Introduction to Nuclear Fusion

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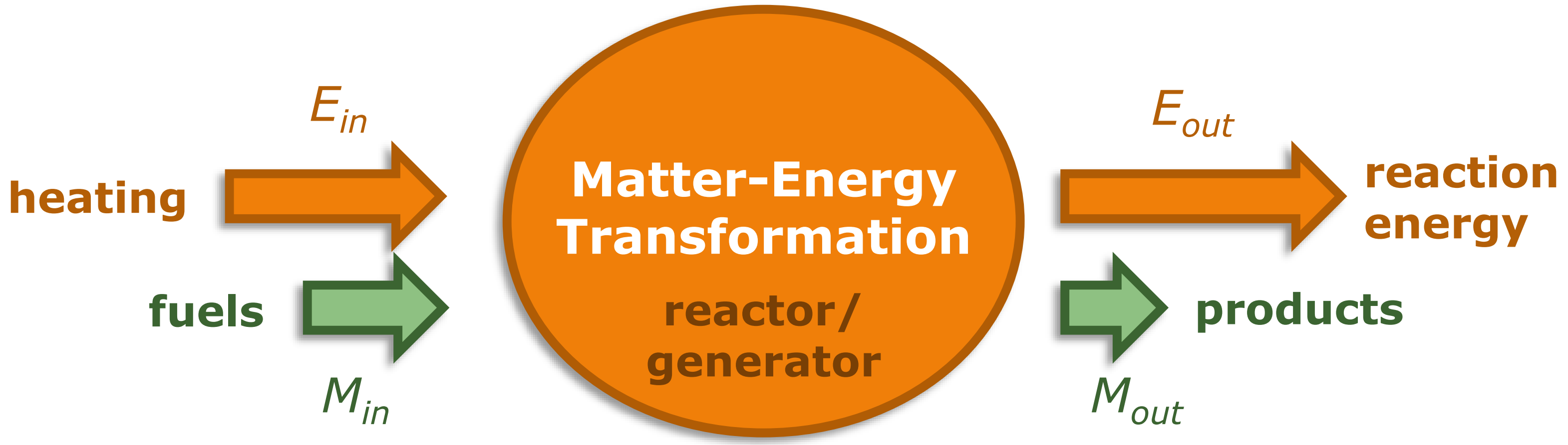
What is nuclear fusion?

Matter and Energy

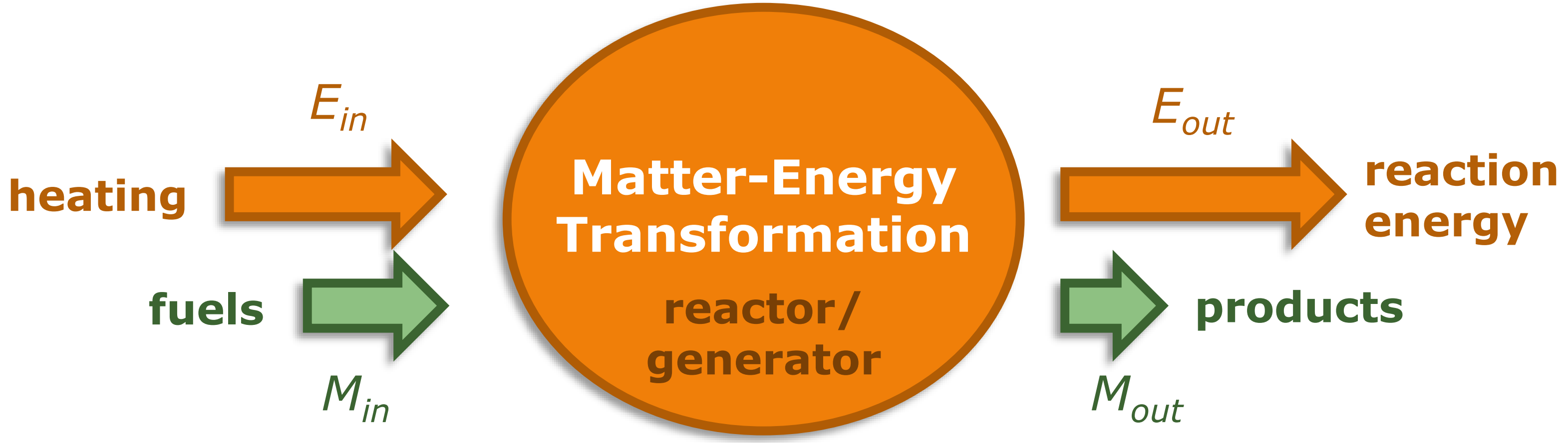


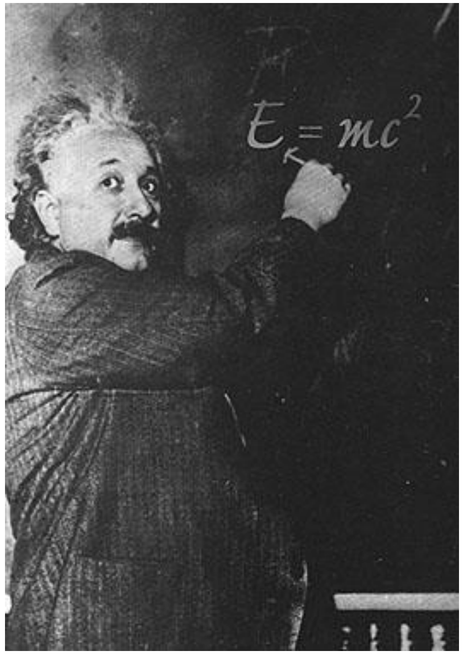
- **Chemical reactions (combustion)**

$$H_2 + \frac{1}{2} O_2 \rightarrow H_2O + 3\,eV$$

$$KT = 1eV \longrightarrow 1\,eV = \frac{1.6\times10^{-19}\,J}{1.38\times10^{-23}\,J/K} = 11600K$$

Matter and Energy





http://www.meteoweb.eu/2011/09/e-possibile-superare-la-velocita-della-luce-teoria-della-relativita-a-rischio/88437/, Dec, 2014

- **Chemical reactions (combustion)**
- **Fission process**
- **Fusion process**

- **Total energy conservation including rest mass energy**

$$E_{in} + M_{in} \rightarrow E_{out} + M_{out}$$

- **If $\Delta m = M_{out} - M_{in} < 0$, then we can get $E_{out} > E_{in}$**

Mass Defect Energy of Nuclear Reaction

reactants **products**

$$a + b \rightarrow d + e$$

$$E^*_{before} = E^*_{after}$$ **Total energy**

$$(E_{k,a} + m_a c^2) + (E_{k,b} + m_b c^2) = (E_{k,d} + m_d c^2) + (E_{k,e} + m_e c^2)$$

For $E_{k,a} + E_{k,b} << Q_{ab}$

$$Q_{ab} = [(m_a + m_b) - (m_d + m_e)]c^2$$
$$= (-\Delta m_{ab})c^2$$

$$\Delta m_{ab} = (m_d + m_e) - (m_a + m_b)$$

$$(m_a c^2 + m_b c^2) \approx (m_d c^2 + m_e c^2) + E_{k,d} + E_{k,e}$$

$$Q_{ab} \approx E_{k,d} + E_{k,e} = \frac{1}{2} m_d v_d^2 + \frac{1}{2} m_e v_e^2$$

Mass Defect Energy of Nuclear Reaction

$$Q_{ab} \approx E_{k,d} + E_{k,e} = \frac{1}{2} m_d v_d^2 + \frac{1}{2} m_e v_e^2$$

$$H_2 + \frac{1}{2} O_2 \rightarrow H_2O + 3\,eV$$

Momentum conservation for reactions with CM at rest

$$m_d v_d = m_e v_e$$

Derive!

$$\frac{1}{2} m_d v_d^2 = \frac{m_e}{m_d + m_e} \left(\frac{m_d + m_e}{m_e} \frac{1}{2} m_d v_d^2 \right)$$

$$= \frac{m_e}{m_d + m_e} \left(\frac{1}{2} \frac{m_d^2}{m_e} v_d^2 + \frac{1}{2} m_d v_d^2 \right)$$

$$= \frac{m_e}{m_d + m_e} \left(\frac{1}{2} \frac{m_d^2}{m_e} \frac{m_e^2}{m_d^2} v_e^2 + \frac{1}{2} m_d v_d^2 \right)$$

$$E_{k,d} \approx \left(\frac{m_e}{m_d + m_e} \right) Q_{ab}, \quad E_{k,e} \approx \left(\frac{m_d}{m_d + m_e} \right) Q_{ab}$$

Ex) d-t fusion reaction $d + t \rightarrow n + \alpha$

$$Q_{dt} = 17.6 \text{ MeV}$$

$$E_{k,n} \approx \frac{4}{5} Q_{dt} \approx 14.1 \text{ MeV}, \quad E_{k,\alpha} \approx \frac{1}{5} Q_{dt} \approx 3.5 \text{ MeV}$$

$$m_n \approx m_p$$

$$m_p = 1.672621 \times 10^{-27}\, kg$$

$$m_n = 1.674927 \times 10^{-27}\, kg$$

Fusion in Nature

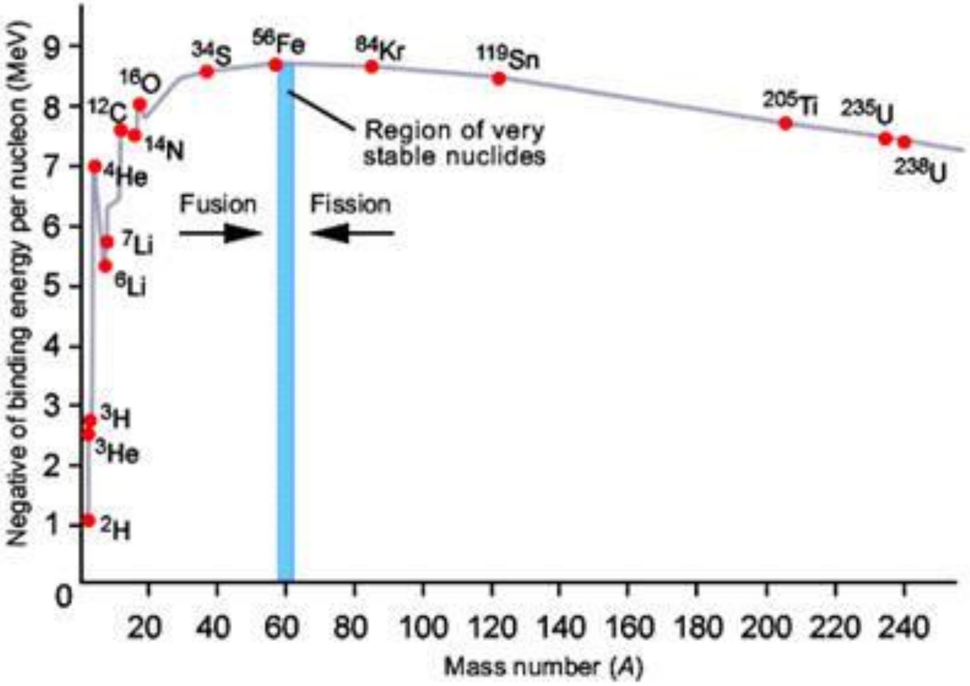
- **Nuclei** are made up of protons and neutron, but the mass of a nucleus is always less than the sum of the individual masses of the protons and neutrons which constitute it.

- **Binding energy**: the amount of energy released when a particular nucleus is formed.

$$Zp+(A-Z)n \rightarrow {}^{A}_{Z}X + B.E.$$

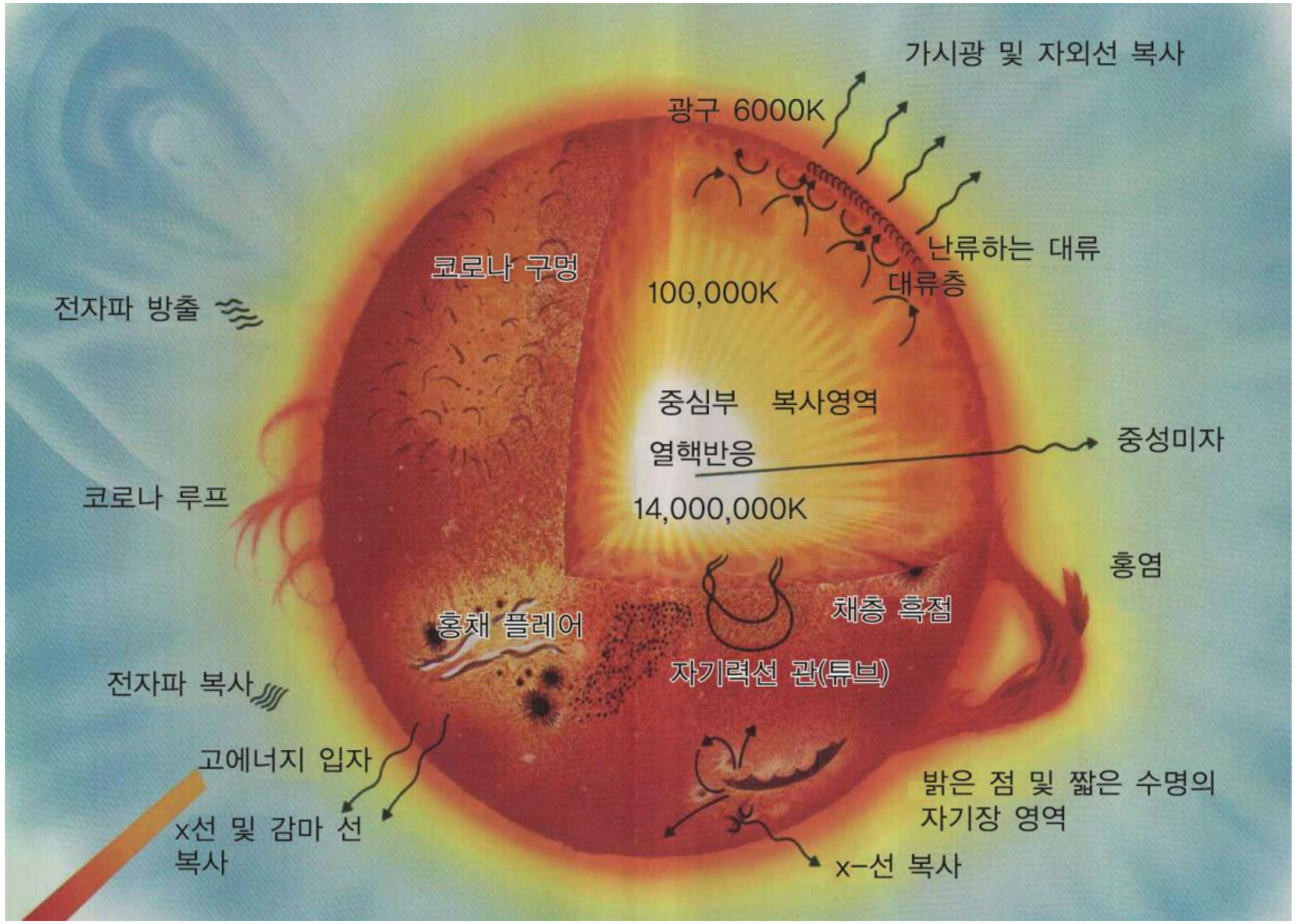
$$B.E. \equiv -[(m_X - Zm_p - (A-Z)m_n)]c^2 = -\Delta mc^2$$

Δm < 0: released energy (exothermic or exoergic)



http://www.daviddarling.info/encyclopedia/B/binding_energy.html

Fusion in Nature



Sun producing 3.8×10^{26} J/s equivalent to 4.3×10^{9} kg

Sun composed of proton (73%), He (25%), etc

How old is the sun?

How long is the sun's lifetime?

Fusion in Nature

- **Fusion reactions by which stars convert hydrogen to helium**

- The PP (proton-proton) chain: in stars the mass of the Sun and less

- The CNO cycle (Bethe-Weizsäcker-cycle): in more massive stars

Nobel prize in physics 1967
"for his contribution to the theory of nuclear reactions, especially his discoveries concerning the energy production in stars"

Hans Albrecht Bethe
(1906. 7. 2 – 2005. 3. 6)

http://www.nobelprize.org/nobel_prizes/physics/laureates/1967/bethe-bio.html

Fusion in Nature

- **Fusion reactions by which stars convert hydrogen to helium**

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- The CNO cycle (Bethe-Weizsäcker-cycle): in more massive stars

> *One of the most impressive discoveries was the origin of the stars, that makes them continue to burn. One of the men who discovered this was out with his girl friend the night after he realized that nuclear reactions must be going on in the stars in order to make them shine. She said "Look at how pretty the stars shine!" He said "Yes, and right now I am the only man in the world who knows why they shine." She merely laughed at him. She was not impressed with being out with the only man who, at that moment, knew why stars shine. Well, it is sad to be alone, but that is the way it is in this world.*

- The Feynman Lectures on Physics I, p.3-7

Fusion in Nature

- **The PP (Proton-Proton) Chain**

Step 1: Smash two protons together to make deuterium

$$p + p \rightarrow d + \beta^{+} + \nu + 1.2\, MeV \qquad p \rightarrow n + \beta^{+} + \nu - 0.782 MeV$$

Positron: antiparticle of the electron-like an electron with charge of +1e
Neutrino: electrically neutral, weakly interacting elementary subatomic particle

http://burro.astr.cwru.edu/Academics/Astr221/StarPhys/ppchain.html

Fusion in Nature

- **The PP (Proton-Proton) Chain**

Step 2: A proton crashes into a deuterium nucleus, making ^3He

$$p + d \rightarrow {}^3He + \gamma + 5.5\,MeV$$

Gamma ray: electromagnetic radiation of an extremely high frequency (very high energy photon)

Do Steps 1 and 2 again so to have two ^3He nuclei.

http://burro.astr.cwru.edu/Academics/Astr221/StarPhys/ppchain.html

Fusion in Nature

- **The PP (Proton-Proton) Chain**

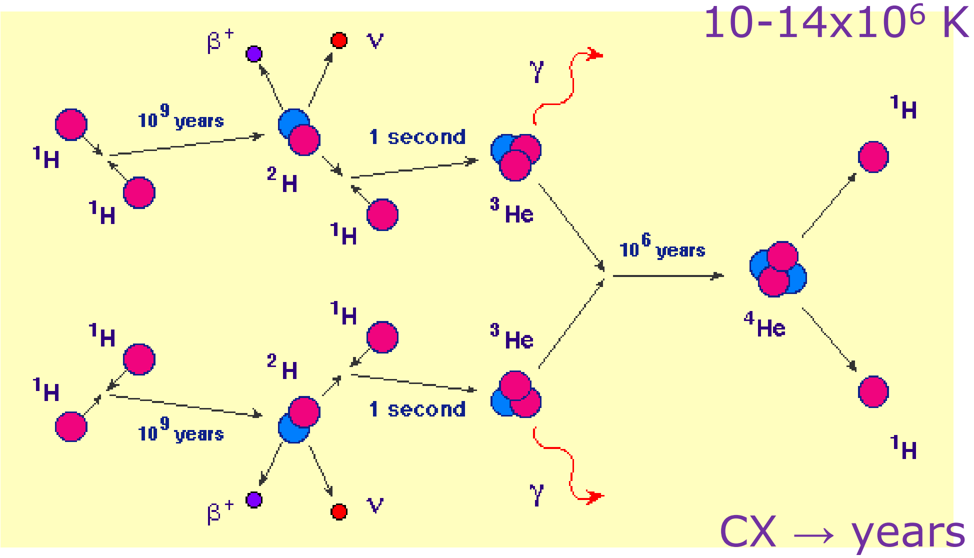
  Step 3: Mash two helium-3 nuclei together to make helium-4

$$^{3}He + {}^{3}He \rightarrow {}^{4}He + 2p + 12.9\ MeV$$

http://burro.astr.cwru.edu/Academics/Astr221/StarPhys/ppchain.html

Fusion in Nature

- **The PP (Proton-Proton) Chain**



10-14x10⁶ K

CX → years

$$p + p \rightarrow d + \beta^+ + \nu + 1.2\ MeV$$

$$p + d \rightarrow {}^3He + \gamma + 5.5\ MeV$$

$$^3He + {}^3He \rightarrow {}^4He + 2p + 12.9\ MeV$$

PPII Chain (14-23x10⁶ K)

$$^3He + {}^4He \rightarrow {}^7Be + \gamma$$

$$^7Be + \beta^- \rightarrow {}^7Li + \nu$$

$$^7Li + p \rightarrow {}^4He + {}^4He$$

PPIII Chain (> 23x10⁶ K)

$$^7Be + p \rightarrow {}^8B + \gamma$$

$$^8B \rightarrow {}^8Be + \beta^+ + \nu$$

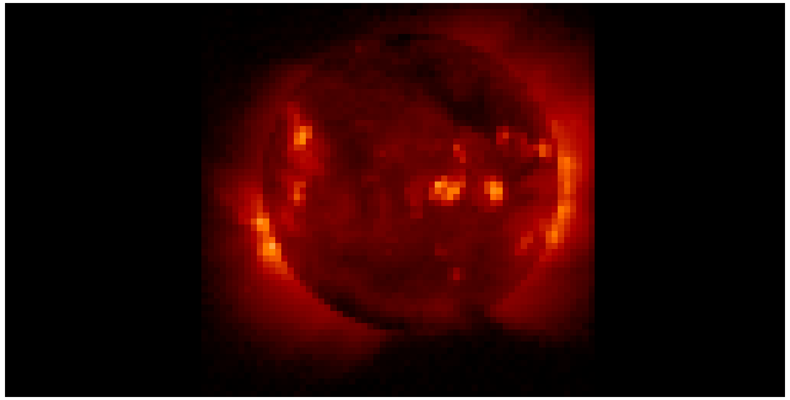
$$^8Be \rightarrow {}^4He + {}^4He$$

nucleosynthesis

http://csep10.phys.utk.edu/astr162/lect/energy/ppchain.html

Fusion in Nature

- **The PP (Proton-Proton) Chain**



$$T_{Sun} \leq 15 \times 10^{6}\, K$$

- Most of ^{4}He nuclei being produced in the Sun are born in the PP chain (98.3%).

http://www.nasa.gov/mission_pages/galex/20070815/f.html

Fusion in Nature

- **The CNO Cycle**

$$^{12}C + p \rightarrow {}^{13}N + 1.9\,MeV$$

$$^{13}N \rightarrow {}^{13}C + \beta^{+} + \nu + 1.5\,MeV$$

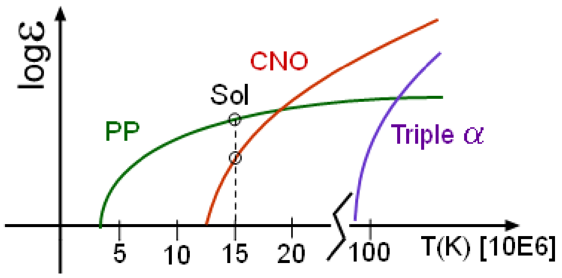
$$^{13}C + p \rightarrow {}^{14}N + 7.6\,MeV$$

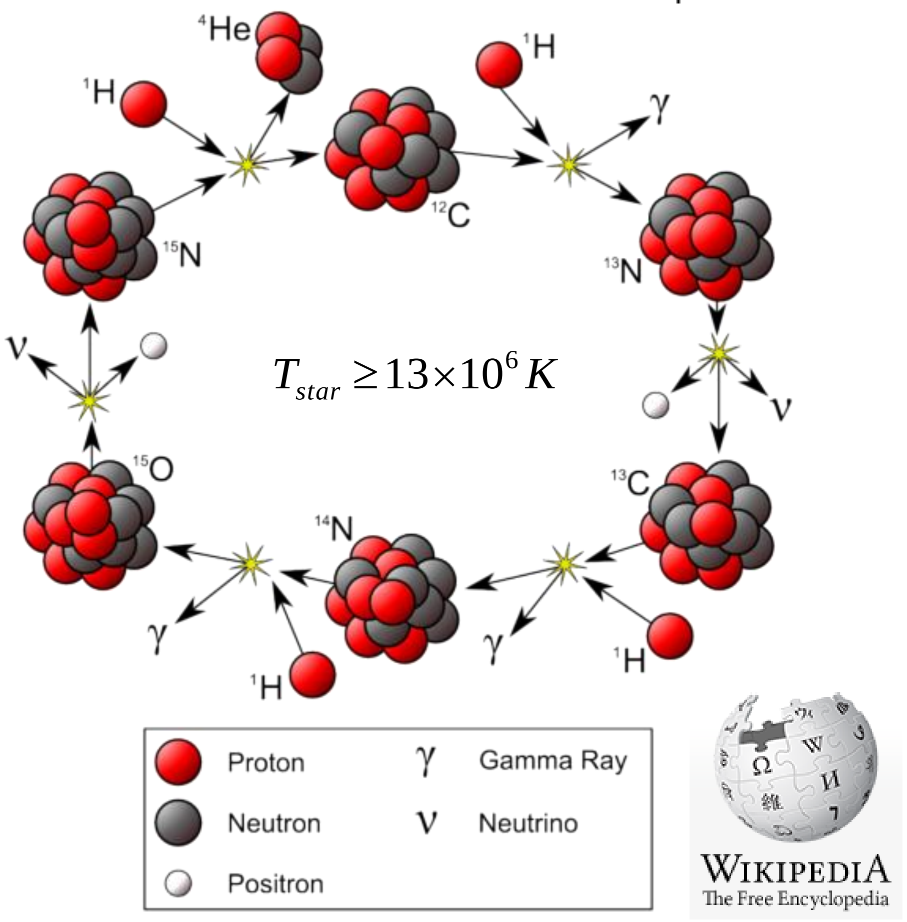
$$^{14}N + p \rightarrow {}^{15}O + 7.3\,MeV$$

$$^{15}O \rightarrow {}^{15}N + \beta^{+} + \nu + 1.8\,MeV$$

$$^{15}N + p \rightarrow {}^{12}C + \alpha + 5.0\,MeV$$

$$4p \rightarrow \alpha + 2\beta^{+} + 2\nu + 25.1\,MeV$$



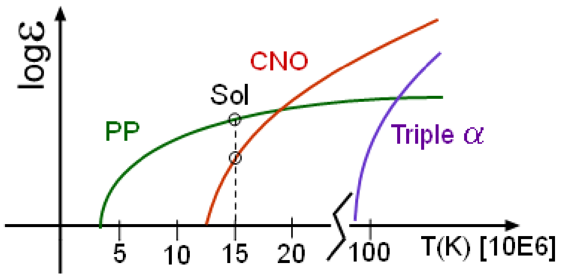


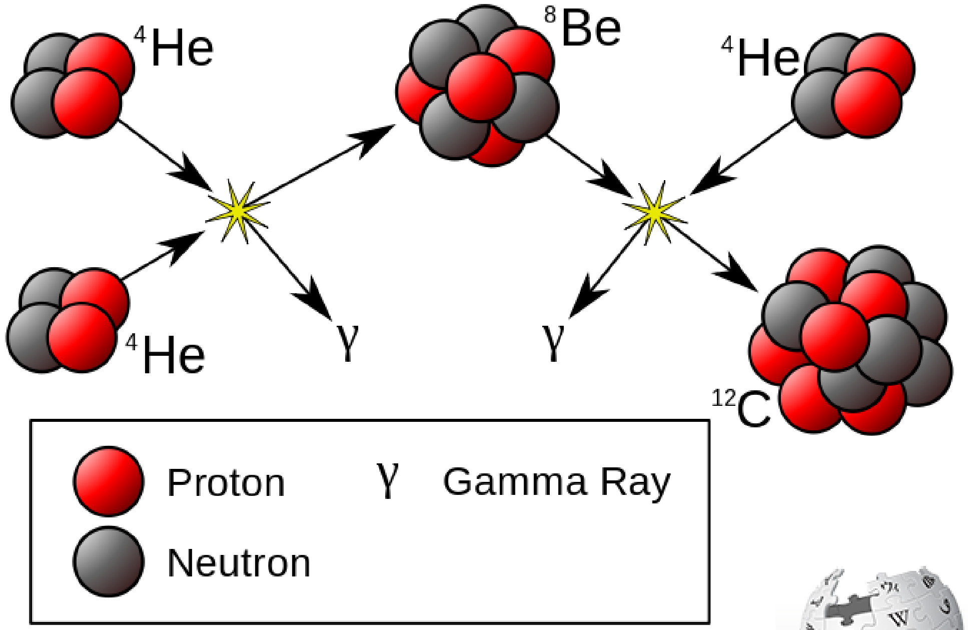
Fusion in Nature

- **The triple alpha process**

$$\alpha + \alpha \rightarrow {}^{8}Be - 92\,keV$$

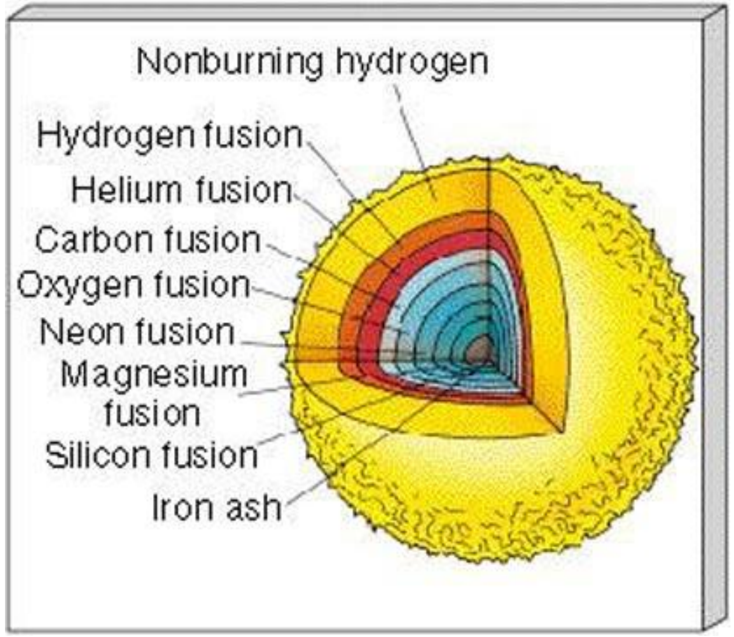
$${}^{8}Be + \alpha \rightarrow {}^{12}C + \gamma + 7.367\,MeV$$

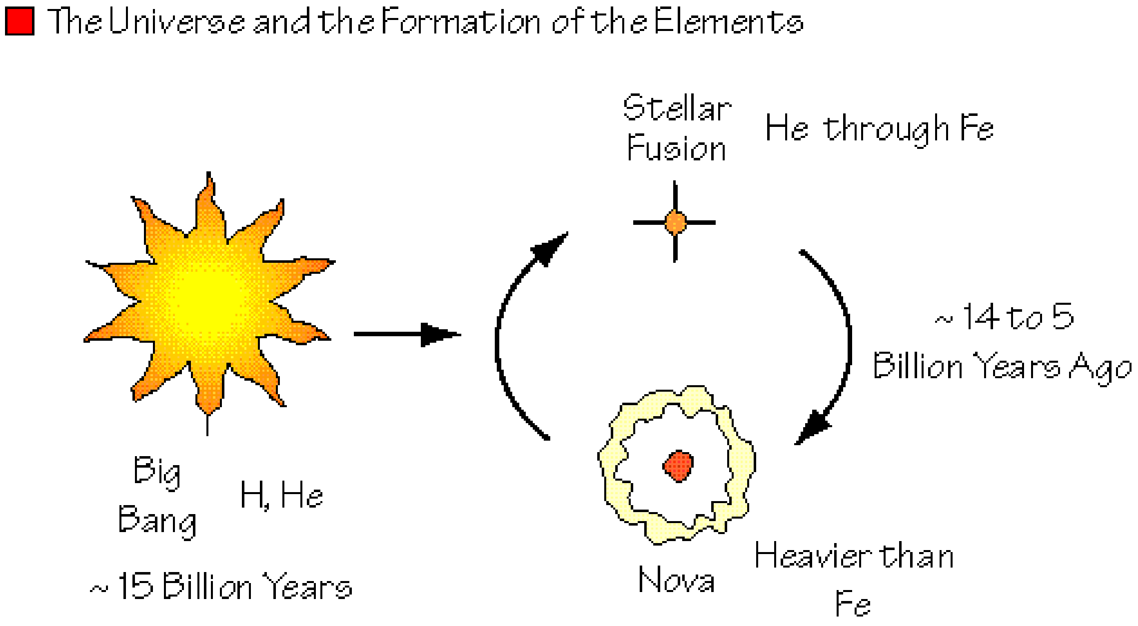




Fusion in Nature

Layers of Fusion in the final stage of a massive star

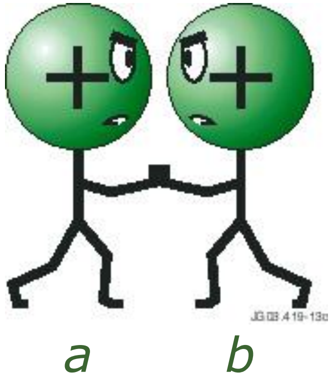




http://jcconwell.wordpress.com/2009/07/20/formation-of-the-elements/
http://eqseis.geosc.psu.edu/~cammon/HTML/Classes/IntroQuakes/Notes/earth_origin_lecture.html

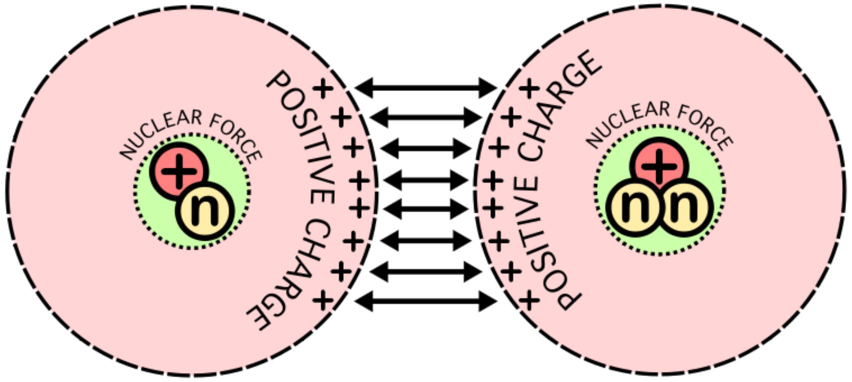
Fusion reaction

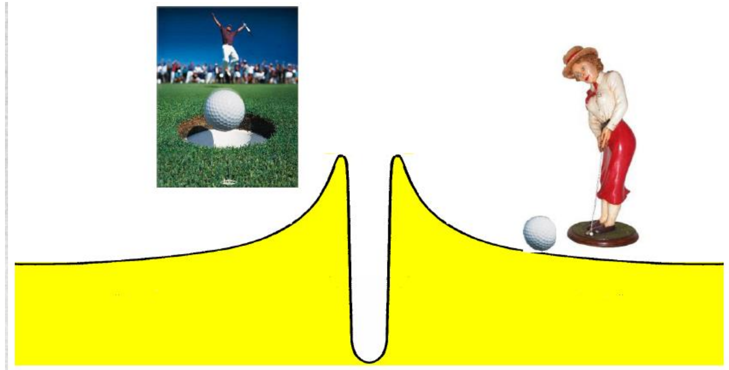
Physical Characterization of Fusion Reaction



$$F_{g,a} = -G\frac{m_a m_b}{r^3}\vec{r}$$

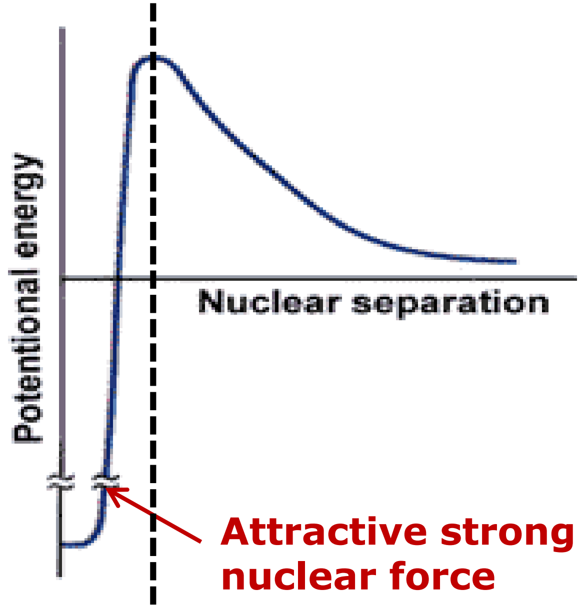
$$F_{c,a} = \frac{1}{4\pi\varepsilon_0}\frac{q_a q_b}{r^3}\vec{r}$$





- **The electrostatic force caused by positively charged nuclei is very strong over long distances, but at short distances the nuclear force is stronger.**
- **As such, the main technical difficulty for fusion is getting the nuclei close enough to fuse.**

Physical Characterization of Fusion Reaction



$$U(R_0) = \frac{1}{4\pi\varepsilon_0}\frac{q_a q_b}{R_0} \sim 0.4\,\text{MeV for } d, t, p$$

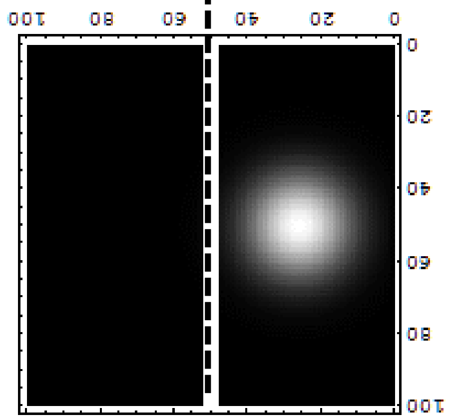
Sun: 1.4 keV

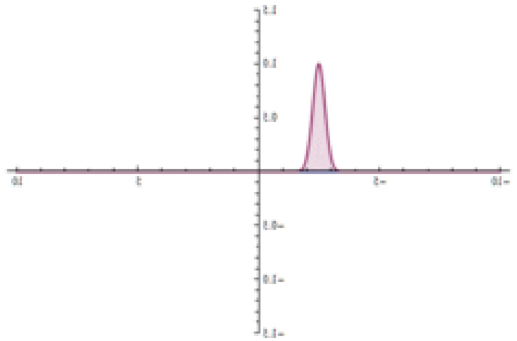
$$R_0 \approx R_p\left(A_a^{1/3} + A_b^{1/3}\right), \quad R_p = (1.3 - 1.7)\times 10^{-15}\,m$$

$$\Pr(tunneling) \propto \frac{1}{v_r}\exp\left[-\gamma \frac{q_a q_b}{v_r}\right]$$

$$E << V_0$$

$$|T|^2 = \frac{(2k_0/\kappa)^2}{(2k_0/\kappa)^2 + \left(\kappa^2 + k_0{}^2\right)^2 \sinh^2 \kappa a} \cong \left(\frac{4k_0\kappa}{\kappa^2 + k_0{}^2}\right)^2 e^{-2\kappa a}$$





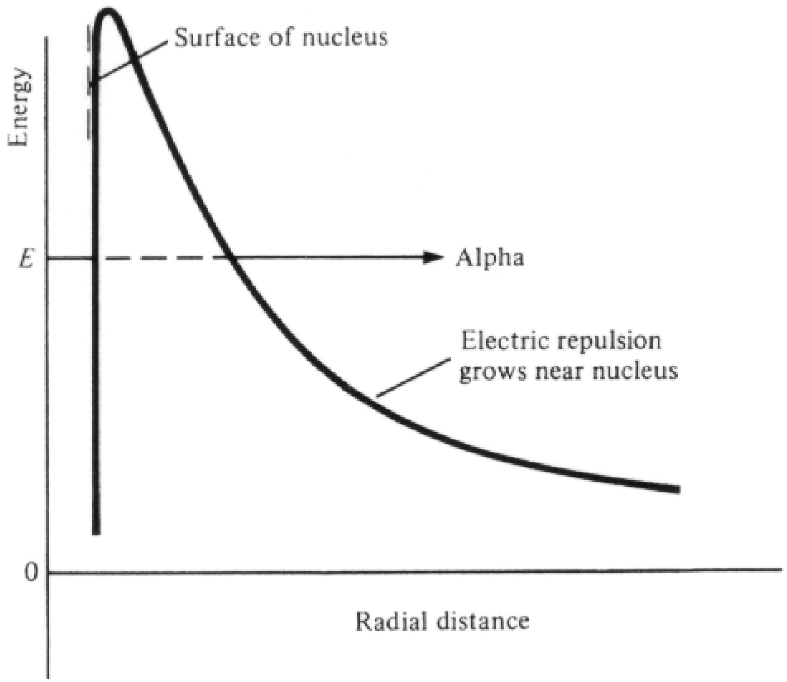
Reflection and tunneling of an electron wave packet directed at a potential barrier

Wikipedia The Free Encyclopedia

A. B. Balantekin and N. Takigawa, 'Quantum tunneling in nuclear fusion', Rev. Mod. Phys. 70, 77 (1998).

Physical Characterization of Fusion Reaction

Potential barrier around a uranium nucleus presented to an alpha particle. The central well is due to the average nuclear attraction of all the nucleons and the hill is due to the electric repulsion of the protons. Alpha particles with energy E trapped inside the nuclear well may still escape to become alpha rays, by quantum mechanically tunnelling through the barrier.



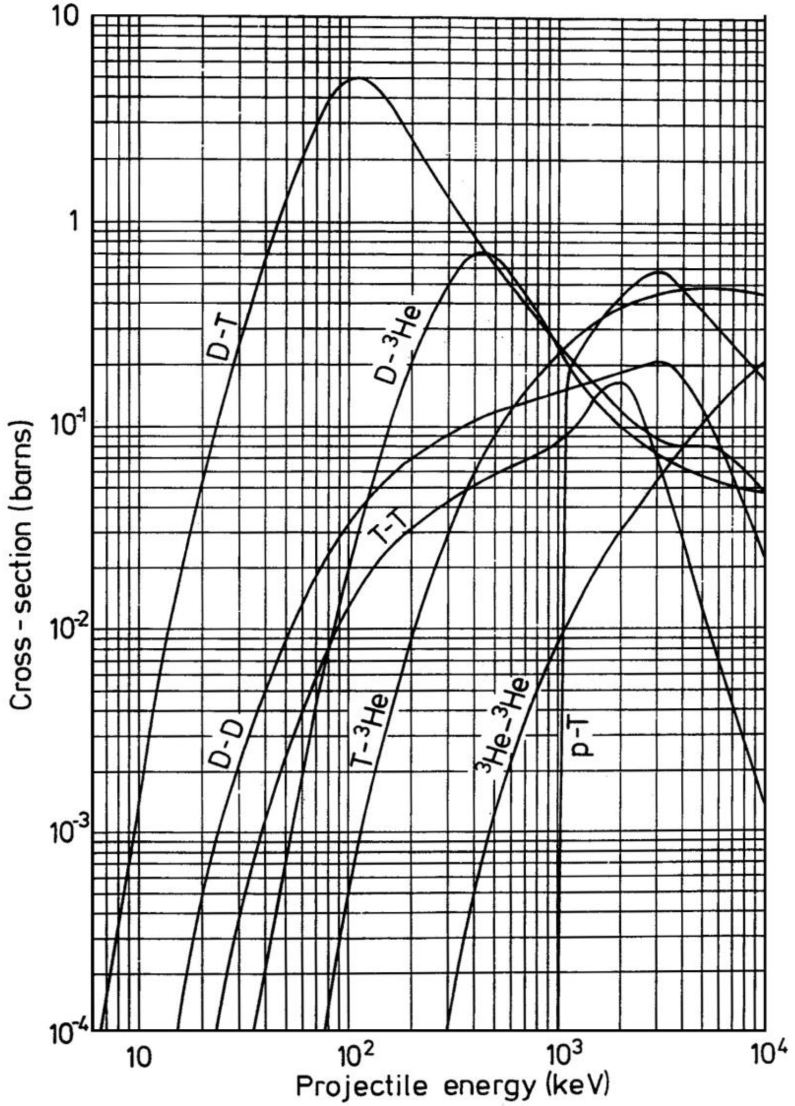
George Gamow (1904-1968)

Max Born (1882-1970)

- By 1928, George Gamow had solved the theory of the alpha decay of a nucleus via tunneling. After attending a seminar by Gamow, Max Born recognized the generality of quantum-mechanical tunneling.

(Max Born, Nobel Prize in Physics 1954)

Fusion Reaction Cross Sections



- Fusion cross section for low energy $E_{CM} < U(R_0)$ by quantum mechanical tunneling process:

$$\sigma_{ab}(E) = \frac{A}{E} e^{-B/\sqrt{E}}$$

Gamow theory (1938)

$$A = const., \quad B = 2^{-1/2} \pi m_r^{1/2} Z_a Z_b e^2 / h\varepsilon_0$$

$$1 barn = 10^{-24} cm^2 = 10^{-28} m^2$$

Fusion Reaction Rate Parameter (Reactivity)

- **Fusion reaction rate density**

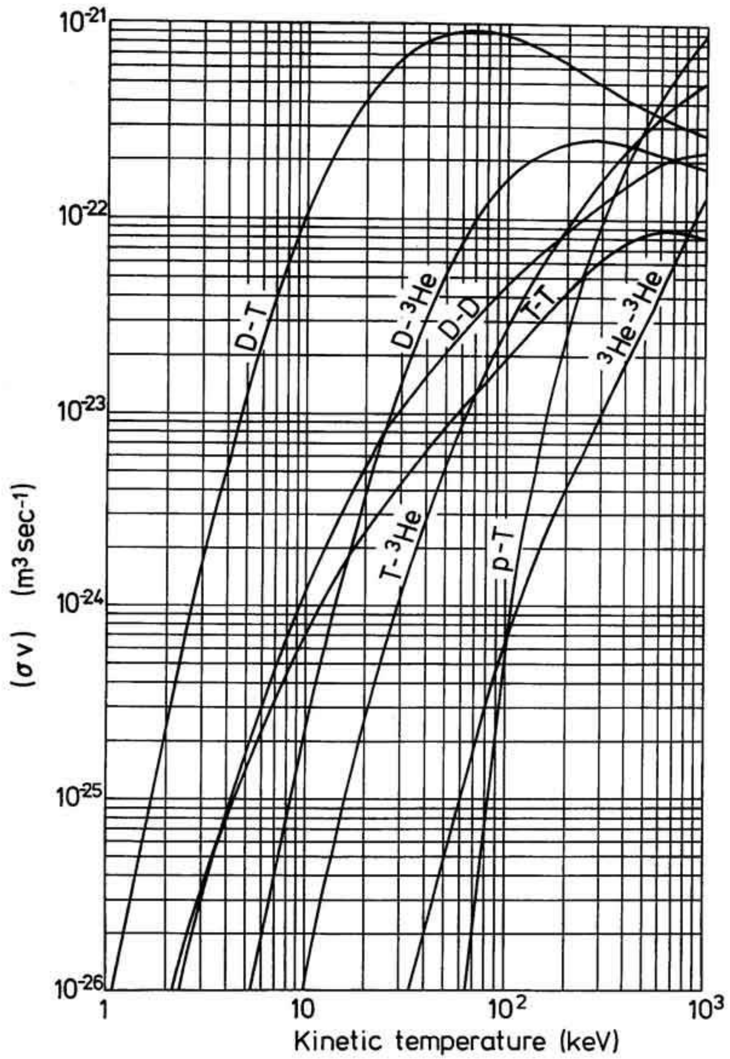
$$R_{fu} \propto N_a N_b v_r \qquad v_r = |\vec{v}_a - \vec{v}_b|$$

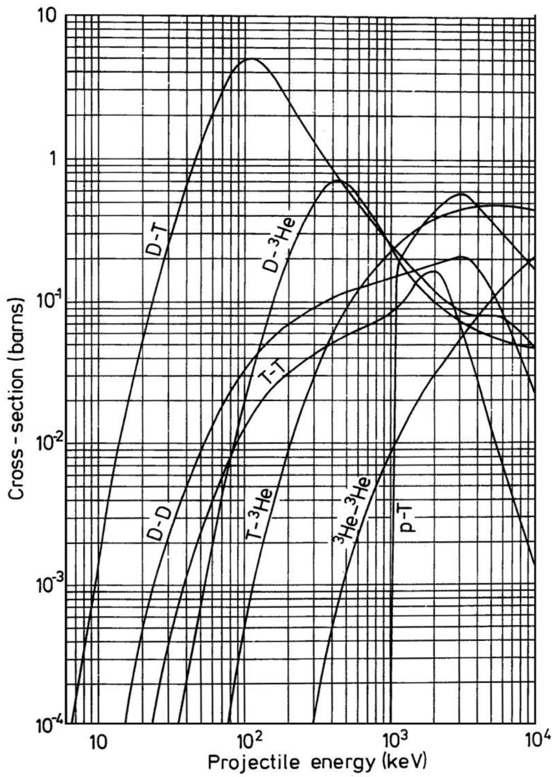
$$R_{fu} = N_a N_b < \sigma v_r >_{ab}$$

- **σ-*v* parameter**

$$< \sigma v >_{ab} = \int_{\vec{v}_a} \int_{\vec{v}_b} \sigma_{ab}\left(|\vec{v}_a - \vec{v}_b|\right) |\vec{v}_a - \vec{v}_b| F_a(\vec{v}_a) F_b(\vec{v}_b) d^3 v_a d^3 v_b$$

Fusion Reaction Rate Parameter (Reactivity)





- Thermodynamic equilibrium $F_x(v_x) \rightarrow M_x(v_x)$
- Both species at the same temperatures

http://www.scienceall.com/jspJavaPopUp.do?classid=CS000140&articleid=611&bbsid=146&popissue=java

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- **σ-*v* parameter**

$$< \sigma v >_{ab} = \int_{\vec{v}_a}\int_{\vec{v}_b} \sigma_{ab}\left(\left|\vec{v}_a - \vec{v}_b\right|\right)\left|\vec{v}_a - \vec{v}_b\right| F_a(\vec{v}_a) F_b(\vec{v}_b) d^3 v_a d^3 v_b$$

- **Fusion power density**

$$P_{fu} = R_{fu} Q_{fu} = N_a N_b < \sigma v >_{ab} Q_{fu}$$