

Air pollution IV

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- Adiabatic lapse rate in the atmosphere
- Air stability
- Modeling atmospheric dispersion of air pollutants emitted from a point source

Adiabatic lapse rate

- When air moves vertically in the atmosphere, how does its temperature change?
 - This is close to the adiabatic expansion; the air adjusts to the decreasing atmospheric pressure by expanding in volume, with a negligible exchange of heat between it and the surrounding air



Adiabatic lapse rate

- First law of thermodynamics

$$dU = dQ - dW$$

change in internal
energy of the
system

Heat transfer to
the system

Change in energy
due to work done
by the system

For an adiabatic system, $dQ = 0$

$$dW = p dV \quad p = \text{pressure}; V = \text{volume}$$

$$dU = C_v dT \quad C_v = \text{heat capacity}; T = \text{temperature}$$

$$\text{So: } C_v dT = -p dV$$

As air moves up, $V \uparrow \Rightarrow T \downarrow$

Adiabatic lapse rate and air stability

- Dry adiabatic lapse rate

$$\Gamma = -\frac{dT}{dz} = 1^{\circ}C / 100\ m$$

- Air stability based on dry adiabatic lapse rate

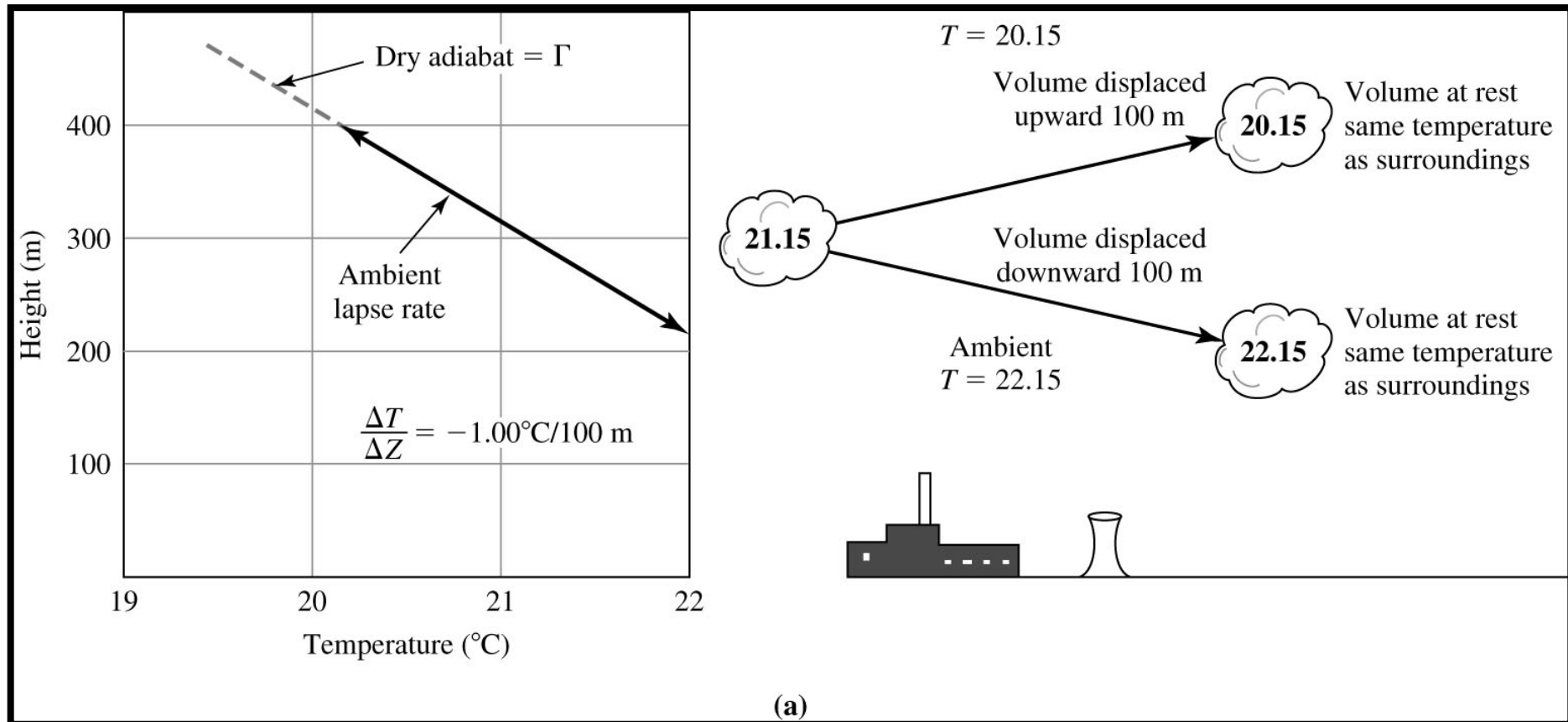
$\Lambda = \Gamma$ Neutral stability

$\Lambda > \Gamma$ Unstable atmosphere

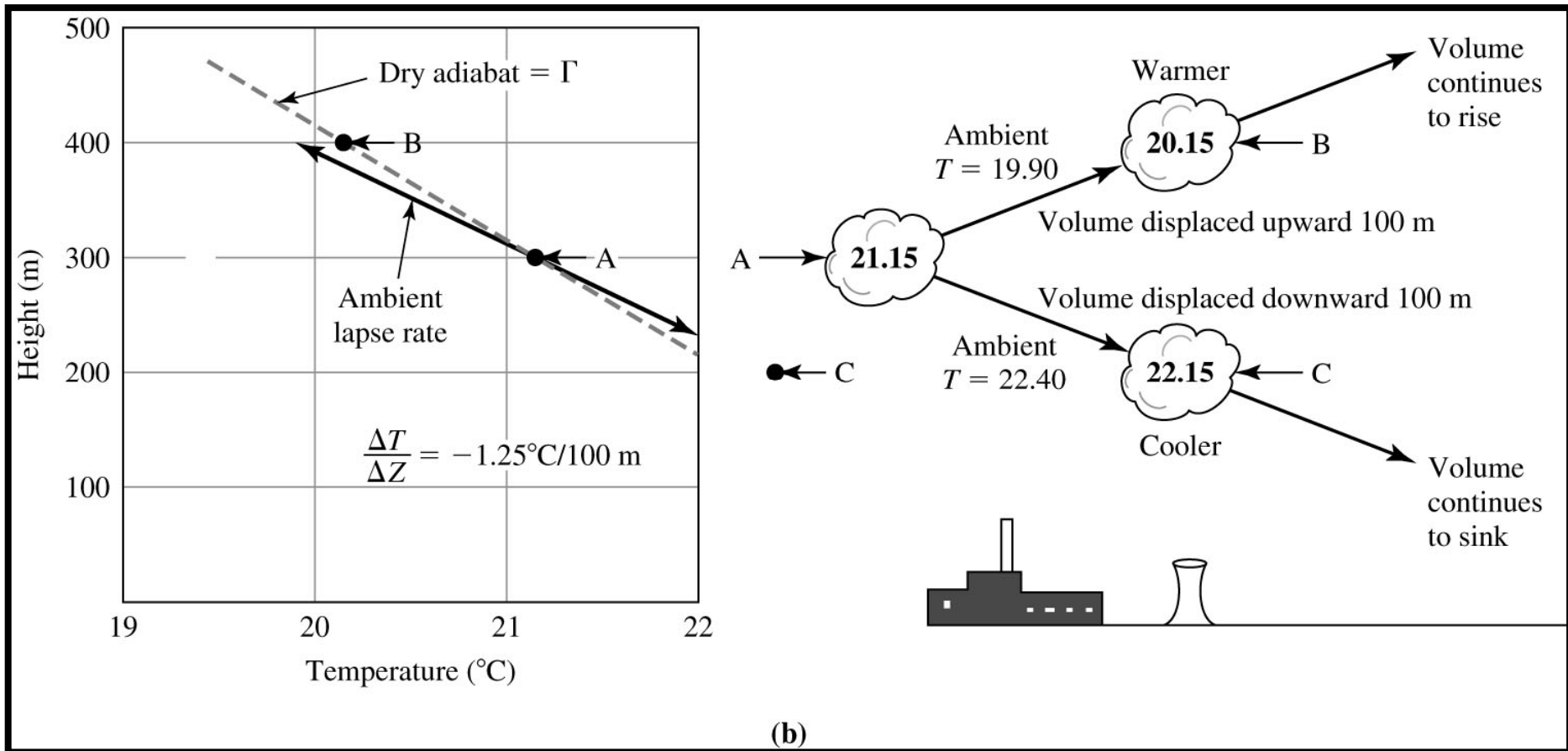
$\Lambda < \Gamma$ Stable atmosphere

$\Lambda = \text{actual lapse rate of the atmosphere}$

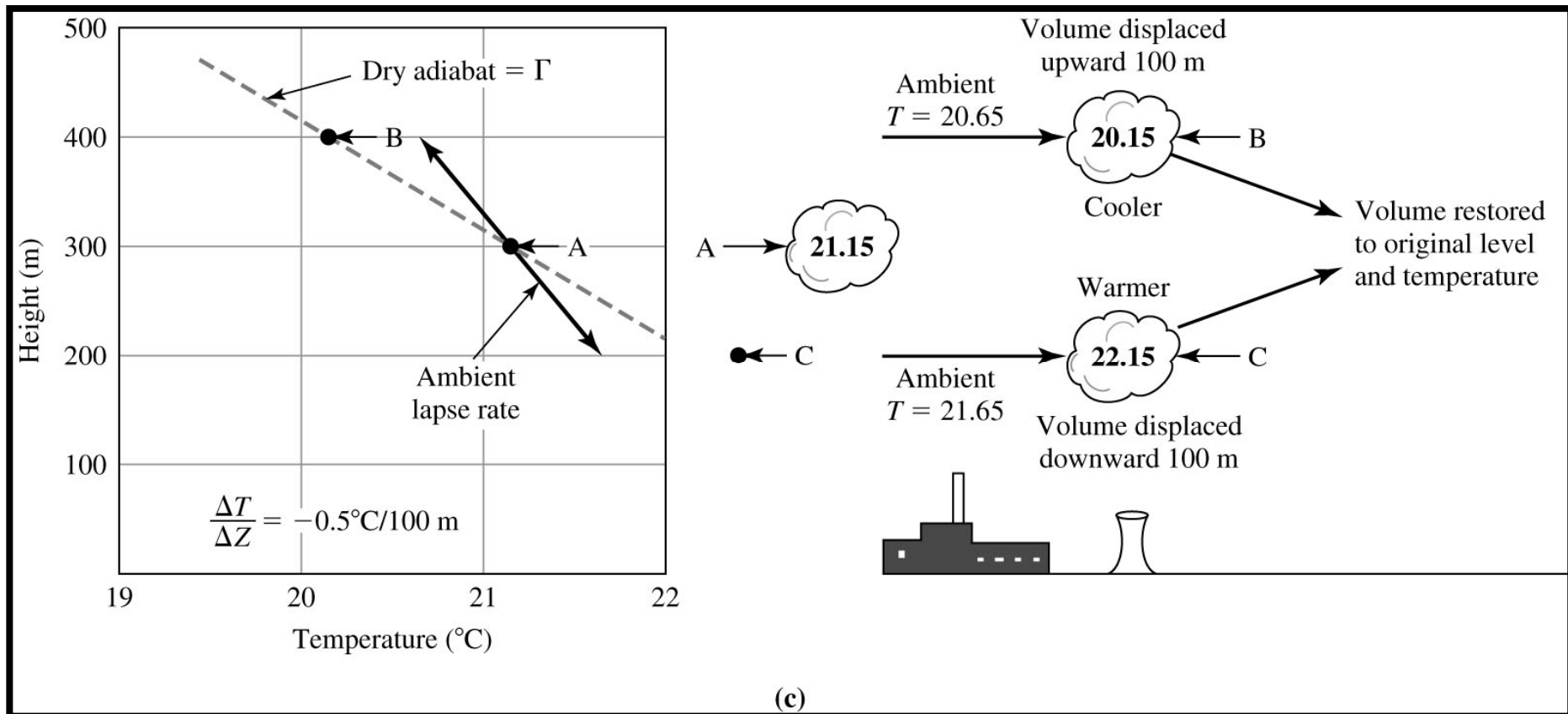
Neutral stability



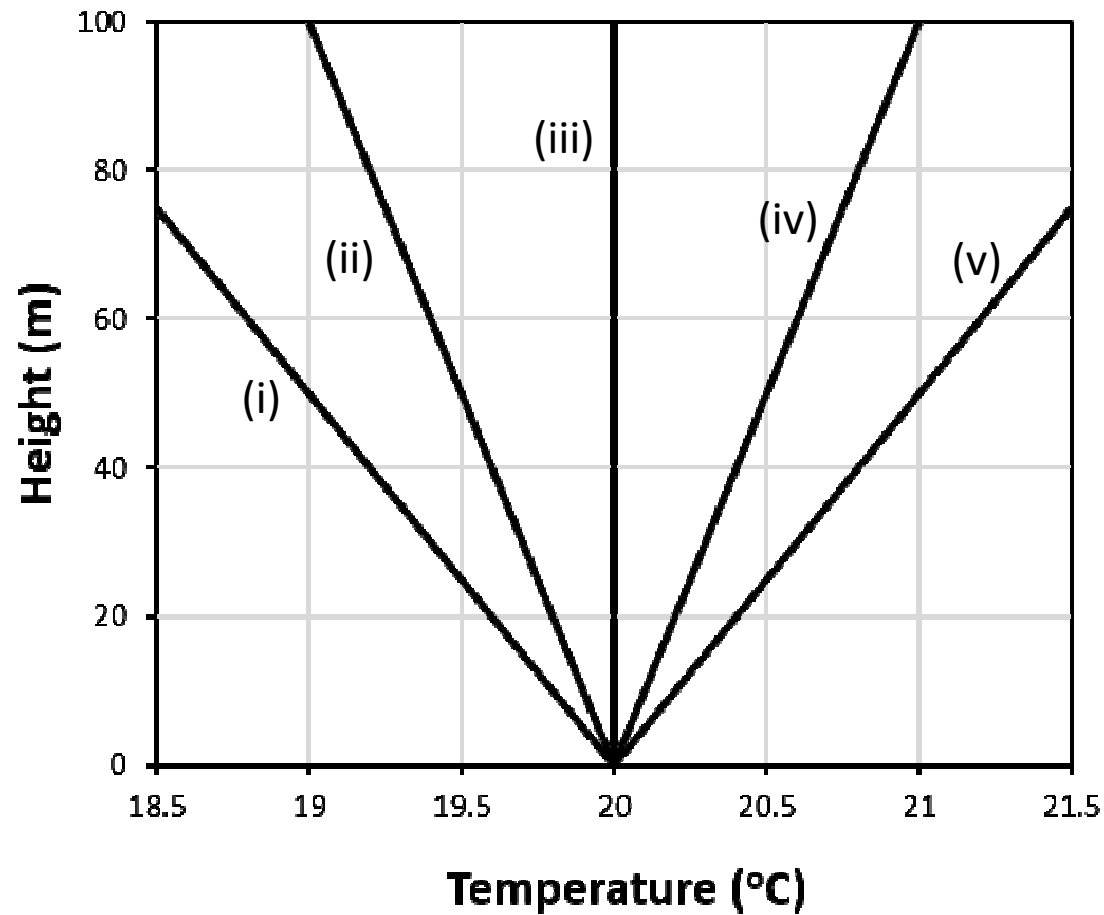
Unstable



Stable



Air stability



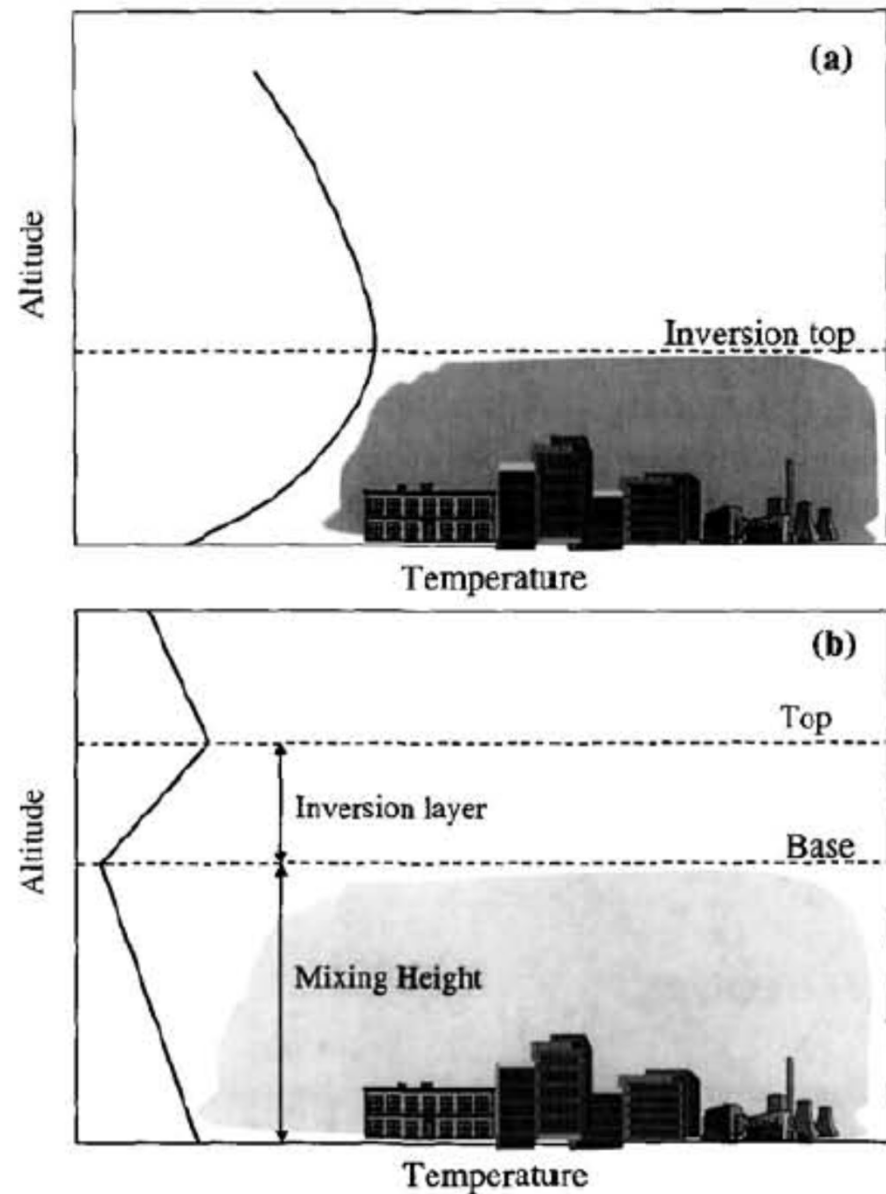
Neutral:

Unstable:

Stable:

Inversion layer

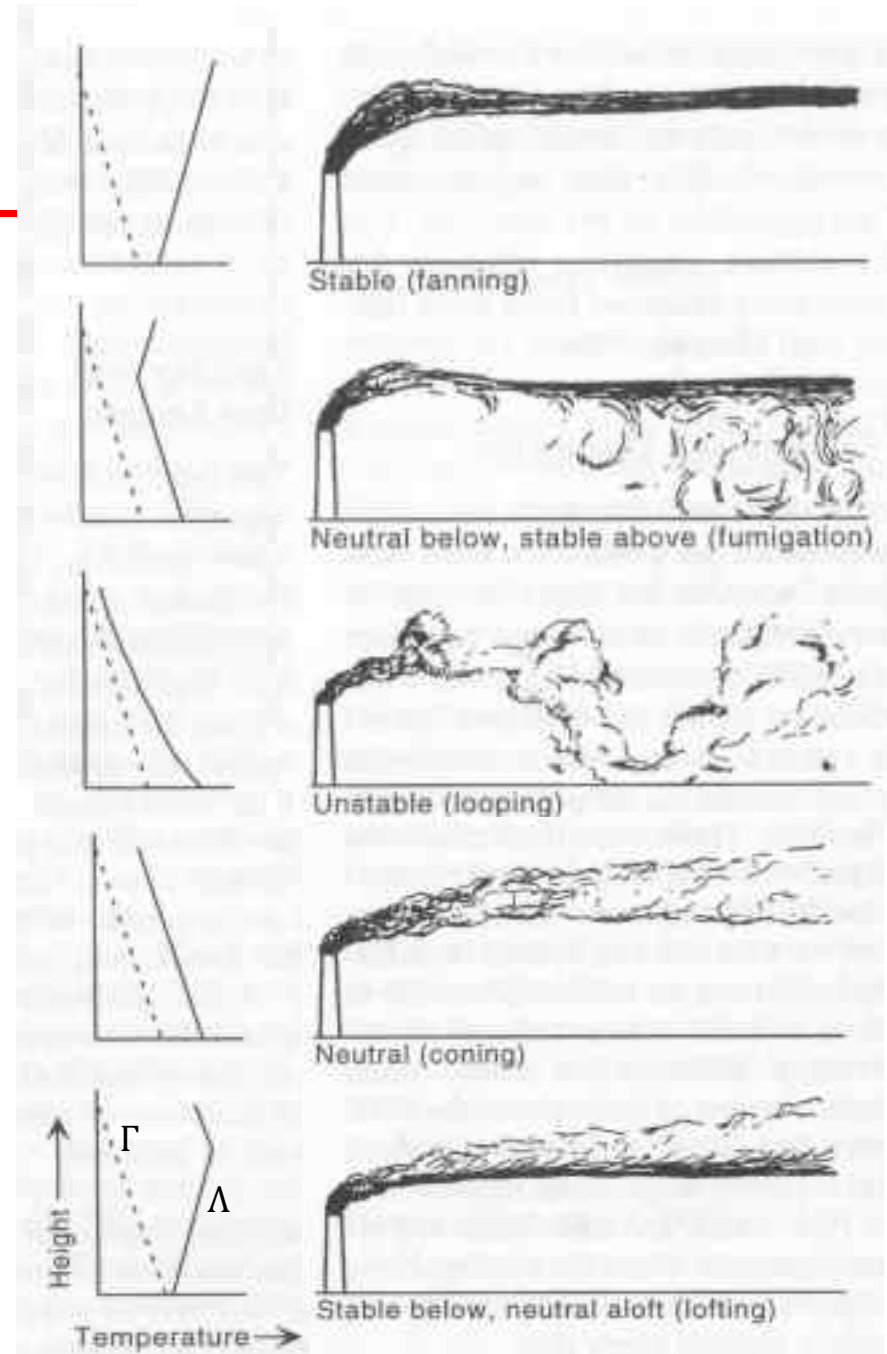
- Inversion: an increase in atmospheric temperature with height
- Radiation inversion: form by nighttime cooling of the ground
- Subsidence inversion: form by sinking (→ warming) of air



Plume behavior

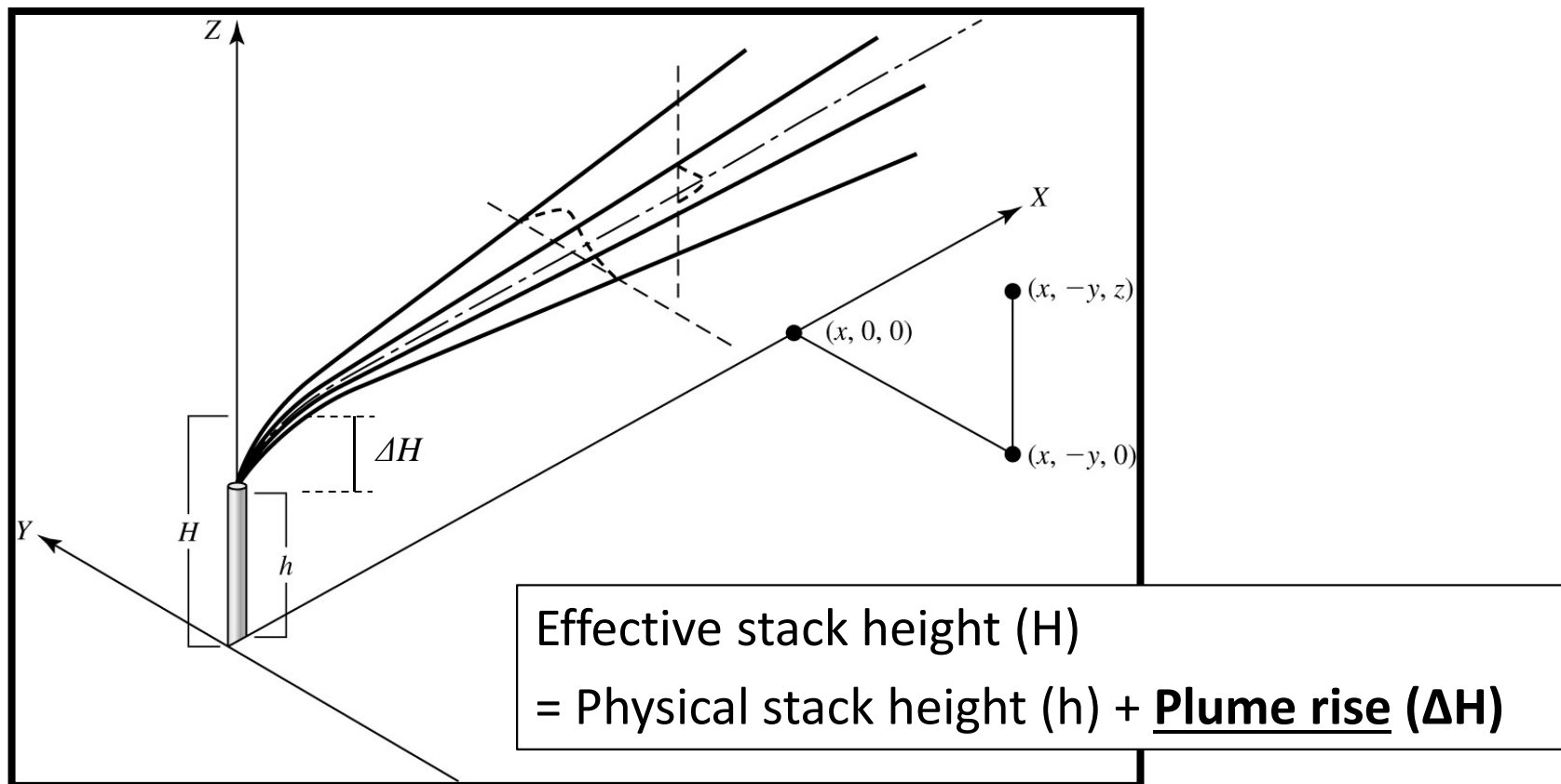
- Effect of atmospheric lapse rates on plume behavior

Slade, 1967



Atmospheric dispersion of air pollutants

- Coordination system



Plume rise

- Plume rise, ΔH
 - Due to buoyancy of the hot gas and inertia of the gas leaving the stack

$$\Delta H = \frac{v_s d}{u} \left[1.5 + \left(2.68 \times 10^{-2} \cdot P \cdot \left(\frac{T_s - T_a}{T_s} \right) \cdot d \right) \right]$$

v_s = stack velocity (m/s)

d = stack diameter (m)

u = wind speed (m/s)

P = pressure (kPa)

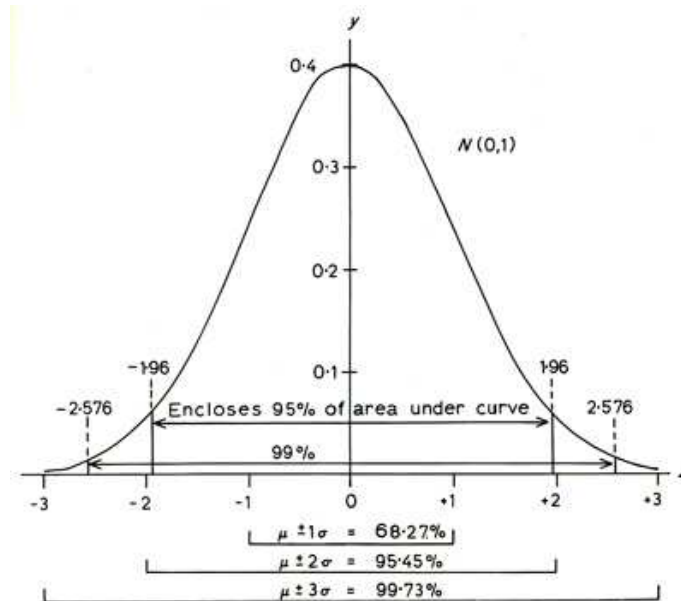
T_s = stack temperature (K)

T_a = air temperature (K)

Normal distribution

- We assume Gaussian (normal) distribution of pollutants

Gaussian distribution



$$y = \frac{1}{\sigma(2\pi)^{1/2}} \exp \left[\frac{-(x - \mu)^2}{2\sigma^2} \right]$$

When the area of the curve is 1

$\mu = \text{mean}$

$\sigma = \text{standard deviation}$

Applying normal distribution for prediction

... But in two directions (y and z) at a certain distance downwind

So, for a continuous point source of an air pollutant located at (x, y, z) = (0, 0, H):

$$C(x, y, z) = \left(\frac{E}{u}\right) \times \left(\frac{1}{\sigma_y(2\pi)^{1/2}} \exp\left[\frac{-(y-0)^2}{2\sigma_y^2}\right]\right) \times \left(\frac{1}{\sigma_z(2\pi)^{1/2}} \exp\left[\frac{-(z-H)^2}{2\sigma_z^2}\right]\right)$$

Total mass of pollutant
between x m and $(x+1)$ m

C = concentration (g/m^3)
 E = emission rate (g/s)
 u = wind speed (m/s)

General equation

Therefore,

$$C(x, y, z) = \left(\frac{E}{2\pi u \sigma_y \sigma_z} \right) \left[\exp \left(\frac{-y^2}{2\sigma_y^2} \right) \right] \left[\exp \left(\frac{-(z - H)^2}{2\sigma_z^2} \right) \right]$$

where σ_y and σ_z are functions of x and atmospheric conditions (wind speed, stability, etc.)

When the plume touches the ground

How do we account for the ground?

1) No reflection: apply the equation we derived!

$$C(x, y, z) = \left(\frac{E}{2\pi u \sigma_y \sigma_z} \right) \left[\exp \left(\frac{-y^2}{2\sigma_y^2} \right) \right] \left[\exp \left(\frac{-(z-H)^2}{2\sigma_z^2} \right) \right]$$

2) Total reflection: assume a virtual source at $(x, y, z) = (0, 0, -H)$

$$C(x, y, z) = \left(\frac{E}{2\pi u \sigma_y \sigma_z} \right) \left[\exp \left(\frac{-y^2}{2\sigma_y^2} \right) \right] \times \left(\left[\exp \left(\frac{-(z-H)^2}{2\sigma_z^2} \right) \right] + \left[\exp \left(\frac{-(z+H)^2}{2\sigma_z^2} \right) \right] \right)$$

Equation for total reflection at the ground

Concentration of a pollutant at the ground level with total reflection:

$$C(x, y, 0) = \left(\frac{E}{\pi u \sigma_y \sigma_z} \right) \left[\exp \left(\frac{-y^2}{2\sigma_y^2} \right) \right] \left[\exp \left(\frac{-H^2}{2\sigma_z^2} \right) \right]$$

Eq. (12-18) of the textbook

Now, how do we obtain σ_y and σ_z (functions of x and atmospheric stability)?

Key to stability categories

Surface wind speed (at 10 m) (m/s)	Day ^a			Night ^a	
	Incoming solar radiation			Thinly overcast or	
	Strong	Moderate	Slight	$\geq 1/2$ Low cloud	$\leq 3/8$ Cloud
<2	A	A-B	B	—	—
2-3	A-B	B	C	E	F
3-5	B	B-C	C	D	E
5-6	C	C-D	D	D	D
>6	C	D	D	D	D

^aThe neutral class, D, should be assumed for overcast conditions during day or night. Note that “thinly overcast” is not equivalent to “overcast.”

Notes: Class A is the most unstable and class F is the most stable class considered here. Night refers to the period from one hour before sunset to one hour after sunrise. Note that the neutral class, D, can be assumed for overcast conditions during day or night, regardless of wind speed.

“Strong” incoming solar radiation corresponds to a solar altitude greater than 60° with clear skies; “slight” insolation corresponds to a solar altitude from 15° to 35° with clear skies. Table 170, Solar Altitude and Azimuth, in the Smithsonian Meteorological Tables, can be used in determining solar radiation. Incoming radiation that would be strong with clear skies can be expected to be reduced to moderate with broken (5/8 to 7/8 cloud cover) middle clouds and to slight with broken low clouds.

(Source: Turner, 1967.)

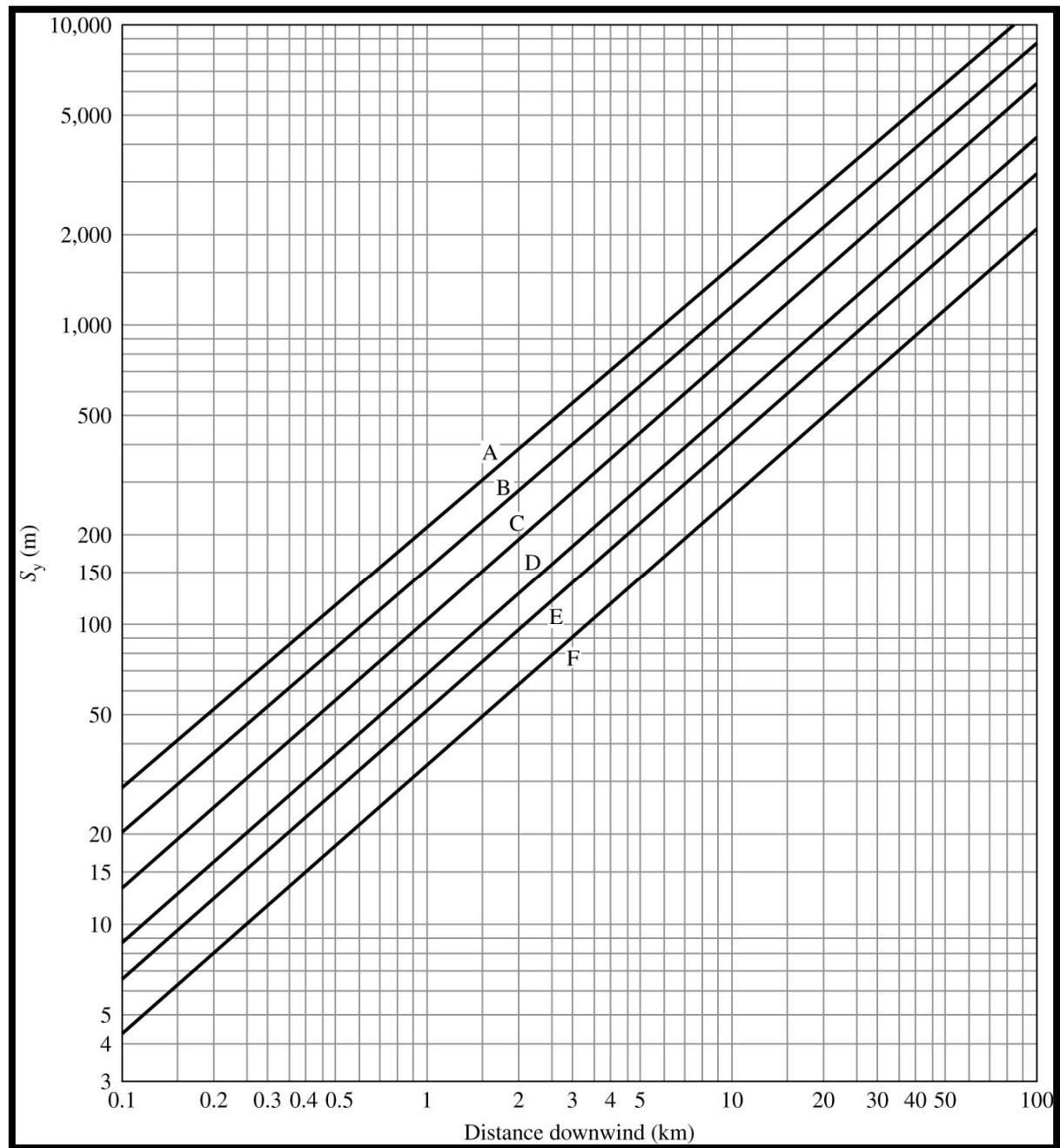
$$\sigma_y = ax^{0.894}$$

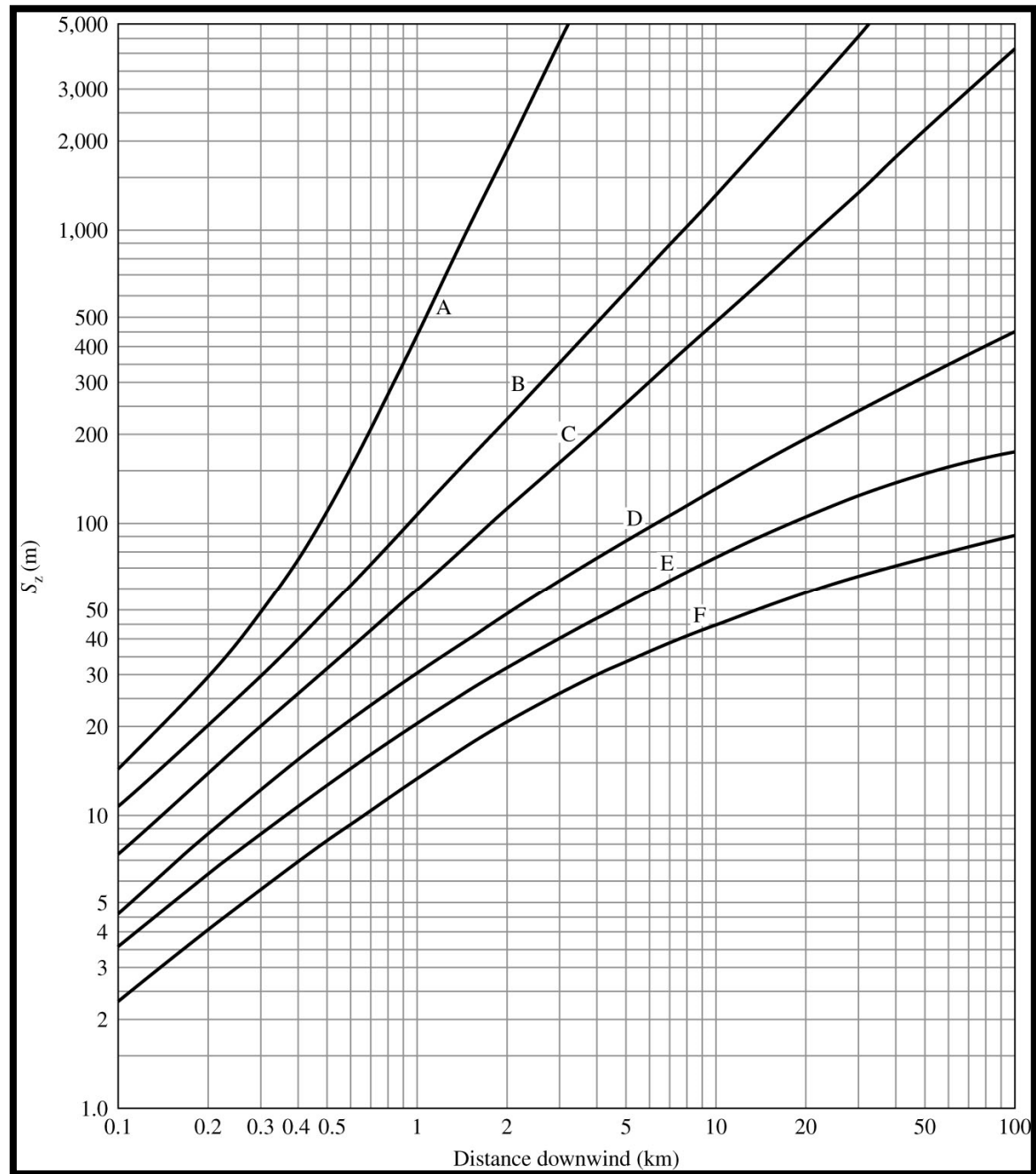
$$\sigma_z = cx^d + f \quad x = \text{downwind distance (km)}$$

Values of a , c , d , and f for calculating s_y and s_z

Stability class	a	$x \leq 1 \text{ km}$			$x > 1 \text{ km}$		
		c	d	f	c	d	f
A	213	440.8	1.941	9.27	459.7	2.094	-9.6
B	156	100.6	1.149	3.3	108.2	1.098	2
C	104	61	0.911	0	61	0.911	0
D	68	33.2	0.725	-1.7	44.5	0.516	-13.0
E	50.5	22.8	0.678	-1.3	55.4	0.305	-34.0
F	34	14.35	0.74.0	-0.35	62.6	0.18	-48.6

(Source: Martin, 1976.)





Atmospheric dispersion of air pollutants

Q: A coal-fired power plant emits SO_2 at a rate of 1656 g/s. At 3 km downwind on an overcast summer afternoon, what is the concentration of SO_2 on the ground? The following parameters apply.

Stack parameters:

Height = 120 m

Diameter = 1.2 m

Exit velocity = 10 m/s

Temperature = 315°C

Atmospheric conditions:

Pressure = 95.0 kPa

Temperature = 25°C

Wind speed = 4.5 m/s

Reading assignment

Textbook Ch 12 p. 621-631

Atmospheric dispersion of air pollutants

Slide#23 solution)

1) Effective stack height

$$\begin{aligned}\Delta H &= \frac{v_s d}{u} \left[1.5 + \left(2.68 \times 10^{-2} \cdot P \cdot \left(\frac{T_s - T_a}{T_s} \right) \cdot d \right) \right] \\ &= \frac{10 \text{ m/s} \cdot 1.2 \text{ m}}{4.5 \text{ m/s}} \left[1.5 + \left(2.68 \times 10^{-2} \cdot 95.0 \text{ kPa} \cdot \left(\frac{588 \text{ K} - 298 \text{ K}}{255 \text{ K}} \right) \cdot 1.2 \text{ m} \right) \right] \\ &= 8.0 \text{ m}\end{aligned}$$

$$H = 120 \text{ m} + 8 \text{ m} = 128 \text{ m}$$

2) Stability class

*“overcast” – at overcast conditions (thick clouds), the stability class is **D** (neutral) at any time of the day*

Atmospheric dispersion of air pollutants

Slide#23 solution – cont'd)

3) Coefficients

$$x = 3 \text{ km}$$

$$a = 68, c = 44.5, d = 0.516, f = -13.0$$

$$\sigma_y = a \cdot x^{0.894} = 68.3 \cdot 3^{0.894} = 181.6 \text{ m}$$

$$\sigma_z = c \cdot x^d + f = 44.5 \cdot 3^{0.516} - 13 = 65.4 \text{ m}$$

4) Concentration

$$C(x, y, 0) = \left(\frac{E}{\pi u \sigma_y \sigma_z} \right) \left[\exp \left(\frac{-y^2}{2\sigma_y^2} \right) \right] \left[\exp \left(\frac{-H^2}{2\sigma_z^2} \right) \right]$$

$$\begin{aligned} C(3,0,0) &= \left(\frac{1656 \text{ g/s}}{\pi \cdot 4.5 \text{ m/s} \cdot 181.6 \text{ m} \cdot 65.4 \text{ m}} \right) [\exp(0)] \left[\exp \left(\frac{-(128 \text{ m})^2}{2 \cdot (65.4 \text{ m})^2} \right) \right] \\ &= 1.5 \times 10^{-3} \text{ g/m}^3 \end{aligned}$$