Enzyme Reactivity and Inhibition

1. Enzyme reactivity - Michaelis-Menten equation

Step 1: enzyme (E) reacts with substrate (S) to form an enzyme-substrate complex (ES)

$$E + S \xrightarrow[k_{-1}]{k_1} ES$$

Step 2: the enzyme-substrate complex breaks down to form free enzyme and products (P)

$$ES \xrightarrow[k_{-2}]{k_{-2}} E + P$$

- Assumptions: 1) Step 2 is essentially irreversible (the rate of ES formation from E and P is very small)
 - 2) [ES] does not change over time (pseudo-steady state)

Then,
$$k_1[E][S] = k_{-1}[ES] + k_2[ES]$$

 $[E]_{total} = [E] + [ES]$
 $k_1[S]([E]_{total} - [ES]) = (k_{-1} + k_2)[ES]$
 $[ES] = \frac{k_1[S][E]_{total}}{k_1 + k_2 + k_1[S]} = \frac{[S][E]_{total}}{[S] + (\frac{k_{-1} + k_2}{k_1})}$

The rate of enzyme reaction, v, can be written as,

$$\begin{split} v = k_2 [ES] \\ = \frac{k_2 [S] [E]_{total}}{[S] + \left(\frac{k_{-1} + k_2}{k_1}\right)} \end{split}$$

* When should the reaction be at its maximum?

When [S] is very large - there are many, many works (substrates) to do (react with enzyme), so the workers (enzymes) are fully working

In other words:

$$[S]\gg rac{k_{-1}+k_2}{k_1}$$
 and $[ES]\approx [E]_{total}$

Then:

$$v_m = k_2 [E]_{total}$$

Therefore:

$$v = \frac{v_m[S]}{K_M + [S]}$$
 where $K_M = \frac{k_{-1} + k_2}{k_1}$

 K_M = Michaelis constant, the concentration at which the velocity of the reaction reaches half the maximum velocity

* What should the shape of the curve look like? (Fig. 1.12)

2. Inhibition of enzyme reactions

- Inhibition: chemical agents can reduce the activity of an enzyme by binding to it
- Reversible vs. irreversible
 - Reversible inhibition: inhibition: an inhibitor binds to enzymes with non-covalent interactions; the effect of enzyme inhibition disappears when the inhibitor is removed
 - · Irreversible inhibition: an inhibitor binds to enzymes with covalent interactions; the enzymes are made permanently inactive

Reversible inhibition models

- a) competitive inhibition: E binds to I
- Master eq. for enzyme reaction:

$$E + S \xrightarrow{k_1} ES \xrightarrow{k_2} E + P \tag{I}$$

- Reaction involving competitive inhibitor, I:

$$E + I \underset{k_{-3}}{\overset{k_3}{\longleftrightarrow}} EI \tag{a}$$

- Finding solution

from (I): $k_1[E][S] = k_{-1}[ES] + k_2[ES]$

$$[E] = \frac{k_{-1} + k_2}{k_1} \cdot \frac{[ES]}{[S]} = K_M \frac{[ES]}{[S]}$$

from (a): $k_3[E][I] = k_{-3}[EI]$

$$\begin{split} [EI] &= \frac{k_3}{k_{-3}} [E][I] \\ [EI] &= \frac{k_3}{k_{-3}} \Big(K_M \frac{[ES]}{[S]} \Big) \cdot [I] = \frac{K_M}{K_I} \cdot \frac{[ES]}{[S]} \cdot [I], \text{ where } K_I = \frac{k_{-3}}{k_3} \end{split}$$

Biological Treatment Processes

Lecture note #3

$$\left[E\right]_{total} = \left[E\right] + \left[EI\right] + \left[ES\right]$$

The rate of enzyme reaction under competitive inhibition:

$$v = v_m \frac{[ES]}{[E]_{total}} = v_m \frac{[ES]}{K_M \cdot \frac{[ES]}{[S]} + \frac{K_M}{K_I} \cdot \frac{[ES]}{[S]} \cdot [I] + [ES]} = v_m \frac{[S]}{K_M^* + [S]}$$

- Effect: increased ${\it K}_{\!M}$ (the graph shifts to the right)

b) noncompetitive inhibition: I binds to both ES and E or to ES only

b-1) uncompetitive inhibition: I binds to ES only

- Reaction involving uncompetitive inhibitor, I:

$$ES+I \xleftarrow{k_4}{k_{-4}} ESI \tag{b}$$

- Finding solution

from (I):

$$[E] = K_M \frac{[ES]}{[S]}$$

from (b):

$$\begin{split} k_4 [ES][I] &= k_{-4} [ESI] \\ [ESI] &= \frac{k_4}{k_{-4}} [ES][I] = \frac{1}{K_I'} [ES][I] \\ [E]_{total} &= [E] + [ESI] + [ES] \end{split}$$

The rate of enzyme reaction under uncompetitive inhibition:

$$v = v_m \frac{[ES]}{[E]_{total}} = v_m \frac{[ES]}{K_M \frac{[ES]}{[S]} + \frac{1}{K_I'} [ES][I] + [ES]} = v_m^* \frac{[S]}{K_M^* + [S]}$$

Lecture note #3

- Effect: decreased v_m and $K_{\!M}$ (the graph shifts down and to the left)

b-2) mixed noncompetitive inhibition: I binds to both ES and E - more commonReaction involving mixed noncompetitive inhibitor, I:

$$E + I \underset{k_{-3}}{\overset{k_3}{\longleftrightarrow}} EI \tag{a}$$

$$ES+I \xleftarrow{k_4}{\overleftarrow{k_{-4}}} ESI \tag{b}$$

- Finding solution

from our previous work on competitive inhibition:

$$[EI] = \frac{K_M}{K_I} \cdot \frac{[ES]}{[S]} \cdot [I]$$

from our previous work on uncompetitive inhibition:

$$[ESI] = \frac{1}{K_I'} [ES][I]$$
$$[E]_{total} = [E] + [EI] + [ESI] + [ESI]$$

The rate of enzyme reaction under mixed noncompetitive inhibition:

$$v = v_m \frac{[ES]}{[E]_{total}} = v_m \frac{[ES]}{K_M \frac{[ES]}{[S]} + \frac{K_M}{K_I} \frac{[ES]}{[S]} [I] \frac{1}{K_I'} [ES] [I] + [ES]}$$

In many cases, $K_{I}\approx K_{I}^{\prime}$ (the tendency of inhibitor binding to E and ES is the same)

Then,

$$v = \frac{v_m}{1 + \frac{[I]}{K_I}} \cdot \frac{[S]}{K_M + [S]} = v_m^* \frac{[S]}{K_M + [S]}$$

- Effect: decreased $\boldsymbol{v}_{\!m}$ (the graph shifts down)

c) Product inhibition: the product, P, binds to ES

- Reaction for product inhibition:

$$ES + P \xrightarrow[k_{-5}]{k_{-5}} ESP$$

Additional assumption: [ESP] does not change over time

Then,
$$k_5[ES][P] = k_{-5}[ESP]$$

- Finding solution

$$[ESP] = \frac{1}{K_P} [ES][P], \text{ where } K_P = \frac{k_{-5}}{k_5}$$
$$[E]_{total} = [E] + [ES] + [ESP]$$

The rate of enzyme reaction under product inhibition:

$$v = v_m \frac{[ES]}{[E]_{total}} = v_m \frac{[ES]}{K_M \frac{[ES]}{[S]} + \frac{1}{K_P} \frac{[ES]}{[S]} [P]} = v_m^* \frac{[S]}{K_M^* + [S]}$$

- Effect: decreased v_{m} and ${\it K}_{\!M}$ (the graph shifts down and to the left)