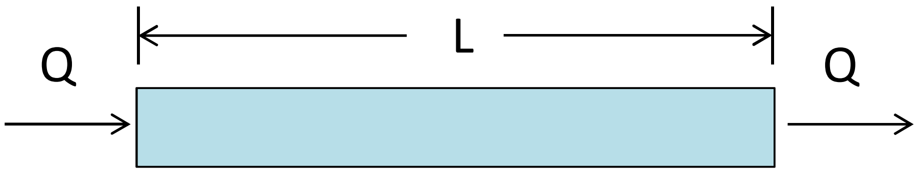
Reactors II

Today's lecture

- **Plug flow reactor**
  - Concept
  - PFR analysis for 1st order reaction
  - PFR analysis for Monod kinetics

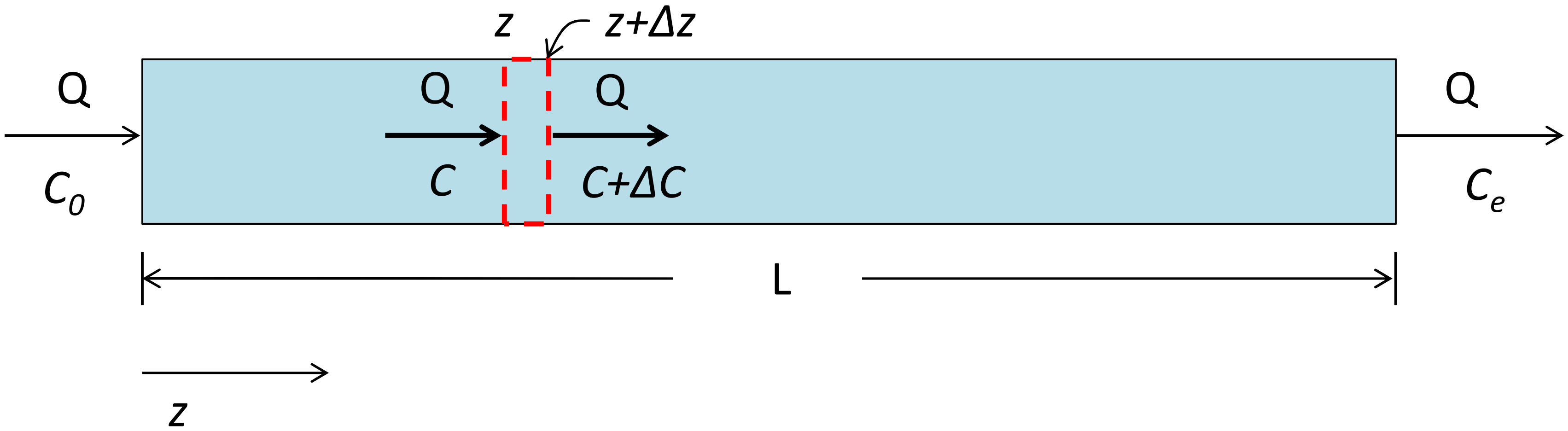
- **Continuous-stirred tank reactor**
  - CSTR analysis for 1st order reaction
  - PFR vs. CSTR
  - CSTR analysis for Monod kinetics

Reactor analysis: PFR



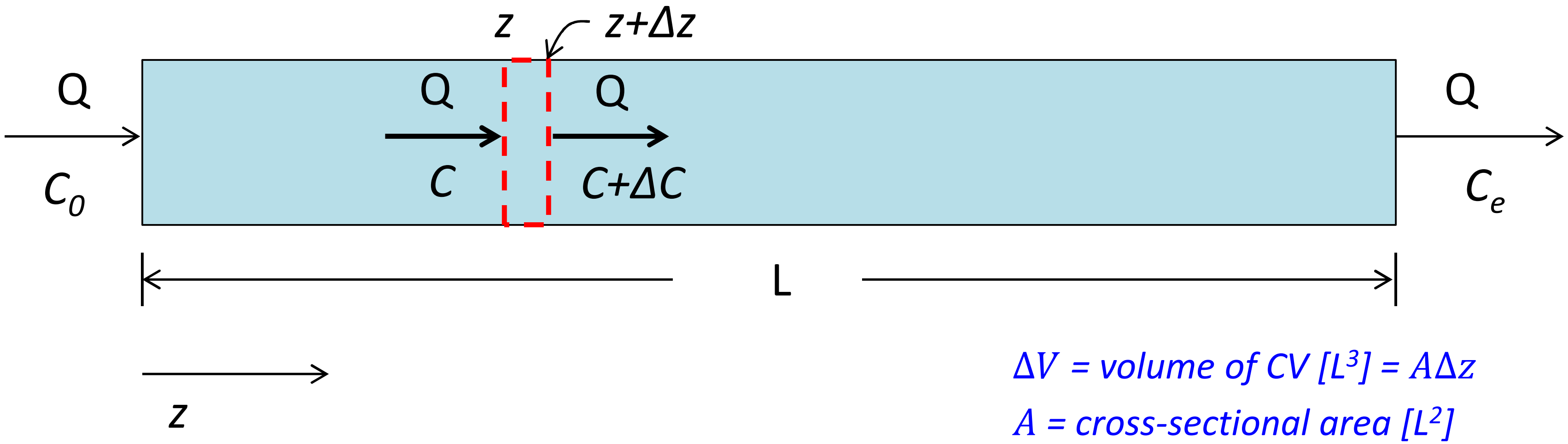
- Plug flow reactor (PFR)
  - assumption: no mixing in the direction of flow & completely mixed in the direction perpendicular to the flow
  - reactors get close to PFR as the length gets longer than the width and depth (e.g., rivers)

PFR, first-order reaction



- Take control volume as a thin plate perpendicular to the flow at $z=z$ with a dimension of Δz in z dir.

PFR, first-order reaction



ΔV = volume of CV [L^3] = $A\Delta z$

A = cross-sectional area [L^2]

$$C/C_0 = e^{-k\theta}$$

→ Same form as the batch reactor (why??)

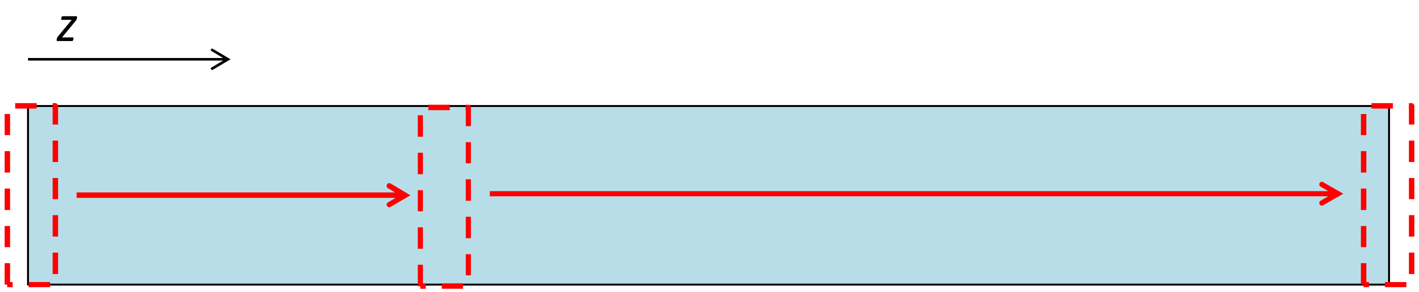
where, $\theta \equiv \dfrac{V}{Q} = \dfrac{A \cdot L}{A \cdot u} = \dfrac{L}{u}$

θ = hydraulic retention time, HRT [T]

V = reactor volume [L^3]

u = flow velocity [LT^{-1}]

What the PFR assumption implies



distance=0
time=0
$C=C_0$

distance=z
time=t
$C=C_0e^{-kt}$
$=C_0e^{-kz/u}$

distance=L
time=Θ
$C=C_0e^{-k\Theta}$
$=C_0e^{-kL/u}$

- We model plug flow reactor as a movement of a "plug"
- The plug has a cross sectional area same as the reactor dimension and an infinitesimal dimension in z-dir (a thin plate)
- Complete mixing within the plug → batch reactor moving in the direction of flow

PFR, Monod kinetics

Recall the batch reaction solution applying Monod kinetics:

$$t = \frac{1}{\hat{q}}\left\{\left(\frac{K}{{X_a}^0 + YS^0} + \frac{1}{Y}\right) ln\left({X_a}^0 + YS^0 - YS\right) - \left(\frac{K}{{X_a}^0 + YS^0}\right) ln\frac{S{X_a}^0}{S^0} - \frac{1}{Y} ln{X_a}^0\right\}$$

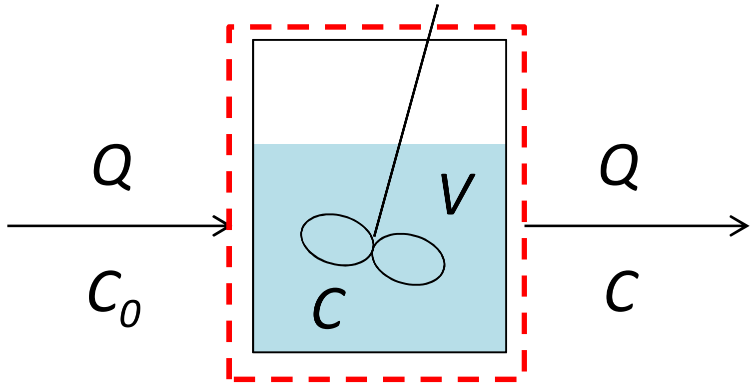
[5.11]

With no doubt, we obtain the PFR solution as:

$$\theta = \frac{1}{\hat{q}}\left\{\left(\frac{K}{{X_a}^0 + YS^0} + \frac{1}{Y}\right) ln\left({X_a}^0 + YS^0 - YS^e\right) - \left(\frac{K}{{X_a}^0 + YS^0}\right) ln\frac{S^e{X_a}^0}{S^0} - \frac{1}{Y} ln{X_a}^0\right\}$$

[5.27]

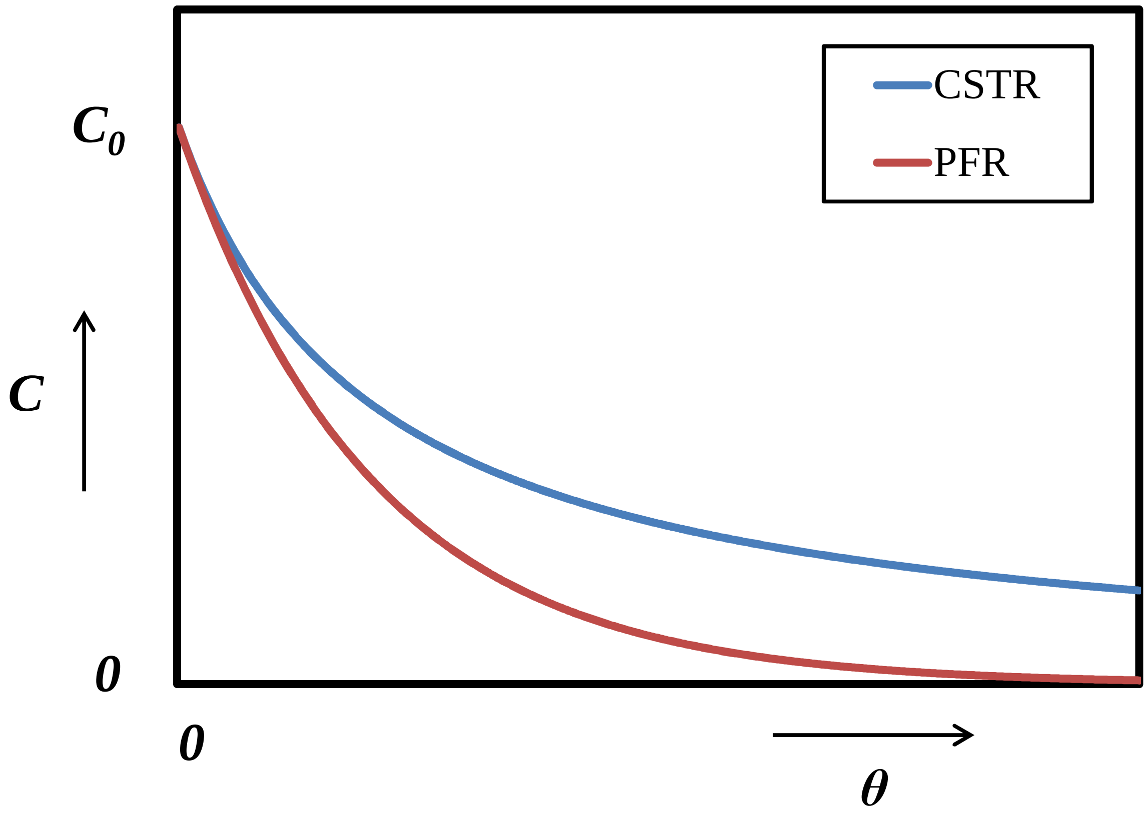
Reactor analysis: CSTR, 1st order, steady state



Steady-state solution:

$$C = \frac{C_0}{1 + k\theta}$$

PFR vs. CSTR



PFR shows better performance esp. at high HRTs

For 1st order reaction,

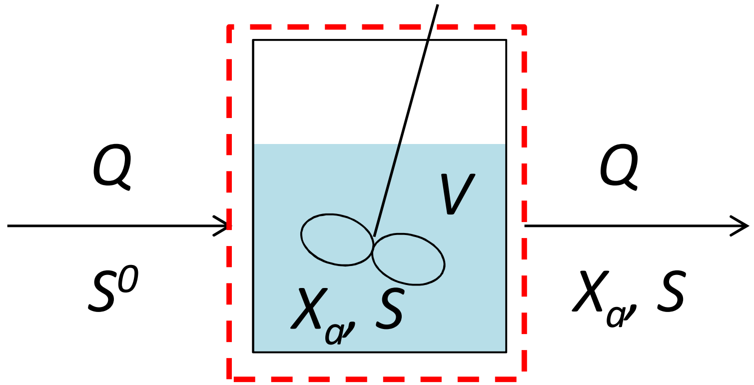
CSTR:

$$C = \frac{C_0}{1 + k\theta}$$

PFR:

$$C = C_0 e^{-k\theta}$$

Reactor analysis: CSTR, Monod, Steady-state



By assuming:

- Steady state
- X_a = 0 in the influent (negligible influent biomass)

The solutions are obtained as:

$$S = K \frac{1 + b\theta}{Y\hat{q}\theta - (1 + b\theta)}$$

-- *surprisingly, not f(S^0)!*

$$X_a = Y \frac{S^0 - S}{1 + b\theta}$$