

Microbial kinetics

Today's lecture

- Monod kinetics
- Addressing decay
- Relating the substrate utilization with the microbial growth

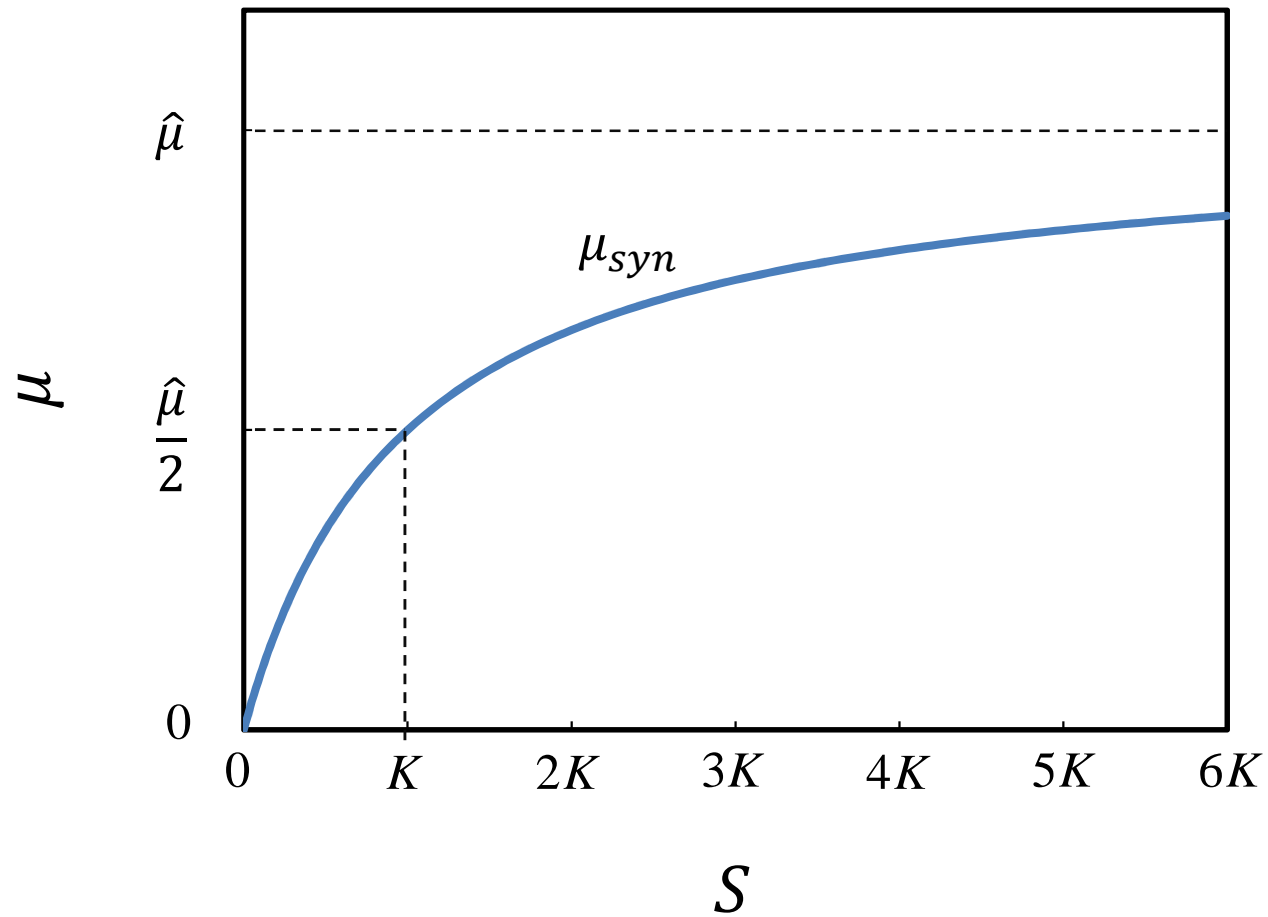
Monod equation

$$\mu_{syn} = \left(\frac{1}{X_a} \cdot \frac{dX_a}{dt} \right)_{syn} = \hat{\mu} \frac{S}{K + S}$$

where

- μ_{syn} = specific growth rate due to synthesis (T^{-1})
- X_a = concentration of active biomass ($M_x L^{-3}$)
- S = concentration of the rate-limiting substrate ($M_s L^{-3}$)
- $\hat{\mu}$ = maximum specific growth rate (T^{-1})
- K = half saturation coefficient ($M_s L^{-3}$)

Monod equation: S vs. μ



Typical values for f_s^o , Y , \hat{q} , and $\hat{\mu}$

Organism Type	Electron Donor	Electron Acceptors	C-Source	f_s^o	Y	\hat{q}	$\hat{\mu}$
Aerobic, Heterotrophs	Carbohydrate BOD	O ₂	BOD	0.7	0.49 gVSS/gBOD _L	27 gBOD _L /gVSS-d	13.2
	Other BOD	O ₂	BOD	0.6	0.42 gVSS/gBOD _L	20 gBOD _L /gVSS-d	8.4
Denitrifiers	BOD	NO ₃ ⁻	BOD	0.5	0.25 gVSS/gBOD _L	16 gBOD _L /gVSS-d	4
	H ₂	NO ₃ ⁻	CO ₂	0.2	0.81 gVSS/gH ₂	1.25 gH ₂ /gVSS-d	1
	S(s)	NO ₃ ⁻	CO ₂	0.2	0.15 gVSS/gS	6.7 gS/gVSS-d	1
Nitrifying Autotrophs	NH ₄ ⁺	O ₂	CO ₂	0.14	0.34 gVSS/gNH ₄ ⁺ -N	2.7 gNH ₄ ⁺ -N/gVSS-d	0.92
	NO ₂ ⁻	O ₂	CO ₂	0.10	0.08 gVSS/gNO ₂ ⁻ -N	7.8 gNO ₂ ⁻ -N/gVSS-d	0.62
Methanotrophs	Acetate BOD	acetate	acetate	0.05	0.035 gVSS/gBOD _L	8.4 gBOD _L /gVSS-d	0.3
	H ₂	CO ₂	CO ₂	0.08	0.45 gVSS/gH ₂	1.1 gH ₂ /gVSS-d	0.5
Sulfide Oxidizing Autotrophs	H ₂ S	O ₂	CO ₂	0.2	0.28 gVSS/gH ₂ S-S	5 gS/gVSS-d	1.4
Sulfate Reducers	H ₂	SO ₄ ²⁻	CO ₂	0.05	0.28 gVSS/gH ₂	1.05 gH ₂ /gVSS-d	0.29
	Acetate BOD	SO ₄ ²⁻	acetate	0.08	0.057 gVSS/gBOD _L	8.7 gBOD _L /gVSS-d	0.5
Fermenters	Sugar BOD	sugars	sugars	0.18	0.13 gVSS/gBOD _L	9.8 gBOD _L /gVSS-d	1.2

Y is computed assuming a cellular VSS_a composition of C₅H₇O₂N, and NH₄⁺ is the N source, except when NO₃⁻ is the electron acceptor; then NO₃⁻ is the N source.

$\hat{\mu}$ has units of d⁻¹.

Source: Environmental Biotechnology textbook

Typical values for K

Process	K (mg substrate/L)
Aerobic: organic mixtures single organics nitrification	50-150 mg COD/L 1-10 mg COD/L 0.4-2 mg $\text{NH}_3\text{-N/L}$
Anaerobic: denitrification methane fermentation: acetate, propionate sewage sludge	0.06-0.20 mg $\text{NO}_3^- \text{-N/L}$ 600-900 mg COD/L 2000-3000 mg COD/L

Addressing decay

- As discussed in the previous lecture, we assume decay is proportional to cell biomass

$$\left(\frac{dX_a}{dt}\right)_{decay} = -bX_a$$

in the form of specific growth rate,

$$\mu_{dec} = \left(\frac{1}{X_a} \cdot \frac{dX_a}{dt}\right)_{decay} = -b$$

where μ_{dec} = specific growth rate due to decay (T^{-1})
 b = decay coefficient (T^{-1})

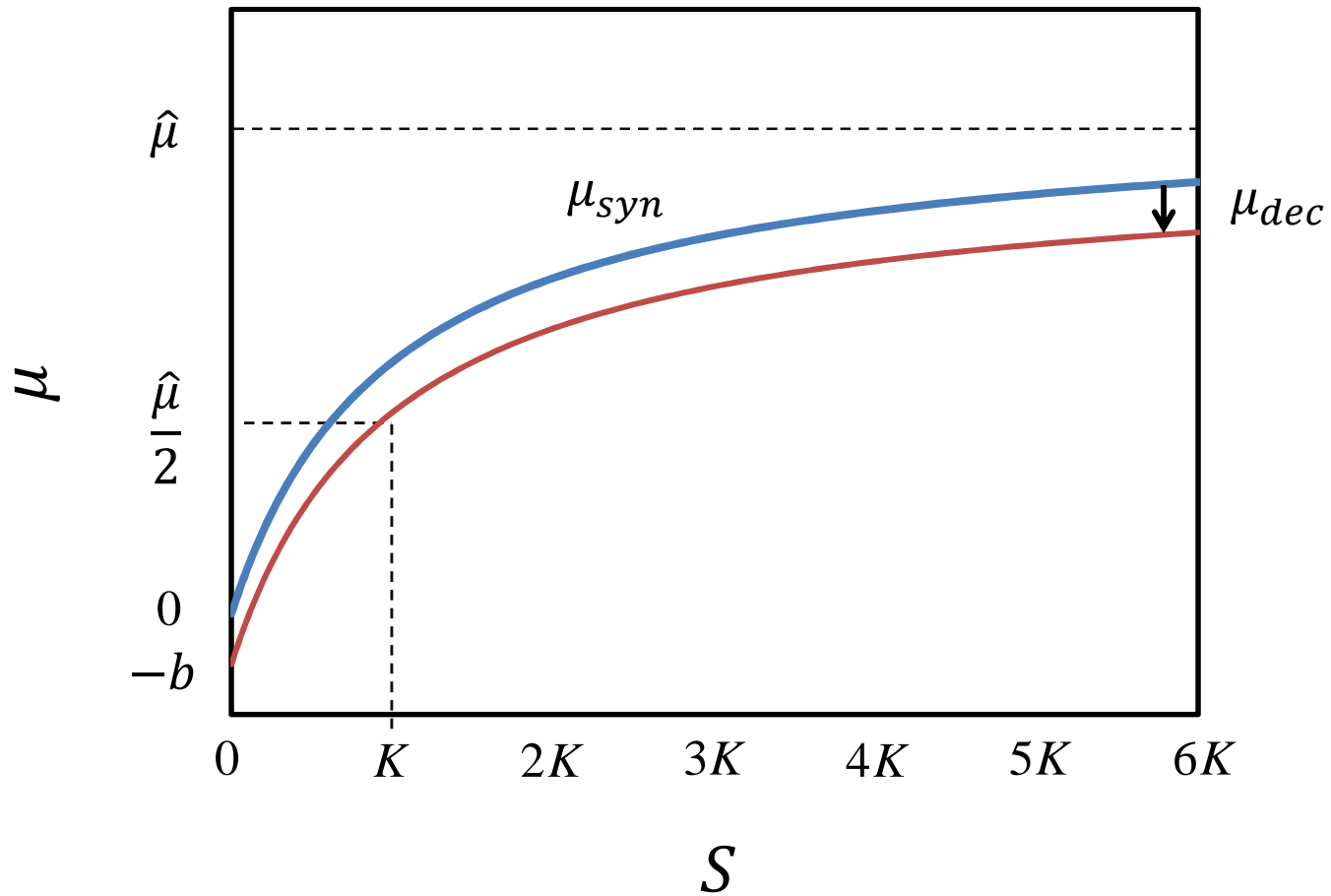
Overall bacterial growth kinetics

(Net growth) = (New growth) + (Decay)

$$\mu = \frac{1}{X_a} \cdot \frac{dX_a}{dt} = \mu_{syn} + \mu_{dec} = \hat{\mu} \frac{S}{K + S} - b$$

where μ = net specific growth rate (T^{-1})

Growth kinetics with decay



More on decay

$$\mu_{dec} = \left(\frac{1}{X_a} \cdot \frac{dX_a}{dt} \right)_{decay} = -b$$

- Most fraction ($f_d \approx 0.8$) is oxidized
- The other fraction ($1 - f_d \approx 0.2$) is accumulated as inert biomass

Rate of oxidation (respiration): $\left(\frac{1}{X_a} \cdot \frac{dX_a}{dt} \right)_{resp} = -f_d b$

Rate of conversion to inert biomass:

$$\left(\frac{1}{X_a} \cdot \frac{dX_a}{dt} \right)_{inert} = -\frac{1}{X_a} \cdot \frac{dX_i}{dt} = -(1 - f_d)b$$

$X_i = \text{inert biomass } (M_x L^{-3})$

Substrate utilization rate

Recall that,

$$Y = \frac{(\text{g new cells produced})}{(\text{g substrate utilized})} = \frac{(dX_a/dt)_{syn}}{-dS/dt}$$

and

$$\mu_{syn} = \left(\frac{1}{X_a} \cdot \frac{dX_a}{dt} \right)_{syn} = \frac{1}{X_a} \left(\frac{dX_a}{dt} \right)_{syn} = \hat{\mu} \frac{S}{K + S}$$

So Monod equation can be also written as:

$$\frac{dS}{dt} = -\frac{1}{Y} \left(\frac{dX_a}{dt} \right)_{syn} = -\frac{\hat{\mu}}{Y} \frac{S}{K + S} X_a$$

Substrate utilization rate, r_{ut} [$M_s L^{-3} T^{-1}$]

$$r_{ut} = \frac{dS}{dt} = -\frac{\hat{q}S}{K + S} X_a$$

$\hat{q} = \hat{\mu}/Y$, max. specific rate of substrate utilization ($M_s M_x^{-1} T^{-1}$)