Topics in IC Design

1.1 Introduction to Jitter

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Outline

- Definition of jitter
- Jitter metrics
- Jitter decomposition
- Bit error rate
- Bathtub curve

Definition of jitter

- SONET specification: "the short-term variations of a digital signal's significant instants from their ideal positions in time"
 - Jitter: timing variations that occur rapidly
 - Wander: those that occur slowly
 - ITU defines the threshold of 10 Hz
 - The exact moments when the transitional signal crosses a chosen amplitude threshold (reference or decision level), e.g. zerocrossing times.

Phase Jitter and Frequency Drift



Jitter metrics

 Clock Jitter: Period jitter, cycle-to-cycle jitter, and time interval error (TIE, absolute jitter, jitter)



Period Jitter Vs. Cycle-Cycle Jitter Vs. Time Internal Error

Period jitter

 Triggered on the first edge, observe the second edge
 Stopped
 126 Acqs
 30 Jul 02 15:39:11



Cycle-to-cycle jitter

- How much the clock period changes between any two adjacent cycles
- Can be found by applying a first-order difference operation to the period jitter
- Shows the instantaneous dynamics

Time interval error (TIE)

- How far each active edge varies from its ideal position
- The ideal edges must be known
- Shows the cumulative effect that even a small amount of period jitter can have over time
- Also called as edge-to-edge jitter, time interval jitter, absolute jitter, phase jitter, or jitter

An example noisy clock

• A nominal period of 1 us, but the actual periods are eight 990 ns followed by eight 1010 ns.



Jitter histogram

• TIE measurement with the total population of 100,000



Eye diagram

 Many short segments of a waveform are superimposed



 The nominal edge locations and voltage levels are aligned



Jitter separation

- Jitter on NRZ data stream
- An analysis technique to predict and reduce timing jitter



Random jitter

- Gaussian distribution
- peak-to-peak value unbounded

$$\rho_x = \frac{1}{\sqrt{2\pi\sigma}} exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$



confidence	1-conf.	+/- Sigma
99.0000000%	1.0E-02	2.733
99.9000000%	1.0E-03	3.419
99.99000000%	1.0E-04	4.003
99.99900000%	1.0E-05	4.517
99.99990000%	1.0E-06	4.983
99.99999000%	1.0E-07	5.411
99.99999900%	1.0E-08	5.810
99.99999990%	1.0E-09	6.184
99.99999999%	1.0E-10	6.538

Periodic jitter

- Repeats in a cyclic fashion
- Caused by external deterministic noise sources coupled into a system
 - switching power-supply noise or a strong local RF carrier
 - Spread-spectrum modulated



Data-dependent jitter

- Correlated with the bit sequence in a data stream
- Often caused by the frequency response of a cable or device
- Also called inter-symbol interference (ISI)

Data-dependent jitter

 Two waveforms with 10101110 and 10101010 sequences are superimposed





Duty-cycle jitter

- Predicted based on whether the associated edge is rising or falling
- Two common causes
 - The slew rate for the rising edges differs from that of the falling edges
 - The decision threshold for a waveform is higher or lower than it should be: Due to DC offset

Duty-cycle jitter

- Unbalanced rise and fall times
- Decision
 threshold higher
 than the 50%
 amplitude point



Composite jitter

• For two or more independent random processes, the distribution that results from the sum of their effects is equal to the convolution of the individual distributions.



Bit error rate (BER)

- The Gaussian probability distribution has unbounded peak-to-peak value.
- Use of rms jitter to estimate the peak-to-peak jitter with the confidence level (BER)

$$Jitter_{P-P} = \propto * Jitter_{RMS}$$

• Q-function (x)

 $Q(z)=\int_z^\infty \frac{1}{\sqrt{2\pi}}e^{-\frac{y^2}{2}}dy$







Two cases of Gaussian Tail

• Bit error occurs when the zero crossing occurs beyond the sampling point.



Bit error rate (BER)

- For any signal containing Gaussian jitter, the eye diagram closes completely after a long enough time.
- In the following eye diagram, the eye is 50% open for 1000 traces. (1 error out of 1000, BER = 10⁻³)
- Or 10 errors out of 10,000, same BER)



Bathtub curve

- Indicates the movement of farthest edges
- Characterizes the eye opening versus the BER.
- For a bit error rate (BER) of 10⁻³ (1 in 1000) the eye is 50% open.
- If one out of 100,000 waveforms crosses, the eye is 25% open with BER of 10⁻⁵.
- A curve connecting the ends of the rulers looks like a bathtub.



Other issues

- What is N-period jitter?
 - Variation of Nth edge.
 - Triggering on the rising edge and see the N-the edge (Intermediate edges ignored)
 - period jitter = 1-period jitter

References

- [1.1] Tektronix, "Understanding and characterizing timing jitter"
- [1.2] MAXIM, "CLK jitter and phase noise conversion"
- [1.3] Tektronix Introduction to Jitter

Topics in IC

1.2 Fourier Transform and Power Spectral Density

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Outline

- Fourier Transform of sinusoidal wave
- Power spectral density of sinusoidal wave
- Fourier Transform of sinusoidal wave with narrowband phase modulation
- <u>Power spectral density</u> of sinusoidal wave with narrowband phase modulation
- Examples: Sinusoidal jitter and white jitter

Why?

- Phase is not directly measurable. Only voltage or current wave can be measured.
- Fourier transform (FFT) is available only for deterministic signals
- For random signals, <u>power spectral density</u> can be measured through spectrum analyzer.

Power Spectral Density (PSD)

- How is the power of a signal or time series distributed with frequency?
- For convenience with abstract signals, power is the squared value of the signal.
- Wiener-Khinchin theorem the PSD of a widesense stationary random process is the Fourier transform of the corresponding autocorrelation function.

Fourier Transform



Power Spectral Density

- For a pure sine wave
- First, calculate autocorrelation function

$$R(\tau) = \overline{y(t)y(t+\tau)} = \frac{1}{2}A_0^2 \cos(2\pi f_0 \tau)$$

$$S_y(f) = \frac{1}{4}A_0^2 \delta(f - f_0) + \frac{1}{4}A_0^2 \delta(f + f_0)$$

$$\xrightarrow{\text{Area}=A_0^{2/4}} A_0$$

$$\xrightarrow{\text{Area}=A_0^{2/4}} A_0$$

Fourier Transform

• For sine wave with narrowband phase modulation, $\cos \varphi(t) \cong 1$, $\sin \varphi(t) \cong \varphi(t)$

$$y(t) = A_0 \sin\left(2\pi f_0 \cdot (t+j(t)) + \varphi_0\right)$$

= $A_0 \sin(2\pi f_0 t + \varphi_0 + \varphi(t))$
 $\cong A_0 \sin(2\pi f_0 t + \varphi_0) + A_0 \varphi(t) \cos(2\pi f_0 t + \varphi_0)$

 Modulating phase appears as modulating amplitude.

$$Y(f) = \frac{1}{2} A_0 e^{i\varphi_0} \delta(f - f_0) - \frac{1}{2} A_0 e^{-i\varphi_0} \delta(f + f_0) + \frac{1}{2} A_0 e^{i\varphi_0} \Phi(f - f_0) + \frac{1}{2} A_0 e^{-i\varphi_0} \Phi(f + f_0)$$

Fourier Transform

Sine wave with narrowband phase modulation

$$Y(f) = \frac{1}{2} A_0 e^{i\varphi_0} \delta(f - f_0) - \frac{1}{2} A_0 e^{-i\varphi_0} \delta(f + f_0) + \frac{1}{2} A_0 e^{i\varphi_0} \Phi(f - f_0) + \frac{1}{2} A_0 e^{-i\varphi_0} \Phi(f + f_0)$$



Power Spectral Density

 Sine wave with narrowband phase modulation, assuming φ(t) is uncorrelated

$$\begin{split} R_{y}(\tau) &= \overline{y(t)y(t+\tau)} \\ &= \frac{1}{2}A_{0}^{2}\cos(2\pi f_{0}\tau) + \frac{1}{2}A_{0}^{2}\overline{\phi(t)\phi(t+\tau)}\cos(2\pi f_{0}\tau) \\ &= \frac{1}{2}A_{0}^{2}\cos(2\pi f_{0}\tau)\left(1 + R_{\phi}(\tau)\right) \\ S_{y}(f) &= \frac{1}{4}A_{0}^{2}\delta\left(f - f_{0}\right) + \frac{1}{4}A_{0}^{2}\delta\left(f + f_{0}\right) \\ &\quad + \frac{1}{4}A_{0}^{2}S_{\phi}\left(f - f_{0}\right) + \frac{1}{4}A_{0}^{2}S_{\phi}\left(f + f_{0}\right) \end{split}$$

Power Spectral Density

• $S_{o}(f)$ are scaled and translated.



Ex 1: Sinusoidal Phase Modulation

Narrowband phase modulation

$$\begin{split} \varphi(t) &= A_{1} \sin(2\pi f_{1}t + \varphi_{1}), A_{1} << 1 \\ y(t) &= A_{0} \sin(2\pi f_{0}t + \varphi_{0} + \varphi(t)) \\ &\cong A_{0} \sin(2\pi f_{0}t + \varphi_{0}) + A_{0}\varphi(t)\cos(2\pi f_{0}t + \varphi_{0}) \\ &= A_{0} \sin(2\pi f_{0}t + \varphi_{0}) \\ &+ A_{0}A_{1} \sin(2\pi f_{1}t + \varphi_{1})\cos(2\pi f_{0}t + \varphi_{0}) \end{split}$$
$$\begin{aligned} Y(f) &= \frac{1}{2}A_{0}e^{i\varphi_{0}}\delta(f - f_{0}) - \frac{1}{2}A_{0}e^{-i\varphi_{0}}\delta(f + f_{0}) \\ &+ \frac{1}{4i}A_{0}A_{1}e^{i\varphi_{0} + i\varphi_{1}}\delta(f - f_{0} - f_{1}) + \frac{1}{4i}A_{0}A_{1}e^{-i\varphi_{0} - i\varphi_{1}}\delta(f - f_{0} + f_{1}) \\ &+ \frac{1}{4i}A_{0}A_{1}e^{-i\varphi_{0} + i\varphi_{1}}\delta(f + f_{0} - f_{1}) + \frac{1}{4i}A_{0}A_{1}e^{i\varphi_{0} + i\varphi_{1}}\delta(f - f_{0} + f_{1}) \end{split}$$
Ex 1: Sinusoidal Phase Modulation

Fourier Analysis

$$Y(f) = \frac{1}{2} A_0 e^{i\varphi_0} \delta(f - f_0) - \frac{1}{2} A_0 e^{-i\varphi_0} \delta(f + f_0) + \frac{1}{4i} A_0 A_1 e^{i\varphi_0 + i\varphi_1} \delta(f - f_0 - f_1) + \frac{1}{4i} A_0 A_1 e^{-i\varphi_0 - i\varphi_1} \delta(f + f_0 + f_1) + \frac{1}{4i} A_0 A_1 e^{-i\varphi_0 + i\varphi_1} \delta(f + f_0 - f_1) + \frac{1}{4i} A_0 A_1 e^{i\varphi_0 + i\varphi_1} \delta(f - f_0 + f_1)$$



Ex 1: Sinusoidal Phase Modulation

Auto-Correlation Function

$$\begin{aligned} R_{y}(\tau) &= \overline{y(t)y(t+\tau)} \\ &= \frac{1}{2}A_{0}^{2}\cos(2\pi f_{0}\tau) + \frac{1}{2}A_{0}^{2}\overline{\phi(t)\phi(t+\tau)}\cos(2\pi f_{0}\tau) \\ &= \frac{1}{2}A_{0}^{2}\cos(2\pi f_{0}\tau) \left(1 + \frac{1}{2}A_{1}^{2}\cos(2\pi f_{1}\tau)\right) \end{aligned}$$

Ex 1: Sinusoidal Phase Modulation

Power Spectral Density

$$S_{y}(f) = \frac{1}{4} A_{0}^{2} \delta(f - f_{0}) + \frac{1}{4} A_{0}^{2} \delta(f + f_{0})$$

$$+ \frac{1}{16} A_{0}^{2} A_{1}^{2} \delta(f - f_{0} - f_{1}) + \frac{1}{16} A_{0}^{2} A_{1}^{2} \delta(f - f_{0} + f_{1})$$

$$+ \frac{1}{16} A_{0}^{2} A_{1}^{2} \delta(f + f_{0} - f_{1}) + \frac{1}{16} A_{0}^{2} A_{1}^{2} \delta(f + f_{0} + f_{1})$$

$$Area = A_{0}^{2/4}$$

$$Area = A_{0}^{2/4}$$

$$Area = A_{0}^{2/4}$$

Example 2: Amplitude Noise

 What if the white Gaussian amplitude noise (not timing jitter) is added on top of the pure sine wave?

Noise floor will not move as we increase the amplitude of the sine wave.

$$y(t) = A_0 \sin(2\pi f_0 t + \varphi_0) + n_v(t)$$

$$R_v(\tau) = \frac{1}{2} A_0^2 \cos(2\pi f_0 \tau) + b$$

$$F_0 = \frac{1}{2} A_0^2 \cos(2\pi f_0 \tau) + b$$

Example 3

What if both amplitude and phase noise is added?



Jitter in Rectangular Wave

What if the wave is rectangular?



Jitter in Rectangular Wave

• With jitter,



Jitter in Rectangular Wave

• With jitter,

$$y(t) = \frac{4}{\pi} A_0 \left(\frac{\sin(2\pi f_0 t + \varphi(t)) + \frac{1}{3}\sin(3 \cdot 2\pi f_0 t + 3 \cdot \varphi(t)) + \frac{1}{5}\sin(5 \cdot 2\pi f_0 t + 5 \cdot \varphi(t)) + \dots \right)$$



Jitter at Divider

 How does PSD change if the frequency of the wave is divided by 2?



• J(t) remains the same.

Jitter at Divider

 How does PSD change if the frequency of the wave is divided by 2?

$$y(t) = A_0 \sin\left(2\pi f_0 \cdot \left(t + j(t)\right) + \varphi_0\right)$$
$$= A_0 \sin(2\pi f_0 t + \varphi_0 + \varphi(t))$$
$$y_{1/2}(t) = A_0 \sin\left(2\pi \left(\frac{f_0}{2}\right) \cdot \left(t + j(t)\right) + \varphi_0\right)$$
$$= A_0 \sin\left(2\pi \left(\frac{f_0}{2}\right) t + \varphi_0 + \frac{1}{2} \cdot \varphi(t)\right)$$

Phase noise amplitude is decreased by 2 and phase noise power is decreased by 4.

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Jitter with Frequency Divider

 How does PSD change if the frequency of the wave is divided by 2?

Its phase noise is decreased by 6dB with the same shape in phase noise PSD.



Jitter with Frequency Multiplier

 How does PSD change if the frequency of the wave is multiplied by N by an ideal frequency multiplier?

> Its phase noise is increased by 20-log₁₀N [dB] with the same shape in phase noise PSD.

References

- [2.1] ADI Phase Noise
- [2.2] Agilent Phase Noise and Jitter

Topics in IC

1.3 Introduction to Phase Noise

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Outline

- Definition of phase noise
- Measuring phase noise

Definition of Phase Noise

- Frequency domain representation of rapid, short-term (> 10Hz), random fluctuations in the phase of a periodic wave
- Cannot be directly measured
- Instead, use power spectral density (PSD)

PSD and **Phase** Noise

S_b(f): symmetric in double sideband

 $y(t) = A_0 \sin(2\pi f_0 t + \varphi(t)), S_y(f) = \frac{1}{4} A_0^2 \delta(f - f_0) + \frac{1}{4} A_0^2 \delta(f + f_0) + \frac{1}{4} A_0^2 S_\phi(f - f_0) + \frac{1}{4} A_0^2 S_\phi(f + f_0)$



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PSD and **Phase** Noise

• Sinusoidal Phase Noise: 🕅

$$\varphi(t) = A_1 \sin(2\pi f_1 t + \varphi_1), A_1 << 1$$



Single Sideband (SSB) Phase Noise Power Spectral Density

 Characterizes an oscillator's short term instabilities in the frequency domain.

$$\mathcal{L}(\Delta \boldsymbol{\omega}) = \mathbf{10} \log \left[\frac{P_{sideband}(\boldsymbol{\omega}_0 + \Delta \boldsymbol{\omega}, \mathbf{1} H z)}{P_{carrier}} \right]$$

- $P_{sideband}(\omega_0 + \Delta \omega, 1Hz)$ = the single sideband power at a frequency offset of $\Delta \omega$ from the carrier in a measurement bandwidth of 1 Hz.
- $-P_{carrier}$ = total power under the power spectrum.
- $-\Delta\omega$ = frequency offset from the carrier.
- $S_{SSB,\varphi}(\Delta f) = 2*10^{L(\Delta f)} \rightarrow L(\Delta f) = Log[1/2 S_{SSB,\varphi}(\Delta f)]$

Single Sideband (SSB) Phase Noise Power Spectral Density

- Advantage ease of measurement
- Disadvantage shows the sum of both amplitude and phase variations



Measurement of Phase Noise

 The PSD curve S_c(f) results when we connect the signal (clock) to a spectrum analyzer.



• Mathematically, *L(f)* can be written as:

 $L(f - f_c) = 10 \log[S_c(f) / S_c(f_c)] in dBc$

Assuming resolution bandwidth of 1Hz

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Measurement of Phase Noise

- Measuring *L(f)* with a spectrum analyzer directly from the spectrum *S_c(f)* is NOT practical.
 - The value of *L(f)* is usually less than -100dBc which exceeds the dynamic range of most spectrum analyzers.
 - f_c can sometimes be higher than the inputfrequency limit of the analyzer.

Measurement of Phase Noise

- The practical way uses a setup that eliminates the spectrum energy at f_c.
 - Similar to the method of demodulating a passband signal to baseband



Other Issues

• L(∆f): Script Capital L or Script L

References

 [3.1] Kundert - An Introduction to Cyclostationary Noise

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1.4 Jitter and Phase Noise

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Outline

- Introduction
- Definitions of jitter
- Synchronous and accumulating jitter
- Modeling PLLs with jitter
- Simulation and analysis
- Summary

Introduction

- Modeling the jitter of a PLL
 - Predicting the noise of the individual blocks
 - Converting the noise of the block to jitter
 - Building high-level behavioral models
 - Assembling the blocks into a PLL model
 - Simulating the PLL with modeled jitter

Frequency Synthesis

- A PLL-based frequency synthesizer
 - Reference oscillator (OSC)
 - Frequency dividers (FDs)
 - Phase frequency detector (PFD)
 - Charge pump, loop filter (CP, LF)
 - Voltage-controlled oscillator (VCO)



Jitter

- An undesired perturbation or uncertainty in the timing of events
- The noisy signal $v_n(t) = v(t+j(t))$
 - For a noise-free signal ${\cal V}$
 - Displacing time with a stochastic process j
- Converting to phase noise

$$\phi(t) = 2\pi f_0 j(t), \text{ where } f_0 = 1/T$$

$$v_n(t) = v \left(t + \frac{\phi(t)}{2\pi f_0} \right)$$

Jitter Metrics

- Define {t_i} as the sequence of times for positivegoing threshold crossing in v_n
- Edge-to-edge jitter
 - $-J_{ee}(i) = \sqrt{\operatorname{var}(\delta t_i)}$
 - The variation in the delay between a triggering event (ideal timing) and a response event
 - Same as Time Interval Error (TIE)

Jitter Metrics

k-cycle or long-term jitter

$$-J_k(i) = \sqrt{\operatorname{var}(t_{i+k} - t_i)}$$

- Uncertainty in the length of k cycles
- For a single period, period jitter $J = J_1$
- Cycle-to-cycle jitter

$$-J_{\rm cc}(i) = \sqrt{\operatorname{var}(T_{i+1} - T_i)}$$

- Define $T_i = t_{i+1} t_i$ as the period of cycle *i*
- Identifies large adjacent cycle displacement

Jitter metrics

edge-to-edge jitter

$$J_{ee}(i) = \sqrt{\operatorname{var}(\delta t_i)}$$

$$k\text{-cycle jitter}$$

$$J_k(i) = \sqrt{\operatorname{var}(t_{i+k} - t_i)}$$

$$t_i \quad k \text{ cycles}$$

$$t_{i+k}$$

RMS vs. peak-to-peak jitter

- RMS metrics are unbounded when the noise sources have Gaussian distributions
- Peak-to-peak jitter: the magnitude that the jitter exceeds only for a specified "error rate"

 $J_{\rm PP} = \alpha J_{\rm RMS}$

Error Rate vs $\boldsymbol{\alpha}$



Error Rate	α
10 ⁻³	6.180
10-4	7.438
10 ⁻⁵	8.530
10 ⁻⁶	9.507
10 ⁻⁷	10.399
10 ⁻⁸	11.224
10 ⁻⁹	11.996
10^{-10}	12.723
10^{-11}	13.412
10^{-12}	14.069
10 ⁻¹³	14.698
10^{-14}	15.301
10^{-15}	15.883
10^{-16}	16.444

Types of Jitter

- Synchronous jitter
 - A variation in the delay between the received input and the produced output.
 - No memory, no accumulation.
- Accumulating jitter
 - Accumulation of all variations in the delay between an output transition and the subsequent output transition.
 - Timing variation in one cycle is added to sum of all the previous cycles and affects the subsequent cycles.
Types of Jitter

• PM (Phase Modulated) jitter vs FM(Frequency Modulated) jitter

CHARACTERISTICS OF PM AND FM JITTER.

Jitter	Туре	Circuits	J
PM	synchronous	driven (PFD/CP, FD)	$\frac{\sqrt{\operatorname{var}(n_v, t_c)}}{\dot{v}(t_c)}$
FM	accumulating	autonomous (OSC, VCO)	\sqrt{aT}

Synchronous Jitter

- An undesired fluctuation in the delay between the input and the output events
- Exhibited by driven systems (PFD/CP, FD)
- Phase modulated or PM jitter
- The output signal in response to a periodic input sequence of transitions
 - The frequency is exactly that of the input
 - Only the phase fluctuates

Simple synchronous jitter

 Let η be a white Gaussian stationary or Tcyclostationary process, then

 $j_{\text{sync}}(t) = \eta(t) \quad \eta(t)$: 시간축 변화 Process

$$v_{\rm n}(t) = v(t+j_{\rm sync}(t))$$

- $v_n(t)$ exhibits simple synchronous jitter
 - Driven circuits are broadband
 - Noise sources are white, Gaussian and small

Simple synchronous jitter

•
$$J_{ee}(i) = \sqrt{\operatorname{var}(j_{\operatorname{syne}}(t_i))}$$

•
$$J_{k}(i) = \sqrt{\operatorname{var}(t_{i+k} - t_{i})}$$
$$= \sqrt{\operatorname{var}([(i+k)T + j_{\operatorname{sync}}(t_{i+k})] - [iT + j_{\operatorname{sync}}(t_{i})])}$$
$$= \sqrt{2\operatorname{var}(j_{\operatorname{sync}}(t_{i}))}$$
$$= \sqrt{2}J_{\operatorname{ee}}(i)$$

• $j_{sync}(t_i)$ is independent of *i*, so is J_{ee} and J_k are also independent of *i*.

•
$$J_k = J_1 = J$$
 (period jitter)

Simple synchronous jitter

•
$$J_{cc}(i) = \sqrt{\operatorname{var}(T_{i+1} - T_i)}$$
$$= \sqrt{\operatorname{var}(t_{i+2} - t_{i+1} - t_{i+1} + t_i)}$$
$$= \sqrt{6}J_{ee} = \sqrt{3}J$$
 $\leftarrow \text{Error in the paper!}$

 This is valid only under WGN - in the absence of flicker noise.



Extracting synchronous jitter

- Noisy signal $v_n(t) = v(t) + n_v(t)$ Noise to jitter conversion
- For cyclostationary n_{v} ,





Accumulating jitter

- An undesired variation in the time since the previous output transition
- The uncertainty accumulates with every transition
- Exhibited by autonomous systems (OSC and VCO)
- Frequency modulated or FM jitter

Simple accumulating jitter

 Let η be a white Gaussian stationary or Tcyclostationary process, then

 $j_{acc}(t) = \int_{0}^{t} \eta(\tau) d\tau$ $\eta(t)$: 단위 시간 동안의 시간축 변화 Process $v_{n}(t) = v(t+j_{acc}(t))$

• $v_n(t)$ exhibits simple accumulating jitter – If noise sources are white, Gaussian and small

Simple accumulating jitter

 Each transition is relative to the previous transition, and the variation in the length of each period is independent.

•
$$J_k = \sqrt{kJ}$$
 for $k = 0, 1, 2, ...$ where
 $J = \sqrt{\operatorname{var}(j_{\operatorname{acc}}(t_i + T) - j_{\operatorname{acc}}(t_i))}$

• $J_{\rm cc} = \sqrt{2}J$

Synchronous vs Accumulating Jitter



Synchronous vs Accumulating Jitter



Extracting accumulating jitter

- Assume that a noisy oscillator exhibits simple accumulating jitter
- η is a white Gaussian T-cyclostationary noise process with
 - A single-sided PSD $S_{\eta}(f) = 2c$
 - An autocorrelation function

$$R_{\eta}(t_1, t_2) = E[\eta(t_1) \cdot \eta(t_2)]$$
$$= c \cdot \delta(t_1 - t_2)$$

Extracting accumulating jitter

• For Wiener process (Brownian Motion)

$$\begin{split} j_{\rm acc}(t) &= \int_0^t \eta_T(\tau) d\tau \qquad R_{j_{\rm acc}}(t_1, t_2) = c \min(t_1, t_2) \\ J^2 &= \operatorname{var}(j_{\rm acc}(t+T) - j_{\rm acc}(t)) \\ &= \operatorname{E}[(j_{\rm acc}(t+T) - j_{\rm acc}(t))^2] \\ &= \operatorname{E}[j_{\rm acc}(t+T)^2 - 2j_{\rm acc}(t+T)j_{\rm acc}(t) + j_{\rm acc}(t)^2] \\ &= \operatorname{E}[j_{\rm acc}(t+T)^2] - 2\operatorname{E}[j_{\rm acc}(t+T)j_{\rm acc}(t)] + \operatorname{E}[j_{\rm acc}(t)^2] \\ &= R_{j_{\rm acc}}(t+T, t+T) - 2R_{j_{\rm acc}}(t+T, t) + R_{j_{\rm acc}}(t, t) \\ &= c(t+T) - 2ct + ct \\ &= cT \\ J &= \sqrt{cT} \end{split}$$

Extracting accumulating jitter

- η is not measurable,
- Phase noise $\phi_{acc}(t) = 2\pi f_0 j_{acc}(t) = 2\pi f_0 \int_0^t \eta(\tau) d\tau$

$$S_{\phi_{\text{acc}}}(\Delta f) = 2c \frac{(2\pi f_{\text{o}})^2}{(2\pi\Delta f)^2} = \frac{2cf_{\text{o}}^2}{\Delta f^2}$$



Normalized Phase Noise

- Measurable quantity $L(\Delta f)$ is defined as
 - the ratio between power at a frequency offset of Δf from the carrier in a bandwidth of 1 Hz and total carrier power.



Normalized Phase Noise

• Therefore

$$\mathcal{L}(\Delta f) = \frac{1}{2}S_{\phi}(\Delta f)$$

$$\mathcal{L}(\Delta f) = \frac{1}{2} S_{\phi_{\text{acc}}}(\Delta f) = \frac{cf_o^2}{\Delta f^2}$$

Extracting the jitter of VCO

- A very low noise oscillator
 - Rael and Abidi in 0.35um CMOS
 - $-f_0 = 1.1$ GHz, a loaded Q = 6



Extracting the jitter of VCO

 $\mathcal{L} = -110 \text{ dBc}$ at 100 kHz offset

$$c = \mathcal{L}(\Delta f) \frac{\Delta f^2}{f_0^2} = 82.6 \times 10^{-21} \text{ UI}^2/\text{Hz}$$

$$J = \sqrt{cT} = \sqrt{\frac{c}{f_0}} = \sqrt{\frac{82.6 \times 10^{-21}}{1.1 \text{ GHz}}} = 8.7 \text{ fs}$$

Other issues

- η(t): Stationary jitter normalized to 1UI
 RMS Period jitter = T*sqrt(E[η²(t)])
- c: Under white stationary noise process
 - Normalized phase noise power $(\phi_{rms}/2\pi)^2$ per 1 Hz
 - <u>Jitter power</u> normalized to 1 UI accumulated for 1s.
 - Rms period jitter $\sigma = sqrt(c^*T)=T^*sqrt(c^*f_0)$

References

- [4.1] Kundert Predicting the phase noise and jitter of PLL-based frequency synthesizers
- [4.2] Zanchi How to calculate period jitter

Topics in IC

1.5 LC Oscillator, Phase Noise, and Jitter

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Outline

- Lossy LC Resonator
- Lossless LC Oscillator
- LC Oscillator with Negative Resistance
- LC Oscillator with VCCS
- LC Oscillator with Nonlinear VCCS
- Noisy LC Oscillator
- Phase Noise in LC Oscillator

Lossy LC Resonator

Oscillation not sustainable

$$f_{0} = \frac{1}{\sqrt{LC}}, Q = \omega_{0}CR$$

$$e^{-\frac{\omega_{0}}{2Q}t}$$



Lossless LC Oscillator

Sustained Oscillation

$$R_{eff} = \left(-R_a \parallel R\right)$$
$$= \frac{\left(-R_a \cdot R\right)}{\left(-R_a + R\right)}$$



If $R_a > R$. Reff > 0, lossy. If $R_a = R$, Reff = ∞ , sustaining oscillation If $R_a < R$, Reff < 0, exponentially expanding

LC Oscillator with VCCS



LC Oscillator with Nonlinear VCCS



LC Oscillator with Nonlinear VCCS



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Noisy LC Oscillator

• White Gaussian current noise is injected.



- A_n(t) is very small. Suppressed by the nonlinear resistor and/or buffer.
- Only $\varphi_n(t)$ is affected by $i_n(t)$.

Phase Noise in LC Oscillator

- Jitter is accumulated.
- Define $\eta(t)$ as a white, Gaussian process with $j_{acc}(t) = \int_0^t \eta(t) dt$



Period Jitter in LC Oscillator

- Jitter is accumulated.
- Define $\eta(t)$ as a white, Gaussian process with

$$j_{acc}(t) = \int_0^t \eta(t) dt$$

• Period Jitter J is calculated as follows:

$$J = \sqrt{cT}$$
$$= \sqrt{\mathcal{L}(\Delta f) \frac{\Delta f^2}{f_0^2} T}$$
$$= \sqrt{\mathcal{L}(f_0) T}$$

How to build a Negative Resistance

Use of cross-couplng



Thus, $R=v/i=(v_1 - v_2)/(g_m v_1)=-2/g_m$

References

- [5.1] Leeson A Simple Model of Feedback Oscillator Noise Spectrum
- [5.2] Lee Oscillator Phase Noise A Tutorial

Topics in IC

1.6. Oscillator Phase Noise

Deog-Kyoon Jeong dkjeong@snu.ac.kr School of Electrical and Computer Engineering Seoul National University 2020 Fall

Outline

- Introduction
- General considerations
- Oscillator phase noise in LTI system
- Phase noise theory in LTV system

Introduction

- Noise in oscillators
 - Amplitude noise and Phase noise
 - Generally amplitude fluctuations are greatly attenuated
 - Phase noise generally dominates
- Practical issues related to "how to perform simulations of phase noise"
- Identify general tradeoffs among key parameters
 - Power dissipation, Oscillator frequency, Resonator Q, circuit noise power
 - At first, these tradeoffs studied qualitatively in a hypothetical ideal oscillator
 - Linearity of the noise-to-phase noise is assumed

Introduction

- Oscillators are linear time-varying systems
- Periodic time variation leads to frequency translation of device noise to produce phasenoise spectra
- Upconversion of 1/f noise into close-in phase noise depends on symmetry properties
 - Symmetry properties are controllable by designers
- Class-C operation of active elements within an oscillator are beneficial
General Considerations

- Assume, the energy restorer is noiseless
 - The tank resistance is the only noise element
 - Signal energy stored in the tank v

$$E_{\text{stored}} = \frac{1}{2}CV_{\text{pk}}^2$$

Mean-square signal voltage

$$\overline{V_{\rm sig}^2} = \frac{E_{\rm stored}}{C}$$

Total mean-square noise voltage : Integrating the resistor's thermal noise density over noise bandwidth

 $\frac{1}{2}R \stackrel{\text{l}}{=} C \bigotimes L$

$$\overline{V_n^2} = 4kTR \int_0^\infty \left| \frac{Z(f)}{R} \right|^2 df = 4kTR \cdot \frac{1}{4RC} = \frac{kT}{C}$$

$$- \text{Noise-to-signal ratio : } \sqrt{\frac{N}{S}} = R \frac{\overline{V_n^2}}{\overline{V_{sig}^2}} = \frac{kT}{E_{stored}}$$

Noiseless Energy Restorer

General Considerations

By considering power consumption and resonator Q

$$Q = \frac{\omega E_{\text{stored}}}{P_{\text{diss}}}$$

- Therefore

$$\frac{N}{S} = \frac{\omega kT}{QP_{\rm diss}}$$

– Noise-to-carrier ratio α 1/(product of Q and Power)

α oscillation frequency

This relation holds approximately for real oscillators

Oscillator Phase Noise (LTI)

- The only source of noise is white noise of the tank conductance (represent as a current source across the tank with a mean-square spectral density) $\frac{\overline{i_n^2}}{\Delta f} = 4kTG$
- For relatively small $\Delta \omega$ (offset frequency) from center frequency $\omega_{o.}$
 - The impedance of an LC tank approximated as

$$Z(\omega_0 + \Delta \omega_0) \approx j \cdot \frac{\omega_0 L}{2\frac{\Delta \omega}{\omega_0}} = \frac{1}{G} \cdot \frac{\omega_0}{2Q\Delta \omega} \quad \text{where} \quad Q = \frac{R}{\omega_0 L} = \frac{1}{\omega_0 GL}$$

Oscillator Phase Noise (LTI)

 Multiply noise current by the squared magnitude of the tank impedance to obtain mean-square noise voltage

$$\frac{\overline{v_n^2}}{\Delta f} = \frac{\overline{i_n^2}}{\Delta f} \cdot |Z|^2 = 4kTR\left(\frac{\omega_0}{2Q\Delta\omega}\right)^2$$

- In idealized LC model, thermal noise affects both amplitude and phase-noise.
- In equilibrium, amplitude and phase-noise power are equal
 - So, the amplitude limiting mechanism present in any oscillator removes half the noise

Oscillator Phase Noise (LTI)

Normalized single-sideband noise spectral density

$$L\{\Delta\omega\} = 10\log\left[\frac{2kT}{P_{sig}} \cdot \left(\frac{\omega_0}{2Q\Delta\omega}\right)^2\right]$$

Lesson's formula



9

- Several phase-noise theories tried to explain certain observations as a consequence of nonlinear behavior
- By injecting a single frequency sinusoidal disturbance into oscillator
- Nonlinear mixing has been proposed to explain the phase noise
- Memoryless nonlinearity cannot explain the discrepancies

- Linearity would be a reasonable assumption as far as the noise-to-phase transfer function is concerned
 - expect doubling the injected noise to produce double the phase disturbance
- Perform linearization around the steady-state solution
 - Which automatically takes the effect of device nonlinearity into account
- Oscillators are fundamentally time-varying systems

• Example to show time invariance fails



 Therefore, an oscillator is linear, but time varying (LTV) system

- An impulsive input produces a step change in the phase noise
 - Impulse response $h_{\phi}(t, \tau) = \frac{\Gamma(\omega_0 \tau)}{q_{\max}} u(t \tau)$ where u(t) is unit step function
 - Dividing by q_{max} makes impulse sensitivity function (ISF) $\Gamma(x)$ independent of amplitude and dimensionless
- ISF : encodes information about the sensitivity of an oscillator to an impulse injected at phase $\omega_0 \tau$
 - ISF has maximum value near zero crossings
 - ISF has a zero value at maxima of the oscillation waveform

Typical shapes of ISF's for LC and Ring oscillator



 Excess phase can be computed once ISF has determined

$$\phi(t) = \int_{-\infty}^{\infty} h_{\phi}(t,\,\tau) i(\tau) \, d\tau = \frac{1}{q_{\max}} \, \int_{-\infty}^{t} \, \Gamma(\omega_{0}\tau) i(\tau) \, d\tau$$

 This computation can be visualized with the equivalent block



- ISF is periodic and expressible as Fourier series $\Gamma(\omega_0 \tau) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 \tau + \theta_n)$ - The excess phase caused
 - by an injected noise current

$$\phi(t) = \frac{1}{q_{\max}} \left[\frac{c_0}{2} \int_{-\infty}^t i(\tau) d\tau + \sum_{n=1}^\infty c_n \int_{-\infty}^t i(\tau) \cos(n\omega_0\tau) d\tau \right].$$



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- A linear, but time-varying system can exhibit similar behavior
 - Example : Inject a sinusoidal current $i(t) = I_m \cos[(m\omega_0 + \Delta\omega)t]$
 - The excess phase

 $\phi(t) \approx \frac{I_m c_m \sin(\Delta \omega t)}{2q_{\max} \Delta \omega} \quad \text{ when } n=m$

– The spectrum of $\Phi(t)$ consists of two equal side-bands

Phase-to-voltage conversation is nonlinear

- Because it involves phase modulation of sinusoid
- Equal-power sideband $P_{\text{SBC}}(\Delta \omega) \approx 10 \cdot \log \left(\frac{I_m c_m}{4q_{\max}\Delta \omega}\right)^2$

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Topics in IC Design

- $P_{\rm SBC}(\Delta\omega) \approx 10 \cdot \log \left(\frac{\frac{i_n^2}{\Delta f} \sum_{m=0}^{\infty} c_m^2}{4q_{\rm max}^2 \Delta \omega^2} \right)$ In case of white noise source
- It indicates both upward and downward frequency translations of noise into the noise near carrier



17

Reference

- [6.1] Hajimiri A General Theory of Phase Noise in Electrical Oscillators
- [6.2] Razavi A Study of Phase Noise
- [6.3] Hajimiri Jitter and Phase Noise in Ring Oscillators







A number of years ago when we at Agilent were Hewlett-Packard one of our engineers represented phase noise measurements as a puzzle with many pieces that are sometimes not so easily connected.



Today, we have new hardware and improved techniques, but phase noise measurements can still be a puzzling question and generally there is not one solution that fits all requirements.

Today we will review some of the basics of phase noise and the three most common measurement techniques and where they apply. Hopefully we can make the puzzle of phase noise measurements a little easier to solve.

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	Bibliography	
	Summary	
	 Comparison of Agilent Phase Noise I 	Measurement Solutions
	 Agilent E5500 Phase Noise Measurement System 	
	 Two-channel cross correlation met 	hod
	 Phase detector techniques 	
	 Direct phase noise measurement (analyzer) 	with a spectrum
	 Phase noise measurement technique 	es
	 What is phase noise? 	
Agenda		

What is Phase Noise?
 The basic concept of phase noise centers around frequency stability, or the characteristic of an oscillator to produce the same frequency over a specified time period.
 Frequency stability can be broken into two components:
 Long-term frequency stability—frequency variations that occur over hours, days, months, or even years
 Short-term frequency stability—describing frequency changes that occur over a period of a few seconds, or less, duration.
 In our discussion of phase noise we will focus on short-term frequency variations in oscillators and other electronic devices like amplifiers
 Phase noise can be described by in many ways, but the most common is single sideband (SSB) phase noise, generally denoted as f(f)
 The U.S. National Institute of Standards and Technology (NIST) defines <u>f</u>(f) as the ratio as the power density at an offset frequency from the carrier to the total power of the carrier signal.
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The first question that is often asked is; What is phase noise? In the main, when we are talking about phase noise we are talking about the frequency stability of a signal. We look at frequency stability from several view points. Many times we are concerned with the long-term stability of an oscillator over the observation time of hours, days, months or even years. Many oscillators that have excellent long-term stability, such as Rubidium oscillators, don't have very good phase noise.

When discussing phase noise we are really concerned with the short-term frequency variations of the signal during an observation time of seconds or less. This short-term stability will be the focal point of our discussion today.

The most common way to describe phase noise is as single sideband (SSB) phase noise which is generally denoted as $\mathcal{L}(f)$ (script L of f). The US National Institute of Standards and Technology defines single sideband phase noise as the ratio of the spectral power density measured at an offset frequency from the carrier to the total power of the carrier signal.



Before we get too far along, let's look at the difference between an ideal signal (a perfect oscillator) and a more typical signal. In the frequency domain, the ideal signal is represented by a single spectral line.

In the real world; however, there are always small, unwanted amplitude and phase fluctuations present on the signal. Notice that frequency fluctuations are actually an added term to the phase angle portion of the time domain equation. Because phase and frequency are related, you can discuss equivalently about unwanted frequency or phase fluctuations.

In the frequency domain, the signal is no longer a discrete spectral line. It is now represented by spread of spectral lines - both above and below the nominal signal frequency in the form of modulation sidebands due to random amplitude and phase fluctuations.



You can also use phasor relationships to describe how amplitude and phase fluctuations affect the nominal signal frequency.



Historically, the most generally used phase noise unit of measure has been the single sideband power within a one hertz bandwidth at a frequency f away from the carrier referenced to the carrier frequency power. This unit of measure is represented as script L(f) in units of dBc/Hz



Traditionally, when measuring phase noise directly with a swept RF spectrum analyzer, the f(f) ratio is the ratio of noise power in a 1 Hz bandwidth, offset from the carrier at the desired offset frequency, relative to the carrier signal power. This is a slight simplification compared to using the total integrated signal power; however, the difference in minimal considering the great differences in power involved.

On modern spectrum analyzers like the Agilent PXA the delta marker can be used to determine the relative signal power and the noise power. Note that the carrier power is measured in dBm using a simple marker and that the noise power is measured using Band/Interval density marker. The band/Interval density marker provides integrated power normalized to a 1 Hz bandwidth.



Thermal noise (kTB) is the mean available noise power per Hz from a resistor at a temperature T. As the temperature of the resistor increases, the kinetic energy of its electrons increases and more power becomes available. Thermal noise is broadband and virtually flat with frequency. However, when displayed on a spectrum analyzer the analyzer's own noise figure increases the measured noise power, limiting the small signal performance of the analyzer.



Thermal noise can limit the extent to which you can measure phase noise. Thermal noise as described by kTB at room temperature is -174 dBm/Hz. Since phase noise and AM noise contribute equally to kTB, the phase noise power portion of kTB is equal to -177 dBm/Hz (3 dB less than the total kTB power).

If the power in the carrier signal becomes a small value, for example -20 dBm, the limit to which you can measure phase noise power is the difference between the carrier signal power and the phase noise portion of kTB (-177 dBm/Hz - (-20 dBm) = -157 dBc/Hz). Higher signal powers allow phase noise to be measured to a lower dBc/Hz level.



If more signal power is the answer, simply add an amplifier-or so you would expect.



But the amplifier itself adds noise. Adding amplification also adds noise, which we need to account for within our measurement.



Amplifiers boost not only the input signal but also the input noise. The input signal-to-noise ratio is only preserved when the amplifier itself does not add noise.

Noise figure is simply the ratio of the signal-to-noise at the input of a two-port device to the signal-to-noise ratio at the output, at a source impedance temperature of 290°K. In other words, noise figure is a measure of the signal degradation as it passes through the device—due to the addition of noise by the device. What does this have to due with phase noise measurements?



The noise power at the output of an amplifier can be calculated if its gain and noise figure are known. The noise at the amplifier output is given by:

N(out) = FGkTB.

The display shows the rms voltages of a signal and noise at the output of the amplifier. We want to see how this noise affects the phase noise of the amplifier.



Using phasor methods, we can calculate the effect of the superimposed noise voltages on the carrier signal. We can see from the phasor diagram that V_{Nrms} produces a $\Delta\Phi$ rms term. For small angles, $\Delta\Phi$ rms = V_{Nrms}/Vs_{peak} . The total $\Delta\Phi$ rms can be found by adding two individual phase components power-wise. Squaring this result and dividing by the bandwidth gives the spectral density of phase fluctuations or phase noise. The phase noise is directly proportional to the thermal noise at the input and the noise figure of the amplifier.

Note that this phase noise component is independent of frequency.

To summarize, amplifiers help boost carrier power signal to levels necessary for successful measurements, but the theoretical phase noise measurement limit is reduced by the noise figure of the amplifier and low signal power.



Due to the random nature of the instabilities, the phase deviation is represented by a spectral density distribution plot. The term spectral density describes the power distribution (mean square deviation) as a continuous function, expressed in units of energy within a given bandwidth. The short term instability is measured as low-level phase modulation of the carrier and is equivalent to phase modulation by a noise source. There are four different units used to quantify spectral density:

 $S_{\phi}(f)$, L(f), $S_{v}(f)$, and $S_{y}(f)$.



A measure of phase instability often used is S_{Φ} (f), the spectral density of phase fluctuations, on a per Hertz basis. If we demodulate the phase modulated signal, using a phase detector, we obtain V_{out} as a function of phase fluctuations of the input signal. Measuring V_{out} on a spectrum analyzer gives $\Delta V_{rms}(f)$ which is proportional to $\Delta \Phi_{rms}(f)$

The term spectral density describes the energy distribution as a continuous function, expressed in units of phase variance (radians) per unit bandwidth. If we use 1 radian(rms)/rt Hz as the phase variance comparison, we can express $S_{\phi}(f)$ in terms of dB.

For large phase variations (>> 1 radian rms/rt Hz), S_{Φ} (f) will be greater than 0 dB. For small phase variations (< 1 radian rms/rt Hz), S_{Φ} (f) will be less than 0 dB.

 $S_{\Phi}(f)$ is a very useful for analysis of the effects of phase noise on systems that have phase sensitive circuits, such as digital communications links.



L(f) is an indirect measure of noise energy easily related to the RF power spectrum observed on a spectrum analyzer. The historical definition is the ratio of the power in one phase modulation sideband per hertz, to the total signal power. L(f) is usually presented logarithmically as a plot of phase modulation sidebands in the frequency domain, expressed in dB relative to the carrier power per hertz of bandwidth [dBc/Hz].

This historical definition is confusing when the phase variations exceed small values because it is possible to have phase noise values that are greater than 0 dB even though the power in the modulation sideband is not greater than the carrier power.

IEEE STD 1139 has been changed to define L(f) as $S_{\Phi}(f)/2$ to eliminate the confusion.



Historical measurements of L(f) with a spectrum analyzer typically measured phase noise when the phase variation was much less than 1 radian. Phase noise measurement systems, however, measure S_{Φ} (f), which allows the phase variation to exceed this small angle restriction. On this graph, the typical limit for the small angle criterion is a line drawn with a slope of -10 dB/decade that passes through a 1 Hz offset at -30 dBc/Hz. This represents a peak phase deviation of approximately 0.2 radians integrated over any one decade of offset frequency.

This plot of L(f) resulting from the phase noise of a free-running VCO illustrates the confusing display of measured results that can occur if the instantaneous phase modulation exceeds a small angle. Measured data, S_{Φ} (f)/2 (dB), is correct, but historical L(f) is obviously not an appropriate data representation as it reaches +15 dBc/Hz at a 10 Hz offset (15 dB more power at a 10 Hz offset than the total power in the signal). The new definition of L(f) = S_{Φ} (f)/2 allows this condition, since S_{Φ} (f) in dB is relative to 1 radian. Exceeding 0 dB simply means than the phase variations being measured are greater than 1 radian.




The direct spectrum method is the oldest phase noise measurement method and probably the simplest to make. The device under test (DUT) is simply connected to the analyzer's input and the analyzer is tuned to the carrier frequency of the DUT. Next the power of the carrier is measured and a measurement of the power spectral density of the oscillator noise, at a specified offset frequency, is referenced to the carrier power. Pretty simple—right, but there may be more here than meets the eye.

For this measurement you may also want to consider making corrections for:

- The noise bandwidth of the analyzer's resolution bandwidth filters. This is a two-step process: First, the RBW filter's 3-dB bandwidth must be normalized to 1 Hertz by taking 10*Log (RBW filter BW/1 Hz). Next a correction factor must be applied to correct the filters noise bandwidth to its 3-dB bandwidth. Generally, most Agilent digital IF spectrum analyzer RBW filters have a noise bandwidth that is 1.0575 wider than the 3-dB bandwidth. For example, if a 300 Hz RBW filter were used you would need to add a correction factor of 10 * log(300 * 1.0757) or 25.09 dB
- 2. Effects of the analyzer's circuitry. These corrections should account for the way that a peak detector responds to noise, under reporting rms noise power by a factor of 1.05 dB. In addition, the logging process in spectrum analyzers tend to amplify noise peaks less than the rest of the noise signal resulting in a reported power that is less than the actual noise power. Combining these two effects results in a noise power measurement that is 2.5 dB below the actual noise power.

Agilent Application Note 150 fully explains the above correction factors.

These considerations are not necessary when making measurements with the analyzer's built in phase noise measurement personality or when using the analyzer's internal band/interval density marker to make the noise spectral density measurement.



As mentioned previously, the direct spectrum method of phase noise measurement is a simple and time proven method of measuring the phase noise of an oscillator, but there are potential problems associated with this measurement. The biggest limiting factor is the quality of the spectrum analyzer being used, but all spectrum analyzer share some common limitations.

As we discussed on the previous slide, the 3-dB bandwidth and the noise bandwidth of the analyzer's resolution bandwidth filters are not identical and correction factors must be used. We also discussed errors associated with measuring the true rms power of noise, caused by the analyzer's detector processing and logarithmic amplifiers. In addition to these errors, other factors limit the analyzer's ability to correctly measure the phase noise of a signal. These factors include:

- The residual FM of the analyzer's local oscillator, and
- The noise sidebands or phase noise of the analyzer's local oscillator. Just like in a receiver, the LO phase noise is added to the signal that is up or down converted by the mixers in the analyzer.
- The noise floor of the spectrum analyzer.

Lastly, spectrum analyzers generally only measure the scalar magnitude of noise sidebands of the signal and are not able to differentiate between amplitude noise and phase noise. In addition, the measurement process is complicated by having to make a noise measurement at each frequency offset of interest, sometimes a very time consuming task.

To a large extent the use of a phase noise measurement personality like the Agilent N9068A Phase Noise measurement application for Agilent X-Series signal analyzes greatly simplifies the measurement task and minimizes the effects of the above mentioned measurement errors.



The N9068A Phase Noise Measurement application provides a simple one-button measurement menu for making quick and accurate phase noise measurements. The application automatically optimizes the measurement in each offset range to give the best possible measurement accuracy. The user can quickly select between a spectrum monitor mode, a spot measurement that shows phase noise verses time at a single offset frequency, or a log-plot view; as shown on this slide.



The N9068A Phase Noise Measurement application not only produces a nice log plot of an oscillator's phase noise, it also provides many common phase noise related measurements through special purpose marker functions. Most of these measurements are accessed through the marker function hard key on the front of the analyzer. Measurements like residual FM, integrated phase noise and oscillator spurs are simple one button measurements.



In this slide an integrated phase noise measurement is shown with marker 4. In the display note the two vertical bars showing the offset region over which phase noise is being integrated. The total integrated phase noise over this region is shown in the marker table as 3.445 millidegrees. Note that integrated phase noise is estimated as for a double sideband measurement.

Other band marker functions are also shown such as residual FM and jitter.



Many times it is desirable to have a calibrated phase noise signal that can be used to verify a given test setup. This is particularly valuable when developing your own phase noise software for a direct phase noise measurement. The method described here can be used with any phase noise measurement technique and can provide valuable insight into the performance of a given phase noise measurement system.

A reliable calibrated phase noise test signal can be created by frequency modulating a signal generator with a uniform noise signal. The slope of the noise sidebands will be constant at -20 dB per decade. The desired sideband level can be selected by changing the deviation of the FM signal.

When using this technique, it is important to ensure that the signal generator's noise output be at least 10 dB lower than the desired calibration signal at your specified offset frequency. The phase noise measurement shown on this slide was produced with a uniform noise signal FM modulated at a rate of 500 Hertz. This produced a phase noise at a 10 kHz offset of -100 dBc/Hz.

In addition to using an Agilent PSG signal generator and FM modulation, an Agilent MXG signal generator could be used with its phase noise impairment mode.

M Noise Deviation	Offset 1 Hz	10 Hz	100 Hz	1 kHz	10 kHz	100 kHz
Hz	-60	-80	-100	-120	-140	-160
6 Hz	-50	-70	-90	-110	-130	-150
0 Hz	-40	-60	-80	-100	-120	-140
58 Hz	-30	-50	-70	-90	-110	-130
00 Hz	-20	-40	-60	-80	-100	-120
.58 kHz	-10	-30	-50	-70	-90	-110
kHz	0	-20	-40	-60	-80	-100
5.8 kHz	10	-10	-30	-50	-70	-90
0 kHz	20	0	-20	-40	-60	-80
Note: For vas selec Hz offset	r the exan ted to pro	nple on the duce a cali	previous sl ibrated phas	ide, an FM se noise of	deviation o -100 dBc/H	f 500 Hz z at a 10





To separate phase noise from amplitude noise, a phase detector is required. This slide shows the basic concept for the phase detector technique. The phase detector converts the phase difference of the two input signals into a voltage at the output of the detector. When the phase difference is set to 90 (quadrature), the voltage output will be zero volts. Any phase fluctuation from quadrature will result in a voltage fluctuation at the output.

Several methods have been developed based upon the phase detector concept. Among them, the reference source/PLL (phase-locked-loop) is one of the most widely used methods. Additionally, the phase detector technique also enables residual/additive noise measurements for two-port devices.



The basis of the Reference Source / PLL method is the double balanced mixer used as a phase detector. Two sources, one from the DUT and the other from the reference source, provide inputs to the mixer. The reference source is controlled such that it follows the DUT at the same carrier frequency (fc) and in phase quadrature (90 out of phase) nominally. The mixer sum frequency (2fc) is filtered out by the low pass filter (LPF), and the mixer difference frequency is 0 Hz (dc) with an average voltage output of 0 V.

Riding on this dc signal are ac voltage fluctuations proportional to the combined (rms-sum) noise contributions of the two input signals. For accurate phase noise measurements on signals from the DUT, the phase noise of the reference source should be either negligible or well characterized. The baseband signal is often amplified and input to a baseband spectrum analyzer.

The reference source/PLL method yields the overall best sensitivity and widest measurement coverage (e.g. the frequency offset range is 0.01 Hz to 100 MHz). Additionally, this method is insensitive to AM noise and capable of tacking drifting sources. Disadvantages of this method include requiring a clean, elec-tronically tunable reference source, and that measuring high drift rate sources requires reference with a wide tuning range.



The frequency discriminator method is another variation of the phase detector technique with the require-ment of a reference source being eliminated.

This slide shows how the analog delay-line discriminator method works. The signal from the DUT is split into two channels. The signal in one path is delayed relative to the signal in the other path. The delay line converts the frequency fluctuation to phase fluctuation. Adjusting the delay line or the phase shifter will determine the phase quadrature of the two inputs to the mixer (phase detector). Then, the phase detector converts phase fluctuations to voltage fluctuations, which can then be read on the baseband spectrum analyzer as frequency noise. The frequency noise is then converted for phase noise reading of the DUT.



Although the analog delay-line discriminator method degrades the measurement sensitivity (at close-in offset frequency, in particular), it is useful when the DUT is a noisier source that has high-level, low-rate phase noise, or high close-in spurious sideband conditions which can pose problems for the phase detector PLL technique.

A longer delay line will improve the sensitivity but the insertion loss of the delay line may exceed the source power available and cancel any further improve-ment. Also, longer delay lines limit the maximum offset frequency that can be measured. This method is best used for free-running sources such as LC oscillators or cavity oscillators.



The heterodyne (digital) discriminator method is a modified version of the

analog delay-line discriminator method and can measure the relatively large phase noise of unstable signal sources and oscillators. This method features wider phase noise measurement ranges than the PLL method and does not need re-connection of various analog delay lines at any frequency. The total dynamic range of the phase noise measurement is limited by the LNA and ADCs, unlike the analog delay-line discriminator method previously described.

This limitation is improved by the cross-correlation technique explained in the next section. The heterodyne (digital) discriminator method also provides very easy and accurate AM noise measurements (by setting the delay time zero) with the same setup and RF port connection as the phase noise measurement.

This method is only available in Agilent's E5052B signal source analyzer.







The two-channel cross-correlation technique combines two duplicate single-channel reference sources/PLL systems and performs cross-correlation operations between the outputs of each channel, as shown in this slide. This method is available only in the E5052B signal source analyzer, among Agilent phase noise measurement solutions.

DUT noises through each channel are coherent and are not affected by the crosscorrelation, whereas, the internal noises generated by each channel are incoherent and are diminished by the cross-correlation operation at the rate of $M^{1/2}$ (M being the number of correlation). This can be expressed as:

Nmeas = N_{DUT} + $(N_1 + N_2)/M^{1/2}$

where, $_{Nmeas}$ is the total measured noise at the display; N_{DUT} the DUT noise; N_1 and N_2 the internal noise from channels 1 and 2, respectively; and M the number of correlations.

The two-channel cross-correlation technique achieves superior measurement sensitivity without requiring exceptional performance of the hardware components. However, the measurement speed suffers when increasing the number of correlations.







The Agilent E5500 Phase Noise Measurement System is a modular system based on the phase detector technique of phase noise measurement. The system is designed to be extremely flexible, allowing for various configurations and interconnecting with a vast array of signal analyzers and signal sources.



The E5500 system allows the most flexible measurements on one-port VCOs, DROs, crystal oscillators, and synthesizers. Two-port devices, including amplifiers and converters, plus CW, pulsed and spurious signals can also be measured. The E5500 measurements include absolute and residual phase noise, AM noise, and low-level spurious signals. The standalone-instrument architecture easily configures for various measurement techniques, including the reference source/PLL and analog delay-line discriminator method.

With a wide offset range capability, from 0.01 Hz to 100 MHz (0.01 Hz to 2 MHz without optional spectrum/signal analyzer), the E5500 provides more information on the DUT's phase noise performance extremely close to and far from the carrier. Depending on the low-noise downconverter selected, the E5500 solution handles carrier frequencies up to 26.5 GHz, which can be extended to 110 GHz with the use of the Agilent 11970 Series millimeter harmonic mixer. The required key components of the E5500 system include a phase noise test set (N5500A) and phase noise mea-surement PC software.

In addition, when confi gured with the programmable delay line the E5500 sys-tem can implement the "Frequency/analog delay-line discriminator" technique that offers good far-out but poor close-in sensitivity, suitable for measuring the free-running sources with a large amount of drift.





Comparison of Agilent Phase Noise Solutions

PN Measurement Technique	Advantages	Disadvantages	
Direct Spectrum Measurement	Easy operation Cuick check of phase-locked signals Instrument is not dedicated to phase noise can be used for general purpose also.	Difficult to measure close-in PN of quiet signal sources like crystal oscillators Cannot measure PN of drifty signal sources, such as free- running VCDs.	
Phase detector (reference source (PLL)	Applicable to broad offset range Can measure very low PN at close- in offsets with a good LO Measure PN for pulsed carriers as well as CW Can separate PN from AM noise.	PN noise is limited by LO noise Complicated set up and calibration required.	
Phase detector (analog delay- line discriminator)	Can measure very low PN at far-out offset/requencies Suitable for measuring relatively dirty sources. like YIG oscillators	Not applicable to close-in PN measurements due to gain degradation by discriminator Complicated set up and calibration required Difficult to obtain the right delay line at an arbitrary frequency.	
PLL method and heterodyne digital discriminator with two- channel cross-correlation	Easy operation, setup, and cal. Measure very low PN at broad offsets Cross-correlation improves PN sensitivity Consequence M and SN increases	Long measurement time for extremely low PN at close-in offset frequencies.	
	PN Measurement Technique DirectSpectrum Measurement Phase detector (reference source / PLL) Phase detector (analog detey- like discriminator) PLL method and heterodyne digital discriminator with two- channel cross-correlation	PN Measurement Technique Advantages DirectSpectrum Measurement • Easy operation • Quick check of phase-locked signals • Instruments not dedicated to phase noise can be used for general purpose also. Phase detector (reference source / PLL) • Applicable to broad offset range • Can measure very low PN at close- in offsets with a good LO • Measure PN for pulsed carriers as well as CW • Can separate PN from AM noise. Phase detector (analog detay- like discriminator) • Can measure very low PN at far-out offset frequencies • Suitable for measuring relatively dirity sources, like YIG oscillators PLL method and heterodyme channel cross-correlation • Easy operation, setup, and cal. • Measure very low PN at broad offsets • Cross-correlation improves PN, sensitivity	





