

# Bacterial energetics

# Today's class

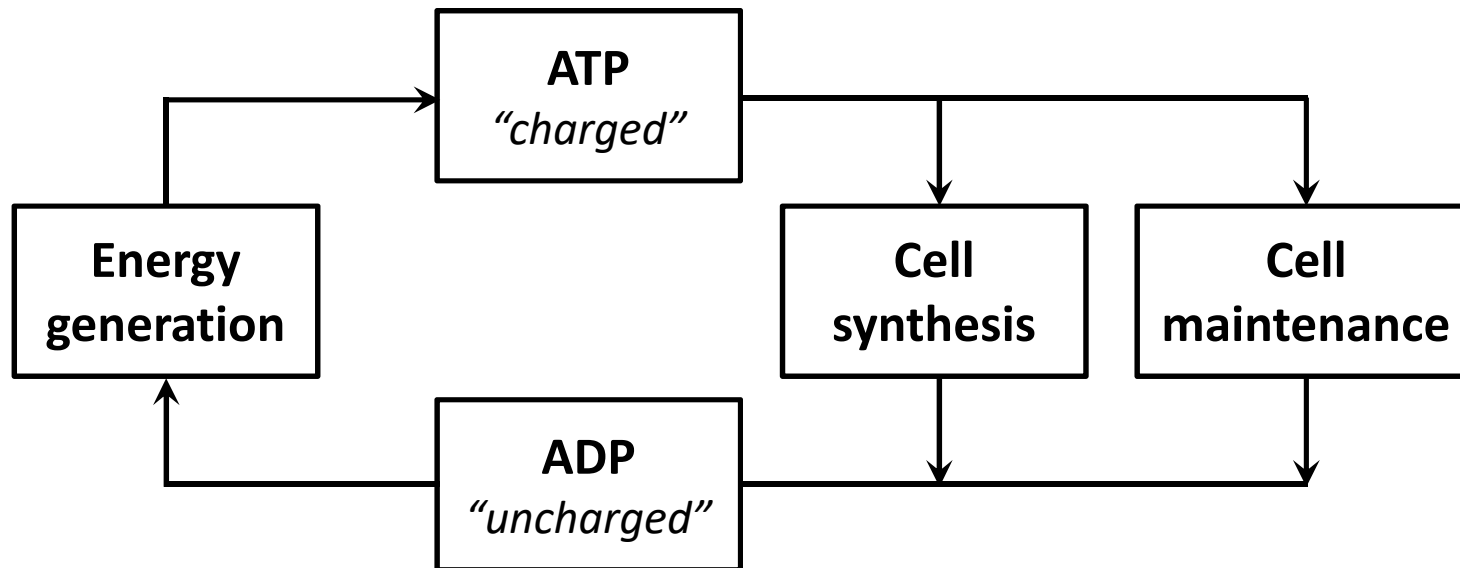
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- Bacterial energetics – overview
- Gibbs free energy of reaction
- Correlating reaction energetics and yield coefficient

# Bacterial energetics

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- Microorganisms carry out redox reactions to obtain energy for growth and cell maintenance



## Organic half-reactions and their Gibb's free energy at pH = 7.0 (1)

Textbook Table 2.2

Reaction Number	Reduced Compounds	Half-reaction		$\Delta G^{0'}$ (kJ/e <sup>-</sup> eq)
O-1	Acetate	$\frac{1}{8}CO_2 + \frac{1}{8}HCO_3^- + H^+ + e^-$	$= \frac{1}{8}CH_3COO^- + \frac{3}{8}H_2O$	27.40
O-2	Alanine	$\frac{1}{6}CO_2 + \frac{1}{12}HCO_3^- + \frac{1}{12}NH_4^+ + \frac{11}{12}H^+ + e^-$	$= \frac{1}{12}CH_3CHNH_2COO^- + \frac{5}{12}H_2O$	31.37
O-3	Benzoate	$\frac{1}{5}CO_2 + \frac{1}{30}HCO_3^- + H^+ + e^-$	$= \frac{1}{30}C_6H_5COO^- + \frac{13}{30}H_2O$	27.34
O-4	Citrate	$\frac{1}{6}CO_2 + \frac{1}{6}HCO_3^- + H^+ + e^-$	$= \frac{1}{18}(COO^-)CH_2COH(COO^-)CH_2COO^- + \frac{4}{9}H_2O$	33.08
O-5	Ethanol	$\frac{1}{6}CO_2 + H^+ + e^-$	$= \frac{1}{12}CH_3CH_2OH + \frac{1}{4}H_2O$	31.18
O-6	Formate	$\frac{1}{2}HCO_3^- + H^+ + e^-$	$= \frac{1}{2}HCOO^- + \frac{1}{2}H_2O$	39.19
O-7	Glucose	$\frac{1}{4}CO_2 + H^+ + e^-$	$= \frac{1}{24}C_6H_{12}O_6 + \frac{1}{4}H_2O$	41.35
O-8	Glutamate	$\frac{1}{6}CO_2 + \frac{1}{9}HCO_3^- + \frac{1}{18}NH_4^+ + H^+ + e^-$	$= \frac{1}{18}COOHCH_2CH_2CHNH_2COO^- + \frac{4}{9}H_2O$	30.93
O-9	Glycerol	$\frac{3}{14}CO_2 + H^+ + e^-$	$= \frac{1}{14}CH_2OHCHOHCH_2OH + \frac{3}{14}H_2O$	38.88
O-10	Glycine	$\frac{1}{6}CO_2 + \frac{1}{6}HCO_3^- + \frac{1}{6}NH_4^+ + H^+ + e^-$	$= \frac{1}{6}CH_2NH_2COOH + \frac{1}{2}H_2O$	39.80

- low concentration of inhibiting compounds

## Inorganic half-reactions and their Gibb's free energy at pH = 7.0

Textbook Table 2.1

Reaction Number	Reduced-oxidized Compounds	Half-reaction	$\Delta G^{0'}$ (kJ/e <sup>-</sup> eq)
I-1	Ammonium-nitrate	$\frac{1}{8}NO_3^- + \frac{5}{4}H^+ + e^- = \frac{1}{8}NH_4^+ + \frac{3}{8}H_2O$	-35.11
I-2	Ammonium-nitrite	$\frac{1}{6}NO_2^- + \frac{4}{3}H^+ + e^- = \frac{1}{6}NH_4^+ + \frac{1}{3}H_2O$	-32.93
I-3	Ammonium-Nitrogen	$\frac{1}{6}N_2 + \frac{4}{3}H^+ + e^- = \frac{1}{3}NH_4^+$	26.70
I-4	Ferrous-Ferric	$Fe^{3+} + e^- = Fe^{2+}$	-74.27
I-5	Hydrogen-H <sup>+</sup>	$H^+ + e^- = \frac{1}{2}H_2$	39.87
I-6	Nitrite-Nitrate	$\frac{1}{2}NO_3^- + H^+ + e^- = \frac{1}{2}NO_2^- + \frac{1}{2}H_2O$	-41.65
I-7	Nitrogen-Nitrate	$\frac{1}{5}NO_3^- + \frac{6}{5}H^+ + e^- = \frac{1}{10}N_2 + \frac{3}{5}H_2O$	-72.20
I-8	Nitrogen-Nitrite	$\frac{1}{3}NO_2^- + \frac{4}{3}H^+ + e^- = \frac{1}{6}N_2 + \frac{2}{3}H_2O$	-92.56
I-9	Sulfide-Sulfate	$\frac{1}{8}SO_4^{2-} + \frac{19}{16}H^+ + e^- = \frac{1}{16}H_2S + \frac{1}{16}HS^- + \frac{1}{2}H_2O$	20.85
I-10	Sulfide-Sulfite	$\frac{1}{6}SO_3^{2-} + \frac{5}{4}H^+ + e^- = \frac{1}{12}H_2S + \frac{1}{12}HS^- + \frac{1}{2}H_2O$	11.03
I-11	Sulfite-Sulfate	$\frac{1}{2}SO_4^{2-} + H^+ + e^- = \frac{1}{2}SO_3^{2-} + \frac{1}{2}H_2O$	50.30
I-12	Sulfur-Sulfate	$\frac{1}{6}SO_4^{2-} + \frac{4}{3}H^+ + e^- = \frac{1}{6}S + \frac{3}{2}H_2O$	19.15
I-13	Thiosulfate-Sulfate	$\frac{1}{4}SO_4^{2-} + \frac{5}{4}H^+ + e^- = \frac{1}{8}S_2O_3^{2-} + \frac{5}{8}H_2O$	23.58
I-14	Water-Oxygen	$\frac{1}{4}O_2 + H^+ + e^- = \frac{1}{2}H_2O$	-78.72

# Energetics and growth: thoughts

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- Bacterial cells grow more rapidly when:
  - More energy is obtained by oxidation of (an  $e^-$  equivalent of) an  $e^-$  donor
  - More energy is obtained by reduction of (an  $e^-$  equivalent of) an  $e^-$  acceptor
  - Conditions are favorable for rapid utilization of substrates
    - abundance of  $e^-$  donor,  $e^-$  acceptor, nutrients, etc.
    - low concentration of inhibiting compounds

# Energetics and growth: math

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$$Y_n = Y - b \frac{X_a}{-dS/dt}$$

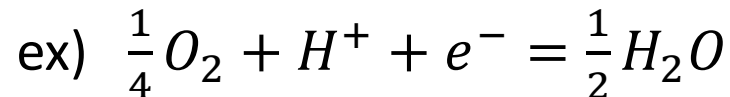
- $Y_n \uparrow$  when  $Y \uparrow$ : favorable e<sup>-</sup> donor and acceptor
- $Y_n \uparrow$  when  $X_a \ll -dS/dt$ : favorable conditions for substrate utilization

*\* We will discuss about the  $-dS/dt$  term in forthcoming lectures!*

# Gibbs free energy: standard conditions

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- “Standard” free energy
  - $\Delta G^0$ , free energy at 25°C, 1 atm, and unit activity for any chemicals involved
  - $\Delta G^{0'}$ , standard free energy adjusted to pH=7



$\Delta G^0$  is for

$$\begin{aligned}\{O_2\} &= P_{O_2} = 1 \text{ atm} \\ \{H^+\} &= 1 \text{ (pH = 0)} \\ \{H_2O\} &= 1\end{aligned}$$

$\Delta G^{0'}$  is for

$$\begin{aligned}\{O_2\} &= P_{O_2} = 1 \text{ atm} \\ \text{pH=7} \quad (\{H^+\} &= 10^{-7}) \\ \{H_2O\} &= 1\end{aligned}$$

$\{H_2O\} = 1$  applies to any dilute aqueous solutions



# Free energy of formation, $\Delta G_f$

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- Gibbs free energy that accompanies the formation of 1 mole of the substance from its component elements

# Gibbs free energy change of reaction

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- Free energy change of reaction,  $\Delta G_r$   
 $\Sigma(\text{product } \Delta G_f) - \Sigma(\text{reactant } \Delta G_f)$

*For a generic reaction written as*

$$0 = \sum_{i=1}^n v_{ir} A_i$$

*$v_{ir}$  = stoichiometric coefficient, (-) for reactants, (+) for products*

*$A_i$  = reaction constituent, reactants or products*

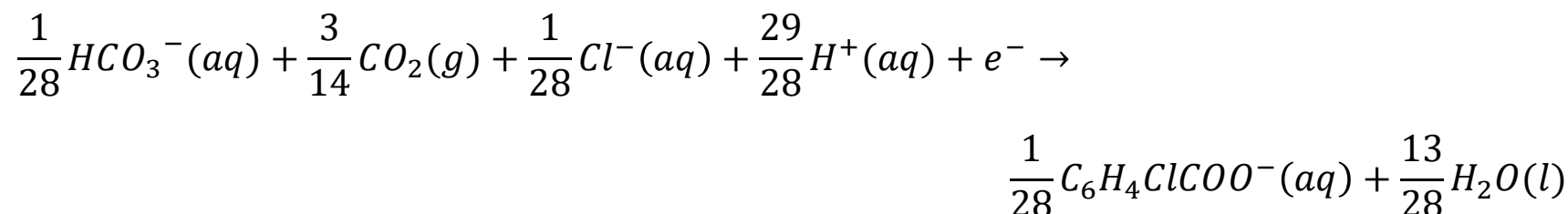
$\Delta G_r$  can be written as

$$\Delta G_r = \sum_{i=1}^n v_{ir} \Delta G_f$$

# Reaction free energy

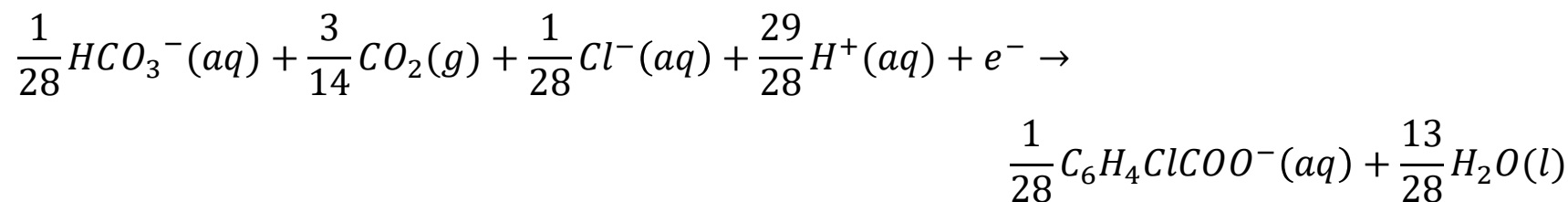
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**Q1:** Calculate the standard free energy adjusted pH 7 for the half reaction of 2-chlorobenzoate formation as given below.



Use the following values of free energy of formation (in textbook Appendix A):

Species	HCO <sub>3</sub> <sup>-</sup> (aq)	CO <sub>2</sub> (g)	Cl <sup>-</sup> (aq)	H <sup>+</sup> (aq, 10 <sup>-7</sup> )	C <sub>6</sub> H <sub>4</sub> ClCOO <sup>-</sup> (aq)	H <sub>2</sub> O(l)
ΔG <sub>f</sub> <sup>0'</sup> @ 25°C (kJ/mole)	-586.85	-394.36	-31.35	-39.87	-237.9	-237.18



Species	HCO <sub>3</sub> <sup>-</sup> (aq)	CO <sub>2</sub> (g)	Cl <sup>-</sup> (aq)	H <sup>+</sup> (aq, 10 <sup>-7</sup> )	C <sub>6</sub> H <sub>4</sub> ClCOO <sup>-</sup> (aq)	H <sub>2</sub> O(l)
ΔG <sub>f</sub> <sup>0'</sup> @ 25°C (kJ/mole)	-586.85	-394.36	-31.35	-39.87	-237.9	-237.18

$$\Delta G_r = \sum_{i=1}^n v_{ir} \Delta G_f$$

*v<sub>ir</sub> = stoichiometric coefficient, (-) for reactants, (+) for products*

$$\Delta G_r^{0'} = \left( - \right) \left\{ \frac{1}{28} \times (-586.85) + \frac{3}{14} \times (-394.36) + \frac{1}{28} \times (-31.35) + \frac{29}{28} \times (-39.87) \right\} + \left\{ \frac{1}{28} \times (-237.9) + \frac{13}{28} \times (-237.18) \right\}$$

$$= 29.26 \text{ kJ/e}^- \text{ eq}$$

# Reaction free energy

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**Q2:** Calculate the standard free energy adjusted pH 7 for overall energy reaction with ethanol as an  $e^-$  donor and oxygen as an  $e^-$  acceptor. Use the half reactions listed in Table 2.2 and 2.3.

$$R_d: \text{O-5} \quad \Delta G_r^{0'} = 31.18 \text{ kJ}/e^- \text{ eq}$$

$$R_a: \text{I-14} \quad \Delta G_r^{0'} = -78.72 \text{ kJ}/e^- \text{ eq}$$

$$R_e = R_a - R_d \quad \Delta G_r^{0'}(R_e) = \Delta G_r^{0'}(R_a) - \Delta G_r^{0'}(R_d)$$

$$R_a: \quad \frac{1}{4}O_2 + H^+ + e^- = \frac{1}{2}H_2O \quad = -78.72 - 31.18$$

$$-R_d: \quad \frac{1}{12}C_2H_5OH + \frac{1}{4}H_2O = \frac{1}{6}CO_2 + H^+ + e^- \quad = -109.90 \text{ kJ}/e^- \text{ eq}$$

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$$R_e: \quad \frac{1}{12}C_2H_5OH + \frac{1}{4}O_2 = \frac{1}{6}CO_2 + \frac{1}{4}H_2O$$

# Reaction free energy: general conditions

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- For nonstandard conditions,

$$\Delta G_r = \Delta G_r^0 + RT \sum_{i=1}^n v_{ir} \ln a_i$$

$a_i$  = activity of constituent  $A_i$

$R$  = gas constant,  $8.314 \times 10^{-3}$  kJ/mole – K

$T$  = absolute temperature, K

### Caution:

-  $v_{ir}$  is negative for reactants and positive for products

-  $\Delta G_r^0$  is for standard conditions -- pH=0

From  $\Delta G_r^{0'}$ , we can calculate  $\Delta G_r^0$  by:

$$\Delta G_r^0 = \Delta G_r^{0'} - RT v_{H^+} \ln[10^{-7}]$$

# Reaction free energy

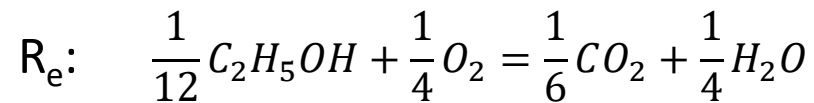
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**Q3:** Calculate the free energy of reaction for aerobic ethanol degradation at the following conditions:  $T = 20^{\circ}\text{C}$ ,  $\text{pH} = 5.0$ ,  $[\text{C}_2\text{H}_5\text{OH}] = 2 \times 10^{-3} \text{ M}$ ,  $P_{\text{CO}_2} = 3 \times 10^{-4} \text{ atm}$ ,  $P_{\text{O}_2} = 0.21 \text{ atm}$ .



From Q2,

$$\Delta G_r^{0'} = -109.90 \text{ kJ}/e^- \text{ eq}$$



$$\Delta G_r^0 = \Delta G_r^{0'} - RTv_H + \ln[10^{-7}] = G_r^{0'} - 0 = G_r^{0'}$$

$$\Delta G_r = \Delta G_r^0 + RT \sum_{i=1}^n v_{ir} \ln a_i$$

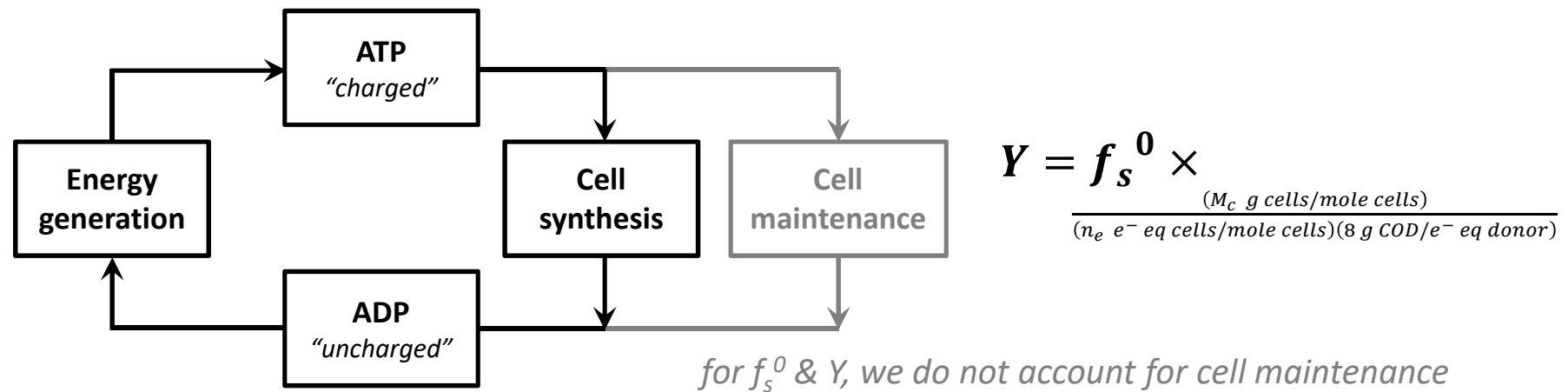
$$= -109.90 \text{ kJ}/e^- \text{ eq} + (8.314 \times 10^{-3} \text{ kJ}/\text{mole} - K) \times (293 \text{ K})$$

$$\times \left\{ -\frac{1}{12} \ln(2 \times 10^{-3}) - \frac{1}{4} \ln(0.21) + \frac{1}{6} \ln(3 \times 10^{-4}) + \frac{1}{4} \ln(1) \right\}$$

$$= -110.98 \text{ kJ}/e^- \text{ eq}$$

*$a_{H_2O} = 1$  for aqueous solutions; you may omit the water term*

# Deriving $f_s^0$ or $Y$ by thermodynamic principles



- To determine  $f_s^0$  or  $Y$  we consider the energy balance for cell synthesis
  - energy is needed (ATP  $\rightarrow$  ADP) to synthesize cells [1]
  - energy reaction ( $R_e$ ) supplies the required energy (ADP  $\rightarrow$  ATP) [2]
  - At steady state,  
energy required for [1] = energy supplied by [2]

# Two steps of cell synthesis, the efficiency

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- We conceptualize the cell synthesis mechanism into two steps

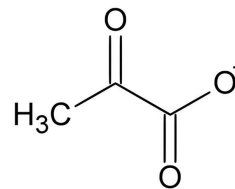
## C-source

Organics (heterotrophs)  
 $CO_2/HCO_3^-$  (autotrophs)

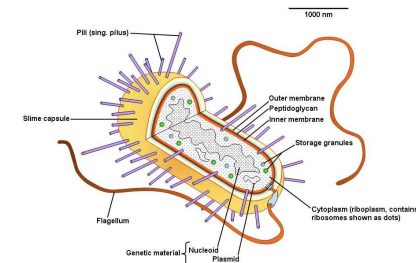


## Building block

e.g., pyruvate ( $CH_3COCOO^-$ )



## Cells



- 1) Carbon source is metabolized to form a building block
- 2) The building block is synthesized to a cell

- We should consider that bacteria are not 100% efficient engines -- each step involves energy loss!

## $\Delta G_p$ (C-source $\rightarrow$ pyruvate), $\Delta G_{pc}$ (pyruvate $\rightarrow$ cells)

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- Energy required to convert carbon source to pyruvate,  $\Delta G_p$  (heterotrophic bacteria, ammonia as N source):

$$\Delta G_p = 35.09 - \Delta G_c^{0'}$$

(in kJ/e<sup>-</sup> eq)

*35.09 = reaction free energy for formation of pyruvate from CO<sub>2</sub>*

*$\Delta G_c^{0'}$  = reaction free energy for formation of carbon source from CO<sub>2</sub>*

- Energy required to convert pyruvate to cells,  $\Delta G_{pc} = 18.8$  kJ/e<sup>-</sup> eq  
(estimated value for a cell formula of C<sub>5</sub>H<sub>7</sub>O<sub>2</sub>N using NH<sub>4</sub><sup>+</sup> as a N source)

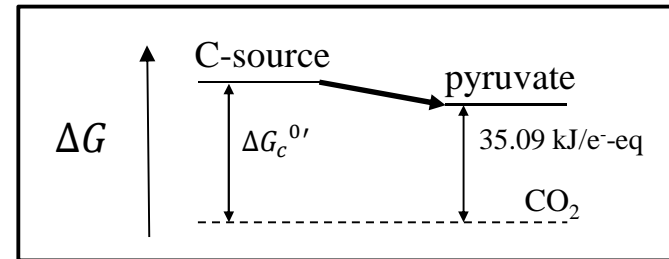
# $\Delta G_S$ (C-source $\rightarrow$ cells)

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- Energy required for cell synthesis from the carbon source,  $\Delta G_S$ :

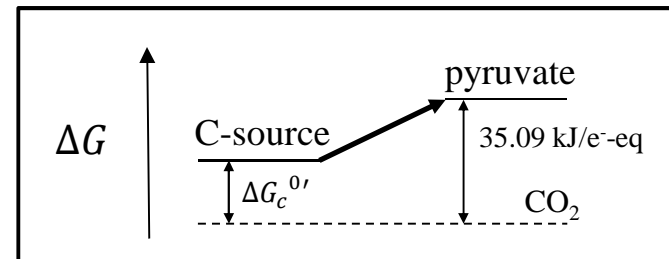
if  $\Delta G_p < 0$  (C-source is at higher energy state than pyruvate)

$$\Delta G_S = \varepsilon \Delta G_p + \frac{\Delta G_{pc}}{\varepsilon}$$



if  $\Delta G_p > 0$  (C-source is at lower energy state than pyruvate)

$$\Delta G_S = \frac{\Delta G_p}{\varepsilon} + \frac{\Delta G_{pc}}{\varepsilon}$$



$\varepsilon$  = energy transfer efficiency

# Energy balance of overall reaction

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- The energy obtained by an energy reaction is used to synthesize cells
- Energy obtained = Energy used (at steady state)
- Keep in mind that bacteria cannot use 100% of the energy available by an energy reaction (energy transfer efficiency,  $\varepsilon$ )
- Therefore we write

$$A\varepsilon\Delta G_r + \Delta G_s = 0$$

*A = e<sup>-</sup> equivalent of e<sup>-</sup> donor used for energy production per e<sup>-</sup> equivalent of cells formed*

*$\Delta G_r$  = reaction free energy for energy reaction =  $\Delta G_a - \Delta G_d$  (per e<sup>-</sup>-eq)*

*$\Delta G_s$  = energy required for cell synthesis from the carbon source (per e<sup>-</sup>-eq)*

# Deriving $f_s^0$ or $Y$ by thermodynamic principles

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- Solving for  $A$ :
$$A = -\frac{\Delta G_p / \varepsilon^n + \Delta G_{pc} / \varepsilon}{\varepsilon \Delta G_r}$$

$n = -1$  for  $\Delta G_p < 0$   
 $+1$  for  $\Delta G_p > 0$

- From  $A$ , we can calculate  $f_s^0$  and  $f_e^0$  as:

$$f_s^0 = \frac{1}{1 + A} \quad f_e^0 = 1 - f_s^0 = \frac{A}{1 + A}$$

- Energy transfer efficiency,  $\varepsilon$ 
  - 55-70% under optimal conditions
  - Use 0.6 for ordinary cases

## Deriving $f_s^0$ or $Y$ by thermodynamic principles

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**Q4:** Estimate  $f_s^0$  and  $Y$  for aerobic oxidation of acetate assuming  $\epsilon=0.4$  and  $0.6$  at standard conditions except for a pH of 7.0. Ammonia is available for cell synthesis.



From O-1,  $\Delta G_c^{0'} = 27.40 \text{ kJ}/e^- \text{ eq}$

$$\Delta G_p = 35.09 - 27.40 = 7.69 \text{ kJ}/e^- \text{ eq}$$

$$\Delta G_r = \Delta G_a - \Delta G_d = -78.72 - 27.40 = -106.12 \text{ kJ}/e^- \text{ eq}$$

$$\Delta G_d = 27.40 \text{ kJ}/e^- \text{ eq}, \quad \Delta G_a = -78.72 \text{ kJ}/e^- \text{ eq} \quad (\text{from I-14})$$

$$\Delta G_p > 0 \rightarrow n = +1$$

$$A = -\frac{\frac{\Delta G_p}{\epsilon^n} + \frac{\Delta G_{pc}}{\epsilon}}{\epsilon \Delta G_r} = -\frac{\frac{7.69}{\epsilon^{+1}} + \frac{18.8}{\epsilon}}{-106.12\epsilon}$$

$\epsilon = 0.4 : A = 1.56$	$f_s^0 = \frac{1}{1 + A}$	$f_s^0 = 0.39, Y = 0.28 \text{ g cells/g COD}$
$\epsilon = 0.6 : A = 0.69$	$\xrightarrow{\hspace{10em}}$	$f_s^0 = 0.59, Y = 0.42 \text{ g cells/g COD}$
$Y = f_s^0 \frac{M_c}{n_e \cdot (8 \text{ g COD}/e^- \text{ eq})}$		