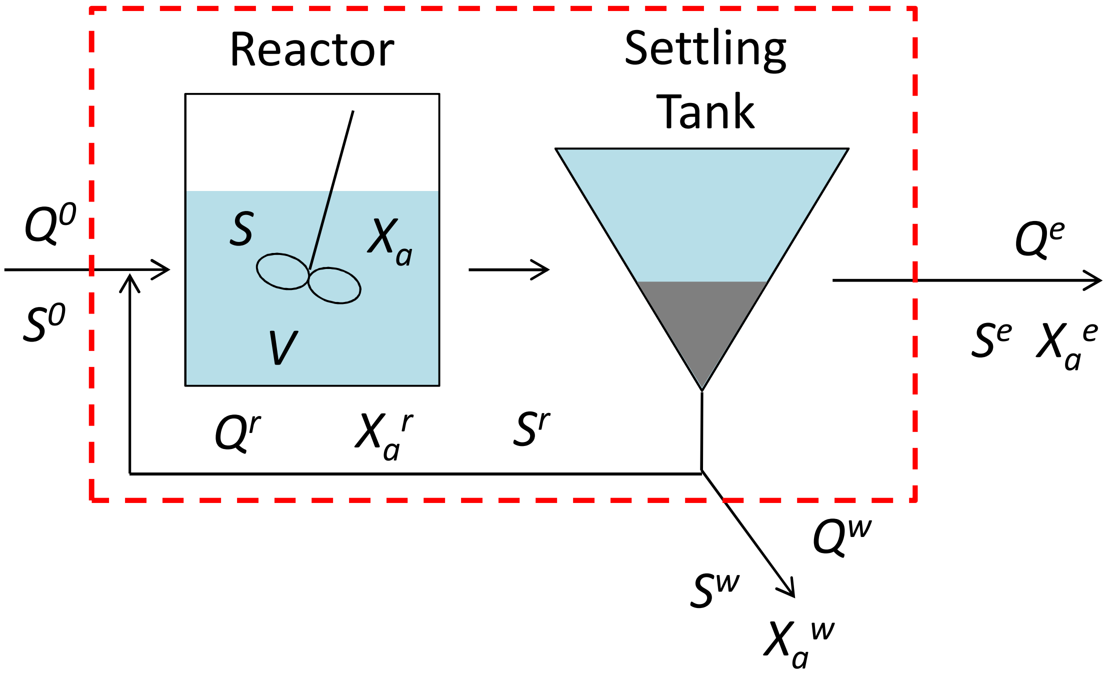
Bioreactor analysis & design III

Today's lecture

- CSTR with settling and cell recycling
  - Deriving solutions for S and X_a
  - Updating other solutions
- Key operational variable -- SRT
- Alternate rate expressions

CSTR with settling and cell recycling



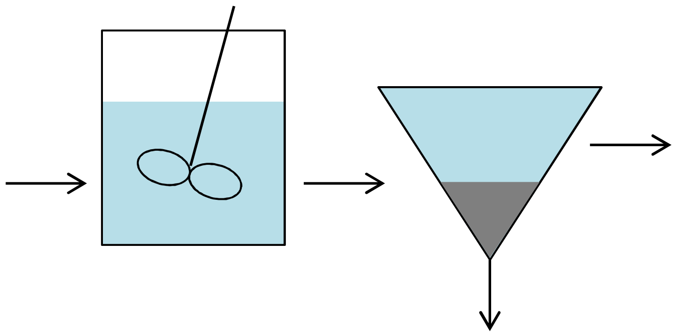
From flow mass balance:
$Q^0 = Q^e + Q^w$

Assumptions

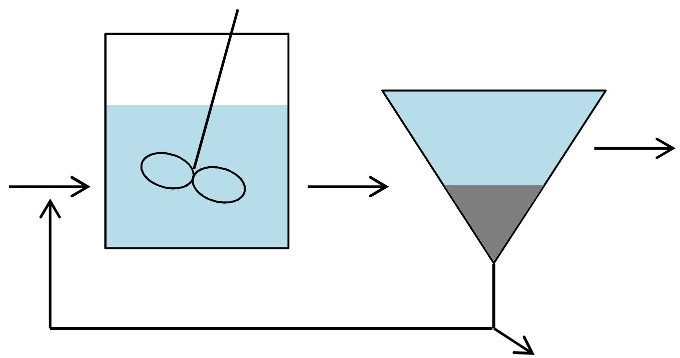
- Biodegradation of soluble substrates in the reactor only, no biodegradation in the settling tank ($S = S^e = S^w = S^r$)
- No active biomass in influent
- Steady state

HRT vs. SRT

- HRT (θ): **H**ydraulic **R**etention **T**ime; the average time the water stays in the system
- SRT (θ_x): **S**olids **R**etention **T**ime (or mean cell residence time, MCRT); the average time the biomass stays in the system

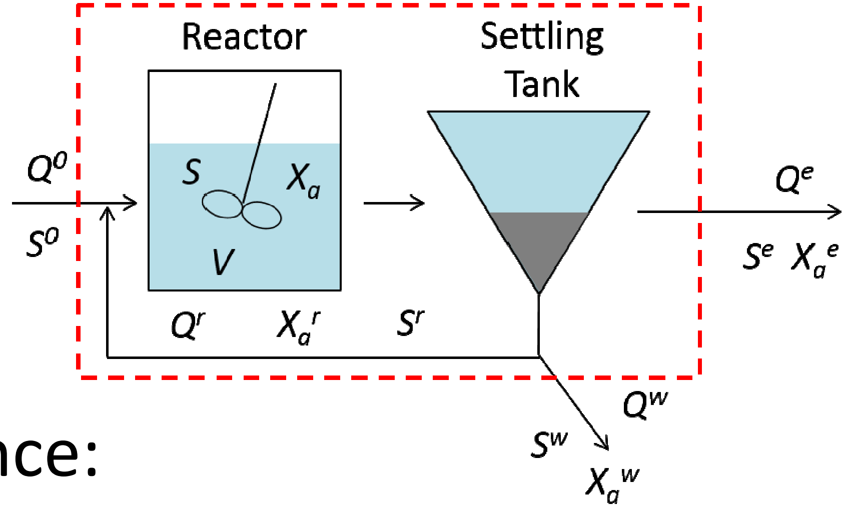


CSTR (chemostat) + clarifier
without sludge return: HRT = SRT



CSTR with sludge return: HRT < SRT

CSTR, cell recycle: Mass balances



- Active biomass mass balance:

$$0 = 0 - (Q^e X_a{}^e + Q^w X_a{}^w) + r_{net} V$$

r_{net} = net rate of active biomass growth ($M_x L^{-3} T^{-1}$)

- Substrate mass balance:

$$0 = Q^0 S^0 - (Q^e S + Q^w S) + r_{ut} V$$

r_{ut} = substrate utilization rate ($M_s L^{-3} T^{-1}$)

CSTR, cell recycle: θ_x & r_{ut}

- To solve the mass balance equations, use the following relationships:

$$\theta_x = \frac{\textit{active biomass in the system}}{\textit{production rate of active biomass}} = \frac{X_a V}{Q^e {X_a}^e + Q^w {X_a}^w}$$

$$r_{ut} = -\frac{\textit{rate of mass substrate utilized}}{\textit{volume of reactor}} = -\frac{Q^0 S^0 - Q^e S - Q^w S}{V}$$

$$= -\frac{Q^0 (S^0 - S)}{V} = -\frac{S^0 - S}{\theta}$$

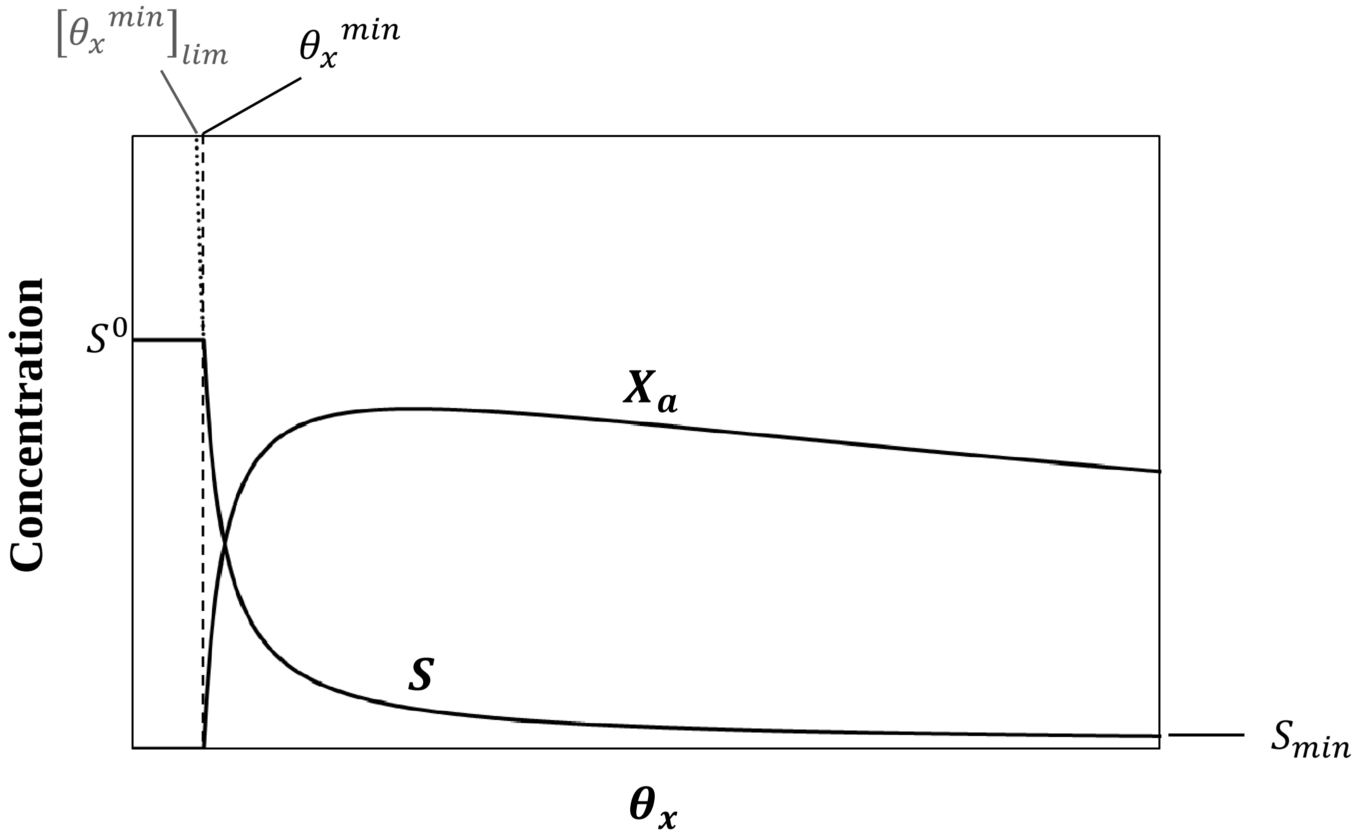
CSTR, cell recycle: Solutions for S & X_a

$$S = K \frac{1 + b\theta_x}{\theta_x(Y\hat{q} - b) - 1}$$

$$X_a = \frac{\theta_x}{\theta} \frac{Y(S^0 - S)}{1 + b\theta_x}$$

- Compare with our solutions for CSTR without cell recycling
- These are **generic solutions** applicable for both with & without cell recycling as long as the bioreactor is a steady state CSTR

S & X_a vs. θ_x: key trends



- $\theta_x \leq \theta_x{}^{min}$: washout ($S^0 \to \infty$: $\theta_x{}^{min} \to [\theta_x{}^{min}]_{lim}$)
- $\theta_x \to \infty$: $S = S_{min}$
- For $\theta_x{}^{min} < \theta_x$, S decreases with increase in θ_x, but X_a peaks at some point

S_{min}, θ_x^{min}, & $[\theta_x^{min}]_{lim}$

(use $S = K\dfrac{1 + b\theta_x}{Y\hat{q}\theta_x - (1 + b\theta_x)}$)

$$\boldsymbol{S_{min}} = \lim_{\theta_x \to \infty} \left\{ K \frac{1 + b\theta_x}{Y\hat{q}\theta_x - (1 + b\theta_x)} \right\} = \boldsymbol{K \frac{b}{Y\hat{q} - b}}$$

$$S^0 = K \frac{1 + b\theta_x{}^{min}}{Y\hat{q}\theta_x{}^{min} - \left(1 + b\theta_x{}^{min}\right)} \quad \Rightarrow \quad \boldsymbol{\theta_x{}^{min} = \frac{K + S^0}{S^0(Y\hat{q} - b) - bK}}$$

$$\boldsymbol{[\theta_x{}^{min}]_{lim}} = \lim_{S^0 \to \infty} \left\{ \frac{K + S^0}{S^0(Y\hat{q} - b) - bK} \right\} = \boldsymbol{\frac{1}{Y\hat{q} - b}}$$

Updated solutions for VSS

- Reactor nbVSS concentration

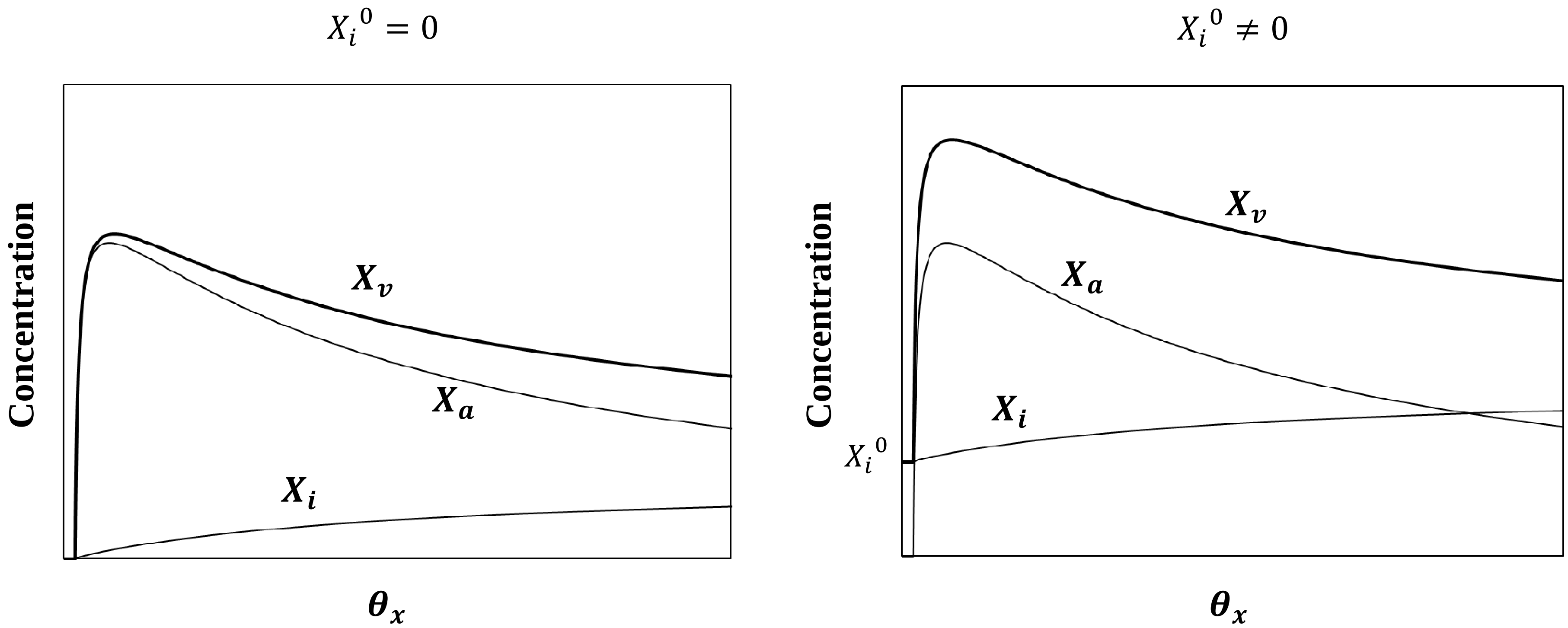
$$X_i = \frac{\theta_x}{\theta}\left[{X_i}^0 + X_a(1 - f_d)b\theta\right]$$

- Reactor total VSS concentration (when ${S_p}^0 = 0$)*

$$X_v = X_a + X_i = \frac{\theta_x}{\theta}\left[{X_i}^0 + \frac{Y(S^0 - S)\{1 + (1 - f_d)b\theta_x\}}{1 + b\theta_x}\right]$$

** Note that in this solution X_v does not account for S_p (biodegradable organic particulates/polymers)*

X's vs. θ_x



- Decay becomes more dominant (as compared to cell synthesis) as θ_x increases

  → For $X_i{}^0 = 0$ case, X_a/X_v increases as θ_x increases

Solids production rate [M_x/T]

- Active biomass production rate (r_{abp})

$$r_{abp} = \frac{X_a V}{\theta_x}$$

- Total VSS production rate (r_{VSS})

$$r_{VSS} = \frac{X_v V}{\theta_x}$$

Again, note that defining
$X_v = X_a + X_i$ *and*
$X_v V/\theta_x$ *= total VSS production rate*
implies that we assume $S_p = 0$

Updated solution for observed yield

$$Y_{obs} = Y\frac{1 + (1 - f_d)b\theta_x}{1 + b\theta_x}$$

because

$$r_{VSS} = \frac{X_v V}{\theta_x} = QX_i{}^0 + \frac{QY(S^0 - S)\{1 + (1 - f_d)b\theta_x\}}{1 + b\theta_x}$$

CSTR, cell recycle: Linking with stoichiometry

$$f_s^{\,0} = Y \frac{(n_e \ \ e^- \ eq \ cells/mole \ cells)(8 \ g \ COD/e^- \ eq \ donor)}{(M_c \ \ g \ cells/mole \ cells)}$$

recall $$Y = f_s^{\,0} \frac{(M_c \ \ g \ cells/mole \ cells)}{(n_e \ \ e^- \ eq \ cells/mole \ cells)(8 \ g \ COD/e^- \ eq \ donor)}$$

$$f_s = f_s^{\,0} \left[\frac{1 + (1 - f_d) b\theta_x}{1 + b\theta_x}\right]$$

because $f_s/f_s^{\,0} = Y_n/Y$

➡ Can obtain full stoichiometry for a given θ_x

if half reactions (R_d, R_e, R_c) & growth parameters ($\hat{q}$, K, Y, b) are known

Alternate rate expressions

- Contois equation

$$r_{ut} = -\frac{\hat{q}S}{BX_a + S}X_a$$

B = constant [M_s/M_x]

When $X_a \rightarrow \infty$, $r_{ut} = -\frac{\hat{q}}{B}S$

(at high biomass concentrations substrate utilization depends on *S*, not X_a)

Alternate rate expressions

- Moser equation

$$r_{ut} = -\frac{\hat{q}S}{K + S^{-\gamma}} X_a$$

γ = constant [unitless]

- Tessier equation

$$r_{ut} = -\hat{q}\left(1 - e^{S/K}\right) X_a$$

*Just **REMEMBER** that Monod Eq. is **NOT** the only option!!!*

Monod equation: extension

$$r_{ut} = -\hat{q}\frac{S}{K+S}\frac{A}{K_A+A}X_a$$

A = e^- acceptor concentration [M_A/L^3]

K_A = half-saturation coefficient for e^- acceptor [M_A/L^3]

- e^- acceptor can also be limiting!
- Can be reduced to single Monod eq. if $A >> K_A$
- Terms for other limiting substances can be added as well (e.g., N, P)