

Fundamentals of biological treatment II

Modeling bacterial growth

- As bacteria consume the substrate, some (specific) portion is used for energy and the other is used to produce new biomass
→ new growth of cells is directly proportional to the substrate utilized
- So, the bacteria growth rate from substrate utilization is expressed as:

$$r_g = \left(\frac{dX_a}{dt} \right)_{growth} = \frac{\mu_m X_a S}{K_s + S}$$

r_g = bacteria growth rate from substrate utilization (g/m³-d)

μ_m = maximum specific bacteria growth rate (1/d)

with $r_g = Yr_{su}$ and $\mu_m = kY$

Y = true yield (g biomass/g substrate utilized)

→ This is the biomass yield we studied!

Modeling bacterial growth

- The substrate utilization rate can be written as:

$$r_{su} = \frac{\mu_m X_a S}{Y(K_s + S)}$$

- Another form of Monod equation:

$$\mu = \frac{1}{X_a} \cdot \left(\frac{dX_a}{dt} \right)_{growth} = \frac{\mu_m S}{K_s + S}$$

$\mu =$ specific bacteria growth rate (1/d)

Biomass decay

- Microorganism concentration decrease when the substrate is depleted
- This is true in the presence of substrates as well!
- Decay (or endogenous decay endogenous respiration)
 - Cell maintenance energy needs
 - Cell lysis due to death or stress from environmental factors
 - Predation (protozoa, etc.)
 - Generally assumed to be proportional to cell concentration:

$$\left(\frac{dX_a}{dt}\right)_{decay} = -bX_a$$

$X_a = \text{active biomass concentration [M/L}^3\text{]}$
 $b = \text{decay coefficient [T}^{-1}\text{]}$

- b in the range of $0.05 \sim 0.20 \text{ d}^{-1}$

Modeling bacterial growth

- **Net biomass growth rate**
(net biomass growth)
= (biomass growth according to substrate utilization) – (biomass decay)

$$\begin{aligned} r_X &= Yr_{su} - bX_a \\ &= Y \frac{kX_a S}{K_s + S} - bX_a \end{aligned}$$

r_X = net biomass growth rate (g VSS/m³-d)

- **Net specific biomass growth rate**

$$\mu_{net} = \frac{r_X}{X_a} = Y \frac{kS}{K_s + S} - b$$

μ_{net} = net specific biomass growth rate (1/d)

Microbial growth kinetics

- Actually, the Monod kinetics can be applied for any growth-limiting substrates
 - Substrates can be e^- donor, e^- acceptor, nutrients, etc.
 - Quite often the e^- donor is limiting while others are available in excess – for growth kinetics, the term substrate generally refers to e^- donor
- **Generalized equation**
 - If factors other than e^- donor can be limiting, include those as well!

Microbial growth kinetics

ex) for aerobic, heterotrophic bacteria; if bsCOD, DO, and ammonia-N are limiting:

$$r_{su} = \left[\frac{\mu_{H,max} S_s}{Y_H (K_s + S_s)} \right] \left(\frac{S_o}{K_o + S_o} \right) \left(\frac{S_{NH}}{K_{NH} + S_{NH}} \right) X_{a,H}$$

$\mu_{H,max}$ = maximum specific growth rate of heterotrophic bacteria (1/d)

Y_H = heterotrophic bacteria synthesis yield coefficient (g VSS/g COD used)

$X_{a,H}$ = active heterotrophic bacteria concentration (g VSS/m³)

S_i = concentration for variable i (i = substrate, DO, ammonia-nitrogen) (g/m³)

K_i = half-velocity constant for variable i (g/m³)

here, the term "substrate" is used for bsCOD (the e⁻ donor)

Microbial growth kinetics

Typical range/values of kinetic coefficients for activated sludge process

Coefficient	Unit	Value ^a	
		Range	Typical
k	g bsCOD/g VSS-d	4-12	6
K_s	mg/L BOD	20-60	30
	mg/L bsCOD	5-30	15
Y	mg VSS/mg BOD	0.4-0.8	0.6
	mg VSS/mg COD	0.4-0.6	0.45
b	1/d	0.06-0.15	0.10

^aAt 20°C, from Metcalf & Eddy / Aecom

Oxygen uptake rate & temperature effect

- **Rate of oxygen uptake**

- Recall that the COD of biomass was:

1.42 g COD/g VSS for $C_5H_7O_2N$

$$r_o = r_{su} - 1.42r_X$$

r_o = oxygen uptake rate (g O_2 /m³-d)

r_{su} = substrate utilization rate (g bsCOD/m³-d)

r_X = net biomass growth rate (g VSS/m³-d)

- **Effect of temperature**

- Recall the modified van't Hoff-Arrhenius relationship:

$$k_T = k_{20}\theta^{(T-20)}$$

*θ = temperature correction factor,
range from 1.02 to 1.25*

Total VSS and active biomass

- Note that total VSS includes not only active biomass but also:
 - Cell debris resulting from endogenous decay
 - Non-biodegradable VSS (nbVSS) in the influent wastewater
- During cell death, some portion dissolves into the liquid for consumption by other bacteria, and the other portion remains as non-biodegradable material
 - 10~15% of original cell weight is converted to nbVSS
 - This is referred to as cell debris
- **Rate of production of cell debris**

$$r_{X,i} = f_d bX$$

$r_{X,i}$ = rate of cell debris production (g VSS/m³-d)

f_d = fraction of biomass that remains as cell debris,

0.10-0.15 g VSS/g biomass VSS depleted by decay

Total VSS and active biomass

- The **total VSS production rate** in the aeration tank of an activated sludge process

$$r_{X,VSS} = Yr_{su} - bX_a + f_d bX_a + QX_i^0 / V$$

*net biomass
VSS from
bsCOD*

*nbVSS from
cells*

*nbVSS in
influent*

$r_{X,VSS}$ = total VSS production rate (g/m³-d)

Q = influent flowrate (m³/d)

X_i^0 = influent nbVSS concentration (g/m³)

V = volume of aeration tank (m³)

- The **active fraction of biomass**

$$F_{X,act} = (Yr_{su} - bX_a) / r_{X,VSS}$$

$F_{X,act}$ = active fraction of biomass in MLVSS, g VSS/g VSS

Total VSS and active biomass

- **Net biomass yield**

- The ratio of the net biomass growth rate and the substrate utilization rate

$$Y_{bio} = r_X / r_{su}$$

Y_{bio} = net biomass yield, g biomass/g substrate used

- **Observed yield**

- The ratio of the actual solids production rate and the substrate utilization rate for a system

$$Y_{obs} = r_{X,VSS} / r_{su}$$

Y_{obs} = observed yield, g VSS produced/g substrate removed

Total VSS and active biomass

Q: An aerobic complete-mix treatment process is used to treat wastewater. The amount of bsCOD in the influent wastewater is 300 g/m^3 and the influent nbVSS concentration is 50 g/m^3 . The influent flowrate is $1000 \text{ m}^3/\text{d}$, the aerobic tank biomass concentration is 2000 g/m^3 , the reactor bsCOD concentration is 2.4 g/m^3 , and the reactor volume is 335 m^3 . If the cell debris fraction (f_d) is 0.10, determine: **i) the net biomass yield, ii) the observed VSS yield, and iii) the active biomass fraction in the MLVSS.** Use the following kinetic parameters.

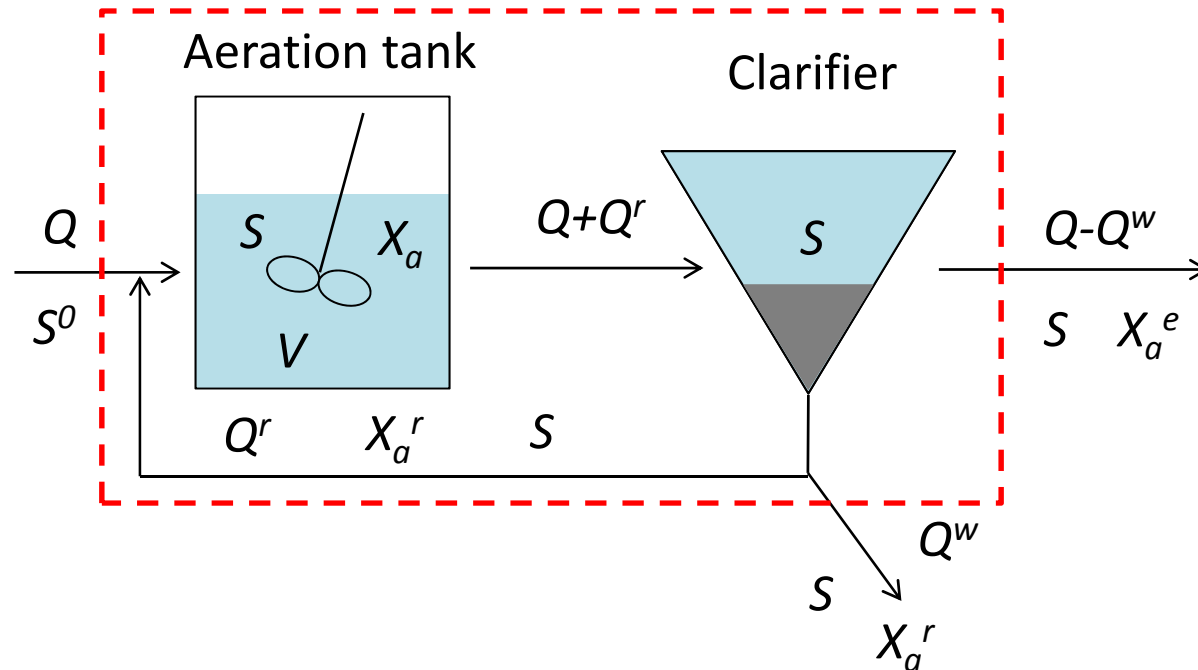
$$k = 6 \text{ g bsCOD/g VSS-d}$$

$$K_s = 15 \text{ g bsCOD/L}$$

$$Y = 0.45 \text{ mg VSS/mg bsCOD}$$

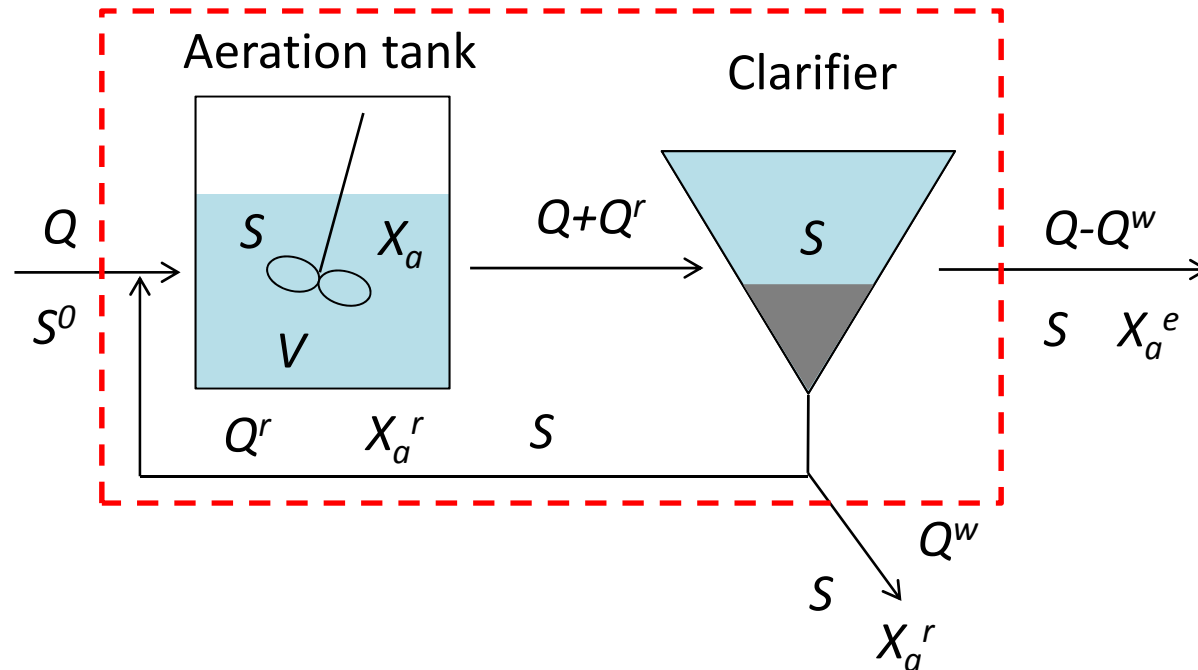
$$b = 0.10 \text{ g VSS/g VSS-d}$$

Suspended growth processes



- The majority of the settled sludge in the secondary clarifier is returned to the aeration tank to obtain high biomass concentration
- Some portion of the settled sludge (due to net growth of biomass) is removed from the system for steady state operation

Suspended growth processes



- General assumptions:
 - Biodegradation of substrate occurs in the aeration tank only, not in the clarifier
 - No active biomass in the influent

Key variable - SRT

- Solids Retention Time (or mean cell residence time)
- The average time the activated sludge solids are in the system
- So SRT can be defined as:
(Amount of solids in the system) / (rate of solids exiting the system)

Assuming that the amount of solids in the clarifier is negligible compared to that in the aeration tank,

$$SRT = \frac{VX_a}{(Q - Q^w)X_a^e + Q^wX_a^r}$$

SRT = solids retention time (d)

V = aeration tank volume (m³)

Q = influent flowrate (m³/d)

X_a = active biomass concentration in the aeration tank (g VSS/m³)

Q^w = waste sludge flowrate (m³/d)

X_a^e = active biomass concentration in the effluent (g VSS/m³)

X_a^r = active biomass concentration in the return activated sludge line (g VSS/m³)