

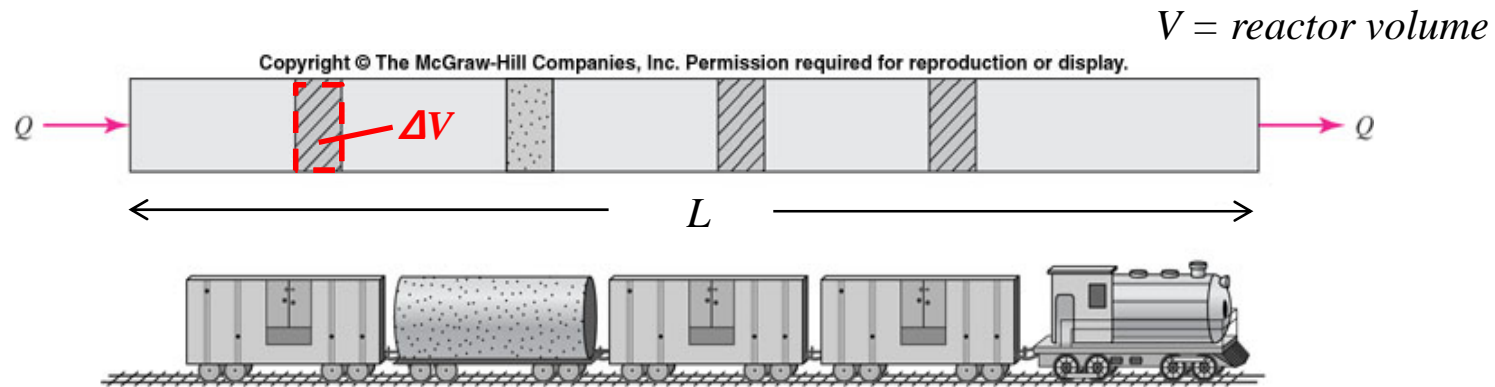
Reactors II

Today's lecture

- Reactor analysis using mass balance
 - Plug flow reactor
 - Completely mixed flow reactor

Reactor analysis – PFR, 1st order reaction

- Plug-flow reactor (PFR)



1) define control volume:

the moving “plug”: a very thin, homogeneous plate moving in the direction of flow

Reactor analysis – PFR, 1st order reaction

2) write a mass balance eq.:

$$\Delta V \frac{dC}{dt} = \frac{d(\text{in})}{dt} - \frac{d(\text{out})}{dt} + R$$

$-kC\Delta V$ (1st order)

3) arrange the equation: $\frac{dC}{dt} = -kC$

integrating over $t=0$ to t_0 ($= L/v_{\text{flow}} = V/Q$):

$$\frac{C_{\text{out}}}{C_{\text{in}}} = e^{-kt_0} \quad \rightarrow \text{same form as the batch reactor!}$$

(why??)

Reactor analysis – PFR, 1st order reaction

- Examples of PFRs



Disinfection



Rivers and streams

Reactor analysis - PFR

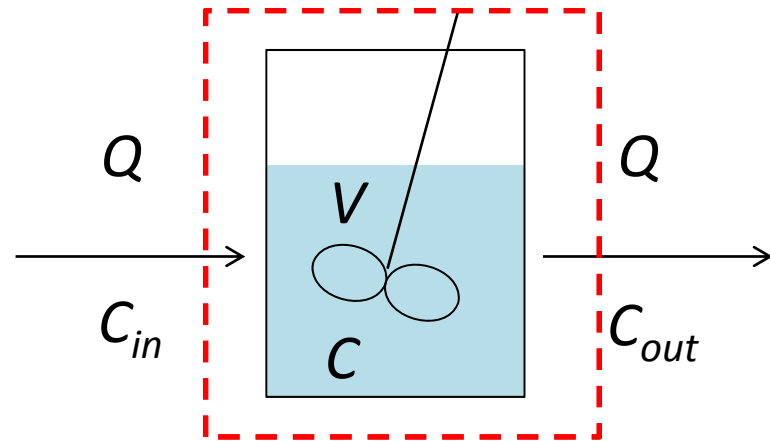
Q: In the U.S., a wastewater treatment plant must disinfect its effluent before discharging the wastewater to a stream. The wastewater contains 4.5×10^5 CFU/L of fecal coliform. The effluent standard for fecal coliform is 2×10^3 CFU/L. Assuming that the disinfection facility is a PFR, determine the length of pipe required if the velocity of the wastewater in the PFR is 0.75 m/s. Assume that the first-order reaction rate constant for destruction of the fecal coliforms is 0.23 min^{-1} .

Reactor analysis – CMFR, 1st order reaction

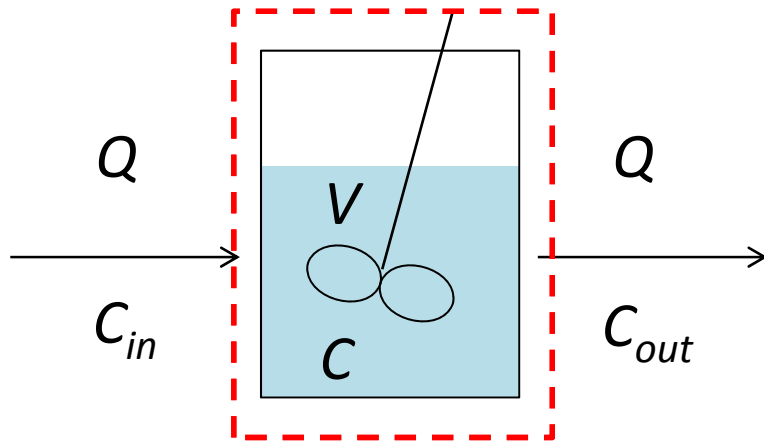
- Completely mixed flow reactor (CMFR)
(continuous-flow stirred tank reactor, CSTR)

- 1) define control volume
- 2) write a mass balance eq.

$$V \frac{dC}{dt} = Q \cdot C_{in} - Q \cdot C_{out} - kCV \text{ (1st order)}$$



Reactor analysis – CMFR, 1st order reaction



Because of homogeneous mixing, $C = C_{out}$

$$V \frac{dC_{out}}{dt} = QC_{in} - QC_{out} - kC_{out}V$$

Reactor analysis – CMFR, 1st order reaction

3) arrange the equation:

$$V \frac{dC_{out}}{dt} = QC_{in} - QC_{out} - kC_{out}V$$

Special case I: No reaction, initial concentration = C_0

$$V \frac{dC_{out}}{dt} = QC_{in} - QC_{out} - kC_{out}V$$

$$\frac{dC_{out}}{dt} = \frac{1}{t_0}(C_{in} - C_{out}) \quad (t_0 = V/Q)$$

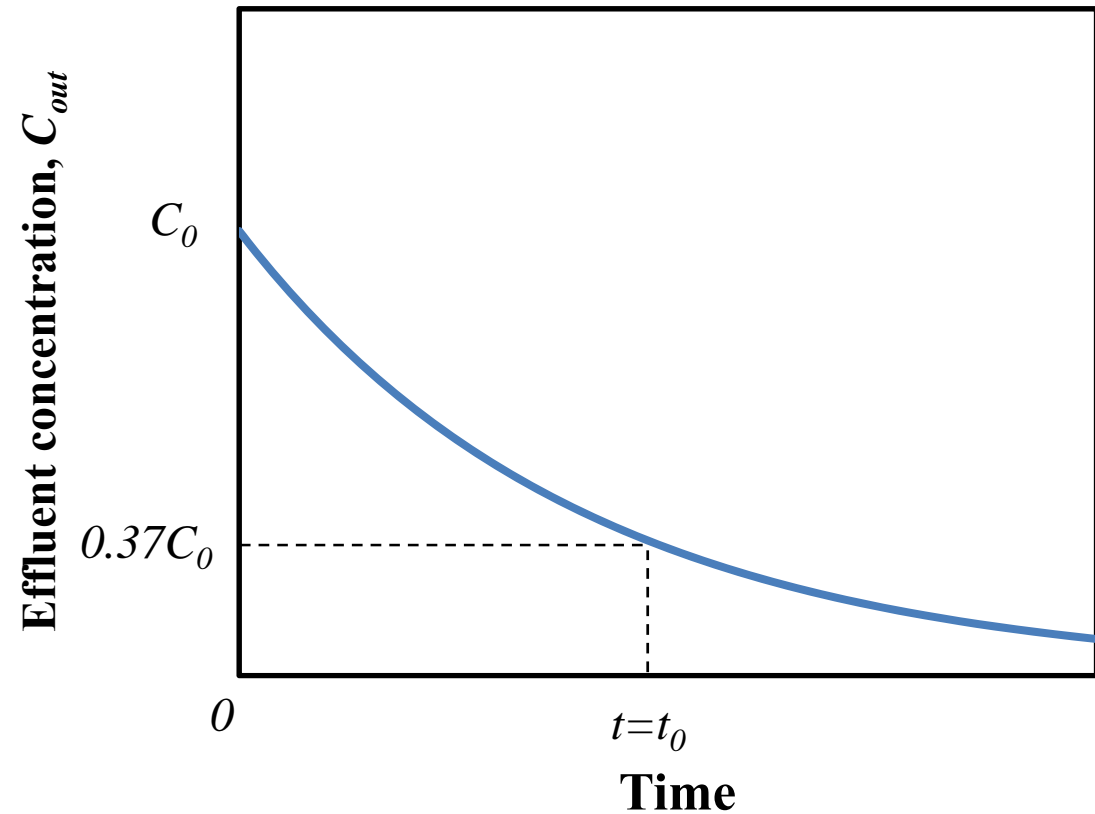
integrating over $t=0$ to t :

$$C_{out,t} = C_0 \left[\exp\left(-\frac{t}{t_0}\right) \right] + C_{in} \left[1 - \exp\left(-\frac{t}{t_0}\right) \right]$$

Reactor analysis – CMFR, 1st order reaction

when $C_{in} = 0$,

$$C_{out,t} = C_o \left[\exp\left(-\frac{t}{t_0}\right) \right]$$



Reactor analysis – CMFR, 1st order reaction

$$V \frac{dC_{out}}{dt} = QC_{in} - QC_{out} - kC_{out}V$$

Special case II: Steady state

$$V \frac{dC_{out}}{dt} = QC_{in} - QC_{out} - kC_{out}V$$

$$C_{out} = \frac{C_{in}}{1 + kt_0} \quad (t_0 = V/Q)$$

→ influent concentration (C_{in}) is reduced in the effluent by a factor of $(1+kt_0)$

Reactor analysis – CMFR, 1st order reaction

- Examples of CMFRs



Biological wastewater
treatment



Lake

Reactor analysis - CMFR

Q: Activated sludge is a key process for most wastewater treatment facilities. The process is often run as a CMFR. Assume a 400 m³-sized CMFR for an activated sludge process receiving 2000 m³/d of wastewater. If a terrorist dumped 10 kg of a non-biodegradable toxic chemical to the reactor, how long will it take for the toxic chemical concentration in the reactor to a safe level (1 mg/L)?

Reactor analysis - CMFR

Q: A chemical degrades in a steady-state CMFR according to first-order reaction kinetics. The upstream concentration of the chemical is 10 mg/L and the downstream concentration is 2 mg/L. Water is being treated at a rate of 29 m³/min. The volume of the tank is 580 m³. What is the rate of decay? What is the rate constant?

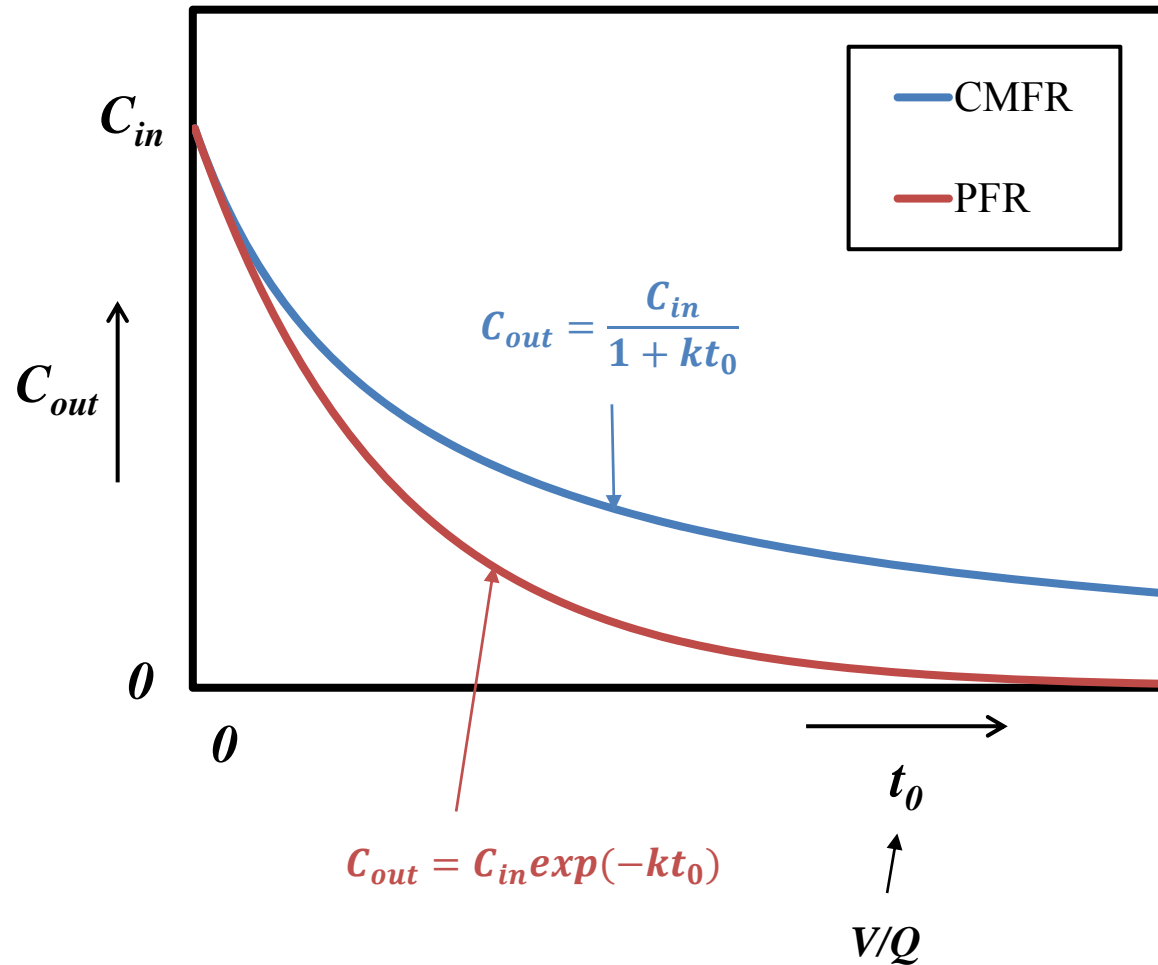
Retention time

- Retention time (detention time), t_0

$$t_0 = V / Q$$

- The average time the fluid particles spend in the reactor
- PFR: the time that fluid particles spend in a reactor is the same ($=t_0$) for all particles
- CMFR: the time that fluid particles spend in a reactor is different

PFR vs. CMFR



- At the same t_0 , PFR shows better performance
- Advantage of using CMFR: less sensitive to shock loads and toxic compounds

Reading assignment

- Textbook Ch4 p. 162-168

Reactor analysis - PFR

Slide#6 solution)

$$\ln \frac{C_{out}}{C_{in}} = -kt_0 = -kL/v_{flow}$$

$$L = -\frac{v_{flow}}{k} \cdot \ln \frac{C_{out}}{C_{in}} = -\frac{0.75 \text{ m/s}}{0.23 \text{ min}^{-1}} \times 60 \text{ s/min} \times \ln \frac{2 \times 10^3 \text{ CFU/mL}}{4.5 \times 10^5 \text{ CFU/mL}}$$

= 1060 m

Reactor analysis - CMFR

Slide#13 solution)

Non-biodegradable → *no reaction*

$$C_{out,t} = C_0 \exp(-t/t_0)$$

$$t = -t_0 \cdot \ln \frac{C_{out,t}}{C_0}$$

$$C_0 = \frac{10 \text{ kg}}{400 \text{ m}^3} \times 10^6 \text{ mg/kg} \times 10^{-3} \text{ m}^3/\text{L} = 25 \text{ mg/L}$$

$$t_0 = \frac{V}{Q} = \frac{400 \text{ m}^3}{2000 \text{ m}^3/\text{d}} = 0.2 \text{ d}$$

$$t = -0.2 \text{ d} \cdot \ln \frac{1 \text{ mg/L}}{25 \text{ mg/L}} = \mathbf{0.64 \text{ day}}$$

Reactor analysis - CMFR

Slide#14 solution)

$$C_{out} = \frac{C_{in}}{1+kt_0}$$

$$k = \frac{1}{t_0} \left(\frac{C_{in}}{C_{out}} - 1 \right)$$

$$t_0 = \frac{580 \text{ m}^3}{29 \text{ m}^3/\text{min}} = 20 \text{ min}$$

$$k = \frac{1}{20 \text{ min}} \left(\frac{10 \text{ mg/L}}{2 \text{ mg/L}} - 1 \right) = \mathbf{0.20 \text{ min}^{-1}}$$

$$\text{rate of decay} = -r = -\left. \frac{dC}{dt} \right|_{\text{reaction}} = k \cdot C_{out}$$

$$= (0.20 \text{ min}^{-1}) \cdot (2 \text{ mg/L}) = \mathbf{0.40 \text{ mg/L} - \text{min}}$$