

# Bioreactor analysis & design II

# Today's lecture

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- Study on additional variables and analyses
  - Soluble microbial products
  - Nutrient & e<sup>-</sup> acceptor consumption
  - Hydrolysis

# Soluble microbial products (SMP)

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- Cell components that released during cell lysis, diffuse through the cell membrane, are lost during synthesis, or are excreted for some purpose
- Does not include intermediates of degradation pathway
- MW = 100s – 1000s
- Biodegradable

# Significance of SMP

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- Appear in all cases
- Constitute the majority of the effluent COD & BOD in many cases
- Can complex metals, foul membranes, & cause color or foaming

# Two types of SMP

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- UAP (substrate-**U**tilization-**A**ssociated **P**roducts):  
produced directly during substrate metabolism

$$r_{UAP} = -k_1 r_{ut}$$

$r_{UAP}$  = rate of UAP formation [ $M_p L^{-3} T^{-1}$ ]  
 $k_1$  = UAP-formation coefficient [ $M_p M_s^{-1}$ ]

- BAP (**B**iomass-**A**ssociated **P**roducts):  
formed directly from biomass as part of  
maintenance and decay

$$r_{BAP} = k_2 X_a$$

$r_{BAP}$  = rate of BAP formation [ $M_p L^{-3} T^{-1}$ ]  
 $k_2$  = BAP-formation coefficient [ $M_p M_x^{-1} T^{-1}$ ]

# SMP biodegradation

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- Assume degradation of both UAP and BAP follow Monod kinetics (with own parameter values for each):

$$r_{deg-UAP} = -\frac{\hat{q}_{UAP}UAP}{K_{UAP} + UAP}X_a$$

$$r_{deg-BAP} = -\frac{\hat{q}_{BAP}BAP}{K_{BAP} + BAP}X_a$$

*UAP = concentration of UAP [M<sub>p</sub>L<sup>-3</sup>]*

*BAP = concentration of BAP [M<sub>p</sub>L<sup>-3</sup>]*

# SMP in a CSTR: mass balance

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- Steady-state mass balance for UAP & BAP:

$$0 = 0 - Q \cdot UAP - k_1 r_{ut} V - \frac{\hat{q}_{UAP} UAP}{K_{UAP} + UAP} X_a V$$

$$0 = 0 - Q \cdot BAP + k_2 X_a V - \frac{\hat{q}_{BAP} BAP}{K_{BAP} + BAP} X_a V$$

# SMP in a CSTR: solutions

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$$UAP = -\frac{\hat{q}_{UAP}X_a\theta + K_{UAP} + k_1r_{ut}\theta}{2} + \frac{\sqrt{(\hat{q}_{UAP}X_a\theta + K_{UAP} + k_1r_{ut}\theta)^2 - 4K_{UAP}k_1r_{ut}\theta}}{2}$$

$$BAP = -\frac{K_{BAP} + (\hat{q}_{BAP} - k_2)X_a\theta}{2} + \frac{\sqrt{(K_{BAP} + (\hat{q}_{BAP} - k_2)X_a\theta)^2 + 4K_{BAP}k_2X_a\theta}}{2}$$

Here, the substrate utilization rate,  $r_{ut}$ , can be computed by

$$r_{ut} = -\frac{S^0 - S}{\theta} = -\frac{\hat{q}S}{K + S}X_a$$



# Observed yield

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The VSS in a CSTR is calculated as:

$$X_v = X_i + X_a = X_i^0 + \underbrace{Y(S^0 - S)}_{\text{substrate utilized in the reactor}} \underbrace{\frac{1 + (1 - f_d)b\theta}{1 + b\theta}}_{\text{VSS gained in the reactor}}$$

*Influent nbVSS*
*substrate utilized in the reactor*
*VSS gained in the reactor*

The net yield in the CSTR is:  $Y \frac{1 + (1 - f_d)b\theta}{1 + b\theta}$

This value is more often called as **Observed Yield ( $Y_{obs}$ )**

# Nutrient consumption

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- For the consumption of nutrients for biomass production:

$$r_n = \gamma_n \cdot Y_{obs} \cdot r_{ut} = \gamma_n \cdot Y \cdot r_{ut} \frac{1 + (1 - f_d)b\theta}{1 + b\theta}$$

*$r_n$  = rate of nutrient consumption [ $M_n L^{-3} T^{-1}$ ]*

*$\gamma_n$  = the stoichiometric ratio of nutrient mass to VSS for biomass [ $M_n M_x^{-1}$ ]*

Using  $C_5H_7O_2N$  as cell formula:

$$\gamma_N = 14 \text{ g N} / 113 \text{ g VSS} = 0.124 \text{ g N/g VSS}$$

$$\gamma_P = 0.2 \times 0.124 = 0.025 \text{ g P/g VSS (assume } g P = 0.2 \times g N)$$

# Nutrient consumption in a CSTR

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- Steady-state mass balance:

$$0 = QC_n^0 - QC_n + r_n V$$

$C_n^0$  = nutrient conc. in the influent [ $M_oL^{-3}$ ]

$C_n$  = nutrient conc. in the reactor [ $M_oL^{-3}$ ]

$$C_n = C_n^0 + r_n \theta$$

if  $C_n < 0$ , nutrient-limiting

# e<sup>-</sup> acceptor consumption

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(e<sup>-</sup> acceptor used in a reactor)  
= [(total O.D. in the influent) - (total O.D. in the effluent)]  
x (conversion factor)

In terms of the use rate for a reactor ( $\Delta S_o/\Delta t$ ):

$$\begin{aligned}\frac{\Delta S_a}{\Delta t} &= \gamma_a [Q(S^0 + 1.42X_v^0) - Q(S + SMP + 1.42X_v)] \\ &= \gamma_a Q [S^0 - S - SMP + 1.42(X_v^0 - X_v)]\end{aligned}$$

*$\gamma_a$  = the stoichiometric ratio of acceptor mass to oxygen demand*

*for oxygen: 1 g O<sub>2</sub>/g COD*

*for nitrate: 0.35 g NO<sub>3</sub><sup>-</sup>N/g COD*

# e<sup>-</sup> acceptor consumption (cont'd)

To estimate the required mass rate of acceptor supply (ex: aeration [O<sub>2</sub>] requirement), the calculated e<sup>-</sup> acceptor use rate,  $\Delta S_a/\Delta t$  can be written as:

$$\frac{\Delta S_a}{\Delta t} = \gamma_a [Q(S^0 + 1.42X_v^0) - Q(S + SMP + 1.42X_v)]$$

$$= Q(S_a^0 - S_a) + R_a$$

$\underbrace{\hspace{10em}}$   
(mass flow rate in) –  
(mass flow rate out)

Requirement of e<sup>-</sup>  
acceptor addition

[M<sub>a</sub>T<sup>-1</sup>]

$S_a^0$  = e<sup>-</sup> acceptor conc. in the  
influent [M<sub>a</sub>L<sup>-3</sup>]

$S_a$  = e<sup>-</sup> acceptor conc. in the  
reactor [M<sub>a</sub>L<sup>-3</sup>]

# Hydrolysis of particulates & polymers

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- Particulates and polymeric substances account for a significant portion of BOD in wastewater
- >50% of BOD in typical sewage is particulates (SS)
- Particulates and large-MW compounds cannot penetrate the cell membrane
  - needs to be hydrolyzed to smaller molecules
- Catalyzed by extracellular enzymes
- The mechanism and kinetics of hydrolysis is not fully understood

# Hydrolysis of particulates & polymers

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One simple way of describing hydrolysis is to assume first-order kinetics for particulates (or polymers):

$$r_{hyd} = -k_{hyd}S_p$$

$r_{hyd}$  = rate of accumulation of particulates ( $=dS_p/dt$ ) [ $M_sL^{-3}T^{-1}$ ]

$k_{hyd}$  = first-order hydrolysis rate coefficient [ $T^{-1}$ ]

$S_p$  = concentration of particulates in the reactor [ $M_sL^{-3}$ ]

In a steady-state CSTR,

$$0 = Q(S_p^0 - S_p) - k_{hyd}S_pV$$

$S_p^0$  = concentration of particulates in the influent [ $M_sL^{-3}$ ]



$$S_p = \frac{S_p^0}{1 + k_{hyd}\theta}$$

# Hydrolysis of particulates & polymers

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- Effect of hydrolysis on dissolved substrates mass balance in a steady-state CSTR

$$0 = Q(S^0 - S) - \frac{\hat{q}S}{K + S}X_aV + k_{hyd}S_pV$$

$$0 = \{(S^0 + k_{hyd}S_p\theta) - S\} - \frac{\hat{q}S}{K + S}X_a\theta$$

→ Increase in  $S^0$  by  $k_{hyd}S_p\theta$

→ Increased biomass, but no change in dissolved substrates in the reactor

So: to include the effect of hydrolysis of particulates,

substitute  $S^0$  with  $(S^0 + k_{hyd}S_p\theta)$