Introduction to Data Mining

Lecture #16: Advertising

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In This Lecture

- Learn the online bipartite matching problem, the greedy algorithm of it, and the notion of competitive ratio
- Learn the problem of web advertising, the adwords problem, and the algorithms for them

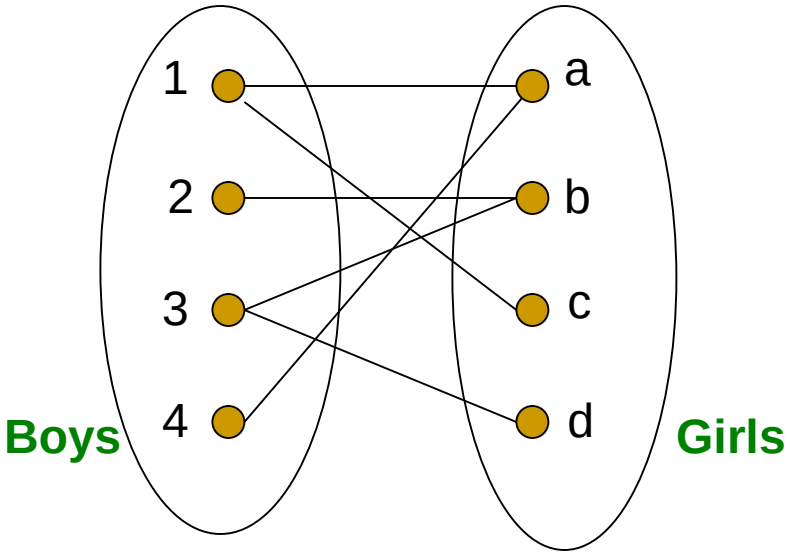
Online Algorithms

- **Classic model of algorithms**
  - You get to see the *entire* input, then compute some function of it
  - In this context, "offline algorithm"
- **Online Algorithms**
  - You get to see the input *one piece at a time*, and need to make irrevocable decisions along the way
  - **Similar to the data stream model**
  - **Why do we care?**

Outline

- **Online Bipartite Matching**
- Web Advertising

Example: Bipartite Matching

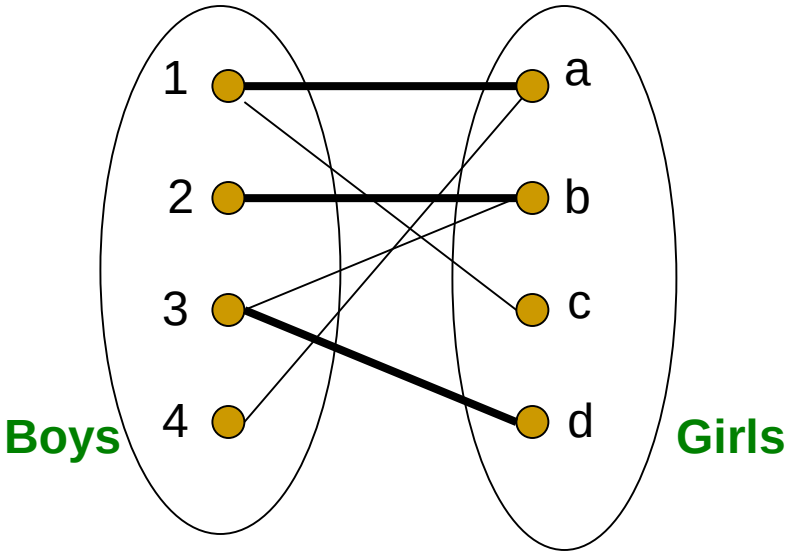


Nodes: Boys and Girls; Edges: Preferences

Goal: Match boys to girls so that maximum number of preferences is satisfied

(but, no person can be matched with >= 2 persons)

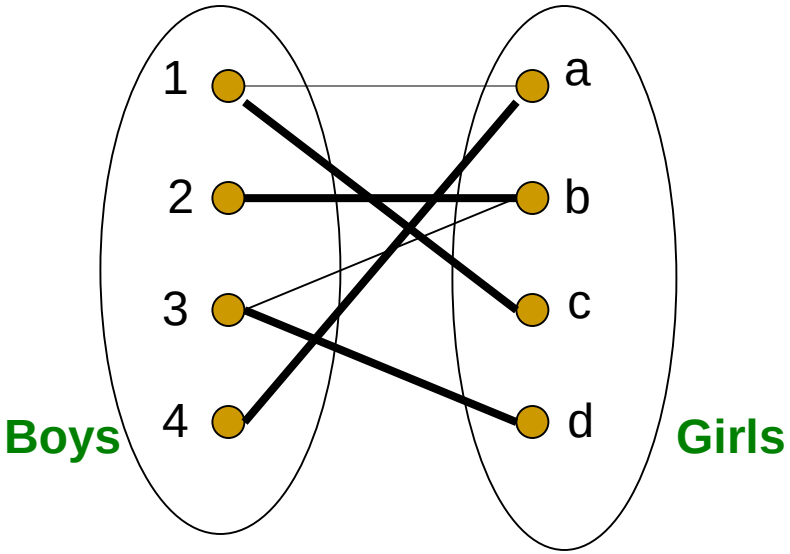
Example: Bipartite Matching



M = {(1,a),(2,b),(3,d)} is a **matching**

Cardinality of matching = |M| = 3

Example: Bipartite Matching



M = {(1,c),(2,b),(3,d),(4,a)} is a **perfect matching**

Perfect matching … all vertices of the graph are matched
Maximal matching … a matching that contains the largest possible number of matches

Matching Algorithm

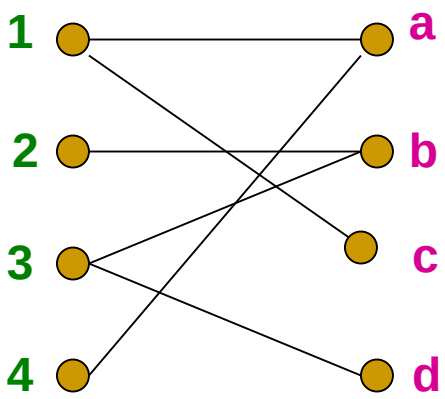
- **Problem: Find a maximal matching for a given bipartite graph**
  - A perfect one if it exists
- There is a polynomial-time offline algorithm based on augmenting paths (Hopcroft & Karp 1973, see http://en.wikipedia.org/wiki/Hopcroft-Karp_algorithm)
- **But what if we do not know the entire graph upfront?**

Online Graph Matching Problem

- Initially, we are given the set **boys**
- In each **round**, **one girl's choices are revealed**
  - That is, girl's **edges** are revealed
- **At that time, we have to decide to either:**
  - Pair the **girl** with a **boy**
  - Do not pair the **girl** with any **boy**

- **Example of application:**
  Assigning tasks to servers (Given a task, and list of servers that can process the task, determine which server to process the task)

Online Graph Matching: Example



(1,a)
(2,b)
(3,d)

Greedy Algorithm

- **Greedy algorithm**
  - An algorithm that follows a heuristic of making the locally optimal choice at each stage with the hope of finding a global optimum
- **Greedy algorithm for the online graph matching problem:**
  - Pair the new girl with **any** eligible boy
    - If there is none, do not pair girl
- **How good is the algorithm?**

Competitive Ratio

- For input ***I***, suppose greedy produces matching M_{greedy} while an optimal matching is M_{opt}

Competitive ratio =

$$min_{\text{all possible inputs } I} (|M_{greedy}|/|M_{opt}|)$$

(what is greedy's worst performance over all possible inputs *I*)

I.e., if competitive ratio is 0.4, we are assured that the greedy algorithm gives an answer which is >= 40% good compared to optimal alg, for *ANY* input.

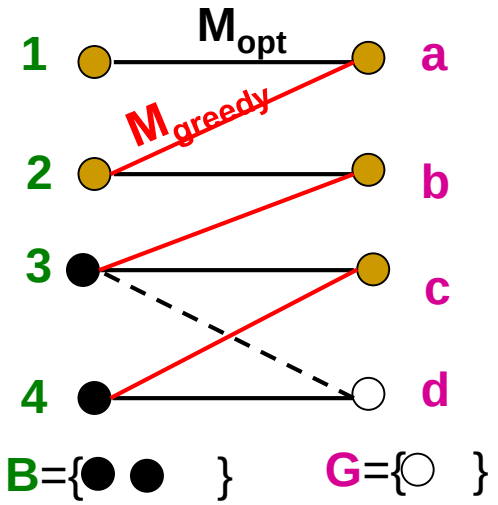
Analyzing the Greedy Algorithm

- Claim: the greedy algorithm for the bipartite matching problem has the competitive ratio 0.5

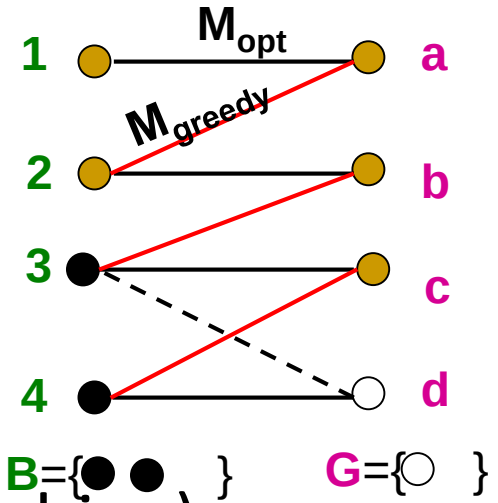
- Proof: (next 2 slides)

Analyzing the Greedy Algorithm

- Consider a case: $M_{greedy} \neq M_{opt}$
- Consider the set G of girls matched in M_{opt} but not in M_{greedy}
- Then every boy B adjacent to girls in G is already matched in M_{greedy}:
  - If there would exist such non-matched (by M_{greedy}) boy adjacent to a non-matched girl then greedy would have matched them
- Since boys B are already matched in M_{greedy} then
  (1) $|M_{greedy}| \geq |B|$

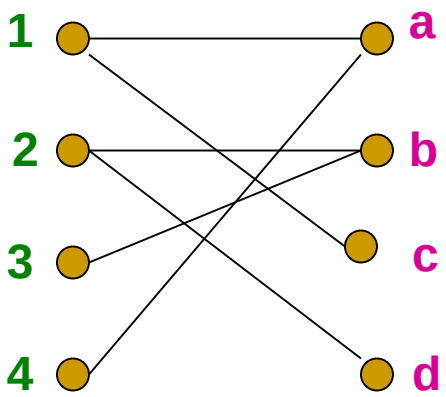


Analyzing the Greedy Algorithm



- **Summary so far:**
  - Girls G matched in M_{opt} but not in M_{greedy}
  - (1) $|M_{greedy}| \geq |B|$
- (2) $|G| \leq |B|$, since G has at least $|G|$ neighbors (at the optimal matching)
  - **So:** $|G| \leq |B| \leq |M_{greedy}|$
- (3) By definition of G also: $|M_{opt}| \leq |M_{greedy}| + |G|$
  - Worst case is when $|G| = |B| = |M_{greedy}|$
- $|M_{opt}| \leq 2|M_{greedy}|$ **then** $|M_{greedy}|/|M_{opt}| \geq 1/2$

Worst-case Scenario



(1,a)
(2,b)

Outline

- ☑ Online Bipartite Matching
- ☐ **Web Advertising**

History of Web Advertising

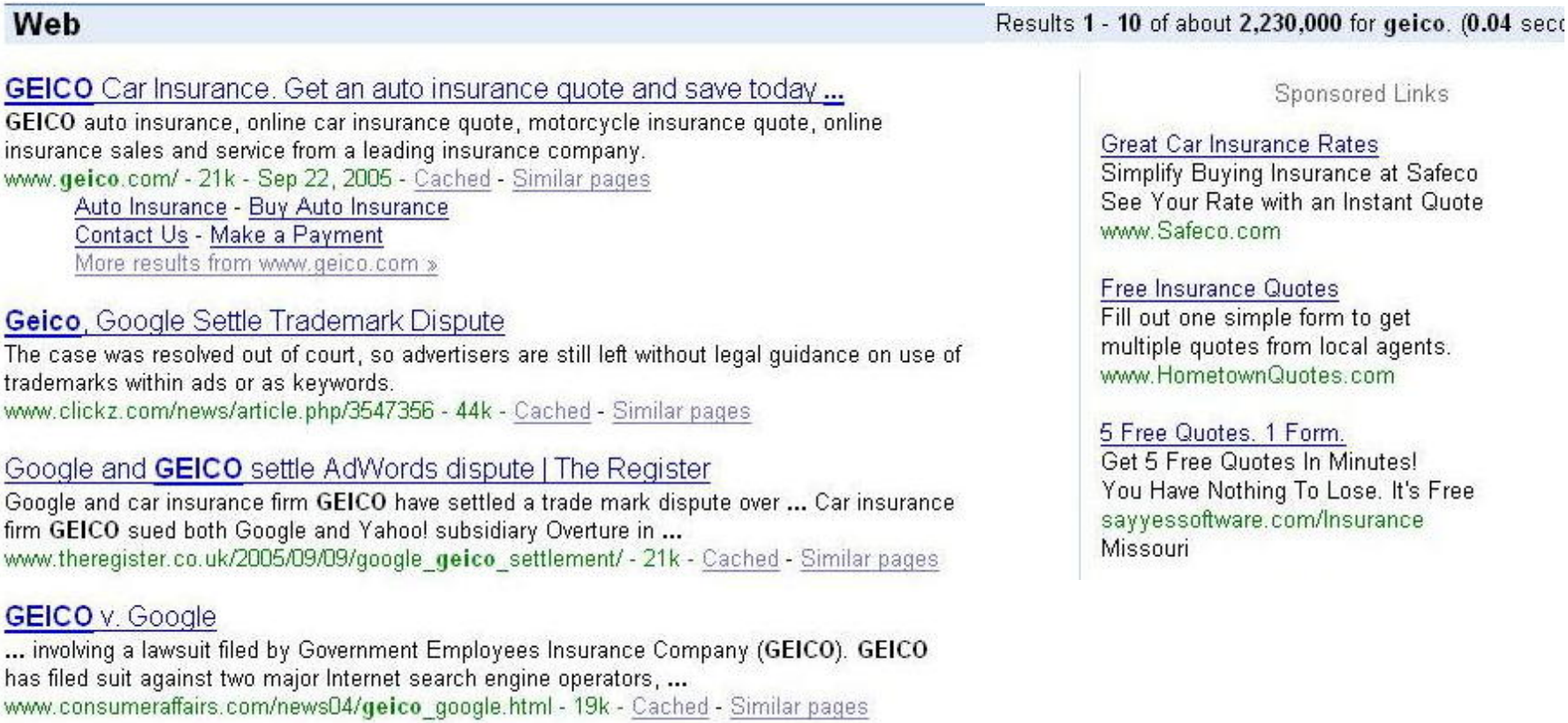
- **Banner ads (1995-2001)**
  - Initial form of web advertising
  - Popular websites charged *X*$ for every 1,000 "impressions" of the ad
    - Called "**CPM**" rate (Cost per thousand impressions)
    - Modeled similar to TV, magazine ads
  - From **untargeted** to **demographically targeted**
  - **Low click-through rates**
    - Low ROI (return on investment) for advertisers

CPM…cost per *mille*
Mille…thousand in Latin

Performance-based Advertising

- **Introduced by Overture around 2000**
  - Advertisers **bid** on **search keywords**
  - When someone searches for that keyword, the **highest bidder's ad is shown**
  - Advertiser is charged only if the ad is clicked on
- Similar model adopted by Google with some changes around 2002
  - Called **Adwords**

Ads vs. Search Results



Web Results **1** - **10** of about **2,230,000** for **geico**. (**0.04** seco

GEICO Car Insurance. Get an auto insurance quote and save today ...
GEICO auto insurance, online car insurance quote, motorcycle insurance quote, online insurance sales and service from a leading insurance company.
www.**geico**.com/ - 21k - Sep 22, 2005 - Cached - Similar pages
Auto Insurance - Buy Auto Insurance
Contact Us - Make a Payment
More results from www.geico.com »

Geico, Google Settle Trademark Dispute
The case was resolved out of court, so advertisers are still left without legal guidance on use of trademarks within ads or as keywords.
www.clickz.com/news/article.php/3547356 - 44k - Cached - Similar pages

Google and **GEICO** settle AdWords dispute | The Register
Google and car insurance firm **GEICO** have settled a trade mark dispute over ... Car insurance firm **GEICO** sued both Google and Yahoo! subsidiary Overture in ...
www.theregister.co.uk/2005/09/09/google_**geico**_settlement/ - 21k - Cached - Similar pages

GEICO v. Google
... involving a lawsuit filed by Government Employees Insurance Company (**GEICO**). **GEICO** has filed suit against two major Internet search engine operators, ...
www.consumeraffairs.com/news04/**geico**_google.html - 19k - Cached - Similar pages

Sponsored Links

Great Car Insurance Rates
Simplify Buying Insurance at Safeco
See Your Rate with an Instant Quote
www.Safeco.com

Free Insurance Quotes
Fill out one simple form to get multiple quotes from local agents.
www.HometownQuotes.com

5 Free Quotes. 1 Form.
Get 5 Free Quotes In Minutes!
You Have Nothing To Lose. It's Free
sayyessoftware.com/Insurance
Missouri

Web 2.0

- **Performance-based advertising works!**
  - Multi-billion-dollar industry
- **Interesting problem:**
  What ads to show for a given query?
  - (Today's lecture)
- **If I am an advertiser, which search terms should I bid on and how much should I bid?**
  - (Not focus of today's lecture)

Adwords Problem

- **Given:**
  - **1.** A set of bids by advertisers for search queries
  - **2.** A click-through rate for each advertiser-query pair
  - **3.** A budget for each advertiser (say for 1 month)
  - **4.** A limit on the number of ads to be displayed with each search query
- **Respond to each search query with a set of advertisers such that:**
  - **1.** The size of the set is no larger than the limit on the number of ads per query
  - **2.** Each advertiser has bid on the search query
  - **3.** Each advertiser has enough budget left to pay for the ad if it is clicked upon

Adwords Problem

- A stream of queries arrives at the search engine: $q_1, q_2, \ldots$
- Several advertisers bid on each query
- When query q_i arrives, search engine must pick a subset of advertisers whose ads are shown

- **Goal:** **Maximize search engine's revenues**
  - **Simple solution:** Instead of raw bids, use the "**expected revenue per click**" (i.e., **Bid*CTR**)
- **Clearly we need an online algorithm!**

The Adwords Innovation

Advertiser	Bid	CTR	Bid * CTR
A	$1.00	1%	1 cent
B	$0.75	2%	1.5 cents
C	$0.50	2.5%	1.125 cents

Click through rate

Expected revenue

The Adwords Innovation

Advertiser	Bid	CTR	Bid * CTR
B	$0.75	2%	1.5 cents
C	$0.50	2.5%	1.125 cents
A	$1.00	1%	1 cent

Complications: Budget

- **Two complications:**
  - **Budget**
  - **CTR of an ad is unknown**

- **Each advertiser has a limited budget**
  - **Search engine guarantees that the advertiser will not be charged more than their daily budget**

Complications: CTR

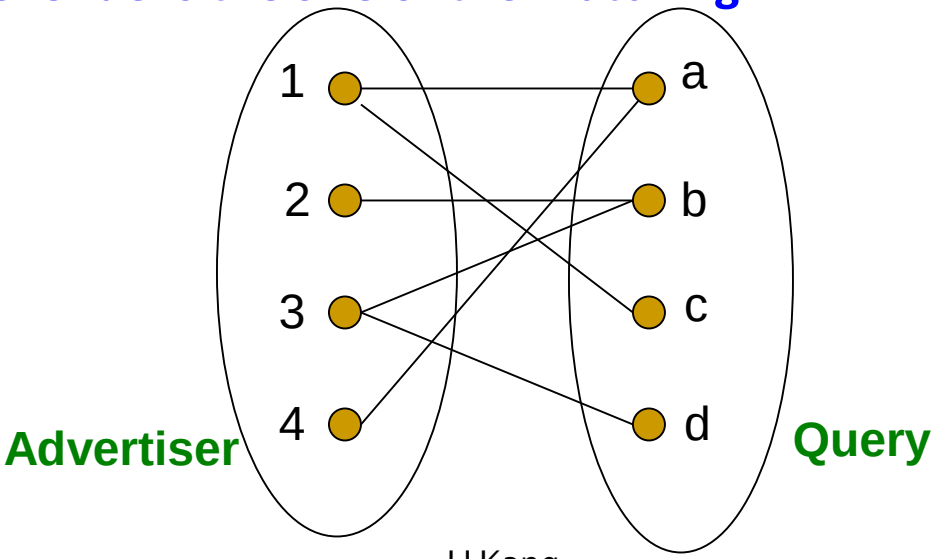
- **CTR: Each ad has a different likelihood of being clicked**
  - **Advertiser 1** bids $2, click probability = 0.1
  - **Advertiser 2** bids $1, click probability = 0.5
  - **Clickthrough rate (CTR)** is measured **historically**
    - **Very hard problem: Exploration vs. exploitation**
      Exploit: Should we keep showing an ad for which we have good estimates of click-through rate
      or
      Explore: Shall we show a brand new ad to get a better sense of its click-through rate

Greedy Algorithm

- **Our setting: Simplified environment**
  - For each query, show only **1** ad
  - All advertisers have the same budget ***B***
  - All ads are equally likely to be clicked
  - Value of each ad is the same (=**1**)
    - Revenue increases by 1 whenever an ad is clicked
- **Simplest algorithm is greedy:**
  - For a query pick any advertiser who has bid **1** for that query
  - **Competitive ratio of greedy is ½**
    - **Why?**

Greedy Algorithm

- **Simplest algorithm is greedy:**
  - For a query pick any advertiser who has bid **1** for that query
  - **Competitive ratio of greedy is ½**
    - **Why? Exactly the same problem as 'bipartite matching'**
    - **The revenue is the size of the matching**



Bad Scenario for Greedy

- **Two advertisers A and B**
  - ***A*** bids on query ***x***, ***B*** bids on ***x*** and ***y***
  - Both have budgets of **$4**
- **Query stream: *x x x x y y y y***
  - Worst case greedy choice: ***B B B B*** _ _ _ _
  - Optimal: **A A A A B B B B**
  - **Competitive ratio = ½**
- **This is the worst case!**
  - **Note:** Greedy algorithm is deterministic – it always resolves draws in the same way

BALANCE Algorithm [MSVV]

- **BALANCE** Algorithm by Mehta, Saberi, Vazirani, and Vazirani
  - **For each query, pick the advertiser with the largest unspent budget**
    - Break ties arbitrarily (**but in a deterministic way**)

Example: BALANCE

- **Two advertisers A and B**
  - **A** bids on query ***x***, **B** bids on ***x*** and ***y***
  - Both have budgets of **$4**
- **Query stream: *x x x x y y y y***
- **BALANCE choice: A B A B B B _ _**
  - Optimal: **A A A A B B B B**

BALANCE on 2 Advertisers

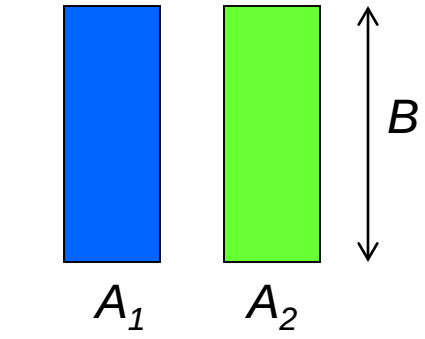
- **Claim:** For **BALANCE** on **2** advertisers
  Competitive ratio = ¾
- **Proof**: (next 3 slides)

BALANCE on 2 Advertisers

- **Consider simple case (w.l.o.g.):**
  - **2** advertisers, $\mathbf{A_1}$ and $\mathbf{A_2}$, each with budget **B** (≥ 1)
  - # of queries: 2**B**
  - (*) Optimal solution exhausts both advertisers' budgets: i.e., a query is assigned to at least an advertiser

- **BALANCE must exhaust at least one advertiser's budget:**
  - If not, there would be some query assigned to neither advertiser, even though the advertisers have some remaining budgets => contradicts (*)
  - Assume BALANCE exhausts $\boldsymbol{A_2}$'s budget, but allocates ***x*** queries fewer than the optimal
  - **Revenue: *BAL = 2B - x***

BALANCE on 2 Advertisers

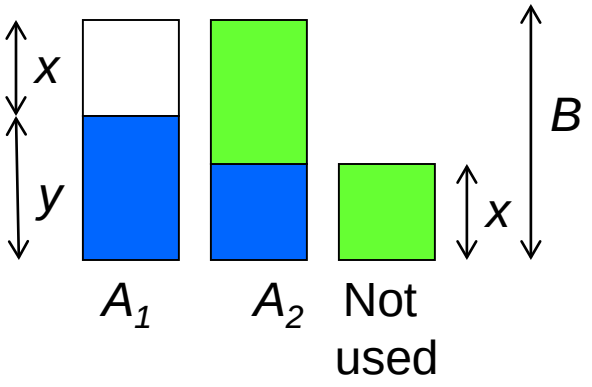


Queries allocated to A_1 in the optimal solution

Queries allocated to A_2 in the optimal solution

Optimal revenue = **2B**

Assume Balance gives revenue **= 2B-x = B+y**

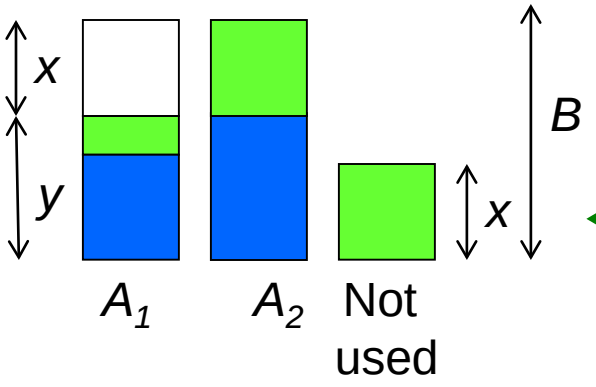


Unassigned queries should be assigned to A_2

(if we could assign to A_1 we would since we still have the budget)

Goal: Show we have **y ≥ x**

Case 1) ≤ ½ of A_1's queries got assigned to A_2 then $y \geq B/2$



Case 2) > ½ of A_1's queries got assigned to A_2 then $x \leq B/2$ **(proof: next slide)**

Balance revenue is minimum for $x = y = B/2$

Minimum Balance revenue = $3B/2$

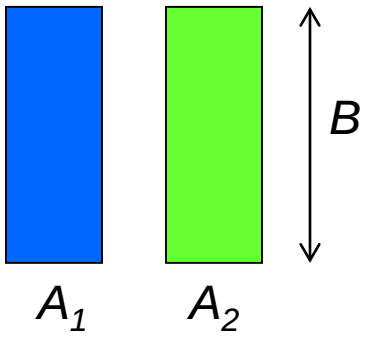
Competitive Ratio = 3/4

BALANCE exhausts A_2's budget

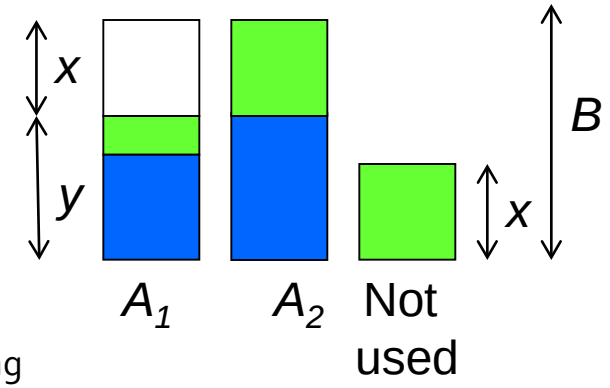
BALANCE on 2 Advertisers

- **Claim: in (Case 2), when** > ½ of **A_1**'s queries got assigned to **A_2** , x ≤ B/2.
  - (Proof)
    - Consider the last query (of A_1) that is assigned to A_2
    - At that time (right before assigned to A_2), Budget of A_2 ≥ Budget of A_1
    - Also, at that time, Budget of A_2 ≤ ½ B
    - Thus, Budget of A_1 ≤ ½ B
    - Since the budget only decreases, x ≤ ½ B

(Optimal)



(Balance)



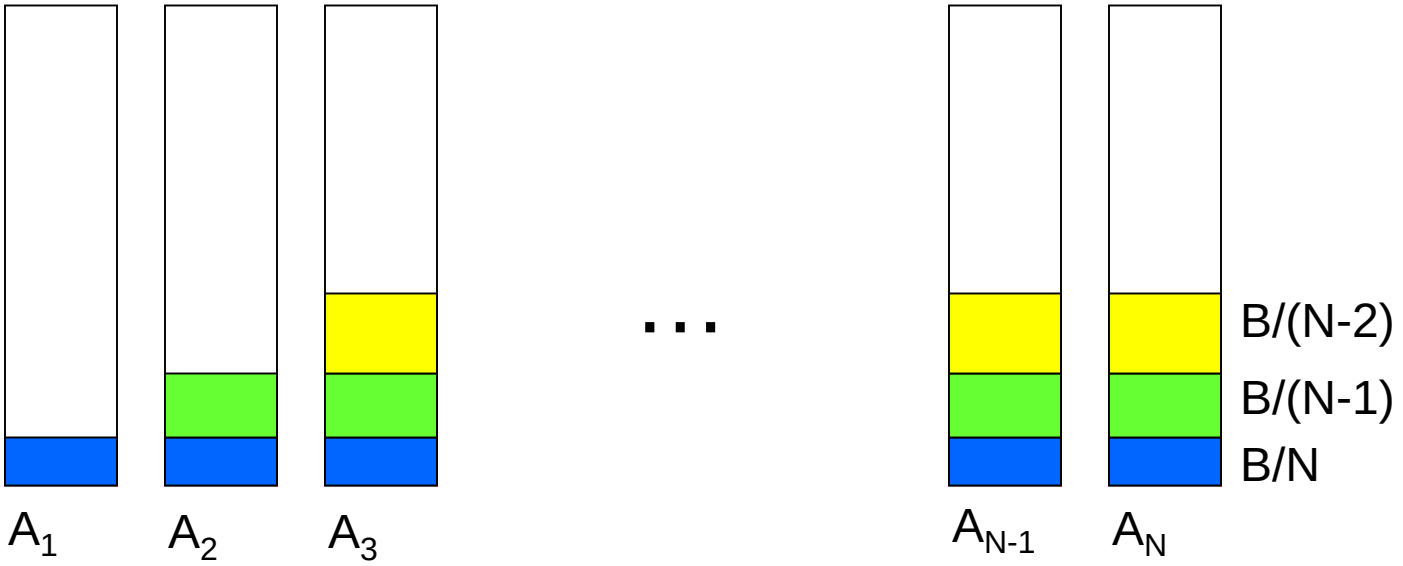
BALANCE: General Result

- **In the general case (many bidders, arbitrary bid, and arbitrary budget), worst competitive ratio of BALANCE is 1–1/e = approx. 0.63**
  - Interestingly, no online algorithm has a better competitive ratio!
- **Let's see the worst case example that gives this ratio**

Worst case for BALANCE

- ***N* advertisers: $A_1, A_2, \ldots A_N$**
  - Each with budget $B > N$
- **Queries:**
  - $N \cdot B$ queries appear in N rounds of B queries each
- **Bidding:**
  - Round **1** queries: bidders $A_1, A_2, \ldots, A_N$
  - Round **2** queries: bidders $A_2, A_3, \ldots, A_N$
  - Round ***i*** queries: bidders $A_i, \ldots, A_N$
- **Optimum allocation:**
  Allocate round ***i*** queries to A_i
  - Optimum revenue $N \cdot B$

BALANCE Allocation

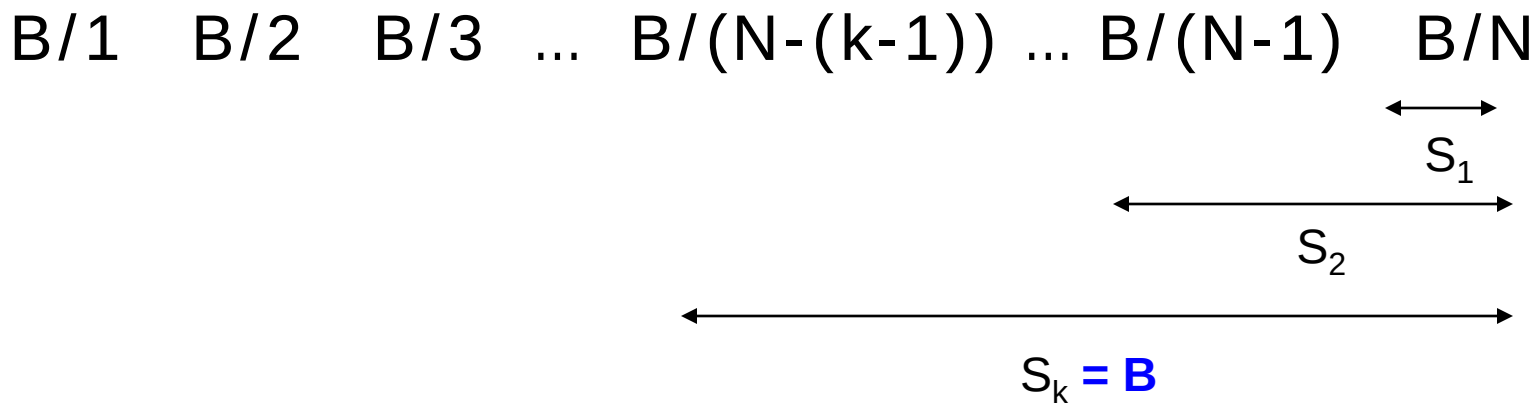


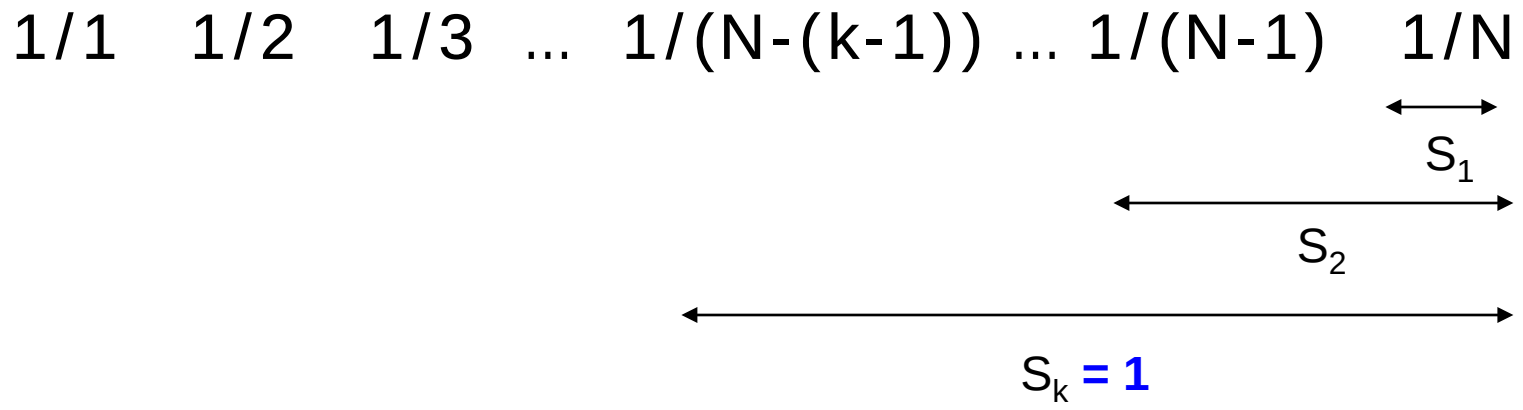
BALANCE assigns each of the queries in round 1 to **N** advertisers. After ***k*** rounds, sum of allocations to each of advertisers $A_k,\ldots,A_N$ is

$$S_k = S_{k+1} = \cdots = S_N = \sum_{i=1}^{k} \frac{B}{N-(i-1)}$$

If we find the smallest *k* such that $S_k \geq B$, then after *k* rounds we cannot allocate any queries to any advertiser

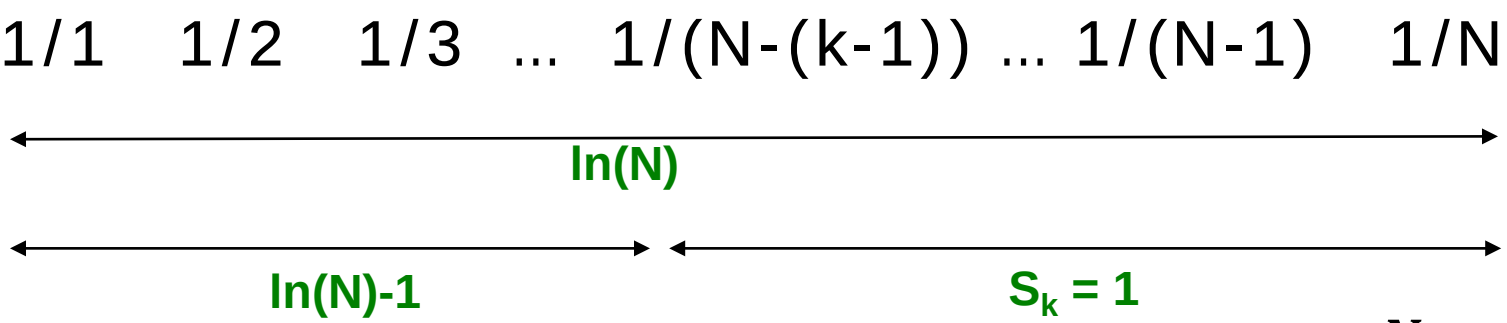
BALANCE: Analysis





BALANCE: Analysis

- **Fact:** $\boldsymbol{H_n = \sum_{i=1}^{n} 1/i \approx \ln(n)}$ for large $\boldsymbol{n}$
  - Result due to Euler



- $\boldsymbol{S_k = 1}$ implies: $\boldsymbol{H_{N-k} = ln(N) - 1 = ln(\frac{N}{e})}$
- We also know: $\boldsymbol{H_{N-k} = ln(N-k)}$
- **So:** $\boldsymbol{N - k = \frac{N}{e}}$
- **Then:** $\boldsymbol{k = N(1 - \frac{1}{e})}$

N terms sum to ln(*N*).
Last *k* terms sum to 1.
First *N-k* terms sum
to ln(*N-k*) but also to ln(*N*)-1

BALANCE: Analysis

- So after the first **k=N(1-1/e)** rounds, we cannot allocate a query to any advertiser
- **Revenue = B·N (1-1/e)**
- **Competitive ratio = 1-1/e**

General Version of the Problem

- **Arbitrary bids and arbitrary budgets!**
- Consider we have 1 query ***q***, advertiser ***i***
  - Bid = x_i
  - Budget = b_i
- **In a general setting BALANCE can be terrible**
  - Consider two advertisers A_1 and A_2
  - A_1: $x_1 = 1$, $b_1 = 110$
  - A_2: $x_2 = 10$, $b_2 = 100$
  - Consider we see **10** instances of **q**
  - BALANCE always selects A_1 and earns **10**
  - Optimal earns **100**

Generalized BALANCE

- **Arbitrary bids:** consider query q, bidder i
  - Bid = x_i
  - Budget = b_i
  - Amount spent so far = m_i
  - Fraction of budget left over $f_i = 1-m_i/b_i$
  - Define $\psi_i(q) = x_i(1-e^{-f_i})$
- Allocate query q to bidder i with largest value of $\psi_i(q)$
- **Same competitive ratio (1-1/e)**

Questions?