Microbial kinetics

Today's class

- Monod kinetics
- Addressing decay
- Relating the substrate utilization with the microbial growth

Monod equation

$$\mu_{syn} = \left(\frac{1}{X_a} \cdot \frac{dX_a}{dt}\right)_{syn} = \hat{\mu}\frac{S}{K + S}$$

where μ_{syn} = *specific growth rate due to synthesis* (T^{-1})

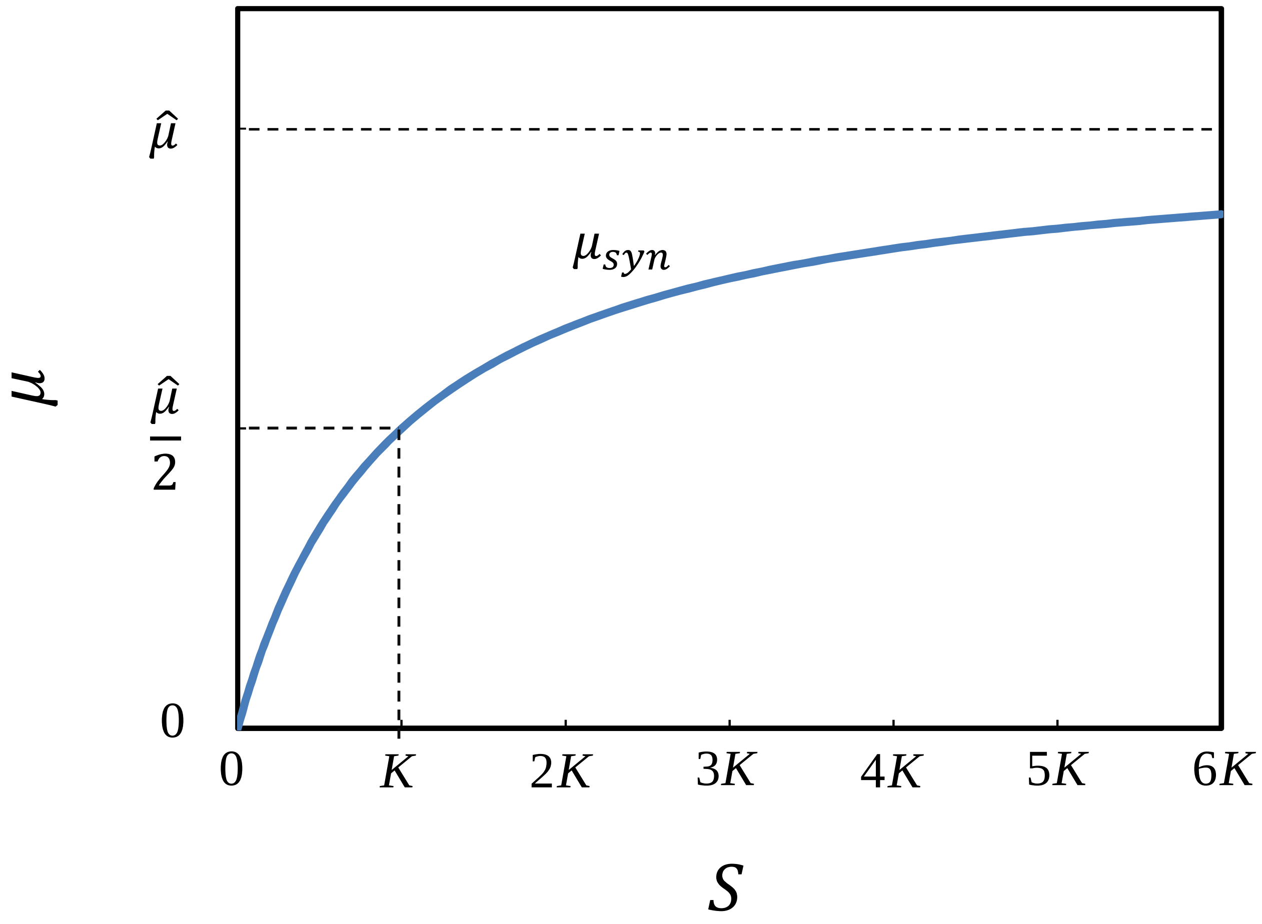
X_a = *concentration of active biomass* ($M_x L^{-3}$)

S = *concentration of the rate-limiting substrate* ($M_s L^{-3}$)

$\hat{\mu}$ = *maximum specific growth rate* (T^{-1})

K = *half saturation coefficient* ($M_s L^{-3}$)

Monod equation: S vs. μ



Typical values for f_s^o, Y, $\hat{q}$, and $\hat{\mu}$

Organism Type	**Electron Donor**	**Electron Acceptors**	**C-Source**	f_s^o	Y	$\hat{q}$	$\hat{\mu}$
Aerobic, Heterotrophs	Carbohydrate BOD Other BOD	O_2 O_2	BOD BOD	0.7 0.6	0.49 g VSS/g BOD_L 0.42 g VSS/g BOD_L	27 g BOD_L/g VSS-d 20 g BOD_L/g VSS-d	13.2 8.4
Denitrifiers	BOD H_2 S(s)	NO_3^- NO_3^- NO_3^-	BOD CO_2 CO_2	0.5 0.2 0.2	0.25 g VSS/g BOD_L 0.81 g VSS/g H_2 0.15 g VSS/g S	16 g BOD_L/g VSS-d 1.25 g H_2/g VSS-d 6.7 g S/g VSS-d	4 1 1
Nitrifying Autotrophs	NH_4^+ NO_2^-	O_2 O_2	CO_2 CO_2	0.14 0.10	0.34 g VSS/g NH_4^+-N 0.08 g VSS/g NO_2^--N	2.7 g NH_4^+-N/g VSS-d 7.8 g NO_2^--N/g VSS-d	0.92 0.62
Methanotrophs	Acetate BOD H_2	acetate CO_2	acetate CO_2	0.05 0.08	0.035 g VSS/g BOD_L 0.45 g VSS/g H_2	8.4 g BOD_L/g VSS-d 1.1 g H_2/g VSS-d	0.3 0.5
Sulfide Oxidizing Autotrophs	H_2S	O_2	CO_2	0.2	0.28 g VSS/g H_2S-S	5 g S/g VSS-d	1.4
Sulfate Reducers	H_2 Acetate BOD	SO_4^{2-} SO_4^{2-}	CO_2 acetate	0.05 0.08	0.28 g VSS/g H_2 0.057 g VSS/g BOD_L	1.05 g H_2/g VSS-d 8.7 g BOD_L/g VSS-d	0.29 0.5
Fermenters	Sugar BOD	sugars	sugars	0.18	0.13 g VSS/g BOD_L	9.8 g BOD_L/g VSS-d	1.2

Y is computed assuming a cellular VSS_a composition of $C_5H_7O_2N$, and NH_4^+ is the N source, except when NO_3^- is the electron acceptor; then NO_3^- is the N source.

$\hat{\mu}$ has units of d^{-1}.

Source: Environmental Biotechnology textbook

Typical values for K

Process	**K (mg substrate/L)**
Aerobic:	
organic mixtures	50-150 mg COD/L
single organics	1-10 mg COD/L
nitrification	0.4-2 mg NH_3-N/L
Anaerobic:	
denitrification	0.06-0.20 mg NO_3^--N/L
methane fermentation:	
acetate, propionate	600-900 mg COD/L
sewage sludge	2000-3000 mg COD/L

Addressing decay

- As discussed in the previous class, we assume decay is proportional to cell biomass

$$\left(\frac{dX_a}{dt}\right)_{decay} = -bX_a$$

in the form of specific growth rate,

$$\mu_{dec} = \left(\frac{1}{X_a} \cdot \frac{dX_a}{dt}\right)_{decay} = -b$$

where μ_{dec} *= specific growth rate due to decay (*T^{-1}*)*

b *= decay coefficient (*T^{-1}*)*

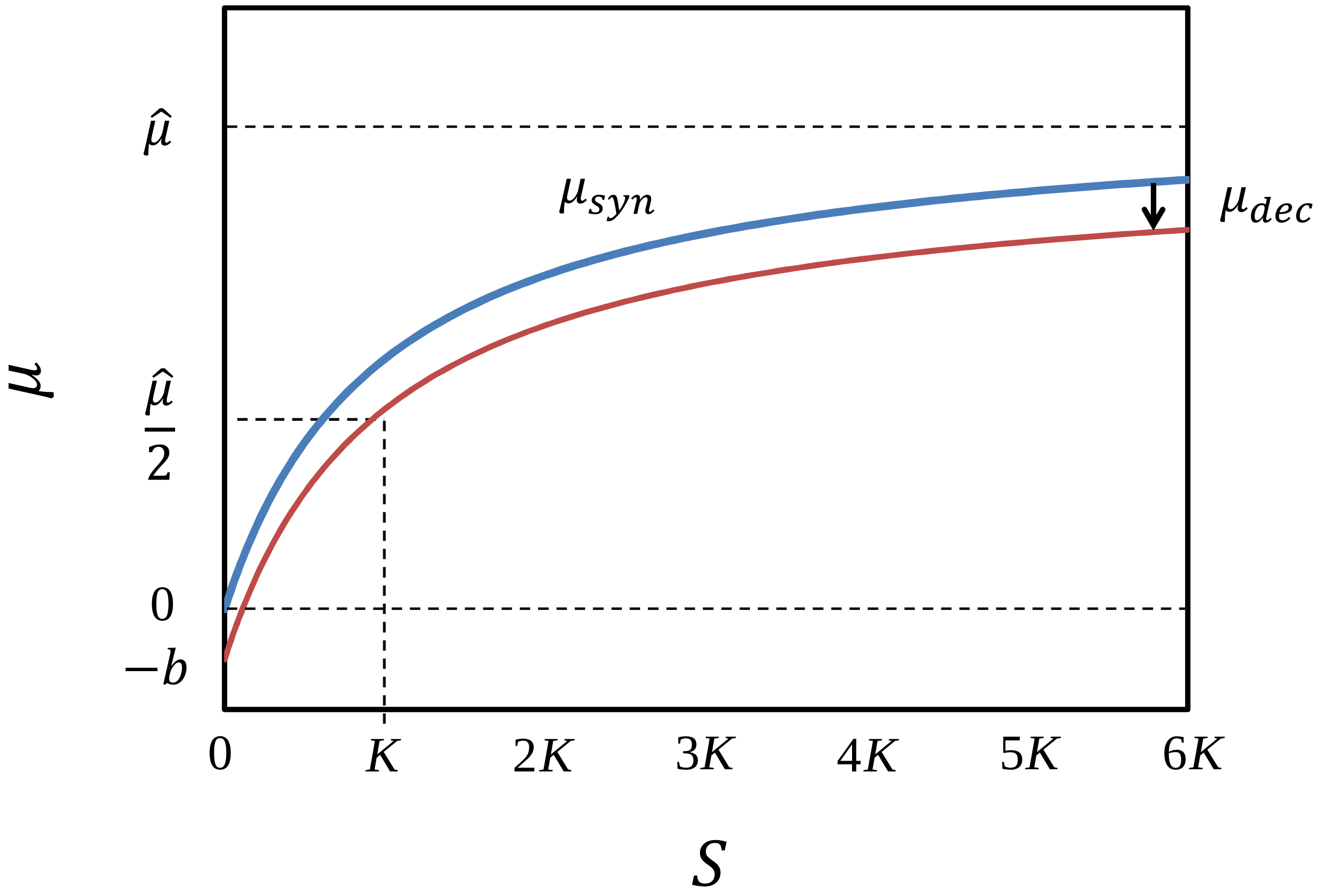
Overall bacterial growth kinetics

(Net growth) = (New growth) + (Decay)

$$\mu = \frac{1}{X_a} \cdot \frac{dX_a}{dt} = \mu_{syn} + \mu_{dec} = \hat{\mu} \frac{S}{K+S} - b$$

where μ *= net specific growth rate (*T^{-1}*)*

Growth kinetics with decay



More on decay

$$\mu_{dec} = \left(\frac{1}{X_a} \cdot \frac{dX_a}{dt}\right)_{decay} = -b$$

- Most fraction (f_d≈0.8) is oxidized
- The other fraction (1-f_d ≈0.2) is accumulated as inert biomass

Rate of oxidation (respiration): $\left(\frac{1}{X_a} \cdot \frac{dX_a}{dt}\right)_{resp} = -f_d b$

Rate of conversion to inert biomass:

$$\left(\frac{1}{X_a} \cdot \frac{dX_a}{dt}\right)_{inert} = -\frac{1}{X_a} \cdot \frac{dX_i}{dt} = -(1 - f_d)b$$

X_i = inert biomass ($M_x L^{-3}$)

Substrate utilization rate

Recall that,

$$Y = \frac{(g\ new\ cells\ produced)}{(g\ substrate\ utilized)} = \frac{(dX_a/dt)_{syn}}{-dS/dt}$$

and

$$\mu_{syn} = \left(\frac{1}{X_a} \cdot \frac{dX_a}{dt}\right)_{syn} = \frac{1}{X_a}\left(\frac{dX_a}{dt}\right)_{syn} = \hat{\mu}\frac{S}{K+S}$$

So Monod equation can be rewritten as:

$$\frac{dS}{dt} = -\frac{1}{Y}\left(\frac{dX_a}{dt}\right)_{syn} = -\frac{\hat{\mu}}{Y}\frac{S}{K+S}X_a$$

Substrate utilization rate, r_{ut} $[M_s L^{-3} T^{-1}]$

$$r_{ut} = \frac{dS}{dt} = -\frac{\hat{q}S}{K+S}X_a$$

$\hat{q} = \hat{\mu}/Y$, *max. specific rate of substrate utilization* $(M_s M_x^{-1} T^{-1})$

Alternate rate expressions

- Contois equation

$$r_{ut} = -\frac{\hat{q}S}{BX_a + S}X_a$$

B = constant $[M_s/M_x]$

When $X_a \rightarrow \infty$, $r_{ut} = -\frac{\hat{q}}{B}S$

(at high biomass concentrations substrate utilization depends on S, not X_a)

Alternate rate expressions

- Moser equation

$$r_{ut} = -\frac{\hat{q}S}{K + S^{-\gamma}}X_a$$

γ = constant [unitless]

- Tessier equation

$$r_{ut} = -\hat{q}\left(1 - e^{S/K}\right)X_a$$

*Just **REMEMBER** that Monod Eq. is **NOT** the only option!!!*

Monod equation: extension

$$r_{ut} = -\hat{q}\frac{S}{K+S}\frac{A}{K_A+A}X_a$$

A = e⁻ acceptor concentration $[M_A/L^3]$

K_A = half-saturation coefficient for e⁻ acceptor $[M_A/L^3]$

- e^- acceptor can also be limiting!
- Can be reduced to single Monod eq. if $A >> K_A$
- Terms for other limiting substances can be added as well (e.g., N, P)