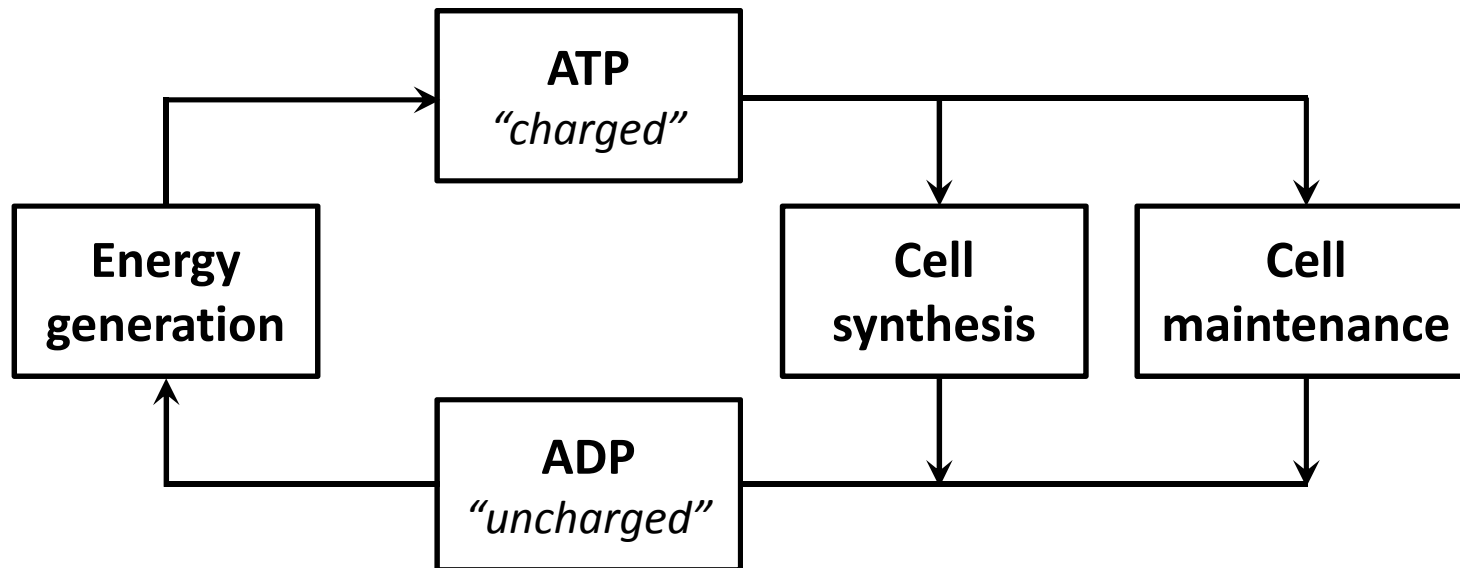# Bacterial energetics

Today's lecture

- Bacterial energetics – overview
- Gibbs free energy of reaction
- Correlating reaction energetics and yield coefficient

Bacterial energetics

- Microorganisms carry out redox reactions to obtain energy for growth and cell maintenance

Energetics and growth: thoughts

- Bacterial cells grow more rapidly when:
    - More energy is obtained by oxidation of (an e^- equivalent of) an e^- donor
    - More energy is obtained by reduction of (an e^- equivalent of) an e^- acceptor
    - Conditions are favorable...

        for rapid utilization of substrates
        - abundance of e^- donor, e^- acceptor, nutrients, etc.
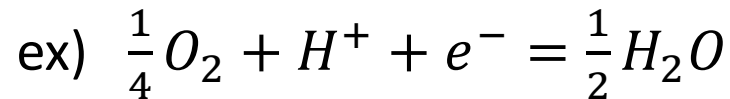   - low concentration of inhibiting compounds

Energetics and growth: math

$$Y_n = Y - b \frac{X_a}{-dS/dt}$$

- $Y_n \uparrow$ when $Y \uparrow$: favorable e⁻ donor and acceptor
- $Y_n \uparrow$ when $X_a << -dS/dt$: favorable conditions for substrate utilization

* We will discuss about the −dS/dt term in forthcoming lectures!

Gibbs free energy: standard conditions

- "Standard" free energy

  - ΔG^0, free energy at 25°C, 1 atm, and unit activity for any chemicals involved

  - $\Delta G^{0\prime}$, standard free energy adjusted to pH=7

ex) $\frac{1}{4}O_2 + H^+ + e^- = \frac{1}{2}H_2O$

ΔG^0 is for $\qquad \{O_2\} = P_{O_2} = 1\ atm$

$\qquad\qquad\qquad \{H^+\} = 1\ $ (pH = 0)

$\qquad\qquad\qquad \{H_2O\} = 1$

$\Delta G^{0\prime}$ is for $\qquad \{O_2\} = P_{O_2} = 1\ atm$

$\qquad\qquad\qquad$ pH=7 $\quad (\{H^+\} = 10^{-7})$

$\qquad\qquad\qquad \{H_2O\} = 1$

> $\{H_2O\} = 1$ applies to any dilute aqueous solutions

Free energy of formation, ΔG_f

- Gibbs free energy that accompanies the formation of 1 mole of the substance from its component elements

Gibbs free energy change of reaction

- Free energy change of reaction, ΔG_r

$$\sum(product \ \Delta G_f) - \sum(reactant \ \Delta G_f)$$

For a generic reaction written as

$$0 = \sum_{i=1}^{n} v_{ir} A_i$$

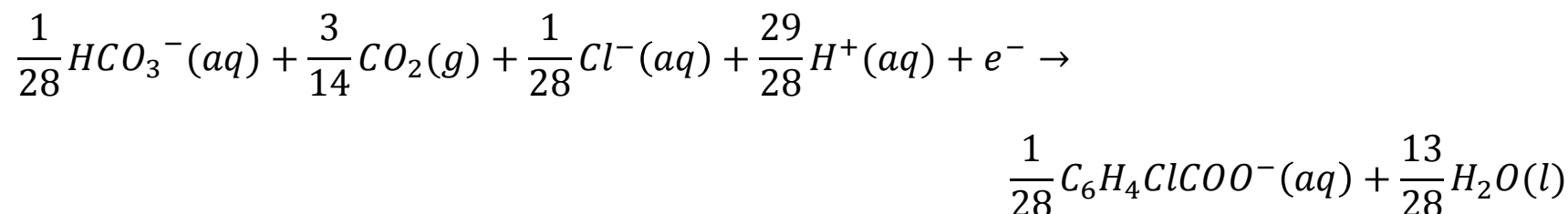v_{ir} = stoichiometric coefficient, (-) for reactants, (+) for products

A_i = reaction constituent, reactants or products

ΔG_r *can be written as*
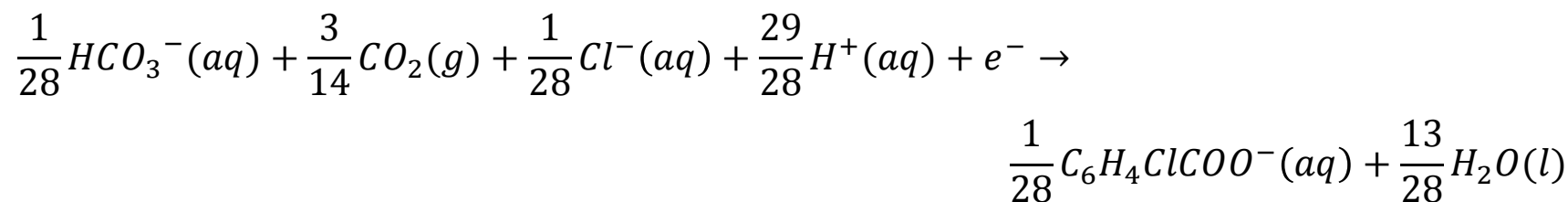
$$\Delta G_r = \sum_{i=1}^{n} v_{ir} \Delta G_f$$

Reaction free energy

Q1: Calculate the standard free energy adjusted pH 7 for the half reaction of 2-chlorobenzoate formation as given below.

$$\frac{1}{28}HCO_3^-(aq) + \frac{3}{14}CO_2(g) + \frac{1}{28}Cl^-(aq) + \frac{29}{28}H^+(aq) + e^- \rightarrow$$

$$\frac{1}{28}C_6H_4ClCOO^-(aq) + \frac{13}{28}H_2O(l)$$

Use the following values of free energy of formation (in textbook Appendix A):

Species	$HCO_3^-(aq)$	$CO_2(g)$	$Cl^-(aq)$	$H^+(aq, 10^{-7})$	$C_6H_4ClCOO^-(aq)$	$H_2O(l)$
$\Delta G_f^{0'}$ @ 25°C (kJ/mole)	-586.85	-394.36	-31.35	-39.87	-237.9	-237.18

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$$\Delta G_r = \sum_{i=1}^{n} v_{ir}\Delta G_f$$

v_{ir} = stoichiometric coefficient, (-) for reactants, (+) for products

$$\Delta G_r^{0\prime} = (-)\left\{\frac{1}{28} \times (-586.85) + \frac{3}{14} \times (-394.36) + \frac{1}{28} \times (-31.35) + \frac{29}{28} \times (-39.87)\right\}$$

$$+ \left\{\frac{1}{28} \times (-237.9) + \frac{13}{28} \times (-237.18)\right\}$$

$$= \mathbf{29.26 \; kJ/e^- \; eq}$$

Reaction free energy

Q2: Calculate the standard free energy adjusted pH 7 for overall energy reaction with ethanol as an e⁻ donor and oxygen as an e⁻ acceptor. Use the half reactions listed in Table 2.2 and 2.3.

R_d: O-5 $\qquad \Delta G_r^{0\prime} = 31.18 \ kJ/e^- \ eq$

R_a: I-14 $\qquad \Delta G_r^{0\prime} = -78.72 \ kJ/e^- \ eq$

$R_e = R_a - R_d$ $\qquad\qquad\qquad \Delta G_r^{0\prime}(R_e) = \Delta G_r^{0\prime}(R_a) - \Delta G_r^{0\prime}(R_d)$

R_a: $\quad \dfrac{1}{4}O_2 + H^+ + e^- = \dfrac{1}{2}H_2O$ $\qquad\qquad = -78.72 - 31.18$

$-R_d$: $\quad \dfrac{1}{12}C_2H_5OH + \dfrac{1}{4}H_2O = \dfrac{1}{6}CO_2 + H^+ + e^-$ $\qquad \mathbf{= -109.90 \ kJ/e^- \ eq}$

R_e: $\quad \dfrac{1}{12}C_2H_5OH + \dfrac{1}{4}O_2 = \dfrac{1}{6}CO_2 + \dfrac{1}{4}H_2O$

Reaction free energy: general conditions

- For nonstandard conditions,

$$\Delta G_r = \Delta G_r{}^0 + RT \sum_{i=1}^{n} v_{ir} \ln a_i$$

a_i = activity of constituent A_i

R = gas constant, 8.314×10^{-3} kJ/mole − K

T = absolute temperature, K

Caution:

- v_{ir} is negative for reactants and positive for products
- $\Delta G_r{}^0$ is for standard conditions -- pH=0

 From $\Delta G_r{}^{0\prime}$, we can calculate $\Delta G_r{}^0$ by:
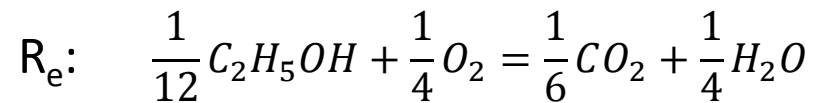
 $$\Delta G_r{}^0 = \Delta G_r{}^{0\prime} - RT v_{H+} \ln[10^{-7}]$$

Reaction free energy

Q3: Calculate the free energy of reaction for aerobic ethanol degradation at the following conditions: T = 20°C, pH = 5.0, $[C_2H_5OH] = 2 \times 10^{-3}$ M, $P_{CO2} = 3 \times 10^{-4}$ atm, $P_{O2} = 0.21$ atm.

From Q2,

$$\Delta G_r^{0'} = -109.90 \ kJ/e^- \ eq$$

R_e: $\quad \dfrac{1}{12} C_2 H_5 OH + \dfrac{1}{4} O_2 = \dfrac{1}{6} CO_2 + \dfrac{1}{4} H_2 O$

$$\Delta G_r^0 = \Delta G_r^{0'} - RT v_{H^+} ln[10^{-7}] = G_r^{0'} - 0 = G_r^{0'}$$

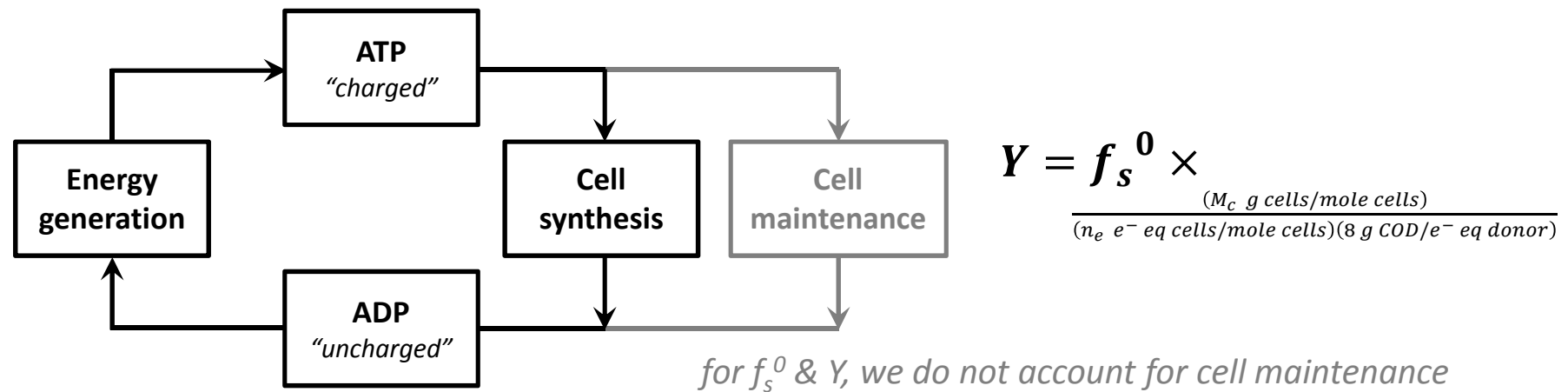$$\Delta G_r = \Delta G_r^0 + RT \sum_{i=1}^{n} v_{ir} ln a_i$$

$$= -109.90 \ kJ/e^- \ eq + (8.314 \times 10^{-3} \ kJ/mole - K) \times (293 \ K)$$

$$\times \left\{ -\frac{1}{12} ln(2 \times 10^{-3}) - \frac{1}{4} ln(0.21) + \frac{1}{6} ln(3 \times 10^{-4}) + \frac{1}{4} ln(1) \right\}$$

$$\boldsymbol{= -110.98 \ kJ/e^- \ eq}$$

$a_{H_2O} = 1$ *for aqueous solutions; you may omit the water term*

Deriving f_s^0 or Y by thermodynamic principles

$$Y = f_s^0 \times \frac{(M_c \; g \; cells/mole \; cells)}{(n_e \; e^- \; eq \; cells/mole \; cells)(8 \; g \; COD/e^- \; eq \; donor)}$$

for f_s^0 & Y, we do not account for cell maintenance

- To determine f_s^0 or Y we consider the energy balance for cell synthesis
  - energy is needed (ATP→ADP) to synthesize cells [1]
  - energy reaction (R_e) supplies the required energy (ADP→ATP) [2]
  - At steady state,
    energy required for [1] = energy supplied by [2]

Two steps of cell synthesis, the efficiency

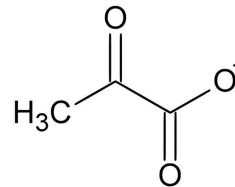- We conceptualize the cell synthesis mechanism into two steps

C-source

Organics (heterotrophs)
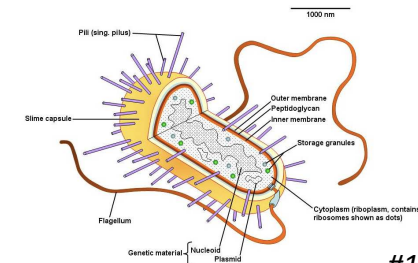
CO_2/HCO_3^- (autotrophs)

Building block

e.g., pyruvate (CH_3COCOO^-)

Cells

#1

1) Carbon source is metabolized to form a building block
2) The building block is synthesized to a cell

- We should consider that bacteria are not 100% efficient engines -- each step involves energy loss!

ΔG_p (C-source → pyruvate), ΔG_{pc} (pyruvate → cells)

- Energy required to convert carbon source to pyruvate, ΔG_p (heterotrophic bacteria, ammonia as N source):

$$\Delta G_p = 35.09 - \Delta G_c^{0\prime}$$

(in kJ/e⁻ eq)

35.09 = reaction free energy for formation of pyruvate from CO_2

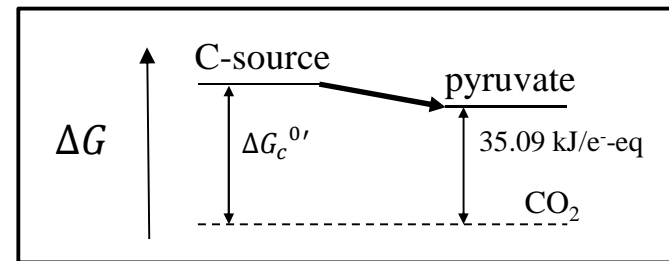*$\Delta G_c^{0\prime}$ = reaction free energy for formation of carbon source from CO_2*

- Energy required to convert pyruvate to cells, $\Delta G_{pc} =$ 18.8 kJ/e⁻ eq

(estimated value for a cell formula of $C_5H_7O_2N$ using NH_4^+ as a N source)

ΔG_s (C-source → cells)

- Energy required for cell synthesis from the carbon source, ΔG_s:
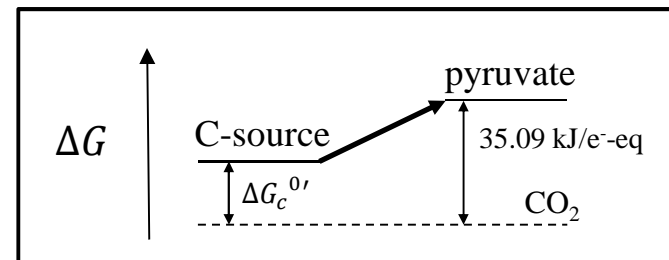
  if $\Delta G_p < 0$ (C-source is at higher energy state than pyruvate)

  $$\Delta G_s = \varepsilon \Delta G_p + \frac{\Delta G_{pc}}{\varepsilon}$$

  if $\Delta G_p > 0$ (C-source is at lower energy state than pyruvate)

  $$\Delta G_s = \frac{\Delta G_p}{\varepsilon} + \frac{\Delta G_{pc}}{\varepsilon}$$

  ε = energy transfer efficiency

Energy balance of overall reaction

- The energy obtained by an energy reaction is used to synthesize cells

- Energy obtained = Energy used (at steady state)

- Keep in mind that bacteria cannot use 100% of the energy available by an energy reaction (energy transfer efficiency, ε)

- Therefore we write

$$A\varepsilon\Delta G_r + \Delta G_s = 0$$

A = e$^-$ equivalent of e$^-$ donor used for energy production per e$^-$ equivalent of cells formed

ΔG_r = reaction free energy for energy reaction = ΔG_a - ΔG_d (per e$^-$-eq)

ΔG_s = energy required for cell synthesis from the carbon source (per e$^-$-eq)

Deriving f_s^0 or Y by thermodynamic principles

- Solving for A:

$$A = -\frac{\Delta G_p/\varepsilon^n + \Delta G_{pc}/\varepsilon}{\varepsilon \Delta G_r}$$

$n = -1$ for $\Delta G_p < 0$

$+1$ for $\Delta G_p > 0$

- From A, we can calculate f_s^0 and f_e^0 as:

$$f_s^{\,0} = \frac{1}{1+A} \qquad f_e^{\,0} = 1 - f_s^{\,0} = \frac{A}{1+A}$$

- Energy transfer efficiency, ε
  - 55-70% under optimal conditions
  - Use 0.6 for ordinary cases

Deriving f_s^0 or Y by thermodynamic principles

Q4: Estimate f_s^0 and Y for aerobic oxidation of acetate assuming ε=0.4 and 0.6 at standard conditions except for a pH of 7.0. Ammonia is available for cell synthesis.

From O-1, $\quad \Delta G_c^{0'} = 27.40 \ kJ/e^- \ eq$

$\Delta G_p = 35.09 - 27.40 = 7.69 \ kJ/e^- \ eq$

$\Delta G_r = \Delta G_a - \Delta G_d \quad = -78.72 - 27.40 = -106.12 \ kJ/e^- \ eq$

$\Delta G_d = 27.40 \ kJ/e^- \ eq, \quad \Delta G_a = -78.72 \ kJ/e^- \ eq$ (from I-14)

$\Delta G_p > 0 \ \rightarrow \ n = +1$

$$A = -\frac{\frac{\Delta G_p}{\varepsilon^n} + \frac{\Delta G_{pc}}{\varepsilon}}{\varepsilon \Delta G_r} = -\frac{\frac{7.69}{\varepsilon^{+1}} + \frac{18.8}{\varepsilon}}{-106.12\varepsilon}$$

$\epsilon = 0.4 : A = 1.56$

$\epsilon = 0.6 : A = 0.69$

$$f_s^{\ 0} = \frac{1}{1 + A}$$

$$Y = f_s^{\ 0} \frac{M_c}{n_e \cdot (8 \ g \ COD/e^- \ eq)}$$

$f_s^{\ 0} = 0.39, \ Y = 0.28 \ g \ cells/g \ COD$

$f_s^{\ 0} = 0.59, \ Y = 0.42 \ g \ cells/g \ COD$

References

#1) https://cronodon.com/BioTech/Bacteria.html