Microbial kinetics

Today's lecture

- Monod kinetics
- Addressing decay
- Relating the substrate utilization with the microbial growth

Monod equation

$$\mu_{syn} = \left(\frac{1}{X_a} \cdot \frac{dX_a}{dt} \right)_{syn} = \hat{\mu} \frac{S}{K + S}$$

where μ_{syn} = specific growth rate due to synthesis (T^{-1})

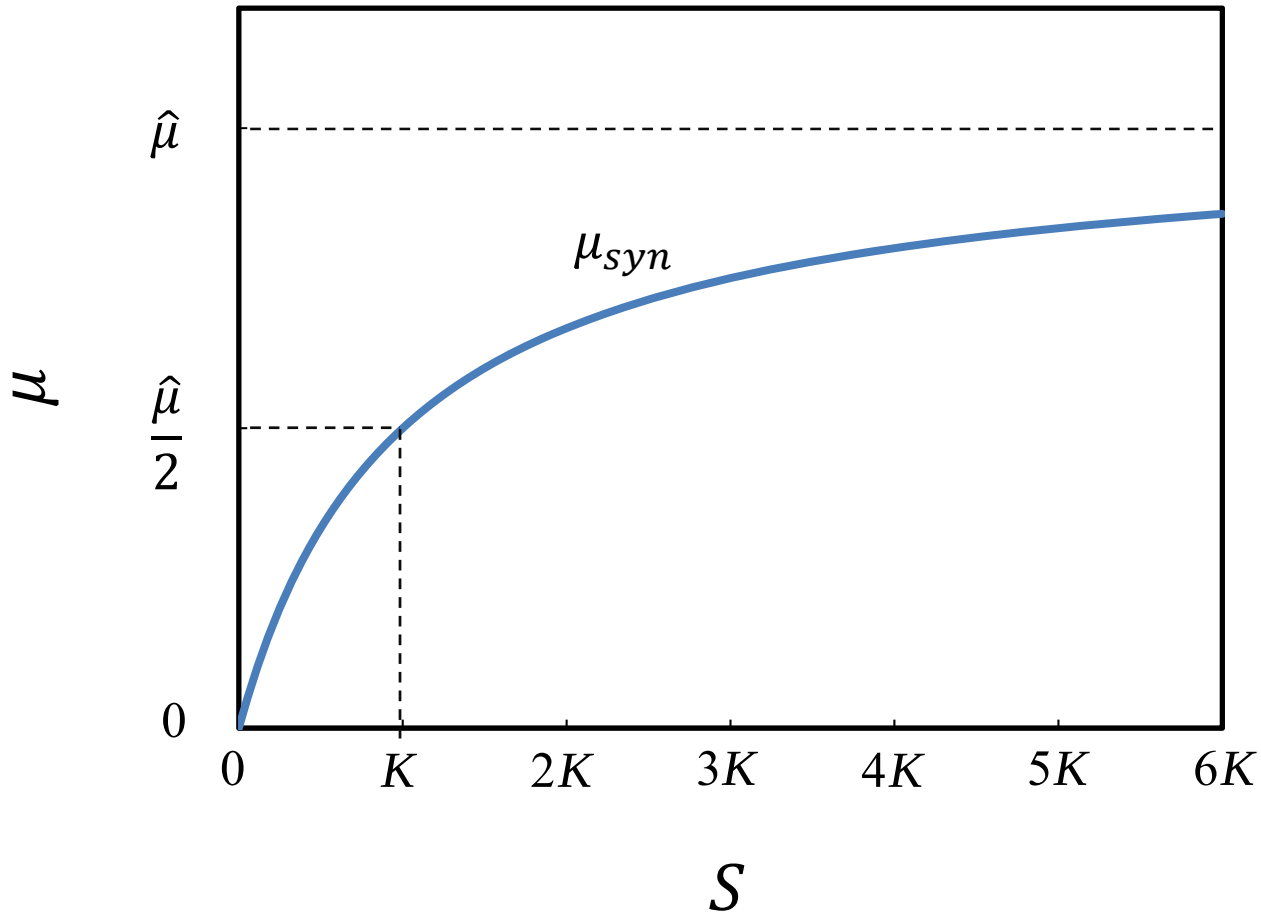
X_a = concentration of active biomass ($M_x L^{-3}$)

S = concentration of the rate-limiting substrate ($M_s L^{-3}$)

$\hat{\mu}$ = maximum specific growth rate (T^{-1})

K = half saturation coefficient ($M_s L^{-3}$)

Monod equation: S vs. μ



Typical values for $f_s{}^o$, Y, $\hat{q}$, and $\hat{\mu}$

Table 3.1 Typical f_s^0, Y, $\hat{q}$, and $\hat{\mu}$ values for key bacterial types in environmental biotechnology

Organism Type	Electron Donor	Electron Acceptors	C-Source	f_s^0	Y	$\hat{q}$	$\hat{\mu}$
Aerobic, Heterotrophs	Carbohydrate BOD	O_2	BOD	0.7	0.49 gVSS/gBOD$_L$	27 gBOD$_L$/gVSS-d	13.2
	Other BOD	O_2	BOD	0.6	0.42 gVSS/gBOD$_L$	20 gBOD$_L$/gVSS-d	8.4
Denitrifiers	BOD	NO_3^-	BOD	0.5	0.25 gVSS/gBOD$_L$	16 gBOD$_L$/gVSS-d	4
	H_2	NO_3^-	CO_2	0.2	0.81 gVSS/gH_2	1.25 gH_2/gVSS-d	1
	S(s)	NO_3^-	CO_2	0.2	0.15 gVSS/gS	6.7 gS/gVSS-d	1
Nitrifying Autotrophs	NH_4^+	O_2	CO_2	0.14	0.34 gVSS/gNH_4^+-N	2.7 gNH_4^+-N/gVSS-d	0.92
	NO_2^-	O_2	CO_2	0.10	0.08 gVSS/gNO_2^--N	7.8 gNO_2^--N/gVSS-d	0.62
Methanogens	acetate BOD	acetate	acetate	0.05	0.035 gVSS/gBOD$_L$	8.4 gBOD$_L$/gVSS-d	0.3
	H_2	CO_2	CO_2	0.08	0.45 gVSS/gH_2	1.1 gH_2/g VSS-d	0.5
Sulfide Oxidizing Autotrophs	H_2S	O_2	CO_2	0.2	0.28 gVSS/gH_2S-S	5 gS/gVSS-d	1.4
Sulfate Reducers	H_2	SO_4^{2-}	CO_2	0.05	0.28 gVSS/gH_2	1.05 gH_2/gVSS-d	0.29
	acetate BOD	SO_4^{2-}	acetate	0.08	0.057 gVSS/gBOD$_L$	8.7 gBOD$_L$/gVSS-d	0.5
Fermenters	sugar BOD	sugars	sugars	0.18	0.13 gVSS/gBOD$_L$	9.8 gBOD$_L$/gVSS-d	1.2

Y is computed assuming a cellular VSS$_a$ composition of $C_5H_7O_2N$, and NH_4^+ is the N source, except when NO_3^- is the electron acceptor; then NO_3^- is the N source. The typical units on Y are presented.

$\hat{q}$ is computed using $\hat{q} = 1e^-$ eq/gVSS$_a$-d.

$\hat{\mu}$ has units of d^{-1}.

Typical values for K

Process	K (mg substrate/L)
Aerobic:	
organic mixtures	50-150 mg COD/L
single organics	1-10 mg COD/L
nitrification	0.4-2 mg NH_3-N/L
Anaerobic:	
denitrification	0.06-0.20 mg NO_3^--N/L
methane fermentation:	
acetate, propionate	600-900 mg COD/L
sewage sludge	2000-3000 mg COD/L

Growth kinetics with decay

- As discussed in the previous lecture, we assume decay is proportional to cell biomass

$$\left(\frac{dX_a}{dt}\right)_{decay} = -bX_a$$

in the form of specific growth rate,

$$\mu_{dec} = \left(\frac{1}{X_a} \cdot \frac{dX_a}{dt}\right)_{decay} = -b$$

where μ_{dec} *= specific growth rate due to decay* (T^{-1})

b *= decay coefficient* (T^{-1})

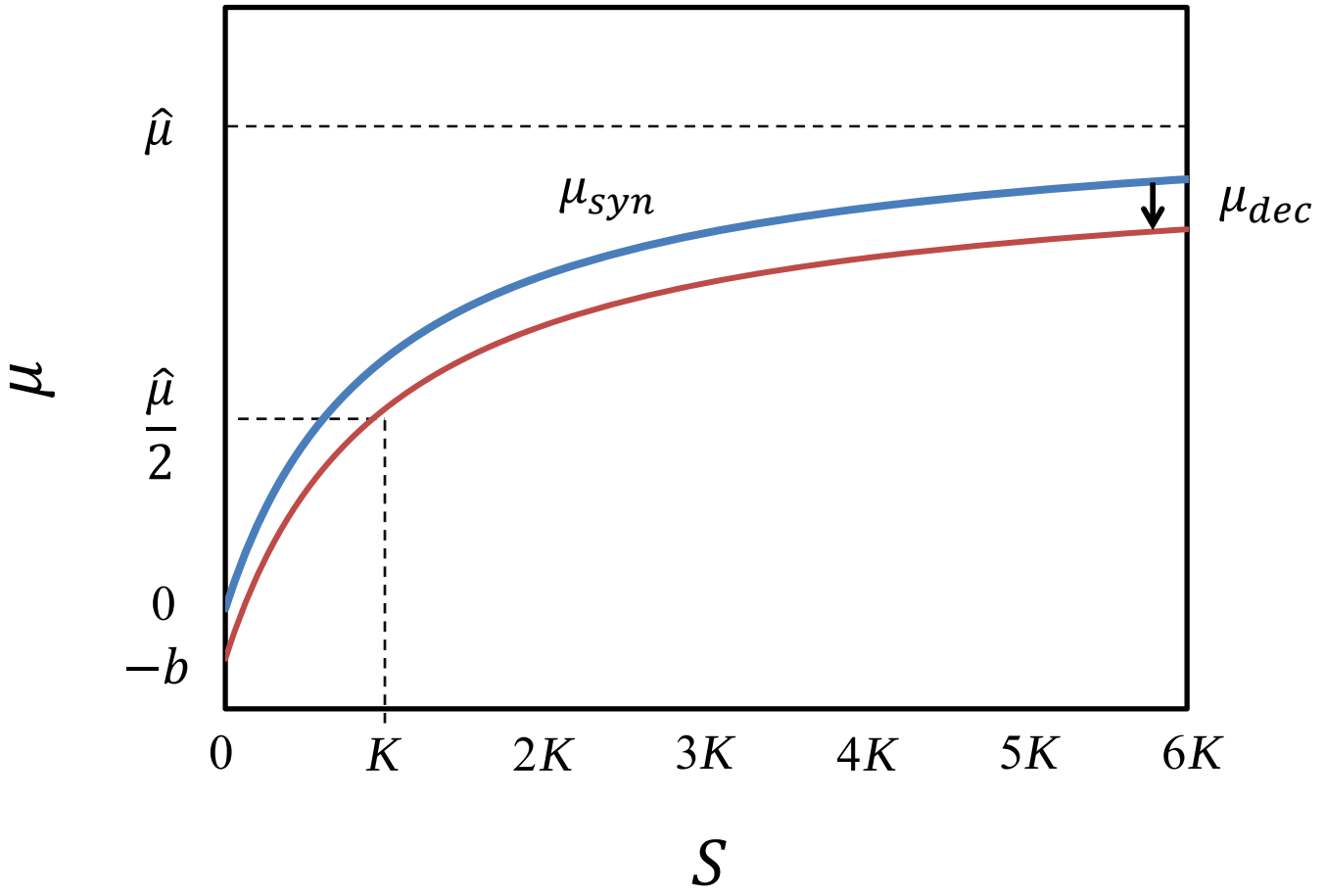
Overall bacterial growth kinetics

(Net growth) = (New growth) + (Decay)

$$\mu = \frac{1}{X_a} \cdot \frac{dX_a}{dt} = \mu_{syn} + \mu_{dec} = \hat{\mu} \frac{S}{K + S} - b$$

where μ *= net specific growth rate* (T^{-1})

Growth kinetics with decay



More on decay

$$\mu_{dec} = \left(\frac{1}{X_a} \cdot \frac{dX_a}{dt}\right)_{decay} = -b$$

- Most fraction (f_d≈0.8) is oxidized
- The other fraction (1-f_d≈0.2) is accumulated as inert biomass

Rate of oxidation (respiration): $\left(\frac{1}{X_a} \cdot \frac{dX_a}{dt}\right)_{resp} = -f_d b$

Rate of conversion to inert biomass:

$$\left(\frac{1}{X_a} \cdot \frac{dX_a}{dt}\right)_{inert} = -\frac{1}{X_a} \cdot \frac{dX_i}{dt} = -(1 - f_d)b$$

X_i = inert biomass ($M_x L^{-3}$)

Substrate utilization rate

Recall that,

$$Y = \frac{(g\ cells\ produced)}{(g\ substrate\ utilized)} = \frac{(dX_a/dt)_{syn}}{-dS/dt}$$

and

$$\mu_{syn} = \left(\frac{1}{X_a} \cdot \frac{dX_a}{dt}\right)_{syn} = \hat{\mu}\frac{S}{K+S}$$

So Monod equation can be also written as:

$$\frac{dS}{dt} = -\frac{1}{Y}\left(\frac{dX_a}{dt}\right)_{syn} = -\frac{\hat{\mu}}{Y}\frac{S}{K+S}X_a$$

Substrate utilization rate

Substrate utilization rate, r_{ut} [$M_s L^{-3} T^{-1}$]

$$r_{ut} = \frac{dS}{dt} = -\frac{\hat{q} S}{K + S} X_a$$

$\hat{q} = \hat{\mu}/Y$, max. specific rate of substrate utilization ($M_s M_x^{-1} T^{-1}$)

Recall that,

$$Y = f_s{}^0 \frac{M_c}{n_e \cdot (8\ g\ COD/e^-\ eq)}$$