Introduction to Data Mining

Lecture #8: Mining Data Streams-3

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Outline

- **Estimating Moments**
- Counting Frequent Items

Generalization: Moments

- **Suppose a stream has elements chosen from a set *A* of *N* values**
- **Let m_i be the number of times value *i* occurs in the stream**
- **The k^{th} *moment* is** $$\sum_{i \in A} (m_i)^k$$
- E.g., for a stream (x, y, x, y, z, z, z, x, z),
  - The 2nd moment is $3^2 + 2^2 + 4^2 = 29$
  - (x appears 3 times, y appears 2 times, z appears 4 times)

Special Cases

$$\sum_{i \in A} (m_i)^k$$

- **0th moment =** number of distinct elements
  - The problem considered in the last lecture
- **1st moment =** count of the numbers of elements = length of the stream
  - Easy to compute
- **2nd moment =** ***surprise number S*** **=** a measure of how uneven the distribution is

Example: Surprise Number

- **Stream of length 100**
- **11 distinct values**

- Item counts: **10, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9** **Surprise** ***S*** **= 910**

- Item counts: **90, 1, 1, 1, 1, 1, 1, 1 ,1, 1, 1** **Surprise** ***S*** **= 8,110**

Problem Definition

- **Q: Given a stream, how can we estimate k-th moments efficiently, with small memory space?**

- **A: AMS method**

AMS Method

- **AMS method works for all moments**
- **Gives an unbiased estimate**
- **We first concentrate on the 2nd moment *S***
- **We pick and keep track of many variables *X*:**
  - For each variable ***X*** we store ***X.el*** and ***X.val***
    - ***X.el*** corresponds to the item ***i***
    - ***X.val*** corresponds to the **count** of item ***i***
  - Note this requires a count in main memory, so number of ***X***s is limited
- **Our goal is to compute** $S = \sum_i m_i^2$

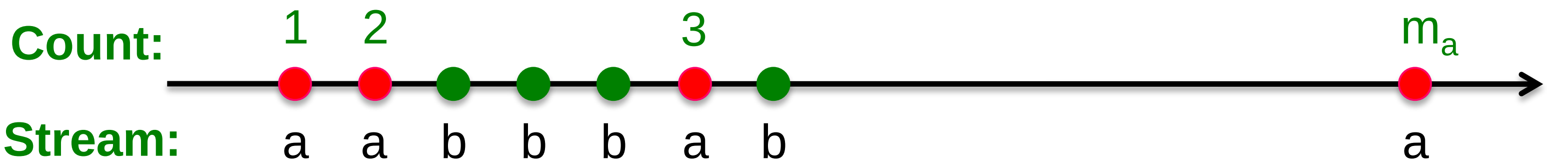
One Random Variable (X)

- **How to set *X.val* and *X.el*?**
  - Assume stream has length ***n*** (we relax this later)
  - Pick some random time ***t*** (***t<n***) to start,
    so that any time is equally likely
  - If the stream have item ***i at*** time ***t***, ***we set X.el = i***
  - Then we maintain count ***c*** (***X.val = c***) of the number of ***i***s i
    n the stream starting from the chosen time ***t***
- **Then the estimate of the 2nd moment ($\sum_i m_i^2$) is:**

$$S = f(X) = n\,(2 \cdot c - 1)$$

  - Note, we will keep track of multiple **X**s, (**X_1, X_2,… X_k**)
    and our final estimate will be $S = \frac{1}{k}\sum_{j=1}^{k} f(X_j)$

Expectation Analysis



- **2nd moment is** $S = \sum_i m_i^2$
- c_t … number of times item at time t appears from time **t** onwards ($c_1 = m_a$, $c_2 = m_a - 1$, $c_3 = m_b$)
- $E[f(X)] = \frac{1}{n}\sum_{t=1}^{n} n(2c_t - 1)$

$$= \frac{1}{n}\sum_i n\,(1 + 3 + 5 + \cdots + 2m_i - 1)$$

m_i … total count of item i in the stream (we are assuming stream has length **n**)

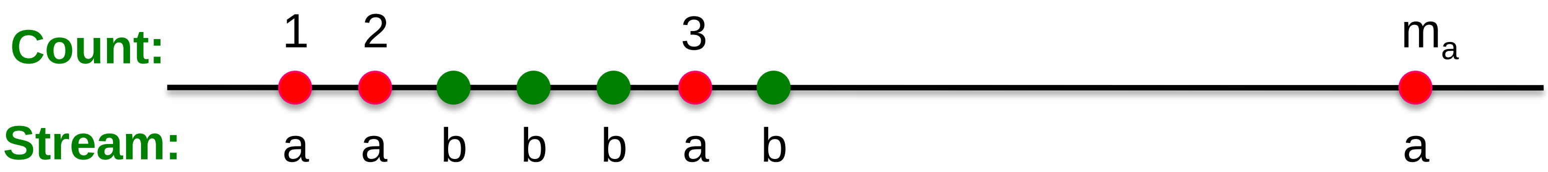
Group times by the value seen

Time t when the last i is seen ($c_t = 1$)

Time t when the penultimate i is seen ($c_t = 2$)

Time t when the first i is seen ($c_t = m_i$)

Expectation Analysis



- $E[f(X)] = \frac{1}{n} \sum_i n\,(1 + 3 + 5 + \cdots + 2m_i - 1)$
  - Little side calculation: $(1 + 3 + 5 + \cdots + 2m_i - 1) = \sum_{i=1}^{m_i} (2i - 1) = 2\frac{m_i(m_i+1)}{2} - m_i = (m_i\ )^2$
- **Then $\boldsymbol{E[f(X)] = \frac{1}{n} \sum_i n\,(m_i)^2}$**
- **So, $\mathbf{E[f(X)]} = \boldsymbol{\sum_i (m_i)^2 = S}$**
- **We have the second moment (in expectation)!**

Higher-Order Moments

- For estimating k[th] moment we essentially use the same algorithm but change the estimate:
  - For **k=2** we used ***n*** **(2·c – 1)**
  - For **k=3** we use: ***n*** **(3·c² – 3c + 1)** (where **c=X.val**)
- Why?
  - **For k=2:** Remember we had $(1 + 3 + 5 + \cdots + 2m_i - 1)$ and we showed terms ***2c-1*** (for **c=1,…,m**) sum to ***m²***
    - $m^2 = \sum_{c=1}^{m} c^2 - \sum_{c=1}^{m} (c-1)^2 = \sum_{c=1}^{m} (2c-1)$
    - **So:** $\mathbf{2c - 1 = c^2 - (c-1)^2}$
  - **For k=3:** $\mathbf{c^3 - (c-1)^3 = 3c^2 - 3c + 1}$
- **Generally:** Estimate $= n\,(c^k - (c-1)^k)$

Combining Samples

- **In practice:**
  - Compute $f(X) = n(2c - 1)$ for as many variables ***X*** as you can fit in memory
  - Average them

- **Problem: Streams never end**
  - We assumed there was a number ***n***, the number of positions in the stream
  - But real streams go on forever, so ***n*** is a variable – the number of inputs seen so far

Streams Never End: Fixups

- **(1)** f(***X***)= n (2c-1) have ***n*** as a factor – keep ***n*** separately; just hold the count c in ***X***
- **(2)** Suppose we can only store ***k*** counts. We must throw some ***X***s out as time goes on:
  - **Objective:** Each starting time ***t*** is selected with probability ***k/n***
  - **Solution: (fixed-size sampling = reservoir sampling!)**
    - Choose the first ***k*** times for ***k*** variables
    - When the ***n***th element arrives (***n > k***), choose it with probability ***k/n***
    - If you choose it, throw one of the previously stored variables **X** out, with equal probability

Outline

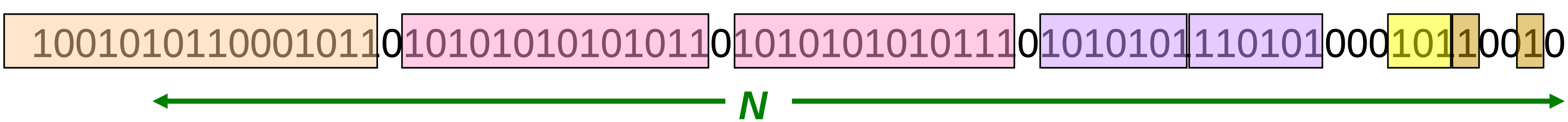
- ☑ Estimating Moments
- ☐ **Counting Frequent Items**

Counting Itemsets

- **New Problem:** Given a stream, how can we find **recent** frequent items (= which appear more than *s* times in the window) efficiently?

Counting Itemsets

- **New Problem:** **Given a stream, which items appear more than *s* times in the window?**
- **Possible solution:** Think of the stream of baskets as one binary stream per item
  - **1** = item present; **0** = not present
  - Use **DGIM** to estimate counts of **1**s for all items



Extensions

- In principle, you could count frequent pairs or even larger sets the same way
  - One stream per itemset
  - E.g., for a basket {i, j, k}, assume 7 independent streams: (i) (j) (k) (i, j) (i, k) (j, k) (i, j, k)
- Drawback:
  - Number of itemsets is way too big

Exponentially Decaying Windows

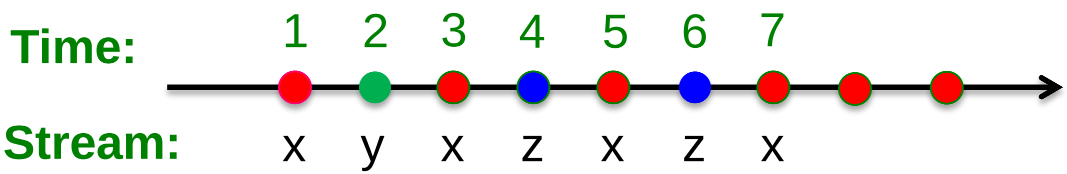
- **Exponentially decaying windows: A heuristic for selecting likely frequent item(sets)**
  - **What are "currently" most popular movies?**
    - Instead of computing the raw count in last ***N*** elements
    - Compute a **smooth aggregation** over the whole stream
- If stream is $a_1, a_2, \ldots$ and we are taking the sum of the stream, take the answer at time t to be: $= \sum_{i=1}^{t} a_i (1-c)^{t-i}$
    - c is a constant, presumably tiny, like 10^{-6} or 10^{-9}
- **When new a_{t+1} arrives:**
  Multiply current sum by **(1-c)** and add a_{t+1}

Example: Counting Items

- If each a_i is an "item" we can compute the **characteristic function** of each possible item x as an Exponentially Decaying Window
  - That is: $\sum_{i=1}^{t} \delta_i \cdot (1-c)^{t-i}$ where $\delta_i = 1$ if $a_i = x$, and **0** otherwise
  - Imagine that for each item x we have a binary stream (**1** if x appears, **0** if x does not appear)
  - New item x arrives:
    - Multiply all counts by **(1-c)**
    - Add **+1** to count for element x
    - ***Remove all items whose weights < s***
- **Call this sum the "weight" of item x**

Note: Assume we are interested in items with weights >= s

Example: Counting Items

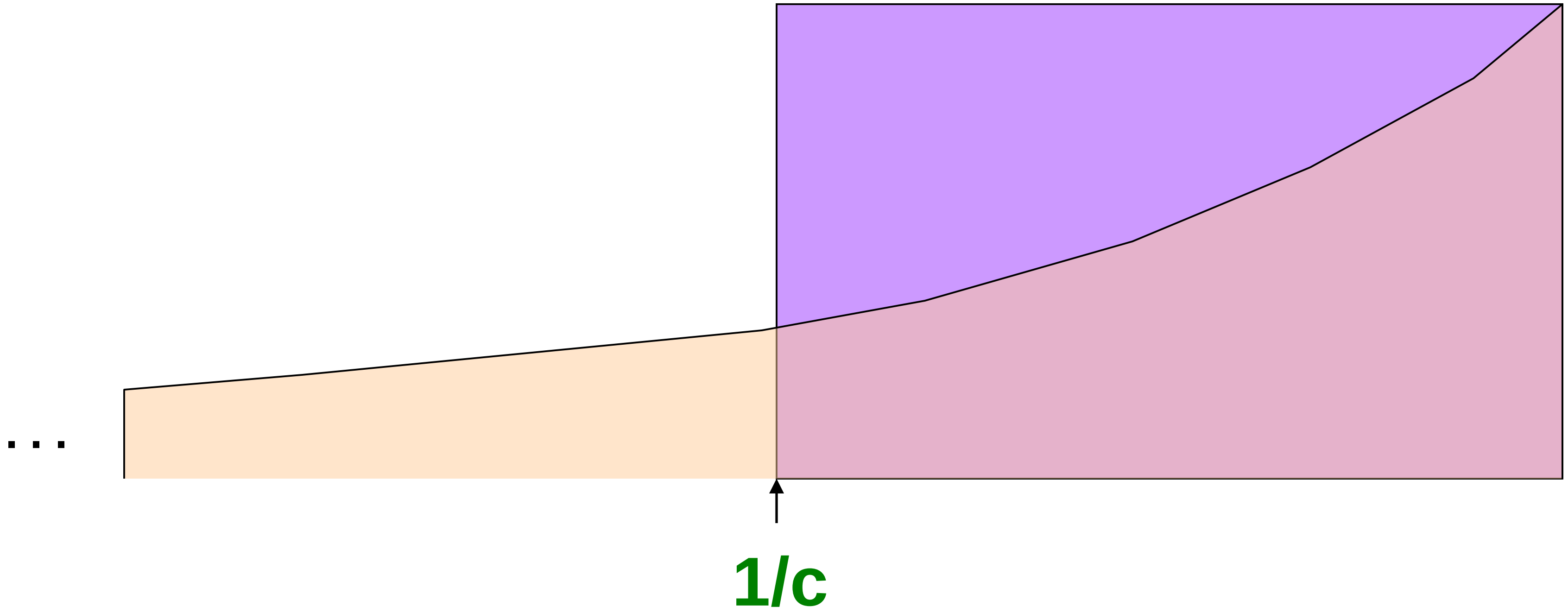


$$\sum_{i=1}^{t} \delta_i \cdot (1-c)^{t-i}$$

Assume c = 0.2,
Keep items with weights >= 1/2

- (T1) x: 1
- (T2) x: 0.8*1, y: 1
- (T3) x: 0.8*0.8 + 1, y: 0.8*1
- (T4) x: 0.8*1.64, y: 0.8*0.8, z = 1
- (T5) x: 1.312+1, y: 0.8*0.64=0.512, z: 0.8*1
- (T6) x: 0.8*2.312, y: 0.8*0.512, z:0.8*0.8
  - Remove y
- (T7) x: 0.8*1.8496+1, z: 0.8*0.64
- ...

Sliding vs. Decaying Windows



- **Important property:** Sum over all weights $\sum_t (1$

Example: Counting Items

- **What are "currently" most popular movies?**
- **Suppose we want to find movies of weight > ½**
  - **Important property:** Sum over all weights $\sum_t (1-c)^t$ is 1/[1 – (1 – c)] **= 1/c**
- **Thus:**
  - There cannot be more than **2/c** movies with weight > **½**
- So, **2/c** is a limit on the number of movies being counted at any time (if we remove movies whose weight <= ½)

Extension to Itemsets

- Assume at each time we are given an itemset
  - E.g., {i, j, k}, {k, x}, {i,j}

- **Count (some) itemsets in an E.D.W.**
  - **What are currently "hot" itemsets?**
  - **Problem:** Too many itemsets to keep counts of all of them in memory

Extension to Itemsets

- **Count (some) itemsets in an E.D.W.**
  - **What are currently "hot" itemsets?**
  - **Problem:** Too many itemsets to keep counts of all of them in memory
- **When a basket B comes in:**
  - Multiply all counts by **(1-c)**
  - For uncounted items in **B**, create new count
  - Add **1** to count of any item in **B** and to any **itemset** contained in **B** that is already being counted
  - **Remove items and itemsets whose counts < ½**
  - Initiate new counts (next slide)

Initiation of New Counts

- Start a count for an itemset $S \subseteq B$ if every proper subset of ***S*** had a count prior to arrival of basket ***B***
  - **Intuitively:** If all subsets of **S** are being counted this means they are "**frequent/hot**" and thus ***S*** has a potential to be "**hot**"
- **Example:**
  - Start counting ***S*=**{i, j}** iff both **i** and **j** were counted prior to seeing ***B***
  - Start counting ***S*=**{i, j, k}** iff **{i, j}**, **{i, k}**, and **{j, k}** were all counted prior to seeing ***B***

Summary – Stream Mining

- Important tools for stream mining
  - Sampling from Data Stream (Reservoir Sampling)
  - Querying Over Sliding Windows (DGIM method for cou nting the number of 1s or sums in the window)
  - Filtering a Data Stream (Bloom Filter)
  - Counting Distinct Elements (Flajolet-Martin)
  - Estimating Moments (AMS method; surprise number)
  - Counting Frequent Itemsets (exponentially decaying w indows)

Questions?