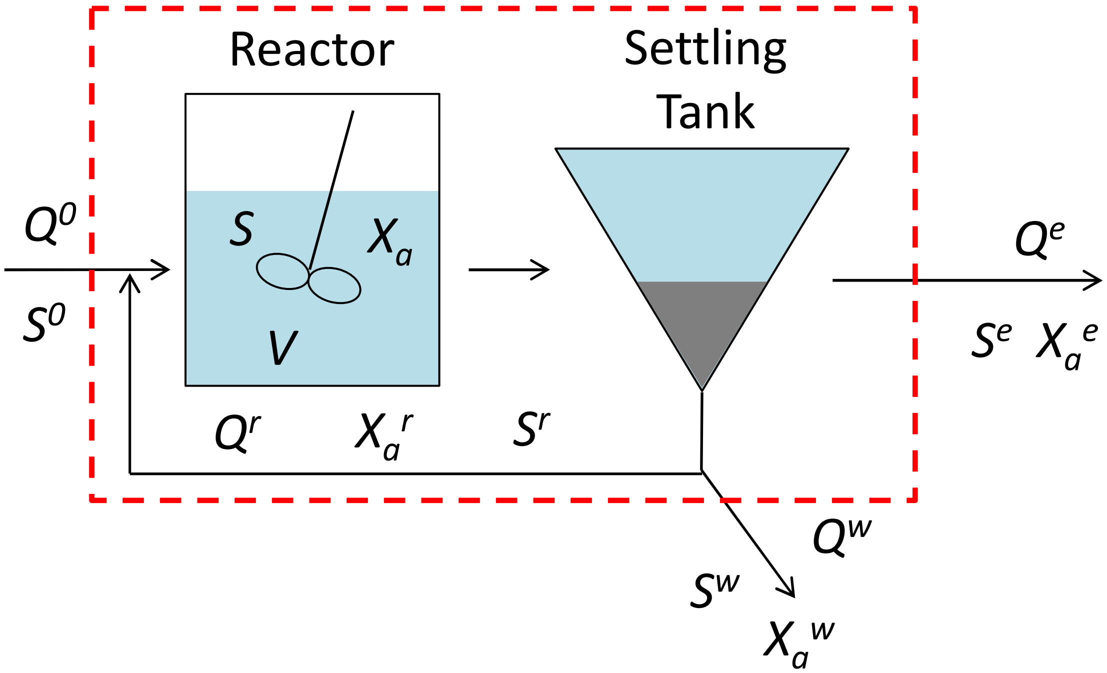
Bioreactor analysis & design III

Today's class

- CSTR with settling and cell recycling
  - Deriving solutions for S and X_a
  - Updating other solutions

- Key operational variable -- SRT

CSTR with settling and cell recycling



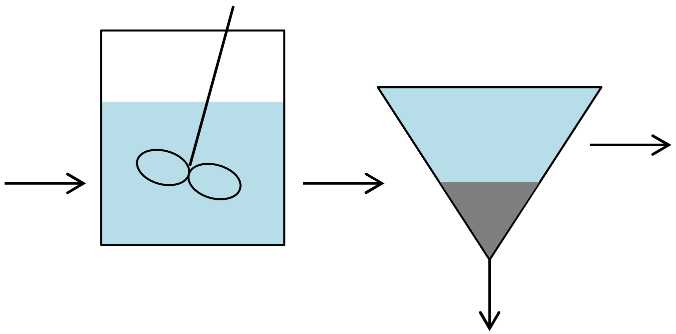
From flow mass balance:
$Q^0 = Q^e + Q^w$

Assumptions

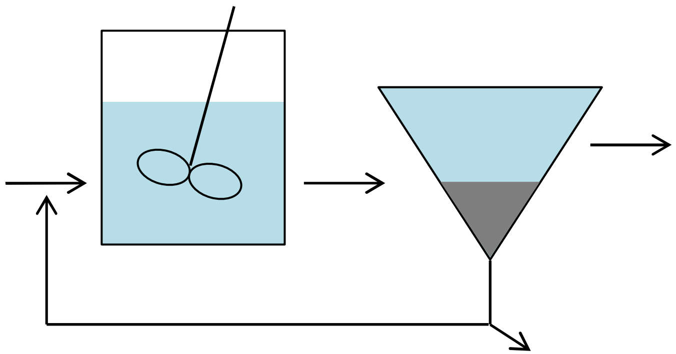
- Biodegradation of soluble substrates in the reactor only, no biodegradation in the settling tank ($S = S^e = S^w = S^r$)
- No active biomass in influent
- Steady state

HRT vs. SRT

- HRT (θ): **H**ydraulic **R**etention **T**ime; the average time the water stays in the system
- SRT (θ_x): **S**olids **R**etention **T**ime (or mean cell residence time, MCRT); the average time the biomass stays in the system

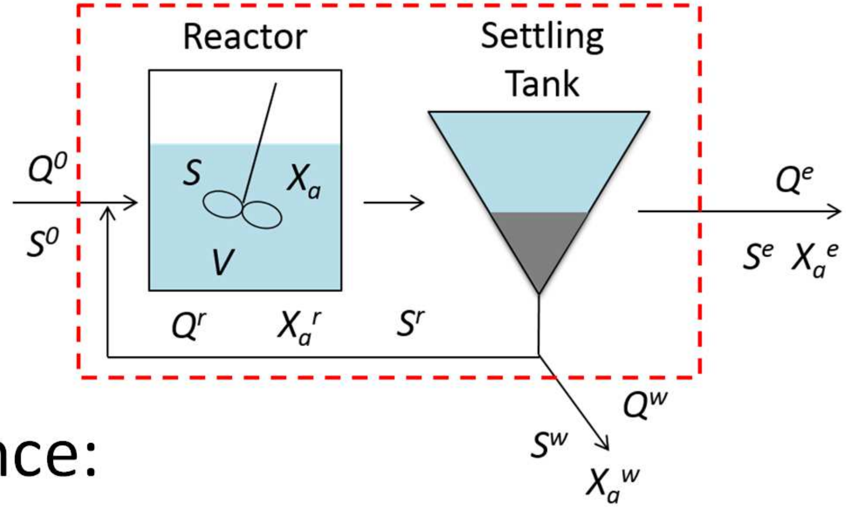


CSTR (chemostat) + clarifier
without sludge return: HRT = SRT



CSTR with sludge return: HRT < SRT

CSTR, cell recycle: Mass balances



- Active biomass mass balance:

$$0 = 0 - (Q^e X_a{}^e + Q^w X_a{}^w) + r_{net} V$$

r_{net} = net rate of active biomass growth ($M_x L^{-3} T^{-1}$)

- Substrate mass balance:

$$0 = Q^0 S^0 - (Q^e S + Q^w S) + r_{ut} V$$

r_{ut} = substrate utilization rate ($M_s L^{-3} T^{-1}$)

CSTR, cell recycle: θ_x & r_{ut}

- To solve the mass balance equations, use the following relationships:

$$\theta_x = \frac{active\ biomass\ in\ the\ system}{production\ rate\ of\ active\ biomass} = \frac{X_a V}{Q^e X_a{}^e + Q^w X_a{}^w}$$

$$r_{ut} = -\frac{rate\ of\ mass\ substrate\ utilized}{volume\ of\ reactor} = -\frac{Q^0 S^0 - Q^e S - Q^w S}{V}$$

$$= -\frac{Q^0(S^0 - S)}{V} = -\frac{S^0 - S}{\theta}$$

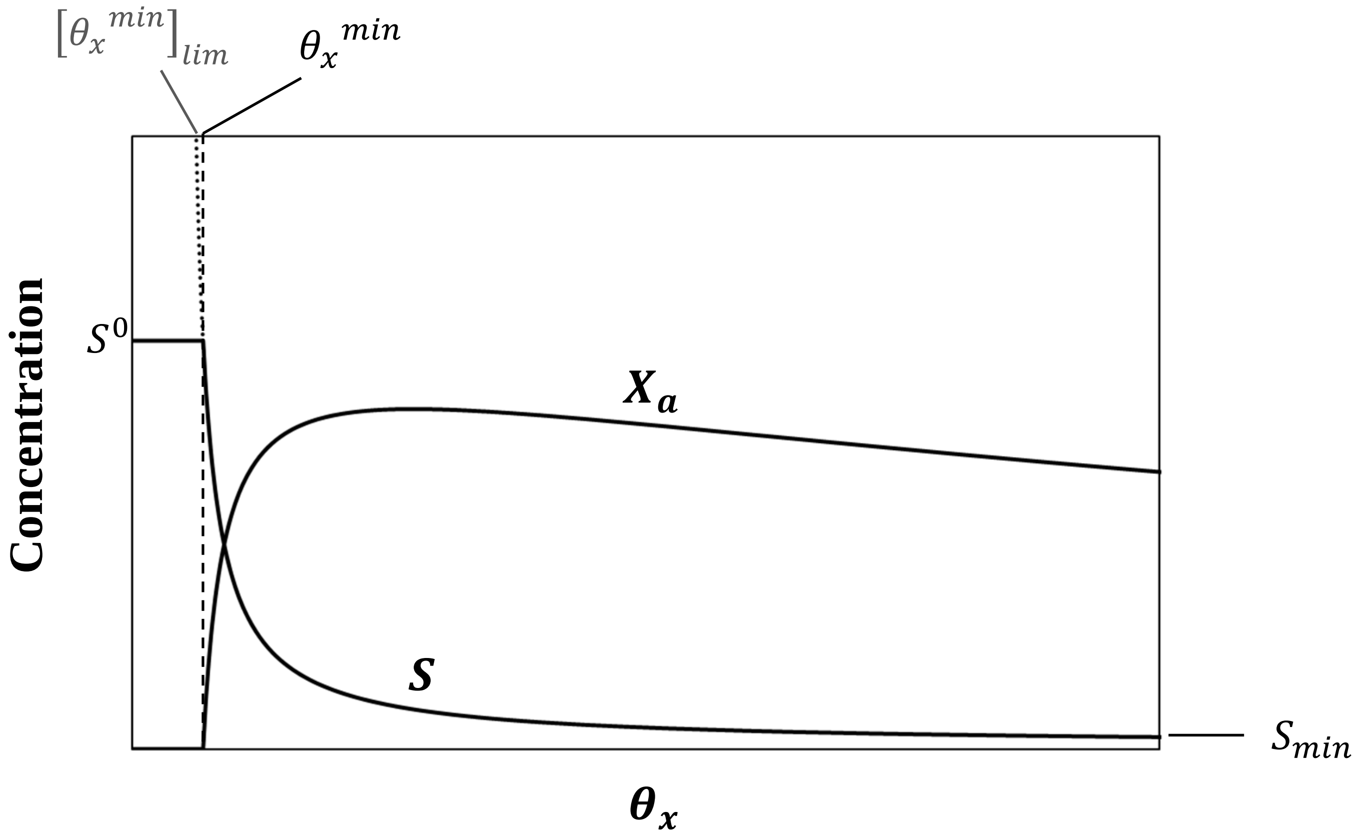
CSTR, cell recycle: Solutions for S & X_a

$$S = K \frac{1 + b\theta_x}{\theta_x(Y\hat{q} - b) - 1}$$

$$X_a = \frac{\theta_x}{\theta} \frac{Y(S^0 - S)}{1 + b\theta_x}$$

- Compare with our solutions for CSTR without cell recycling
- These are **generic solutions** applicable for both with & without cell recycling as long as the bioreactor is a steady state CSTR

S & X_a vs. θ_x: key trends



- $\theta_x \leq \theta_x{}^{min}$: washout $(S^0 \rightarrow \infty\text{: } \theta_x{}^{min} \rightarrow [\theta_x{}^{min}]_{lim})$
- $\theta_x \rightarrow \infty$: $S = S_{min}$
- For $\theta_x{}^{min} < \theta_x$, S decreases with increase in θ_x, but X_a peaks at some point

S_{min}, $\theta_x^{\ min}$, & $[\theta_x^{\ min}]_{lim}$

(use $S = K\dfrac{1+b\theta_x}{Y\hat{q}\theta_x - (1+b\theta_x)}$)

$$\boldsymbol{S_{min}} = \lim_{\theta_x\to\infty}\left\{K\frac{1+b\theta_x}{Y\hat{q}\theta_x - (1+b\theta_x)}\right\} = \boldsymbol{K\frac{b}{Y\hat{q}-b}}$$

$$S^0 = K\frac{1+b\theta_x^{\ min}}{Y\hat{q}\theta_x^{\ min} - \left(1+b\theta_x^{\ min}\right)} \quad \Rightarrow \quad \boldsymbol{\theta_x^{\ min} = \frac{K+S^0}{S^0(Y\hat{q}-b)-bK}}$$

$$\boldsymbol{[\theta_x^{\ min}]_{lim}} = \lim_{S^0\to\infty}\left\{\frac{K+S^0}{S^0(Y\hat{q}-b)-bK}\right\} = \boldsymbol{\frac{1}{Y\hat{q}-b}}$$

Updated solutions for VSS

- Reactor nbVSS concentration

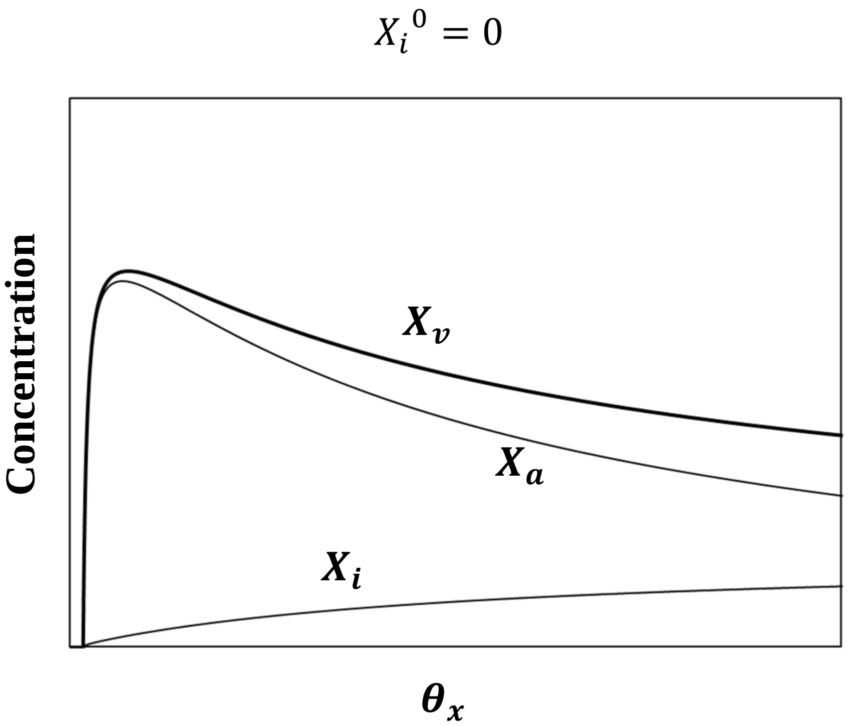
$$X_i = \frac{\theta_x}{\theta}\left[{X_i}^0 + X_a(1 - f_d)b\theta\right]$$

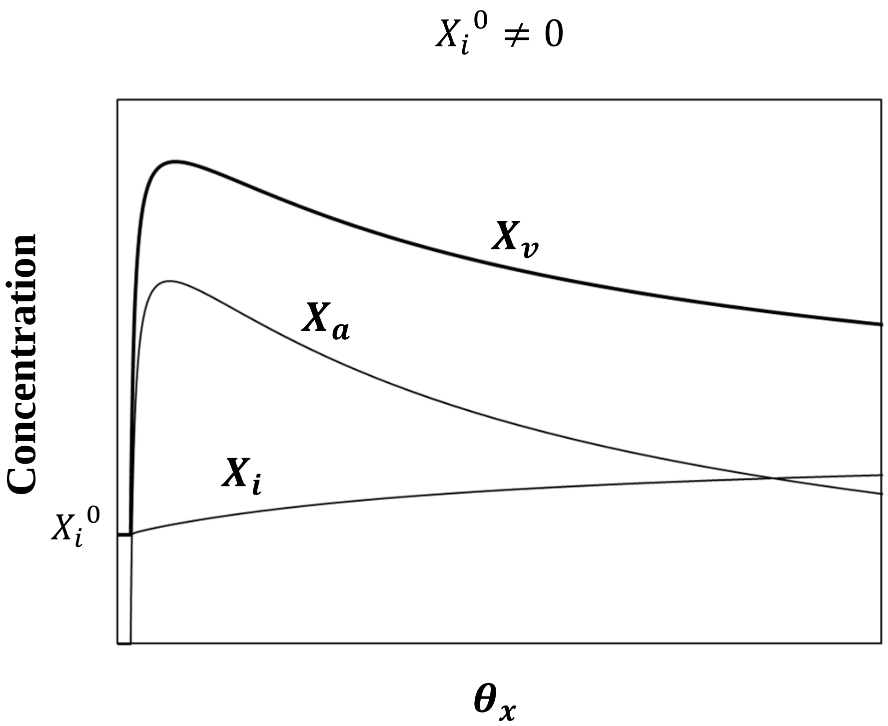
- Reactor total VSS concentration (when ${S_p}^0 = 0$)*

$$X_v = X_a + X_i = \frac{\theta_x}{\theta}\left[{X_i}^0 + \frac{Y(S^0 - S)\{1 + (1 - f_d)b\theta_x\}}{1 + b\theta_x}\right]$$

** Note that in this solution X_v does not account for S_p (biodegradable organic particulates/polymers)*

X's vs. θ_x





- Decay becomes more dominant (as compared to cell synthesis) as θ_x increases
  → For $X_i^{\ 0} = 0$ case, X_a/X_v increases as θ_x increases

Solids production rate [M_x/T]

- Active biomass production rate (r_{abp})

$$r_{abp} = \frac{X_a V}{\theta_x}$$

- Total VSS production rate (r_{VSS})

$$r_{VSS} = \frac{X_v V}{\theta_x}$$

Again, note that defining
$X_v = X_a + X_i$ *and*
$X_v V/\theta_x$ *= total VSS production rate*
implies that we assume $S_p = 0$

Updated solution for observed yield

$$Y_{obs} = Y \frac{1 + (1 - f_d)b\theta_x}{1 + b\theta_x}$$

because

$$r_{VSS} = \frac{X_v V}{\theta_x} = Q{X_i}^0 + \frac{QY(S^0 - S)\{1 + (1 - f_d)b\theta_x\}}{1 + b\theta_x}$$

CSTR, cell recycle: Linking with stoichiometry

$$f_s{}^0 = Y \frac{(n_e \ \ e^- \ eq \ cells/mole \ cells)(8 \ g \ COD/e^- \ eq \ donor)}{(M_c \ \ g \ cells/mole \ cells)}$$

recall $$Y = f_s{}^0 \frac{(M_c \ \ g \ cells/mole \ cells)}{(n_e \ \ e^- \ eq \ cells/mole \ cells)(8 \ g \ COD/e^- \ eq \ donor)}$$

$$f_s = f_s{}^0 \left[\frac{1 + (1 - f_d) b\theta_x}{1 + b\theta_x}\right]$$

because $f_s/f_s{}^0 = Y_n/Y$

➡ Can obtain full stoichiometry for a given θ_x

if half reactions (R_d, R_e, R_c) & growth parameters ($\hat{q}$, K, Y, b) are known