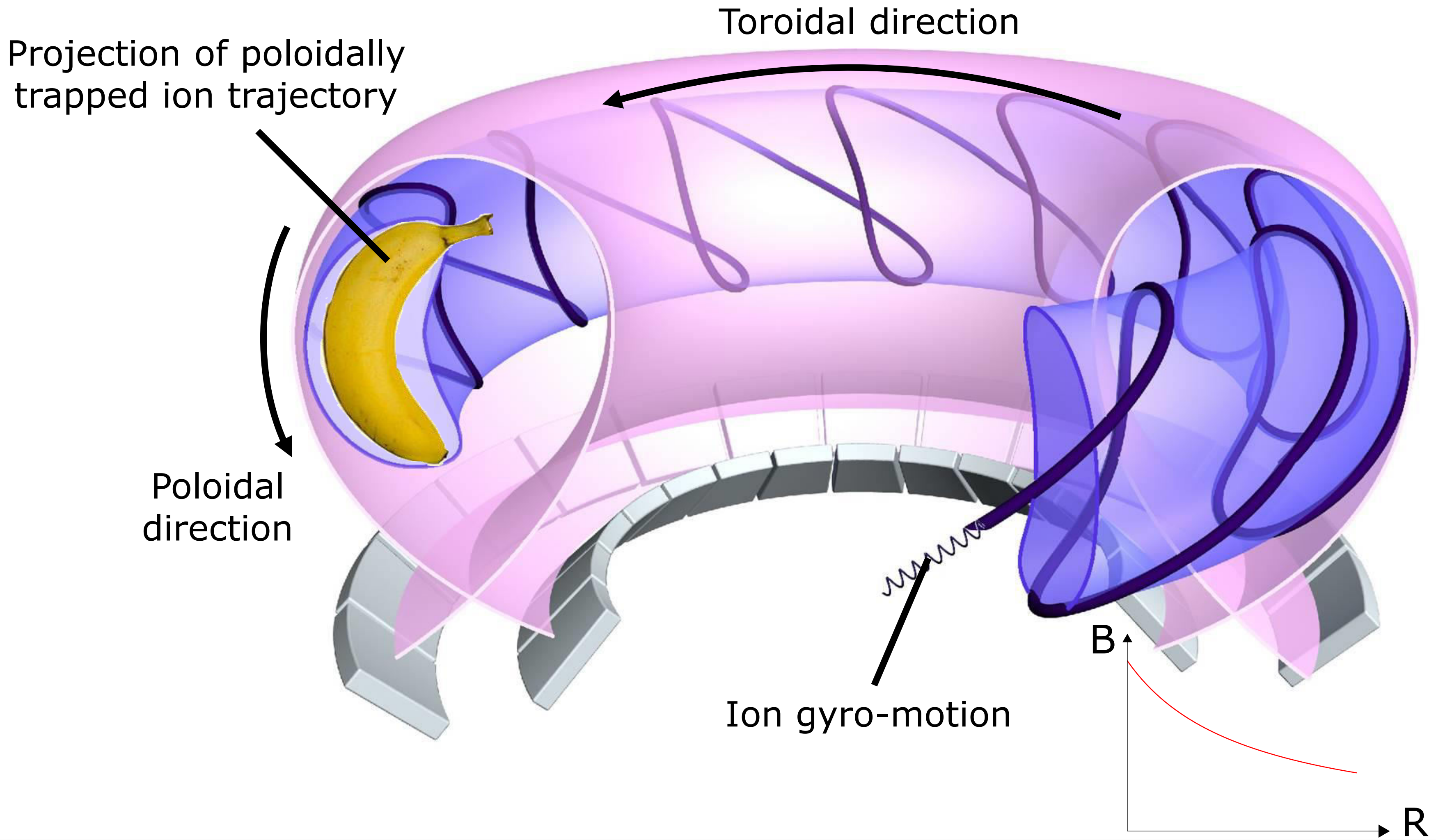


Tokamak Transport

- **Neoclassical Bootstrap current**



Tim Hender, "Neoclassical Tearing Modes in Tokamaks", KPS/DPP, Daejun, Korea, 24 April 2009

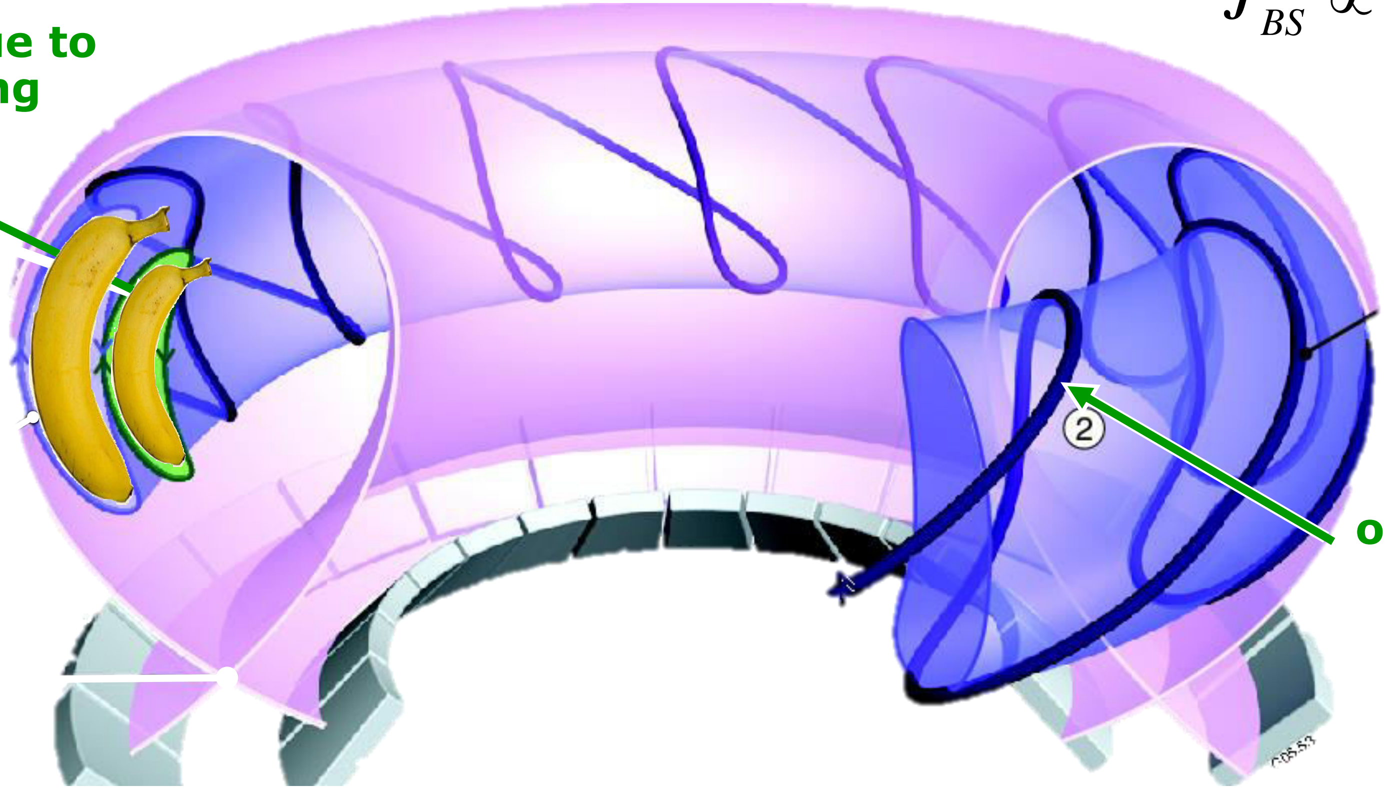
http://tfy.tkk.fi/fusion/research/

Tokamak Transport

- **Neoclassical Bootstrap current**

$$J_{BS} \propto \nabla p$$

Currents due to neighbouring bananas largely cancel



orbits tighter where field stronger

- More & faster particles on orbits nearer the core (green .vs. blue) lead to a net "banana current".
- This is transferred to a helical bootstrap current via collisions.

Tokamak Transport

- **Neoclassical Bootstrap current**

- Named after the reported ability of Baron von Munchausen to lift himself by his bootstraps (Raspe, 1785)
- Suggested with 'Alice in Wonderland' in mind where the heroine managed to support herself in the air by her shoelaces.

http://en.wikipedia.org/wiki/Bootstrapping

Tokamak Transport

- **Bootstrap**

MEANING:
verb tr.: To help oneself with one's own initiative and no outside help.
noun: Unaided efforts.
adjective: Reliant on one's own efforts.

ETYMOLOGY:
While pulling on bootstraps may help with putting on one's boots, it's impossible to lift oneself up like that. Nonetheless the fanciful idea is a great visual and it gave birth to the idiom "to pull oneself up by one's (own) bootstraps", meaning to better oneself with one's own efforts, with little outside help. It probably originated from the tall tales of Baron Münchausen who claimed to have lifted himself (and his horse) up from the swamp by pulling on his own hair.

In computing, booting or bootstrapping is to load a fixed sequence of instructions in a computer to initiate the operating system.
Earliest documented use: 1891.1

Baron Münchausen lifting himself up from the swamp by his own hair
Illustrator: Theodor Hosemann

http://wordsmith.org/words/bootstrap.html

Tokamak Transport

- **Bootstrap**

"I was still a couple of miles above the clouds when it broke, and with such violence I fell to the ground that I found myself stunned, and in a hole nine fathoms under the grass, when I recovered, hardly knowing how to get out again. Looking down, I observed that I had on a pair of boots with exceptionally sturdy straps. Grasping them firmly, I pulled with all my might. Soon I had hoist myself to the top and stepped out on terra firma without further ado."

- With acknowledgement to R. E. Raspe, *Singular Travels, Campaigns and Adventures of Baron Munchausen*, 1786. Edition edited by J. Carswell. London: The Cresset Press, 1948. Adapted from the story on p. 22(???).

http://wordsmith.org/words/bootstrap.html

Tokamak Transport

- **Neoclassical Bootstrap current**

Diffusion Driven Plasma Currents and Bootstrap Tokamak

by

R. J.

UKAEA Res

R

the usual toroidal coordinates. Then in the regime of low collision frequency and in the absence of any driving electric field, steady state diffusion is accompanied by a toroidal current density of magnitude

$$j = -A\left(\frac{r}{R}\right)^{1/2} \frac{1}{B_\theta} \frac{\mathrm{d}p}{\mathrm{d}r} \tag{1}$$

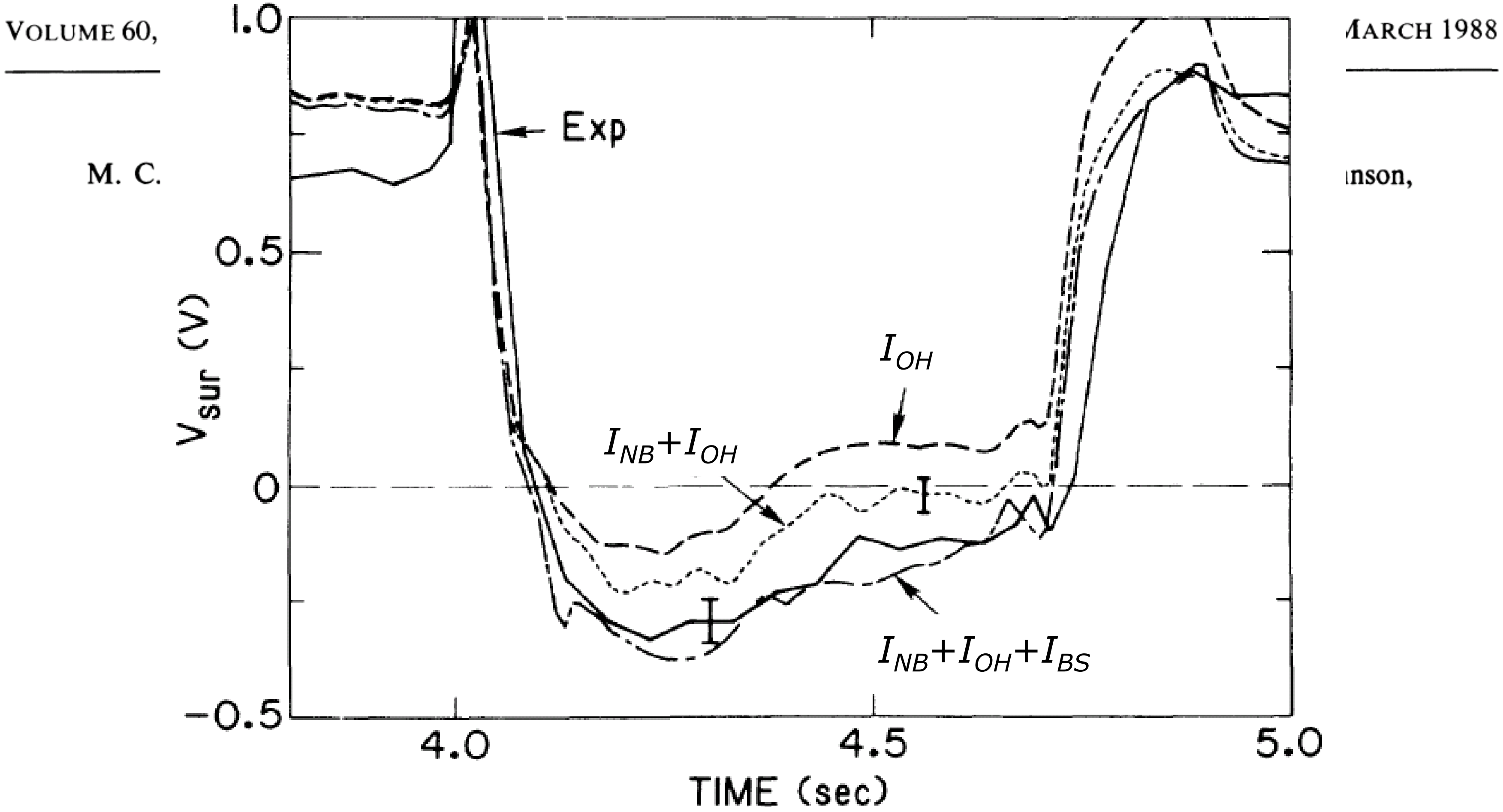
where A is a coefficient whose value depends on the exact collision operator but is of order unity, and p is the plasma pressure.

In tor
ment
toroid
the ma
to ma
curren
of Tokamak machine which operates in a steady state, unlike present pulsed designs.

Nature Physical Science ***229*** *110 (1971)*

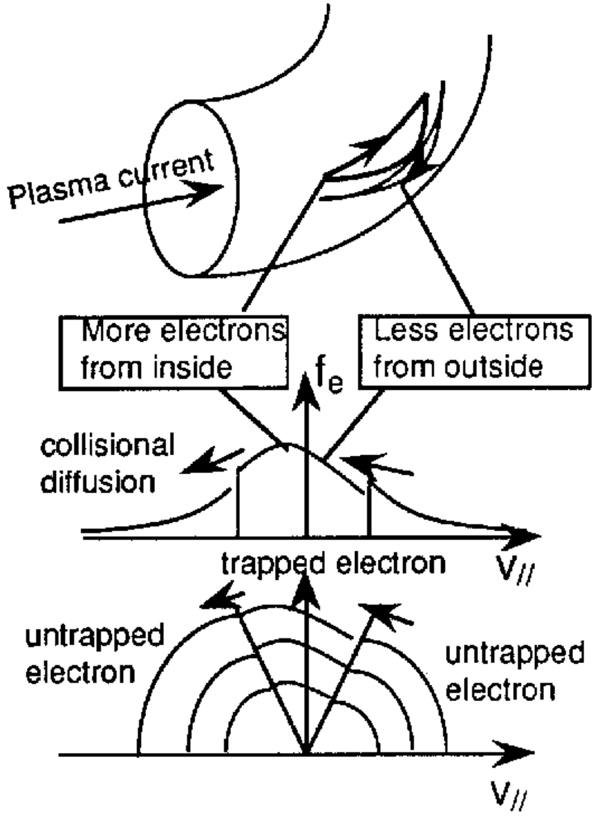
Tokamak Transport

- **Neoclassical Bootstrap current**

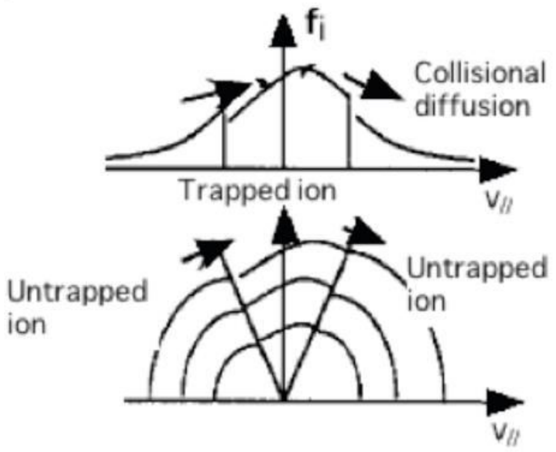


Tokamak Transport

- **Neoclassical Bootstrap current**
  - Trapped-electron orbits and schematics of the velocity distribution function in a collisionless tokamak plasma



Small Coulomb collision smoothes the gap and causes particle diffusion in the velocity space.
Collisional pitch angle scattering at the trapped-untrapped boundary produces unidirectional parallel flow/momentum input and is balanced by the collisional friction force between electrons and ions.



*M. Kikuch et al, PPCF **37** 1215 (1995)*

Tokamak Transport

- **Neoclassical Bootstrap current**

  - Bootstrap current fraction

$$f_B(r) \equiv \frac{J_B}{J_\phi} \approx -1.18 G \varepsilon^{1/2} \beta_P \sim \varepsilon^{1/2} \beta_P \qquad \beta_P = \frac{< p >}{B_p^2 / 2\mu_0}$$

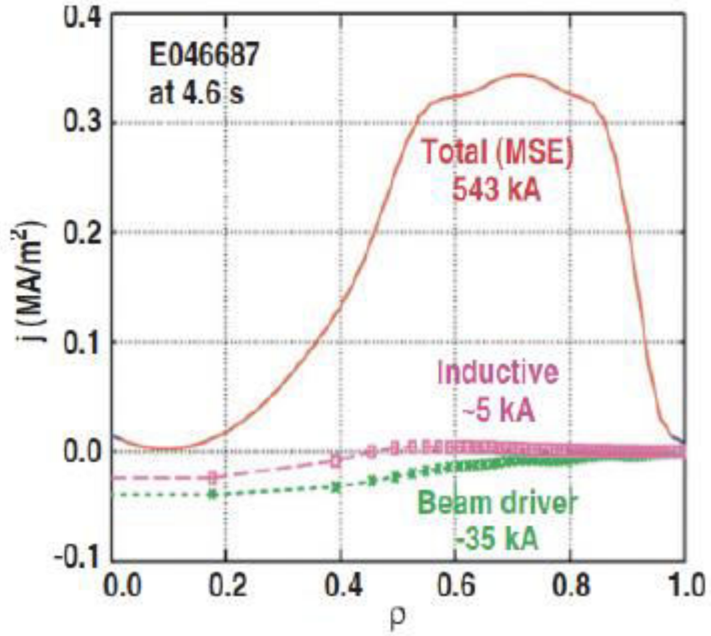
$$G(r) = (\ln n + 0.04 \ln T)' / (\ln r B_\theta)'$$

  - In high-β tokamak, $\beta_p \sim 1/\varepsilon$, implying that $f_B \sim 1/\varepsilon^{1/2}$ >>1:
    The bootstrap current can theoretically overdrive the total current
  - No obvious "anomalous" degradation of J_B due to micro-turbulence
  - The bootstrap current is capable of being maintained in steady state without the need of an Ohmic transformer or external current drive.
    This is indeed a favourable result as it opens up the possibility of steady state operation without the need for excessive amounts of external current drive power.
  - This is critical since bootstrap current fractions on the order of $f_B > 0.7$ are probably required for economic viability of fusion reactors.

Tokamak Transport

- **100% bootstrap discharges**

Y. Takase, IAEA FEC 1996, S. Coda, IAEA FEC 2008

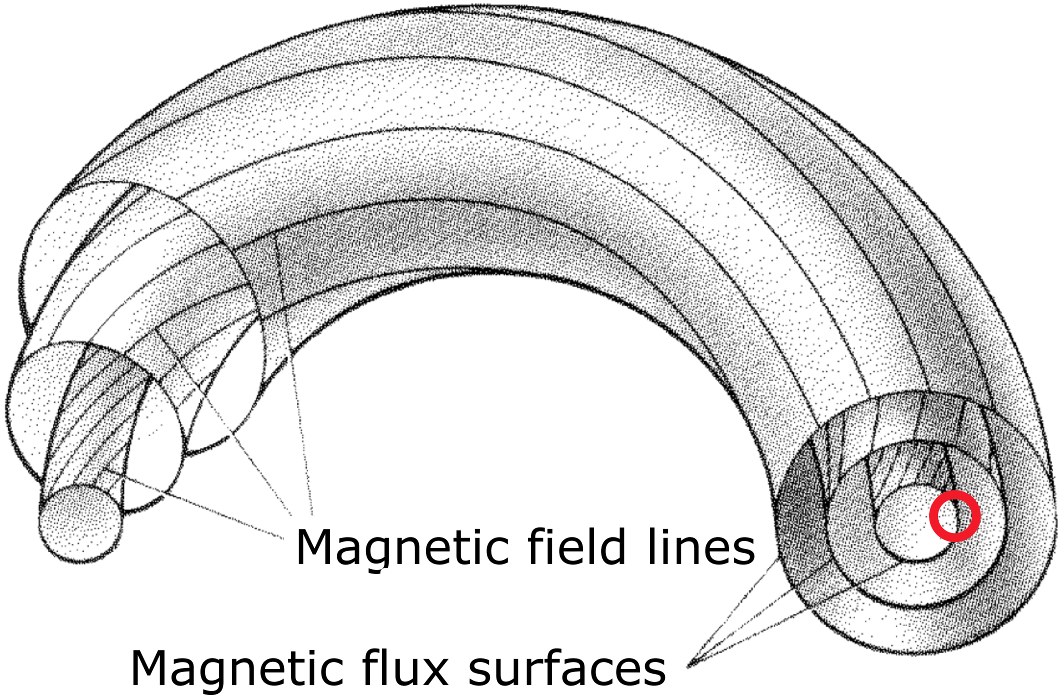


Tokamak Transport

$$D = \frac{(\Delta x)^2}{2\tau}$$: diffusion coefficient (m²/s)

- **Particle Trapping**

  - Collisional excursion across flux surfaces
    untrapped particles: $2r_g$ ($2r_{Li}$)



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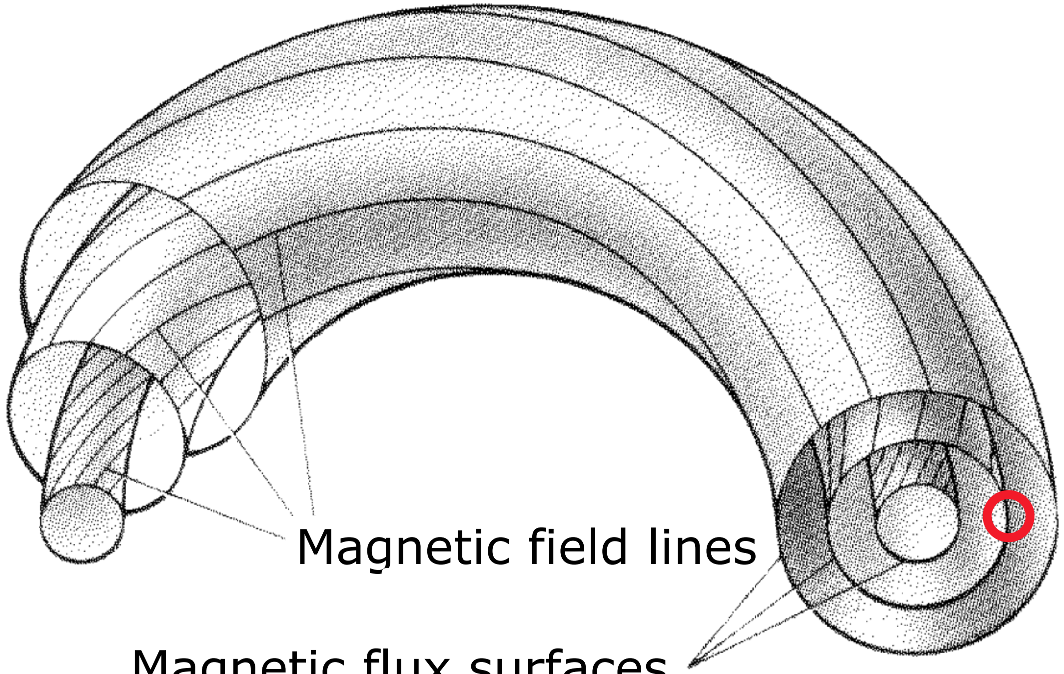


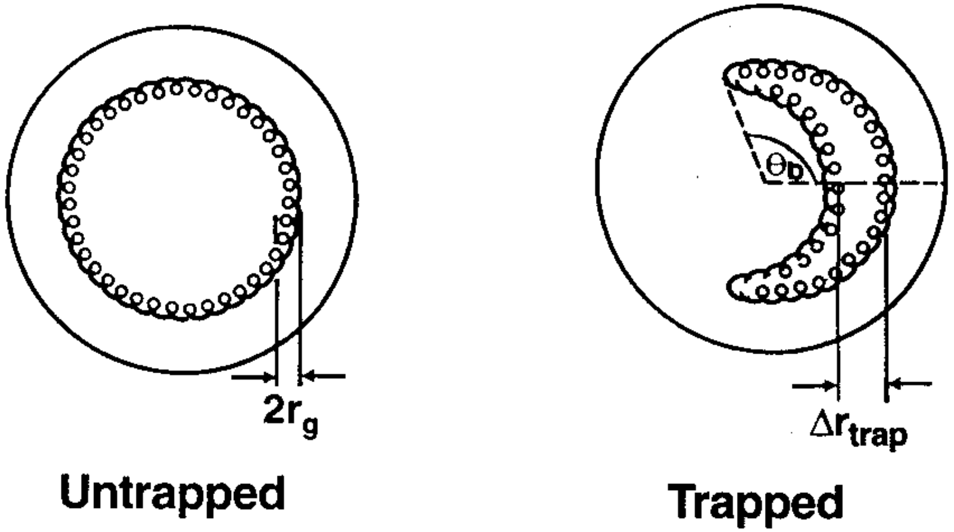
Tokamak Transport

- **Particle Trapping**

  - Collisional excursion across flux surfaces

    untrapped particles: $2r_g$ ($2r_{Li}$)

    trapped particles: $\Delta r_{trap} >> 2r_g$ – enhanced radial diffusion across the confining magnetic field



  - If the fraction of trapped particle is large, this leakage enhancement constitutes a substantial problem in tokamak confinement.

Tokamak Transport

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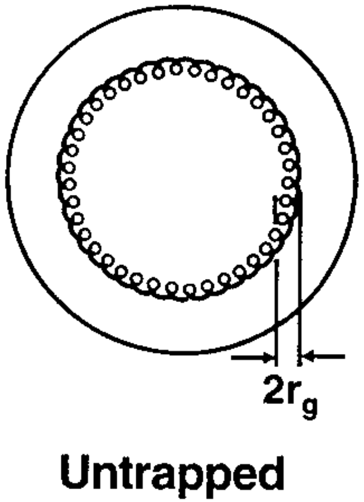

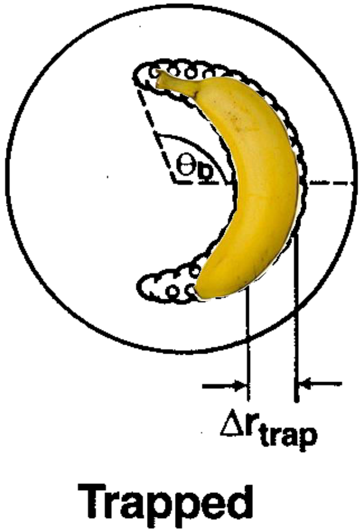

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Tokamak Transport

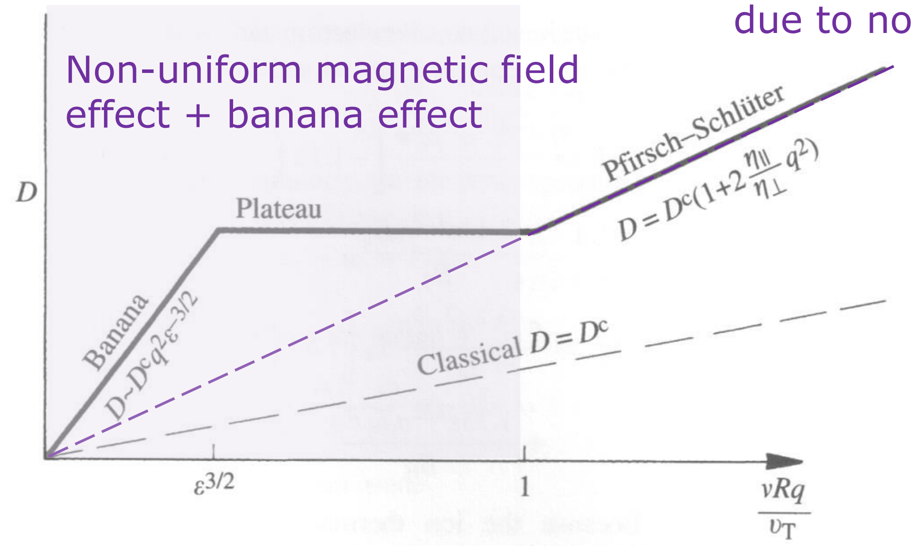
$$\Gamma = -D\nabla n \approx -\frac{(\Delta r)^2}{\tau}\nabla n$$: Fick's law

- **Neoclassical Transports**

  - May increase D, χ up to two orders of magnitude:
    - χ_i 'only' wrong by factor 3-5
    - D, χ_e still wrong by up to two orders of magnitude!

Modified classical diffusion
due to non-uniform magnetic field

Non-uniform magnetic field
effect + banana effect



$$D^c = \frac{\eta_\perp n \sum kT}{B^2}$$

J. Wesson, Tokamaks (2004)