Topics in IC Design

4.1 Introduction to Injection Locked Oscillators

Deog-Kyoon Jeong
dkjeong@snu.ac.kr
School of Electrical and Computer Engineering
Seoul National University

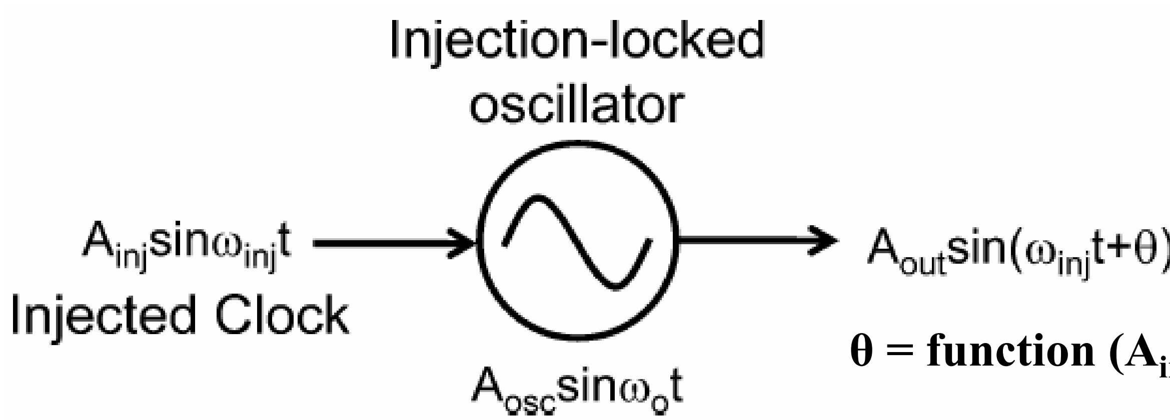
2020 Fall

Outline

- Definition of Injection Locking
- Applications
- Basic Theory with Phasor Diagram
- Lock Range Calculation
- Dynamics

What is Injection Locking?

- **A oscillator is locked to a frequency of an external signal close to its free-running frequency.**
  - **Frequency is locked but not the phase.**



$$\theta = \text{function } (A_{inj}, A_{osc}, \omega_{inj}, \omega_0)$$

Applications

- **Clock generation**
  - **Fundamental injection: $\omega_{out} = \omega_{inj}$**
- **Frequency multiplication**
  - **Subharmonic injection: $\omega_{out} / N = \omega_{inj}$**
- **Frequency division**
  - **Superharmonic injection: $\omega_{out} * N = \omega_{inj}$**
- **Clock recovery from NRZ data stream**

Basic Theory

- **Developed by Adler in 1946**

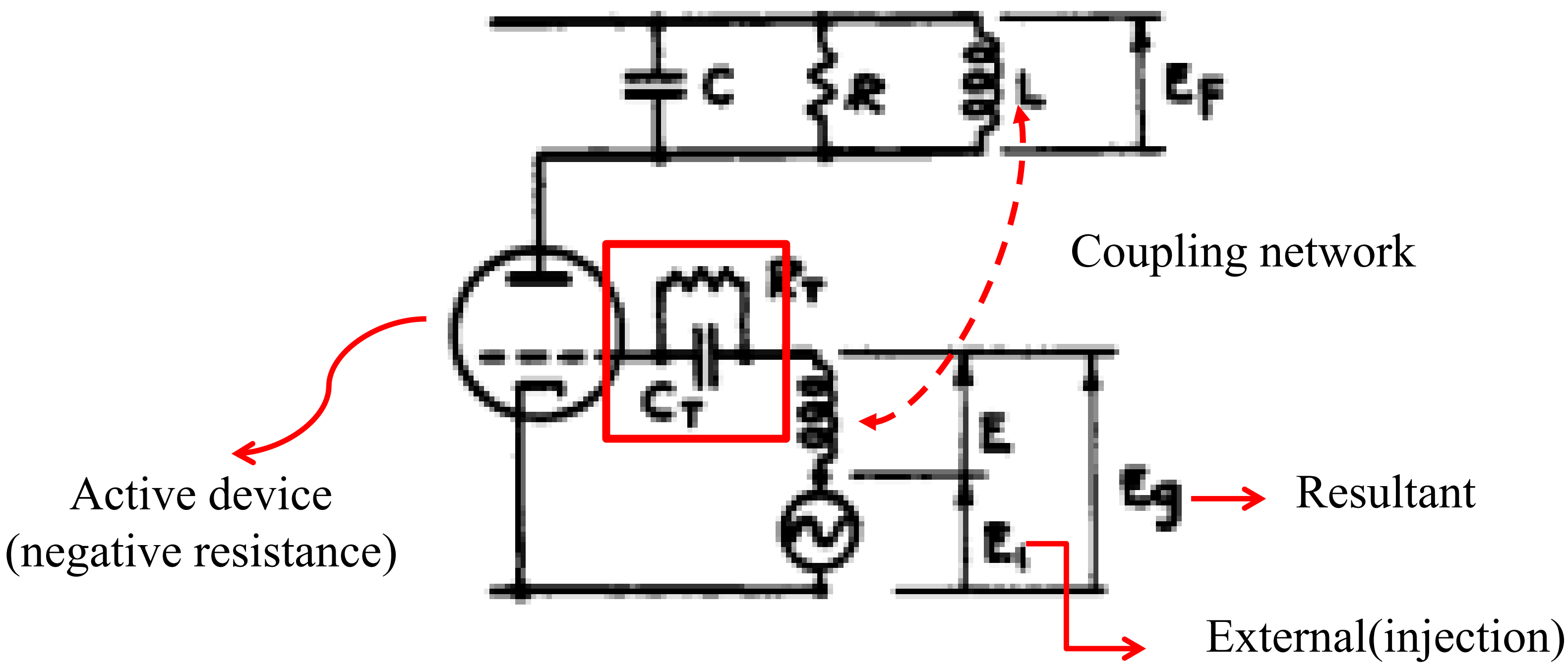


Fig. 1. Oscillator circuit.

Symbols

- ω_0 = free-running frequency
- ω_1 = frequency of external signal (injection)
- $\Delta\omega_0 = \omega_0 - \omega_1$: “undisturbed” beat freq.
- ω = “instantaneous” freq. of oscillator
- $\Delta\omega = \omega - \omega_1$: “instantaneous” beat freq.

- “undisturbed”: injection off
- “instantaneous”: injection on

Three Assumptions

1) $\omega_0 /2Q >> \Delta\omega_0$: injection freq $\cong$ free running freq

2) $T << 1/\Delta\omega_0$: amplitude control mechanism is very fast
   ($T=R_TC_T$)

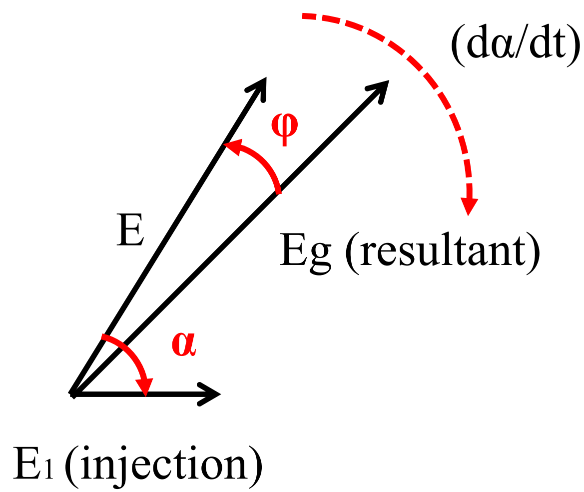
3) $E_1 << E$: weak injection

Phasor Diagram

- **Phasor: Phase Vector**
  - **represents both amplitude and phase**
- **Phasor diagram is used when analyzing linear system with the same frequency**
- **"rotating" phasor is used for expressing signals with slightly different frequency**

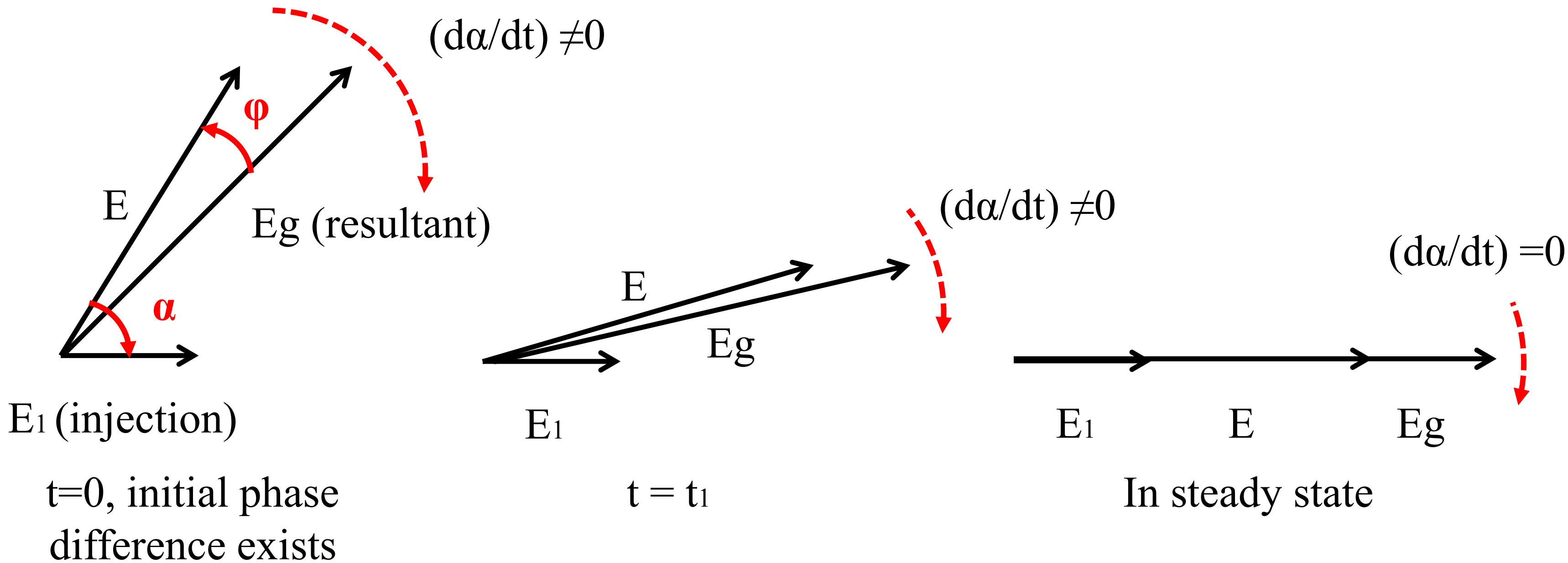
Rotating Phasor

- E_1: Injected signal phasor with <u>fixed</u> frequency (ω_1).
- E: Internally generated signal phasor
  - Rotating clockwise with an angular velocity ($d\alpha/dt$).
  - Actual frequency = $\omega_1 + (d\alpha/dt)$
- E_g: Phasor of sum of internal and externally injected signal



Examples of Rotating Phasor

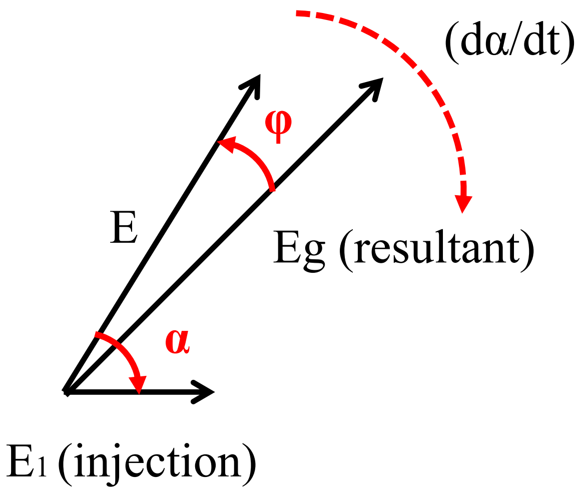
- **When free-running frequency is equal to injected frequency ($\omega_0 = \omega_1$)**



- $\varphi(t) = f(\Delta\omega_0,$ <u>initial phase</u>$)$

Relation between Phasors

- **Vector calculation**



$$\sin\varphi = \frac{E_1}{E_g}\sin(-\alpha) = -\frac{E_1 \sin\alpha}{\sqrt{{E_1}^2 + E^2 + 2E_1 E\cos\alpha}}$$

Under weak injection, $E_1 \ll E$, $\varphi \ll 1$

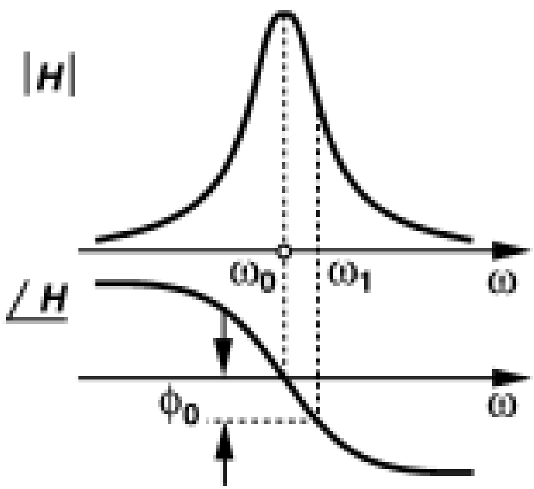
$$\sin\varphi \fallingdotseq \varphi$$

$$-\frac{E_1 \sin\alpha}{\sqrt{{E_1}^2 + E^2 + 2E_1 E\cos\alpha}} \fallingdotseq -\frac{E_1}{E}\sin\alpha$$

$$\varphi = -\frac{E_1}{E}\sin\alpha \quad - ①$$

Phase Shift at Off-resonant Frequency

- **Assuming injection frequency being close to resonant frequency**



$$\varphi = \frac{d\varphi}{d\omega}(\omega - \omega_0)$$

$$= \frac{d\varphi}{d\omega}(\omega - \omega_1 - \omega_0 + \omega_1)$$

$$= \frac{d\varphi}{d\omega}(\Delta\omega - \Delta\omega_0) \quad \leftarrow \quad \Delta\omega = \frac{d\alpha}{dt}$$

$$= \frac{d\varphi}{d\omega}\left(\frac{d\alpha}{dt} - \Delta\omega_0\right) \quad - ②$$

LC Resonant Circuit Phase Response

- **For an RLC tank,**

$$Z = R||sL||\frac{1}{sC}$$
$$= \frac{sRL}{s^2RLC + sL + R}$$
$$= \frac{j\omega RL}{j\omega L + R - \omega^2 RLC}$$

$$\angle Z = \frac{\pi}{2} - \tan^{-1}\left(\frac{\omega L}{R - \omega^2 RLC}\right)$$
$$= \frac{\pi}{2} - \tan^{-1}\left(\frac{\omega L}{R}\frac{\omega_0^2}{\omega_0^2 - \omega^2}\right)$$
$$= \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{Q}\frac{\omega_0}{2(\omega_0 - \omega)}\right)$$
$$= \tan^{-1}\left(\frac{2Q}{\omega_0}(\omega_0 - \omega)\right)$$
$$\therefore \tan\left(\angle Z\right) = \frac{2Q}{\omega_0}(\omega_0 - \omega)$$

LC Resonant Circuit Phase Response

- **For an RLC tank,**

$$\therefore \tan\left(\angle Z\right) = \frac{2Q}{\omega_0}(\omega_0 - \omega)$$

- **In the notation of the Phasor diagram,**

$$\angle Z = -\varphi$$

$$\varphi \ll 1, \quad \tan(\varphi) \doteqdot \varphi$$

$$\varphi = \frac{2Q}{\omega_0}(\omega - \omega_0)$$

$$\rightarrow \frac{d\varphi}{d\omega} = \frac{2Q}{\omega_0}$$

Calculation of Phasors

- Calculation of $\alpha(t)$ from two equations

$$\varphi = -\frac{E_1}{E}\sin\alpha \quad - ①$$

$$\varphi = \frac{d\varphi}{d\omega}\left(\frac{d\alpha}{dt} - \Delta\omega_0\right) \quad - ②$$

$$\rightarrow \; -\frac{E_1}{E}\sin\alpha = \frac{d\varphi}{d\omega}\left(\frac{d\alpha}{dt} - \Delta\omega_0\right) \leftarrow \frac{d\varphi}{d\omega} = \frac{2Q}{\omega_0} \quad : \text{LC resonant circuit}$$

$$\rightarrow \; \frac{d\alpha}{dt} = -\frac{E_1}{E}\frac{\omega_0}{2Q}\sin\alpha + \Delta\omega_0 \quad - ③$$

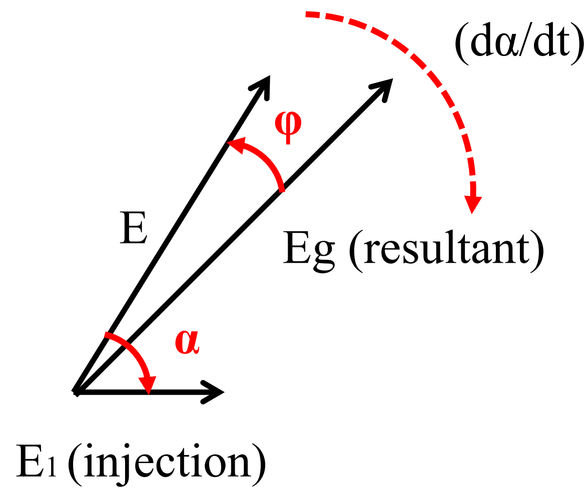
Lock Range Calculation

- **Equation for** α(t) (phase angle between E and E1)

$$\frac{d\alpha}{dt} = -\frac{E_1}{E}\frac{\omega_0}{2Q}\sin\alpha + \Delta\omega_0 \quad - ③$$

- **In steady state,**

$$\frac{d\alpha}{dt} = 0,\ \ 0 = -\frac{E_1}{E}\frac{\omega_0}{2Q}\sin\alpha + \Delta\omega_0 \quad - ③'$$



- **Thus,** $\sin\alpha = 2Q\frac{E}{E_1}\frac{\Delta\omega_0}{\omega_0}$

- **Since** $|\sin\alpha| \le 1,\ \ |2Q\frac{E}{E_1}\frac{\Delta\omega_0}{\omega_0}| \le 1$

- **Normalized lock range is** $\therefore |\frac{\Delta\omega_0}{\omega_0}| \le \frac{1}{2Q}\frac{E_1}{E}$

Note on Lock Range

- **For higher normalized lock range**
  - **higher injection strength (E_1/E)**
  - **low Q resonant circuit (higher bandwidth of the tank)**

$$\therefore \left|\frac{\Delta\omega_0}{\omega_0}\right| \leq \frac{1}{2Q}\frac{E_1}{E}$$

Solution of $\alpha(t)$ when $\Delta\omega_0$=0

- **General solution with $\Delta\omega_0$=0**

$$\frac{d\alpha}{dt} = -\frac{E_1}{E}\frac{\omega_0}{2Q}\sin\alpha + \Delta\omega_0 \quad - ③$$

- **Simplified equation**

$$\frac{d\alpha}{dt} = -B\sin\alpha, \quad B = \frac{E_1}{E}\frac{\omega_0}{2Q}$$

- **Analytical solution**

$$\tan\left(\frac{\alpha}{2}\right) = e^{-Bt}\tan\left(\frac{\alpha_0}{2}\right)$$

- **When α_0 is small, close to a first-order system.**

$$\alpha(t) \sim \alpha_0 e^{-Bt}$$

Solution of $\alpha(t)$ when $\Delta\omega_0$=0

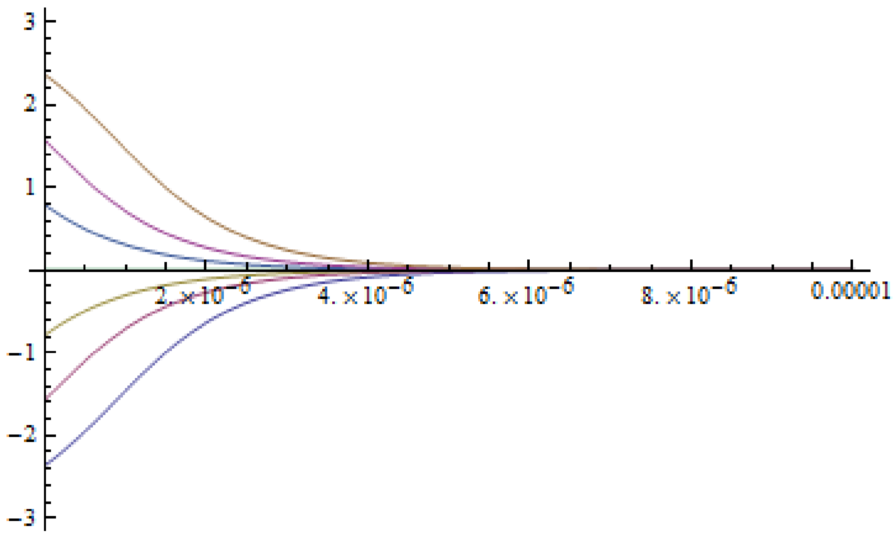
- **In Mathematica,**

```
sol = DSolve[{a'[t] == -B * (Sin[a[t]]), a[0] == C}, a[t], t]
```

$$\left\{\left\{a[t] \to 2\,\mathrm{ArcCot}\left[e^{B\,t}\,\mathrm{Cot}\left[\frac{C}{2}\right]\right]\right\}\right\}$$

```
tab = Table[a[t] /. sol[[1]] /. {B → 10^6, C → c}, {c, -3*Pi/4, 3*Pi/4, Pi/4}];

Plot[Evaluate[tab], {t, 0, 0.00001}, PlotRange → {-Pi, Pi}]
```



Solution of $\alpha(t)$ when $\Delta\omega_0 \neq 0$

- **General solution with $\Delta\omega_0 \neq 0$**

$$\frac{d\alpha}{dt} = -\frac{E_1}{E}\frac{\omega_0}{2Q}\sin\alpha + \Delta\omega_0 \quad - ③$$

- **Simplified equation**

$$\frac{d\alpha}{dt} = -B(\sin\alpha - K) \quad B = \frac{E_1}{E}\frac{\omega_0}{2Q} \quad K = 2Q\frac{E}{E_1}\frac{\Delta\omega_0}{\omega_0}$$

- **Analytical solution when K<1 (Injection locked)**

$$\tan\left(\frac{\alpha}{2}\right) = \frac{1}{K} - \frac{\sqrt{1-K^2}}{K}\tanh\frac{B(t-t_0)}{2}\sqrt{1-K^2}$$

Solution of α(t) when $\Delta\omega_0 \neq 0$

- **Still a first-order system (K<1)**

Note: tan(ix)=i tanh(x)
Arctan(ix)=I Arctanh (x)

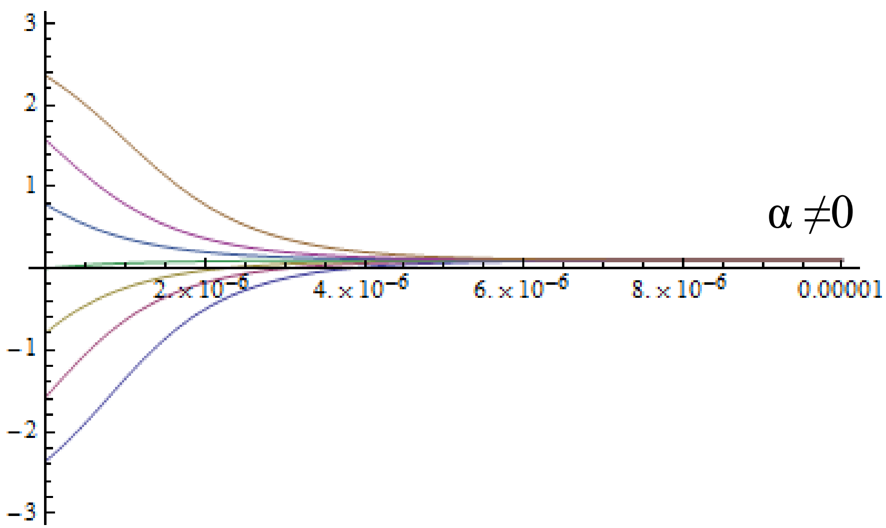
```
In[4]:= sol = DSolve[{a'[t] == -B*(Sin[a[t]] - K), a[0] == C}, a[t], t]
```

$$\text{Out[4]= } \left\{\left\{a[t] \to 2\,\text{ArcTan}\left[\frac{1-\sqrt{-1+K^2}\,\text{Tan}\left[\frac{1}{2}\left(-B\sqrt{-1+K^2}\,t-2\,\text{ArcTan}\left[\frac{-\sqrt{-1+K^2}+K\sqrt{-1+K^2}\,\text{Tan}\left[\frac{C}{2}\right]}{-1+K^2}\right]\right)\right]}{K}\right]\right\}\right\}$$

```
In[5]:= tab = Table[a[t] /. sol[[1]] /. {B → 10^6, K → 0.1, C → c}, {c, -3*Pi/4, 3*Pi/4, Pi/4}];

In[6]:= Plot[Evaluate[tab], {t, 0, 0.00001}, PlotRange → {-Pi, Pi}]
```

Out[6]=



What is its time constant?
What is its final phase error?

Solution of $\alpha(t)$ when $\Delta\omega_0 \neq 0$

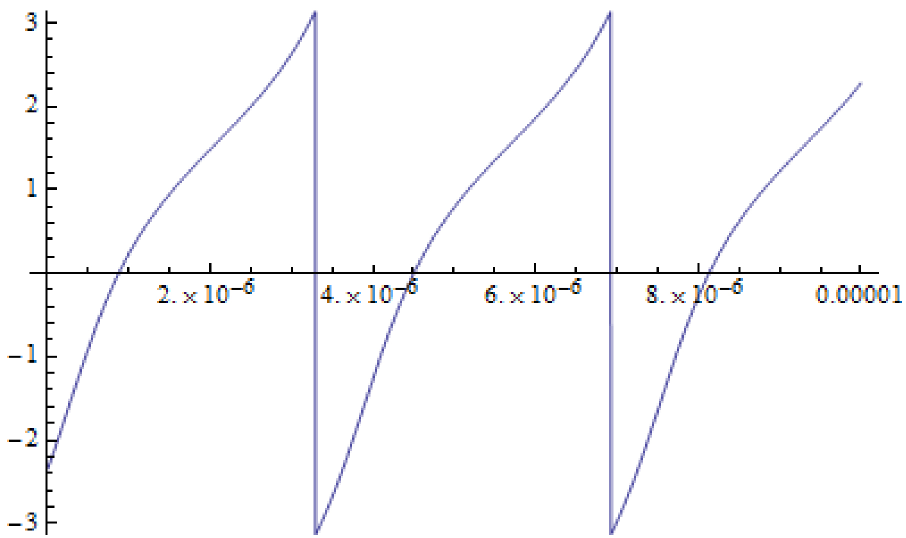
- **Analytical solution when K>1 (Injection pulling, not locked)**

$$\tan\left(\frac{\alpha}{2}\right) = \frac{1}{K} + \frac{\sqrt{K^2-1}}{K}\tan\frac{B(t-t_0)}{2}\sqrt{K^2-1}$$

```
In[12]:= tab2 = Table[a[t] /. sol[[1]] /. {B → 10^6, K → 2.0, C → c}, {c, -3*Pi/4, -3*Pi/4, 0}];

In[13]:= Plot[Evaluate[tab2], {t, 0, 0.00001}, PlotRange → {-Pi, Pi}]
```

Out[13]=



References

- [4.1.1] Adler, “A Study of Locking Phenomena in Oscillators,” Proceedings of the IRE, 1946
- [4.1.2] Razavi, “A Study of Injection Locking and Pulling in Oscillators, JSSC 2004

Topics in IC Design

4.2 Phase Noise in Injection Locked Oscillators

Deog-Kyoon Jeong

dkjeong@snu.ac.kr

School of Electrical and Computer Engineering

Seoul National University

2020 Fall

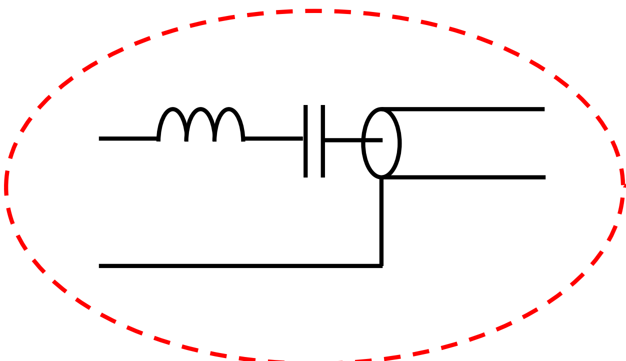
Outline

- **Noise Model**
- **Equation of Signals**
- **Noise in Free-running Oscillators**
- **Noise in Injection-locked Oscillators**
- **Noise in Noisy Injection Signals**

Noise Model

- **Assume Series Resonance**
  - **Ri = internal resistance**
  - **Ro = load resistance**
- **e(t): Injected signal**

$$L\frac{di}{dt} + (R_i + R_0 - R)i + \frac{1}{C}\int i\,dt = e(t). \qquad (1)$$



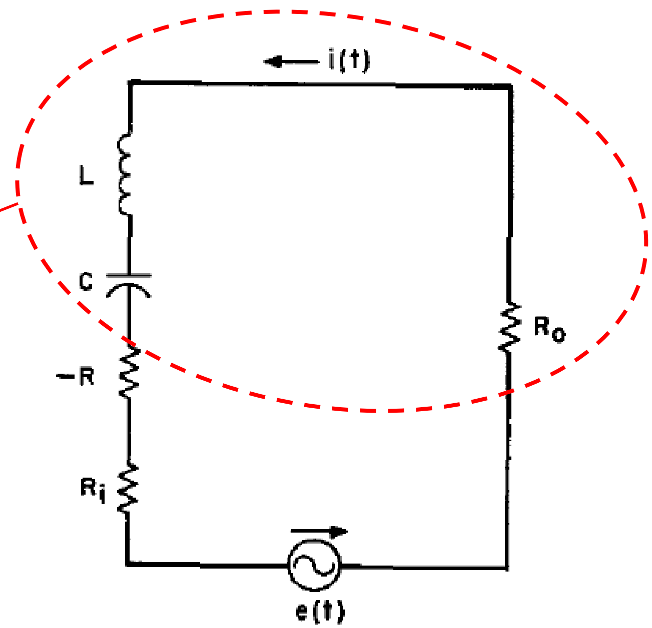


Fig. 1. Equivalent circuit of oscillators.

$$\boxed{Q = \frac{\omega_0 L}{R_0}}$$

Equations of Signals

- **In case of free-running, e(t)=0 and i(t) is periodic with harmonic terms.**

$$i(t) = A_1 \cos(\omega t + \phi_1) + A_2 \cos(2\omega t + \phi_2) + A_3 \cos(3\omega t + \phi_3) + \cdots .$$

- **Suppose small perturbation is given by small e(t)**

$$i(t) = A_1(t) \cos[\omega t + \phi_1(t)] + A_2(t) \cos[2\omega t + \phi_2(t)] + \cdots$$

- **A(t) and φ(t) are slowly varying function compared with ω.**

Equations of Signals

$$i(t) = A_1(t) \cos\left[\omega t + \phi_1(t)\right] + A_2(t) \cos\left[2\omega t + \phi_2(t)\right] + \cdots$$

- **Calculation of di/dt**

$$\frac{di}{dt} = -A_1\left(\omega + \frac{d\phi_1}{dt}\right) \sin\left(\omega t + \phi_1\right) + \frac{dA_1}{dt} \cos\left(\omega t + \phi_1\right) + (\text{slowly varying function}) \sin\left(2\omega t + \phi_2\right) + (\text{slowly varying function}) \cos\left(2\omega t + \phi_2\right) + \cdots$$

Equations of Signals

$$i(t) = A_1(t) \cos[\omega t + \phi_1(t)] + A_2(t) \cos[2\omega t + \phi_2(t)] + \cdots$$

- **Calculation of ∫ i(t) dt**

$$\int i dt = \left(\frac{A_1}{\omega} - \frac{A_1}{\omega^2} \frac{d\phi_1}{dt} \right) \sin(\omega t + \phi_1) + \frac{1}{\omega^2} \frac{dA_1}{dt} \cos(\omega t + \phi_1) + (\text{slowly varying function}) \sin(2\omega t + \phi_2) + (\text{slowly varying function}) \cos(2\omega t + \phi_2) + \cdots$$

- **Use integration by parts and note that A(t), dA(t)/dt, φ(t) and dφ(t)/dt are slowly varying functions.**

$$\int u(x)v'(x)\,dx = u(x)v(x) - \int v(x)u'(x)dx$$

Equations of Signals

- **Substitute di/dt and ∫ i(t) dt into Eq. (1).**

$$L\frac{di}{dt} + (R_i + R_0 - R)i + \frac{1}{C}\int i\,dt = e(t). \qquad (1)$$

$$\frac{di}{dt} = -A_1\left(\omega + \frac{d\phi_1}{dt}\right)\sin(\omega t + \phi_1) + \frac{dA_1}{dt}\cos(\omega t + \phi_1)$$

$$\int i\,dt = \left(\frac{A_1}{\omega} - \frac{A_1}{\omega^2}\frac{d\phi_1}{dt}\right)\sin(\omega t + \phi_1) + \frac{1}{\omega^2}\frac{dA_1}{dt}\cos(\omega t + \phi_1)$$

$$[-LA_1(w + \frac{d\Phi}{dt}) + \frac{1}{C}(\frac{A_1}{w} - \frac{A_1}{w^2}\frac{d\Phi}{dt})]\sin(wt + \Phi(t)) + [L\frac{dA_1}{dt} + (R_i + R_0 - R)A_1 + \frac{1}{Cw^2}\frac{dA_1}{dt}]\cos(wt + \Phi(t)) = e(t)$$

Equations of Signals

- **Multiply sin(ωt+Φ) and cos(ωt+Φ) and integrate over one period.**
  - **Since A and Φ are slowly varying function, they do not change appreciably over one period**

$$[-LA_1(w+\frac{d\Phi}{dt})+\frac{1}{C}(\frac{A_1}{w}-\frac{A_1}{w^2}\frac{d\Phi}{dt})]\sin(wt+\Phi(t))+$$
$$[L\frac{dA_1}{dt}+(R_i+R_0-R)A_1+\frac{1}{Cw^2}\frac{dA_1}{dt}]\cos(wt+\Phi(t)) \quad = e(t)$$

$$\left(-\omega L+\frac{1}{\omega C}\right)-\left(L+\frac{1}{\omega^2 C}\right)\frac{d\phi}{dt}$$
$$=\frac{2}{AT_0}\int_{t-T_0}^{t} e(t)\sin(\omega t+\phi)dt \quad (2)$$

Equation for phase variation

Equations of Signals

- **Multiply sin(wt+*Φ*) and cos(wt+*Φ*) and integrate over one period.**
  - **Since A and *Φ* are slowly varying function, they do not change appreciably over one period**

$$[-LA_1(w+\frac{d\Phi}{dt})+\frac{1}{C}(\frac{A_1}{w}-\frac{A_1}{w^2}\frac{d\Phi}{dt})]\sin(wt+\Phi(t))+$$
$$[L\frac{dA_1}{dt}+(R_i+R_0-R)A_1+\frac{1}{Cw^2}\frac{dA_1}{dt}]\cos(wt+\Phi(t)) \quad = e(t)$$

$$\left(L+\frac{1}{\omega^2 C}\right)\frac{dA}{dt}+(R_i+R_0-\overline{R})A$$
$$=\frac{2}{T_0}\int_{t-T_0}^{t} e(t)\cos(\omega t+\phi)dt \qquad (3)$$

Equation for amplitude variation

Equations of Signals: Injection Locked

- **When the oscillator is injection locked with $e(t) = a_0 \cos w_s t$, oscillation frequency becomes injection frequency ω_s.**

$$\left(-\omega L + \frac{1}{\omega C}\right) - \left(L + \frac{1}{\omega^2 C}\right)\frac{d\phi}{dt} = \frac{2}{AT_0}\int_{t-T_0}^{t} e(t)\sin(\omega t + \phi)dt \qquad (2)$$

- **In steady state,** $\frac{d\Phi}{dt} = 0$ **and right hand side is** $\frac{a_0}{A_0} sin(\Phi)$**.**

- **Let** $w_s = w_0 + \triangle w_0$**, then Eq. (2) becomes**

$$\Delta\omega_0 = \frac{-a_0}{2LA_0}\sin\phi_0. \qquad (5)$$

Lock Range

$$\Delta\omega_0 = \frac{-a_0}{2LA_0}\sin\phi_0. \quad (5)$$

Result of injection on phase: Phase difference between injected signal and oscillation signal

- **Since |sinΦ| < 1, lock range is**

$$|\Delta w_0| < \frac{a_0}{2LA_0}$$

- **Note a_0 is the voltage amplitude, A_0 is the current amplitude.**

$$\left|\Delta\omega_0\right| = \frac{a_0}{2\left(\frac{R_0 Q}{\omega_0}\right)A_0} = \frac{\omega_0}{2Q}\frac{a_0}{R_0 A_0} = \frac{\omega_0}{2Q}\frac{\text{injection signal voltage}}{\text{oscillator internal voltage}}$$

Equations of Signals: Injection Locked

- R is a nonlinear resistor.
- Average resistance over one cycle is $(V/I)_{avg}$.

$$\overline{R} = \frac{2}{AT_0}\int_{t-T_0}^{t} RA\cos^2(\omega t+\phi)dt.$$

- $\overline{R}$ is a function of A. $\overline{R}$ decreases as A increases with the rate of -K.
- $\overline{R} = R_i + R_o - K(A - A_o)$

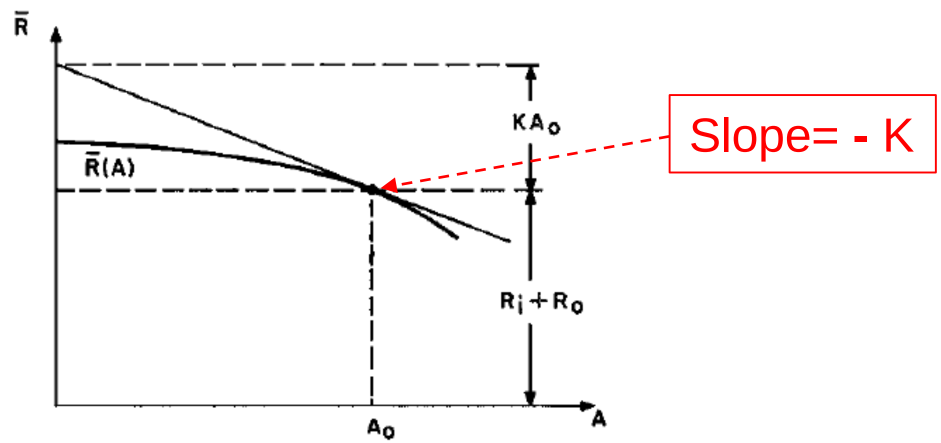


Fig. 2. $\overline{R}$ versus A.

Equations of Signals: Injection Locked

- **When free-running,** $\frac{dA}{dt}=0$ **and** $e(t)=0$**.**

$$\left(L+\frac{1}{\omega^2 C}\right)\frac{dA}{dt}+(R_i+R_0-\overline{R})A = \frac{2}{T_0}\int_{t-T_0}^{t} e(t)\cos(\omega t+\phi)dt \qquad (3)$$

- **Thus,** $\overline{R}=R_i+R_0$ **with** A_0**.**
- **For a small variation** ΔA **,** $\overline{R}$ **varies linearly.**

$$R_i+R_0-\overline{R}=K\Delta A. \qquad (4)$$

Amplitude when Injection Locked

- **When the oscillator is injection locked with** $e(t) = a_0 \cos w_s t$ **oscillation frequency becomes injection frequency ω_s.**

$$\left(L + \frac{1}{\omega^2 C}\right)\frac{dA}{dt} + (R_i + R_0 - \overline{R})A = \frac{2}{T_0}\int_{t-T_0}^{t} e(t)\cos(\omega t + \phi)dt \qquad (3)$$

- **In steady state,** $\frac{dA}{dt} = 0$ **and right hand side is** $a_0 \cos(\Phi_0)$**.**

- **Since** $R_i + R_0 - \overline{R} = K \Delta A_0$**,**

$$\Delta A_0 = \frac{a_0 \cos \phi_0}{K A_0}. \qquad (8)$$

Result of injection on amplitude: Phase difference between injected signal and oscillation signal

Noise in Free-Running Oscillator

- **Suppose e(t) is a Gaussian noise source**

$$\left(-\omega L + \frac{1}{\omega C}\right) - \left(L + \frac{1}{\omega^2 C}\right)\frac{d\phi}{dt} = \frac{2}{AT_0}\int_{t-T_0}^{t} e(t)\sin(\omega t + \phi)dt \quad (2)$$

$$\left(L + \frac{1}{\omega^2 C}\right)\frac{dA}{dt} + (R_i + R_0 - \overline{R})A = \frac{2}{T_0}\int_{t-T_0}^{t} e(t)\cos(\omega t + \phi)dt \quad (3)$$

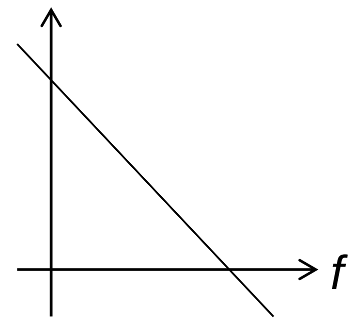
- **Let** $n_1(t) = \frac{2}{T_0}\int_{t-T_0}^{t} e(t)\sin(wt + \Phi)dt$ **and** $n_2(t) = \frac{2}{T_0}\int_{t-T_0}^{t} e(t)\cos(wt + \Phi)dt$
- **In case of free-running, ω= ω_0, and then (2) and (3) become**

$$-2L\frac{d\Phi}{dt} = \frac{1}{A_0}n_1(t) \qquad 2L\frac{d\Delta A}{dt} + KA_0\Delta A = n_2(t)$$

Noise in Free-Running Oscillator

- **Taking Laplace transform on phase noise,**

$$-2L\frac{d\Phi}{dt}=\frac{1}{A_0}n_1(t) \rightarrow \Phi(s) = -\frac{1}{2sLA_0}n_1(s)$$



- **Then**

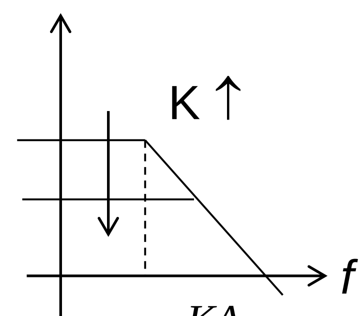
$$|\phi(f)|^2 = \frac{|n_1(f)|^2}{4\omega^2L^2A_0^2} = \frac{2|e|^2}{4\omega^2L^2A_0^2} \qquad (15)$$

- **Taking Laplace transform on amplitude,**

$$2L\frac{d\Delta A}{dt} + KA_0\Delta A = n_2(t) \rightarrow A(s) = \frac{n_2(s)}{2sL+KA_0}$$

- **Then**

$$|\Delta A(f)|^2 = \frac{|n_2(f)|^2}{4\omega^2L^2 + (KA_0)^2} = \frac{2|e|^2}{4\omega^2L^2 + (KA_0)^2}. \qquad (16)$$



$$\omega_c = \frac{KA_0}{2L}$$

Noise in Free-Running Oscillator

- **To combine phase noise and amplitude noise together, calculate autocorrelation function of output current i(t).**

$$R_i(\tau) = \overline{A(t)A(t+\tau)\cos(w_0 t + \Phi(t))\cos(w_0(t+\tau)+\Phi(t+\tau))}$$

$$= (A_0^2 + R_{\triangle A}(\tau)) * \frac{1}{2} cos(w_0\tau)\overline{\cos(\Phi(t+\tau)-\Phi(t))}$$

$$\boxed{R_i(\tau) = \frac{1}{2}\left(A_0{}^2 + \frac{|e|^2}{2LKA_0}\exp\left[-\frac{KA_0}{2L}|\tau|\right]\right) \cdot \exp\left[-\frac{|e|^2}{4L^2A_0{}^2}|\tau|\right]\cos\omega_0\tau.}$$

Noise in Free-Running Oscillator

- **Using** $e^{-\alpha|t|} \Leftrightarrow \dfrac{2\alpha}{\alpha^2 + w^2}$ **on**

$$R_i(\tau) = \frac{1}{2}\left(A_0^2 + \frac{|e|^2}{2LKA_0}\exp\left[-\frac{KA_0}{2L}|\tau|\right]\right) \cdot \exp\left[-\frac{|e|^2}{4L^2A_0^2}|\tau|\right]\cos\omega_0\tau.$$

- **Power Spectral Density becomes**

$$|i(f)|^2 = \frac{|e|^2}{8L^2}\left[\frac{1}{(w-w_0)^2 + (\frac{|e|^2}{4L^2A_0^2})^2} + \frac{1}{(w+w_0)^2 + (\frac{|e|^2}{4L^2A_0^2})^2}\right]$$

$$+ \frac{|e|^2}{8L^2}\left[\frac{1}{(w-w_0)^2 + (\frac{KA_0}{2L})^2} + \frac{1}{(w+w_0)^2 + (\frac{KA_0}{2L})^2}\right]$$

Lorentizian

Due to phase noise

Due to amplitude noise

Noise in Free-Running Oscillator

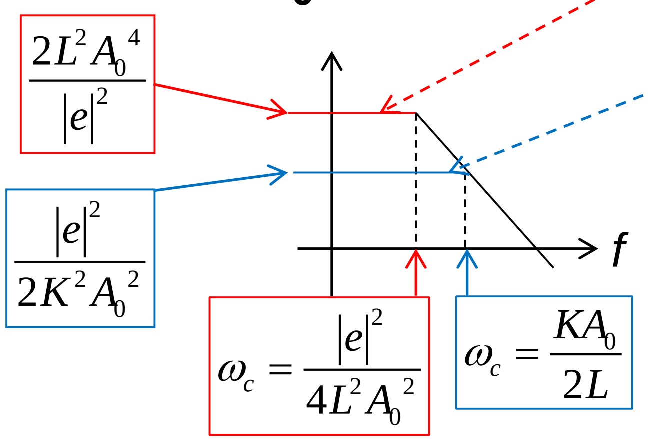
- **Power Spectral Density becomes**

$$
\boxed{
\begin{aligned}
|i(f)|^2 = \frac{|e|^2}{8L^2}\Big[&\frac{1}{(w-w_0)^2+\left(\frac{|e|^2}{4L^2A_0^2}\right)^2}+\frac{1}{(w+w_0)^2+\left(\frac{|e|^2}{4L^2A_0^2}\right)^2}\Big] \\
+\frac{|e|^2}{8L^2}\Big[&\frac{1}{(w-w_0)^2+\left(\frac{KA_0}{2L}\right)^2}+\frac{1}{(w+w_0)^2+\left(\frac{KA_0}{2L}\right)^2}\Big]
\end{aligned}
}
$$

FM noise

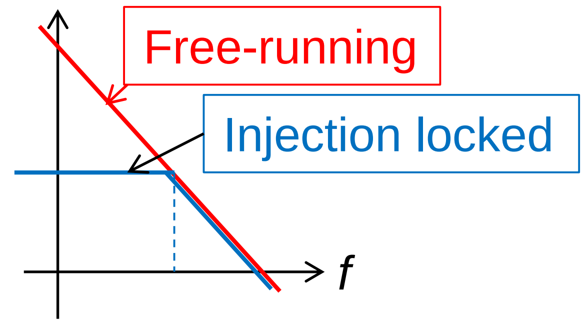
AM noise

- **When ω is near ω_0, first term dominates.**



Noise in Injection-Locked Oscillator

- e(t) has both noise and injection signal.
- Phase noise is calculated as



$$| \Delta\phi(f) |^2 = \frac{| n_1(f) |^2}{4\omega^2 L^2 A_0{}^2 + a_0{}^2 \cos^2 \phi_0}$$

$$= \frac{2 | e |^2}{4\omega^2 L^2 A_0{}^2 + a_0{}^2 \cos^2 \phi_0} \qquad (19)$$

- When compared with free-running case,

$$| \phi(f) |^2 = \frac{| n_1(f) |^2}{4\omega^2 L^2 A_0{}^2} = \frac{2 | e |^2}{4\omega^2 L^2 A_0{}^2} \qquad (15)$$

- Phase fluctuation is considerably reduced when $|w| < \dfrac{a_0 \cos(\Phi_0)}{2LA_0}$

- Noise down when ω is within the lock range $|\triangle w_0| < \dfrac{a_0}{2LA_0}$

Noise in Injection-Locked Oscillator

- **Equation for amplitude noise is calculated as**

$$
\begin{aligned}
|\Delta A(f)|^2 &= \frac{|n_2(f)|^2}{4\omega^2 L^2 + (KA_0)^2} \\
&\quad + \frac{a_0^2 \sin^2 \phi_0}{4\omega^2 L^2 + (KA_0)^2} \frac{|n_1(f)|^2}{4\omega^2 L^2 A_0^2 + a_0^2 \cos^2 \phi_0} \\
&= \frac{2|e|^2}{4\omega^2 L^2 + (KA_0)^2} \frac{4\omega^2 L^2 A_0^2 + a_0^2}{4\omega^2 L^2 A_0^2 + a_0^2 \cos^2 \phi_0}. \qquad (20)
\end{aligned}
$$

- **Compared with free-running case,**

$$
\begin{aligned}
|\Delta A(f)|^2 &= \frac{|n_2(f)|^2}{4\omega^2 L^2 + (KA_0)^2} \\
&= \frac{2|e|^2}{4\omega^2 L^2 + (KA_0)^2}. \qquad (16)
\end{aligned}
$$

- **Amplitude noise is increased.**

Noise in Injection-Locked Oscillator

- **Combining phase noise and amplitude noise together,**

$$R_i(\tau) = \tfrac{1}{2}(A_0 + \Delta A_0)^2(1 - \overline{\Delta\phi^2})\cos\omega_s\tau$$

$$+ \tfrac{1}{2}R_{\Delta A}(\tau)\cos\omega_s\tau + \tfrac{1}{2}A_0{}^2 R_{\Delta\phi}(\tau)\cos\omega_s\tau$$

$$+ \tfrac{1}{2}A_0\sin\omega_s\tau(\overline{\Delta A(\tau)\Delta\phi(0)} - \overline{\Delta A(0)\Delta\phi(\tau)}).$$

Noise in Injection-Locked Oscillator

- **At the edge of the lock range($\cos \Phi_0 = 0$), this injection locking fails and this equation doesn't hold.**

$$|i(f)|^2 = \tfrac{1}{4}(A_0 + \Delta A_0)^2\left(1 - \frac{|e|^2}{2LA_0}\frac{1}{a_0\cos\phi_0}\right)\{\delta(f-f_s)+\delta(f+f_s)\}$$ Pure sinusoid

$$+\frac{|e|^2}{8L^2}\left\{\frac{1}{(\omega-\omega_s)^2+\dfrac{a_0^2\cos^2\phi_0}{4L^2A_0^2}} + \text{similar term with } (\omega+\omega_s)\right\}$$ FM noise

$$+\frac{|e|^2}{8L^2}\left\{\frac{1}{(\omega-\omega_s)^2+\left(\dfrac{KA_0}{2L}\right)^2}\;\frac{(\omega-\omega_s)^2+\dfrac{a_0^2}{4L^2A_0^2}}{(\omega-\omega_s)^2+\dfrac{a_0^2\cos^2\phi_0}{4L^2A_0^2}} + \text{similar term with } (\omega+\omega_s)\right\}$$ AM noise

$$+\frac{|e|^2}{8L^2}\left\{\frac{\omega-\omega_s}{(\omega-\omega_s)^2+\left(\dfrac{KA_0}{2L}\right)^2}\;\frac{\dfrac{a_0\sin\phi_0}{LA_0}}{(\omega-\omega_s)^2+\dfrac{a_0^2\cos^2\phi_0}{4L^2A_0^2}} - \text{similar term with } (\omega+\omega_s)\right\}. \qquad (21)$$ AM noise

Comparison

- **FM noise**

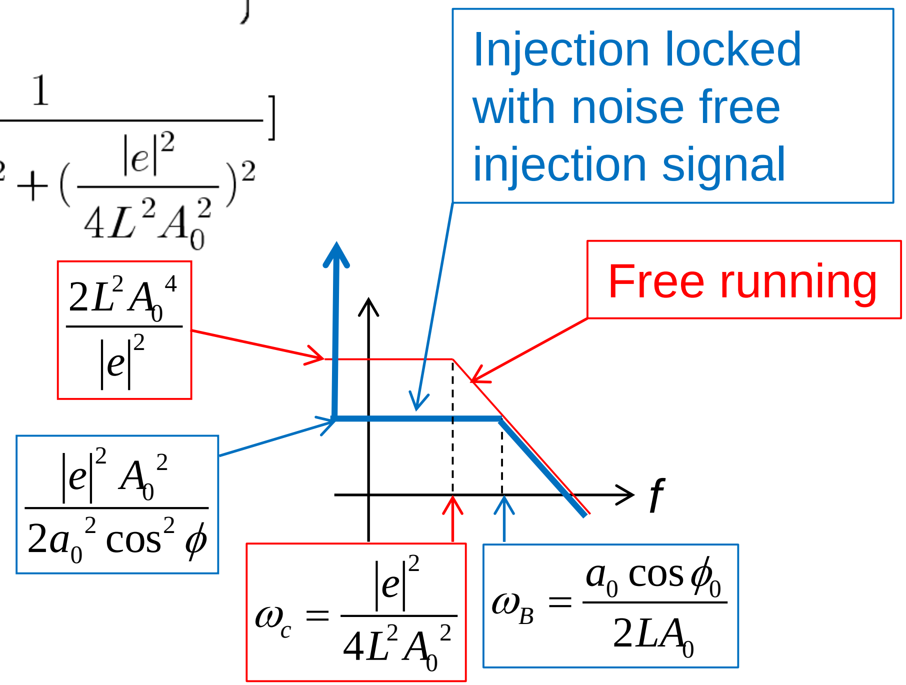
$$|i(f)|^2 = \tfrac{1}{4}(A_0 + \Delta A_0)^2\left(1 - \frac{|e|^2}{2LA_0}\frac{1}{a_0\cos\phi_0}\right)\{\delta(f-f_s)+\delta(f+f_s)\}$$ Pure sinusoid

$$+\frac{|e|^2}{8L^2}\left\{\frac{1}{(\omega-\omega_s)^2+\dfrac{a_0{}^2\cos^2\phi_0}{4L^2A_0{}^2}} + \text{similar term with } (\omega+\omega_s)\right\}$$ FM noise

$$|i(f)|^2 = \frac{|e|^2}{8L^2}\left[\frac{1}{(w-w_0)^2+\left(\dfrac{|e|^2}{4L^2A_0^2}\right)^2}+\frac{1}{(w+w_0)^2+\left(\dfrac{|e|^2}{4L^2A_0^2}\right)^2}\right]$$

$$\omega_B = \frac{a_0\cos\phi_0}{2LA_0} = \sqrt{\omega_L^2-\Delta\omega_0^2}$$

where $\omega_L = \dfrac{a_0}{2LA_0}$ and $\Delta\omega_0 = \dfrac{a_0\sin\phi_0}{2LA_0}$



Noise in Noisy Injection Signal

- **When injected by noisy injection signal, phase noise is**

$$|(\phi - \phi_0)(f)|^2 = \frac{1}{4\omega^2 L^2 A_0^2 + a_0^2 \cos^2\phi_0} \cdot \left\{ \frac{2a_0^2 \cos^2\phi_0}{4\omega^2 L_s^2 A_s^2} |e_s|^2 + 2|e|^2 \right\}. \quad (23)$$

Injection locked with noisy input

- **Compare with free-running and noiseless injection cases,**

$$|\phi(f)|^2 = \frac{|n_1(f)|^2}{4\omega^2 L^2 A_0^2} = \frac{2|e|^2}{4\omega^2 L^2 A_0^2} \quad (15)$$

Free running

$$|\Delta\phi(f)|^2 = \frac{|n_1(f)|^2}{4\omega^2 L^2 A_0^2 + a_0^2 \cos^2\phi_0} = \frac{2|e|^2}{4\omega^2 L^2 A_0^2 + a_0^2 \cos^2\phi_0} \quad (19)$$

injection locked with noise-free input

Noise in Noisy Injection Signal

- **When injected by noisy injection signal, amplitude noise is**

$$|\Delta A(f)|^2 = \frac{1}{4\omega^2L^2 + (KA_0)^2}\left\{\frac{a_0{}^2\sin^2\phi_0}{4\omega^2L^2A_0{}^2 + a_0{}^2\cos^2\phi_0}\right.$$
$$\left.\cdot\left(\frac{LA_0}{L_sA_s}2|e_s|^2 + 2|e|^2\right) + 2|e|^2\right\}. \qquad (24)$$

- **Compare with free-running and noiseless injection cases,**

$$|\Delta A(f)|^2 = \frac{|n_2(f)|^2}{4\omega^2L^2 + (KA_0)^2} = \frac{2|e|^2}{4\omega^2L^2 + (KA_0)^2}.$$

$$|\Delta A(f)|^2 = \frac{|n_2(f)|^2}{4\omega^2L^2 + (KA_0)^2} + \frac{a_0{}^2\sin^2\phi_0}{4\omega^2L^2 + (KA_0)^2}\,\frac{|n_1(f)|^2}{4\omega^2L^2A_0{}^2 + a_0{}^2\cos^2\phi_0}$$
$$= \frac{2|e|^2}{4\omega^2L^2 + (KA_0)^2}\,\frac{4\omega^2L^2A_0{}^2 + a_0{}^2}{4\omega^2L^2A_0{}^2 + a_0{}^2\cos^2\phi_0}. \qquad (20)$$

Noise in Noisy Injection Signal

- **Hard to calculate R(τ) and Power Spectral Density with**

$$\left|(\phi-\phi_0)(f)\right|^2 = \frac{1}{4\omega^2L^2A_0{}^2 + a_0{}^2\cos^2\phi_0} \cdot \left\{\frac{2a_0{}^2\cos^2\phi_0}{4\omega^2L_s{}^2A_s{}^2}\left|e_s\right|^2 + 2\left|e\right|^2\right\}. \quad (23)$$

$$\left|\Delta A(f)\right|^2 = \frac{1}{4\omega^2L^2 + (KA_0)^2}\left\{\frac{a_0{}^2\sin^2\phi_0}{4\omega^2L^2A_0{}^2 + a_0{}^2\cos^2\phi_0} \cdot \left(\frac{LA_0}{L_sA_s}2\left|e_s\right|^2 + 2\left|e\right|^2\right) + 2\left|e\right|^2\right\}. \quad (24)$$

Noise in Noisy Injection Signal

- **Shape of PSD with noisy injection signal**
- **ω_0: Free-running Frequency**
- **ω_d: Frequency difference**

$$\omega_d = \omega_0 - \omega_{inj}$$

- **ω_L: Lock-in Frequency**

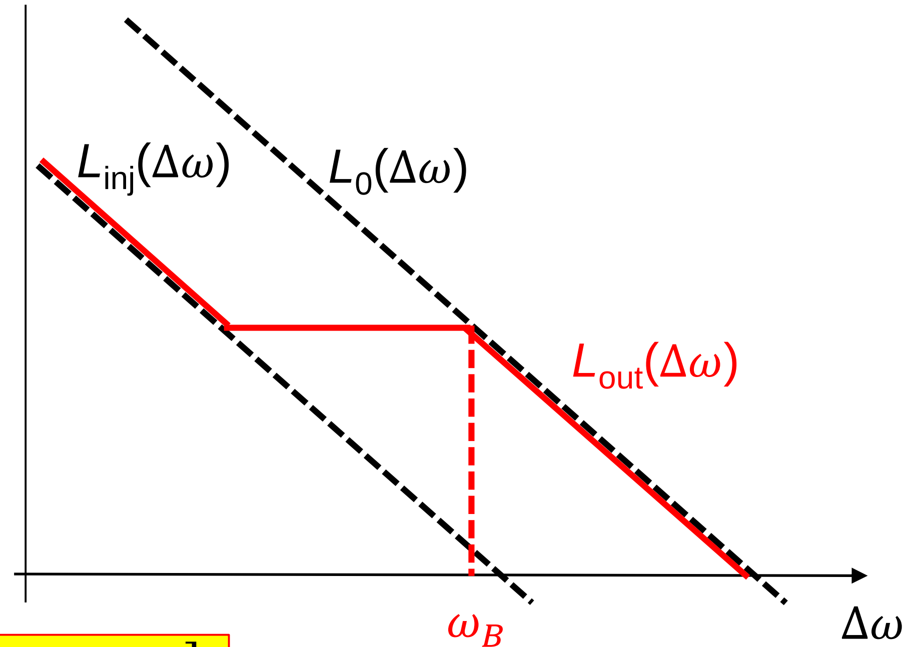
$$\omega_L = \frac{\omega_0}{2Q}\frac{I_{inj}}{I_{osc}}$$

- **ω_B: Beat Frequency**

$$\omega_B = \sqrt{\omega_L^2 - \omega_d^2} \quad \boxed{= \frac{a_0 \cos\phi_0}{2LA_0}}$$

- **PSD**

$$\begin{aligned} L_{\text{out}}(\Delta\omega) &= \text{MAX}\left[L_{\text{inj}}(\Delta\omega), \text{MIN}[L_0(\Delta\omega), L_0(\omega_B)]\right] \\ &= \text{MAX}\left[L_{\text{inj}}(\Delta\omega), \frac{\Delta\omega^2}{\omega_B^2+\Delta\omega^2} L_0(\Delta\omega)\right] \\ &= \frac{\omega_B^2 L_{\text{inj}}(\Delta\omega) + \Delta\omega^2 L_0(\Delta\omega)}{\omega_B^2+\Delta\omega^2} \end{aligned}$$



Effect of Flicker Noise

- **ILO attenuates jitter with 1-st order filtering**
- **ω_0: Free-running Frequency**
- **ω_d: Frequency difference**

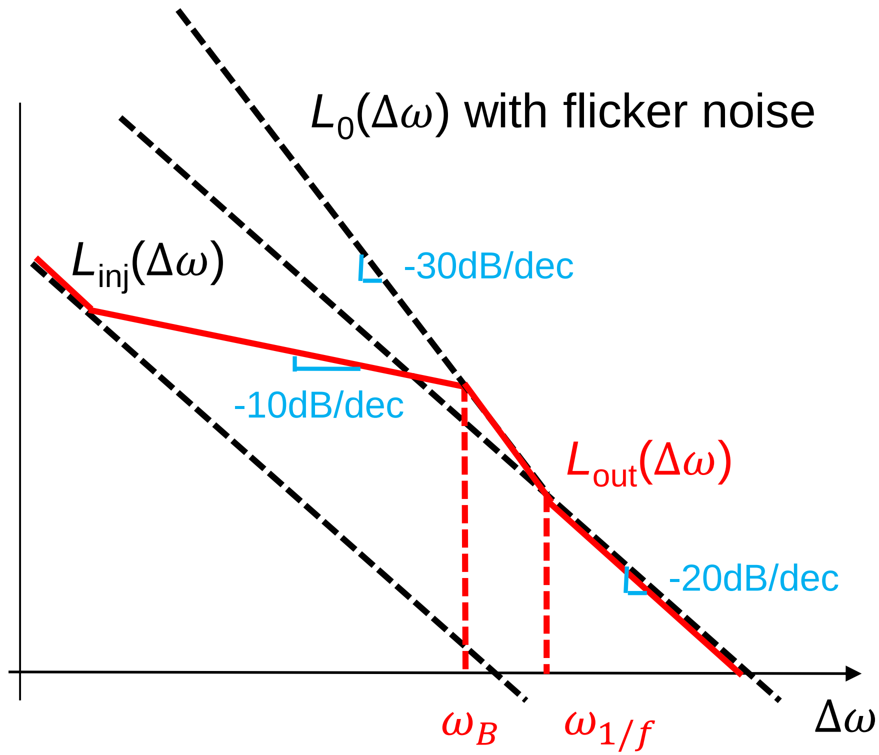
$$\omega_d = \omega_0 - \omega_{inj}$$

- **ω_L: Lock-in Frequency**

$$\omega_L = \frac{\omega_0}{2Q}\frac{I_{inj}}{I_{osc}}$$

- **ω_B: Beat Frequency**

$$\omega_B = \sqrt{\omega_L^2 - \omega_d^2} \quad \boxed{= \frac{a_0 \cos\phi_0}{2LA_0}}$$



- **PSD**

$$L_{out}(\Delta\omega) \;?\!= \frac{\omega_B^2 L_{inj}(\Delta\omega) + \Delta\omega^2 L_0(\Delta\omega)}{\omega_B^2 + \Delta\omega^2}$$

References

- **[4.2.1] K. Kurokawa, "Noise in Synchronized Oscillators", IEEE Transactions on Microwave Theory and Techniques, pp. 234 – 240, 1968**

Topics in IC Design

4.3 Injection Locked Frequency Multipliers

Deog-Kyoon Jeong

dkjeong@snu.ac.kr

School of Electrical and Computer Engineering

Seoul National University

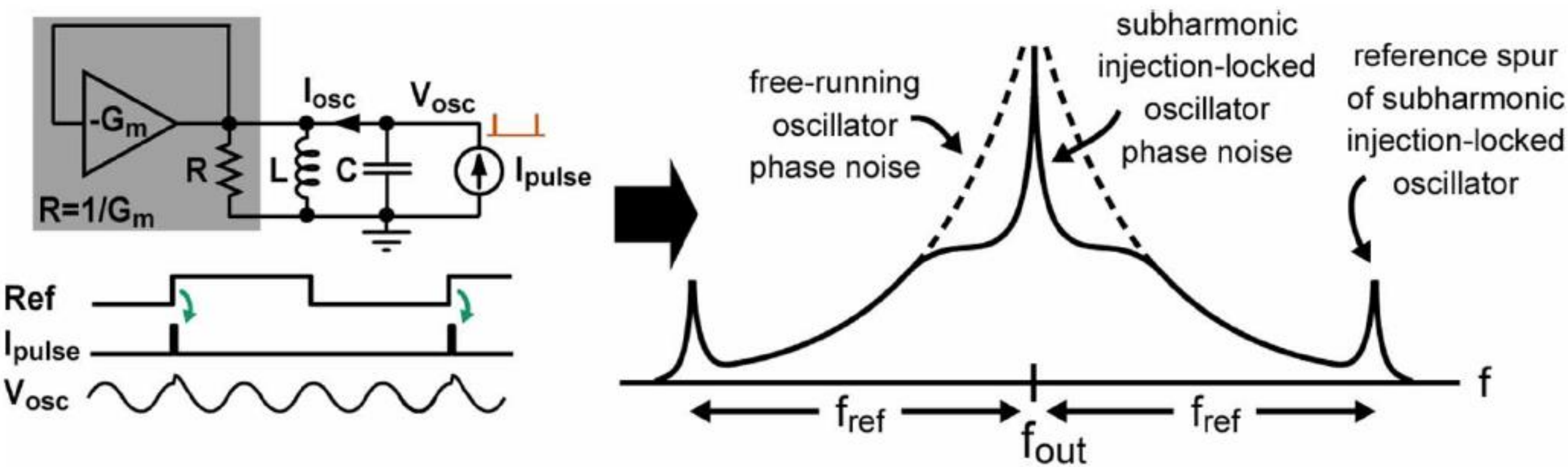
2020 Fall

Outline

- N-th Harmonic Generation
- Pulse injection
- Injection-locked LC Oscillator
- Injection-locked Ring Oscillator

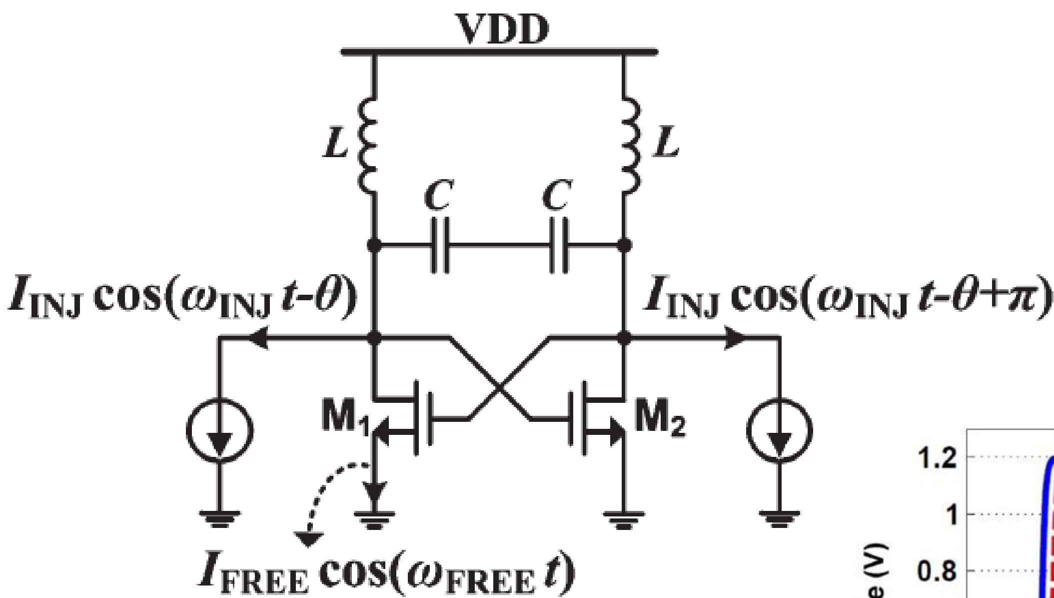
Subharmonic IL LC oscillator

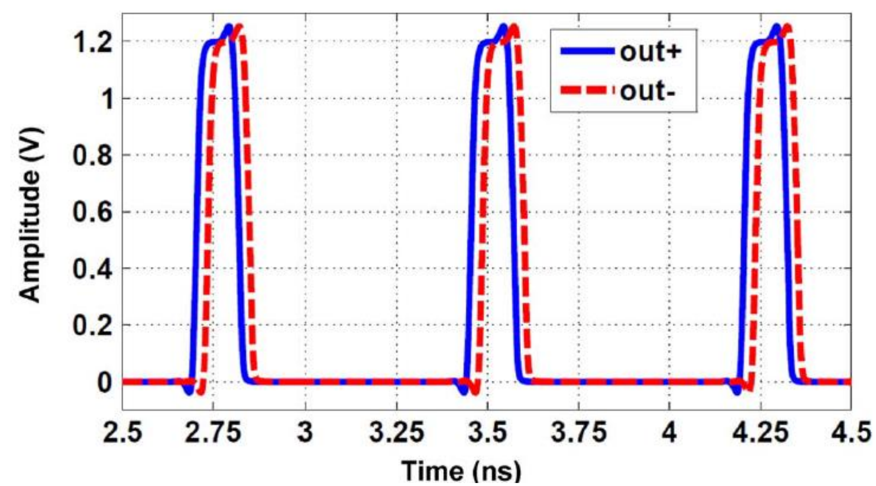
- Subharmonic Injection-locked LC oscillator



Injection-Locked LC Oscillator

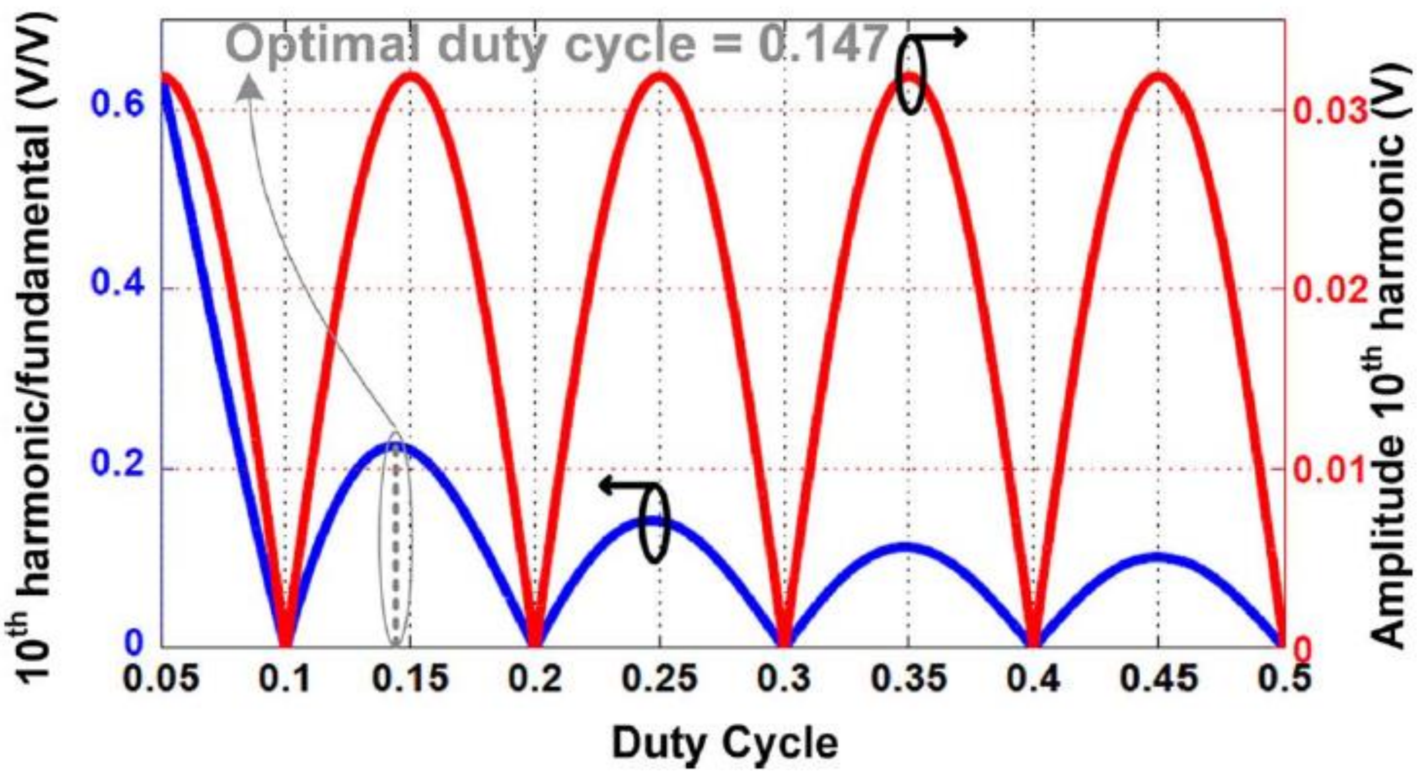
- **Pulses are injected with N-th harmonic**





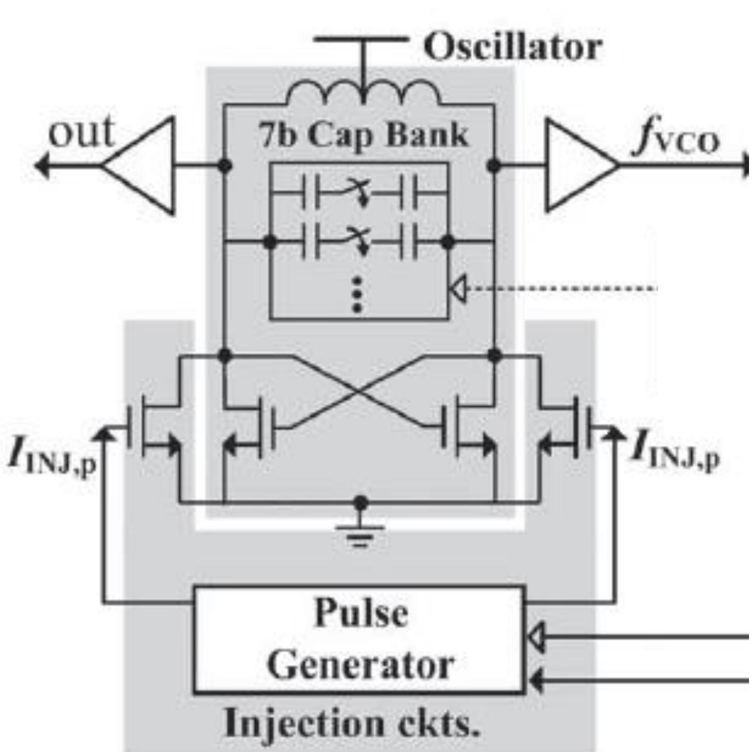
N-th Harmonic Generation

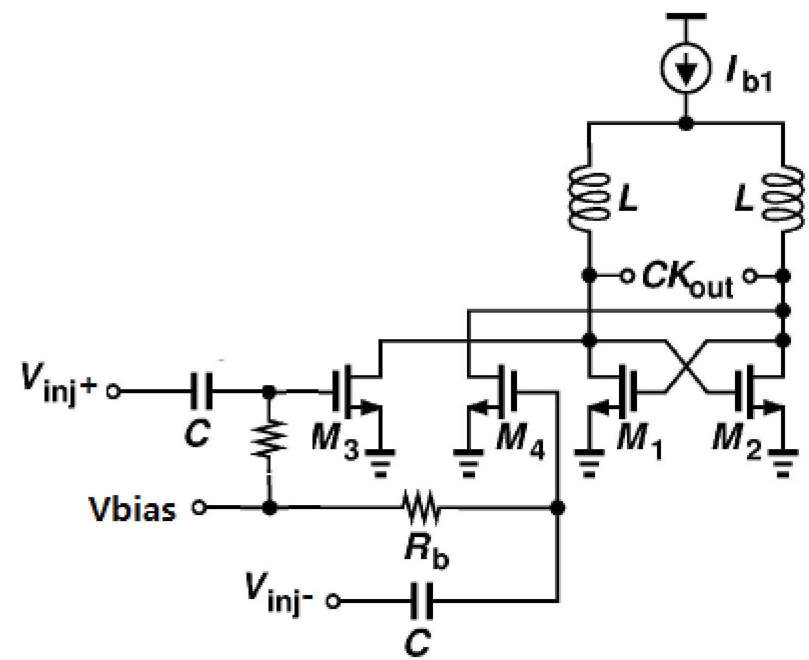
- **Pulse with short duty cycle has a lot of harmonics**
  - **Equation?**



Injection-Locked LC Oscillator

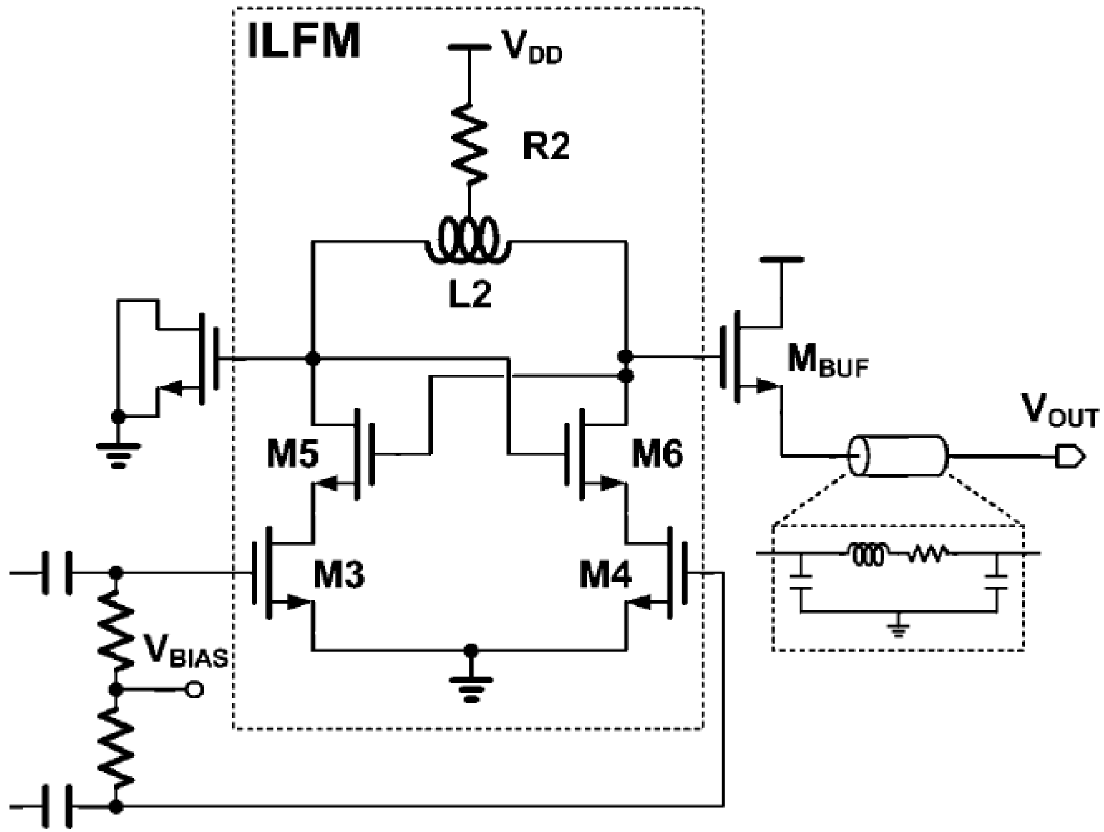
- **N-th harmonic is injected by a pulse generator or by a MOS transistor biased in class-C**





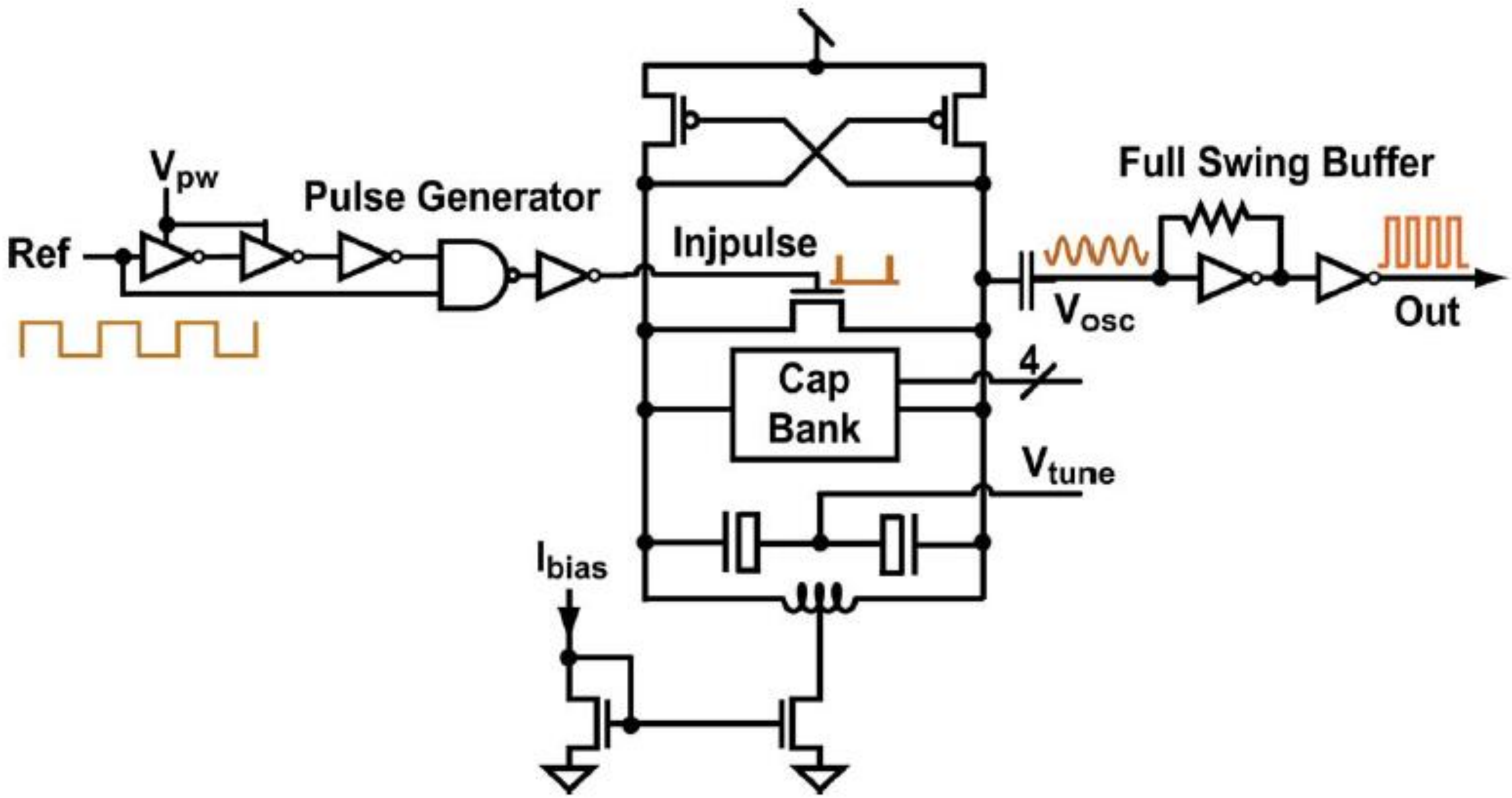
Injection-Locked LC Oscillator

- **Injection to the source of the MOS transistors**



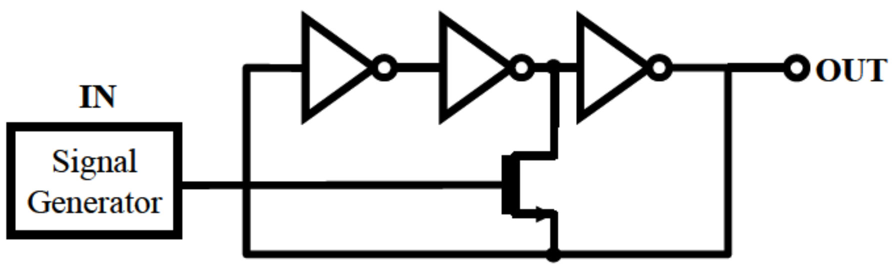
Pulse Injection by Shorting

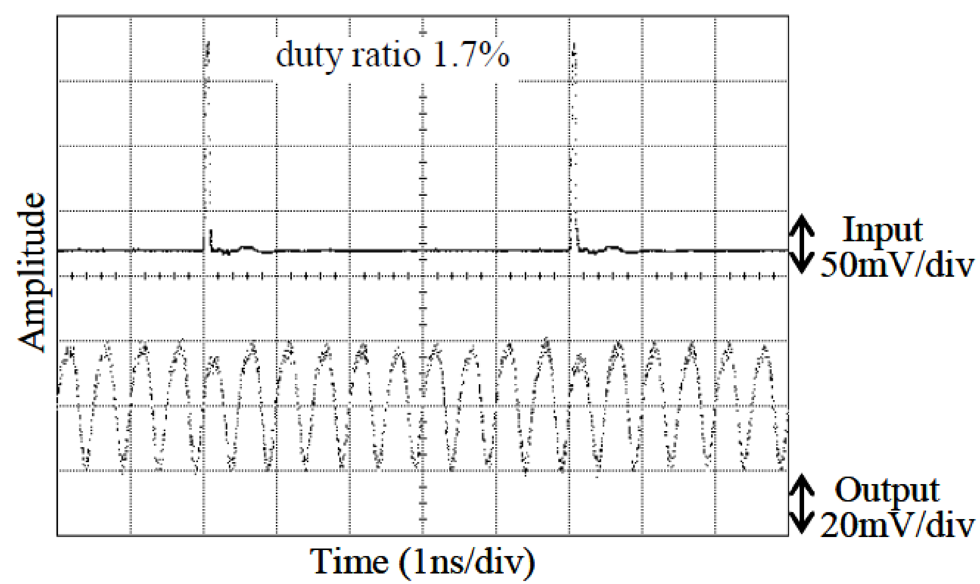
- **Injection by shorting**



Injection-Locked Ring Oscillator

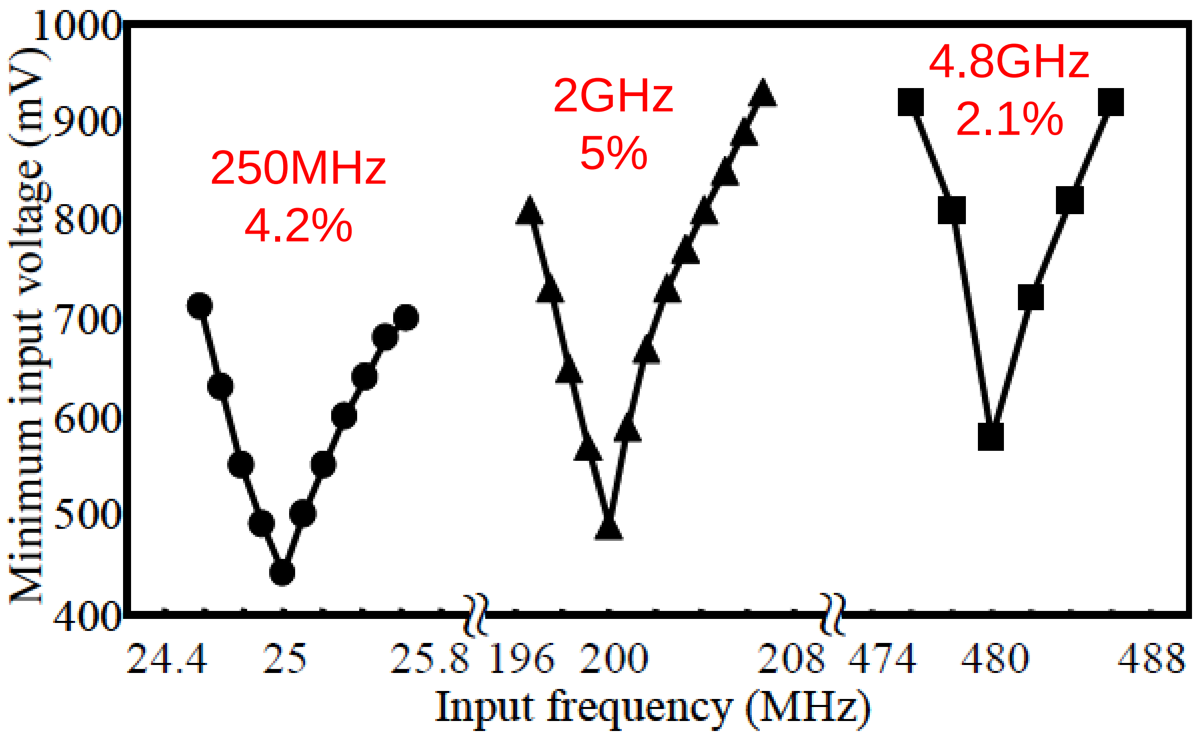
- Pulse-injection-locked frequency multiplier (PILFM)
- 20-MHz input with Sharp pulse with 1.7% duty
- 200-MHz output (10x multiplication)





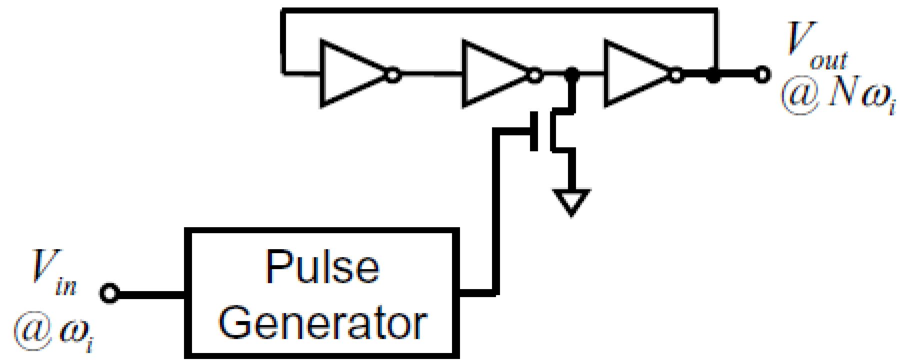
Subharmonic Pulse-Injection Locking

- **Input Sensitivity as a function of output frequency**
  - **1.7% duty ratio maintained**
  - **X10 multiplication with constant tuning range**

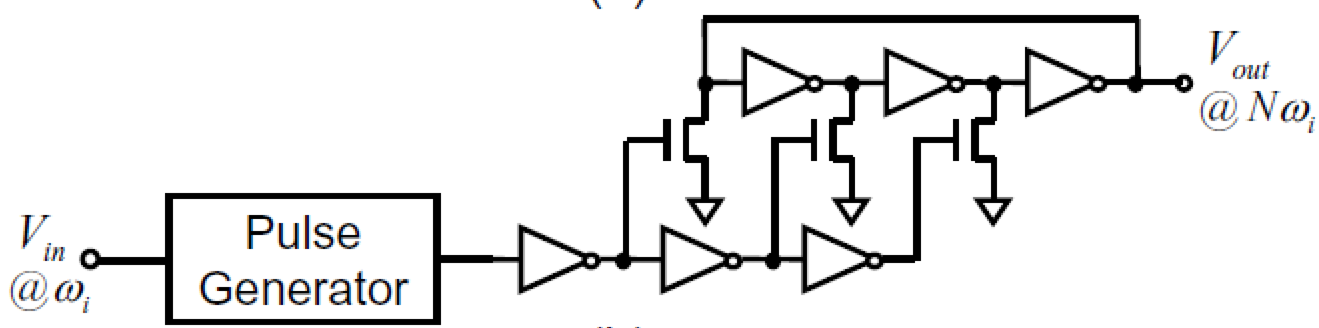


Multiphase ILRO

- **Wide lock range**



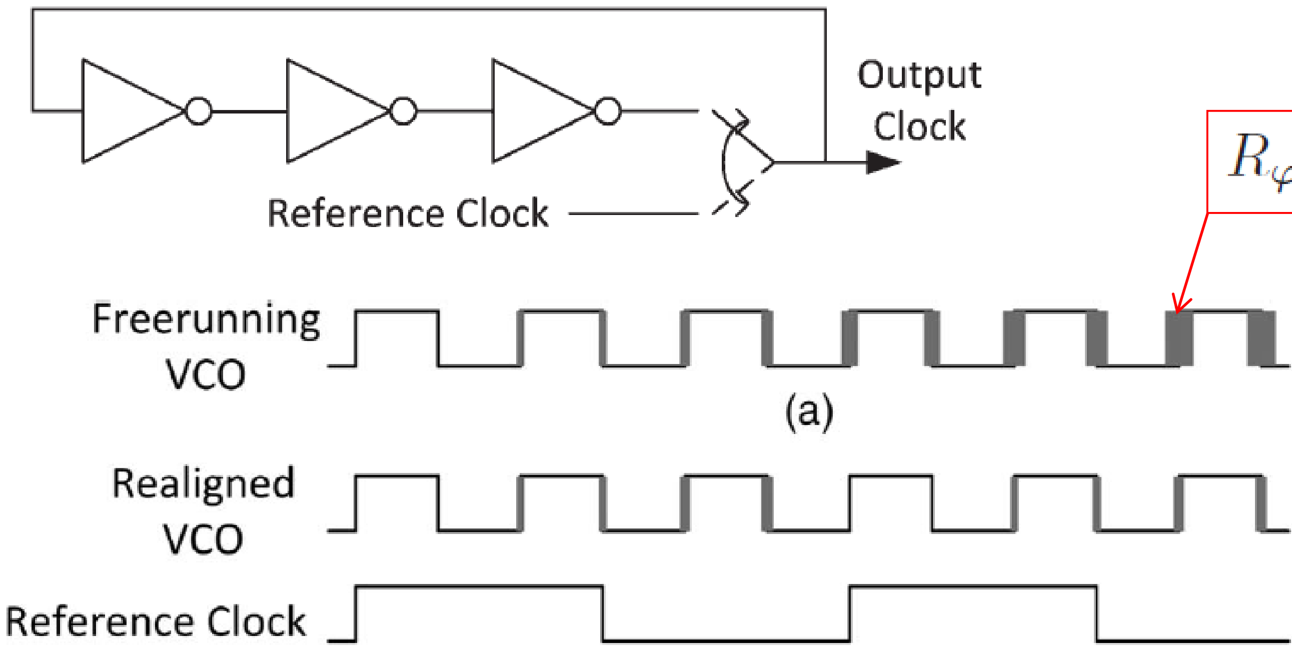
(a)



(b)

Phase Noise

- **Edges are realigned in every N cycles.**

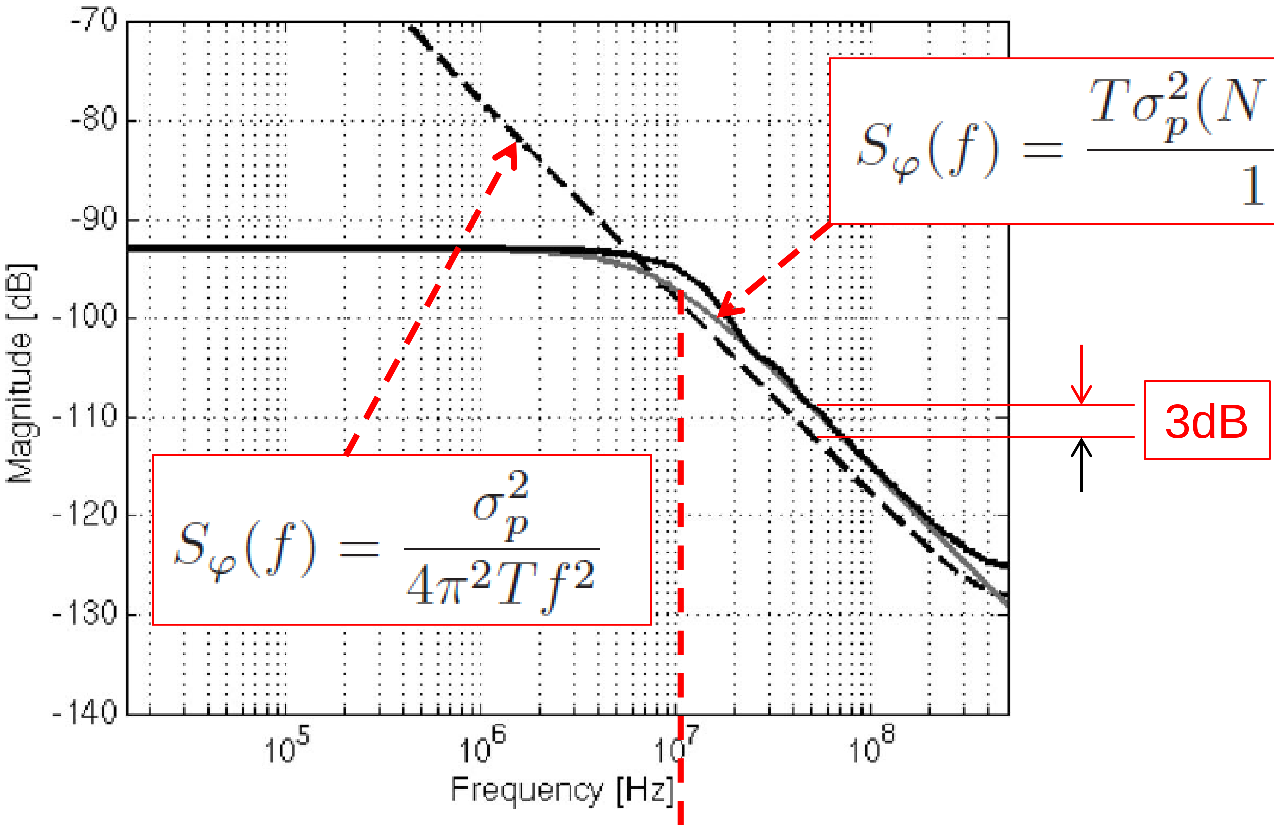


$$R_\varphi(n,0) = n\sigma_p^2$$

$$\bar{R}_\varphi(m) = \begin{cases} \dfrac{\sigma_p^2}{2N}(N-|m|-1) \\ \quad \times(N-|m|) & \text{for } -(N-1)\le m \le N-1 \\ 0, & \text{otherwise.} \end{cases}$$

Phase Noise

- **Edges are realigned in every N cycles.**



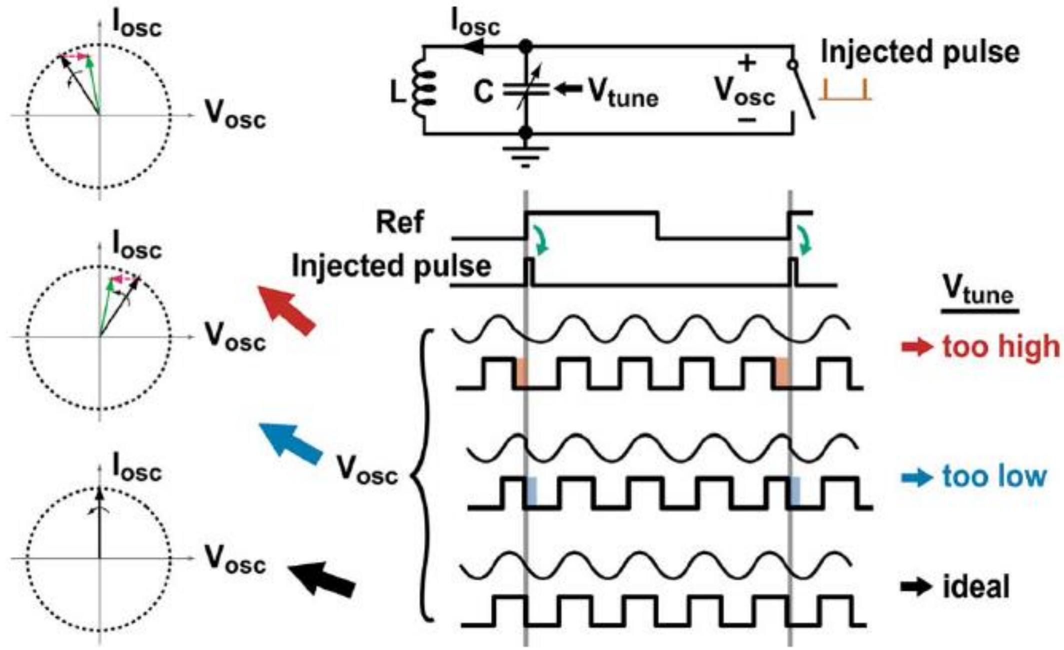
$$S_\varphi(f) = \frac{T\sigma_p^2(N-1)(2N-1)/6}{1+(f/f_c)^2}$$

$$S_\varphi(f) = \frac{\sigma_p^2}{4\pi^2 T f^2}$$

$$f_c = \sqrt{6/(N(N-1))}/(2\pi T) \approx 0.4/(NT)$$

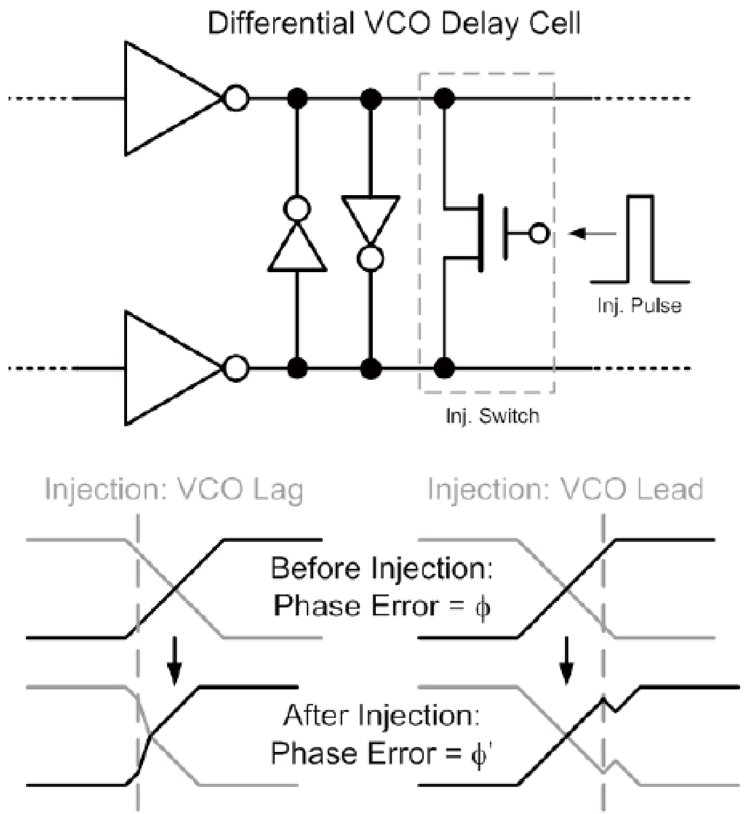
Spur and Frequency Tuning

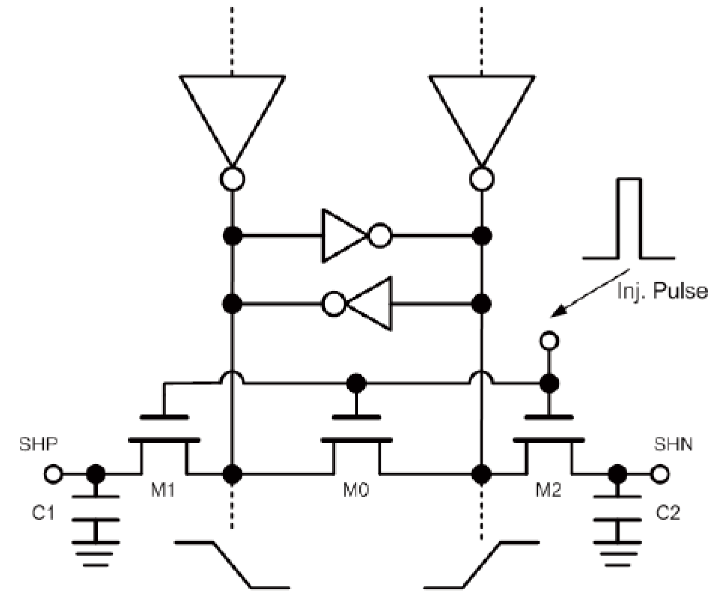
- Impact of frequency tuning and reference spur generation
- When not tuned: $|f_{\mathrm{out}} - f_o| = \alpha \cdot f_{\mathrm{out}}$
- Error in the period of injection cycle: $\Delta \approx (N-1)\alpha \cdot T_{\mathrm{out}} \approx \alpha \cdot T_{\mathrm{ref}}$
- Reference spur: $\approx 20\log_{10}(\Delta / T_{\mathrm{out}}) \approx 20\log_{10}(N \cdot \alpha)$



Tuning of IL Oscillator

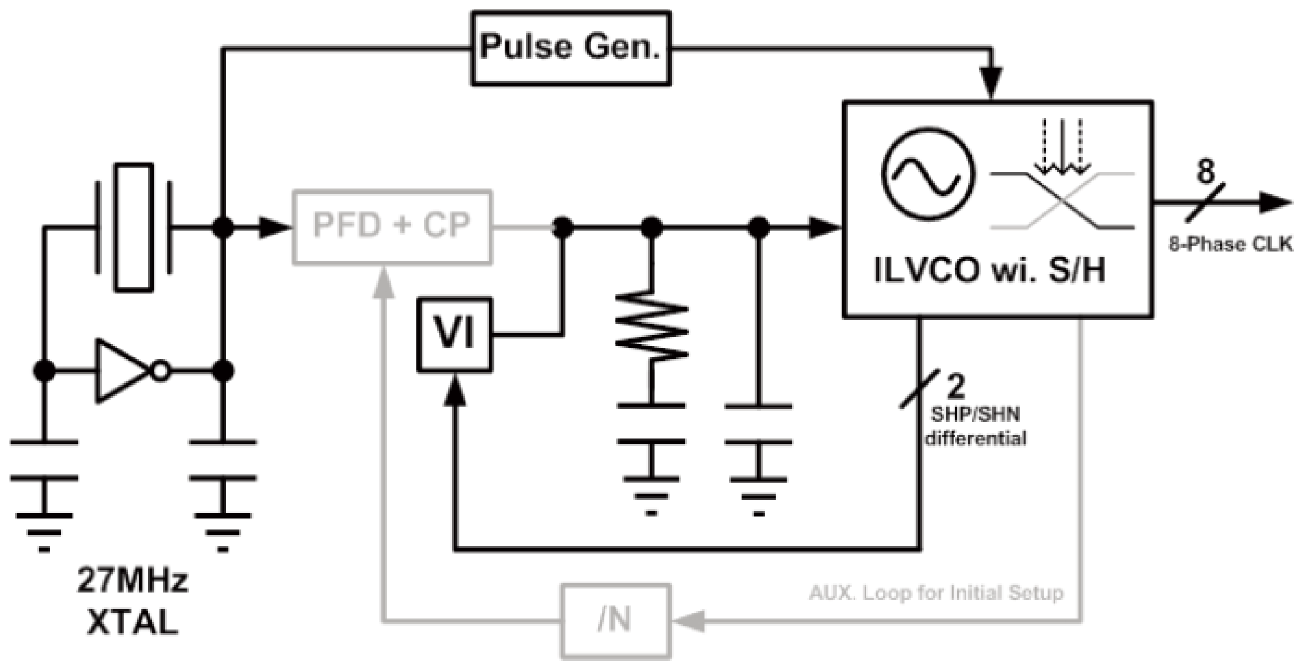
- **Pulse on Injection Switch**
  - **Sample and hold detects phase error**





Tuning of IL Oscillator

- **Tuning of the free-running frequency**
  - **Reference spur is reduced**



References

- [4.3.1] J Lee, et al., “Subharmonically Injection-Locked PLLs for Ultra-Low-Noise Clock Generation,” ISSCC 2009
- [4.3.2] B. Helal, et al., “A Low Jitter Programmable Clock Multiplier Based on a Pulse Injection-Locked Oscillator With a Highly-Digital Tuning Loop,” JSSC 2009
- [4.3.3] C. Liang et al., “An Injection-Locked Ring PLL with Self-Aligned Injection Window, ISSCC 2011
- [4.3.4] C. Wu, et al., “A Phase-Locked Loop With Injection-Locked Frequency Multiplier in 0.18-um CMOS for V-Band Applications,” TMTT, No. 7, 2009
- [4.3.5] Choi et al., “Ultralow In-Band Phase Noise Injection-Locked Frequency Multiplier Design Based on Open-Loop Frequency Calibration,” TCAS-2, 2014
- [4.3.6] K. Takano, et al., “4.8GHz CMOS Frequency Multiplier with Subharmonic Pulse-Injection Locking”, ASSCC, 2007
- [4.3.7] Da Dalt, “An Analysis of Phase Noise in Realigned VCOs,” TCAS-II, 2014
- [4.3.8] Xu, et al., “A 4-15GHz Ring Oscillator based Injection Locked Frequency Multiplier with built-in Harmonic Generation,” CICC 2013

Topics in IC Design

4.4 Injection Locked Frequency Dividers

Deog-Kyoon Jeong

dkjeong@snu.ac.kr

School of Electrical and Computer Engineering

Seoul National University

2020 Fall

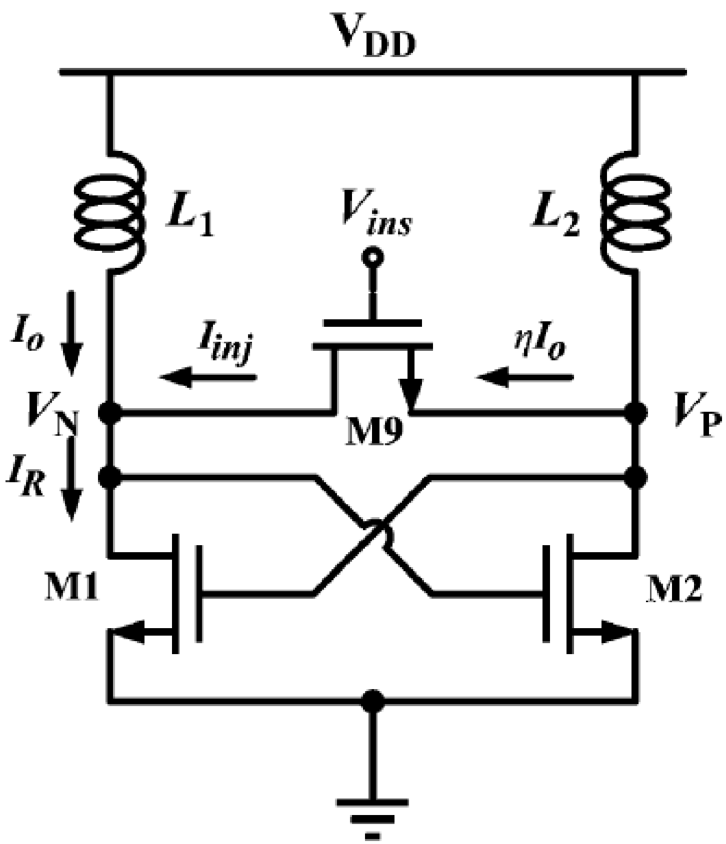
Outline

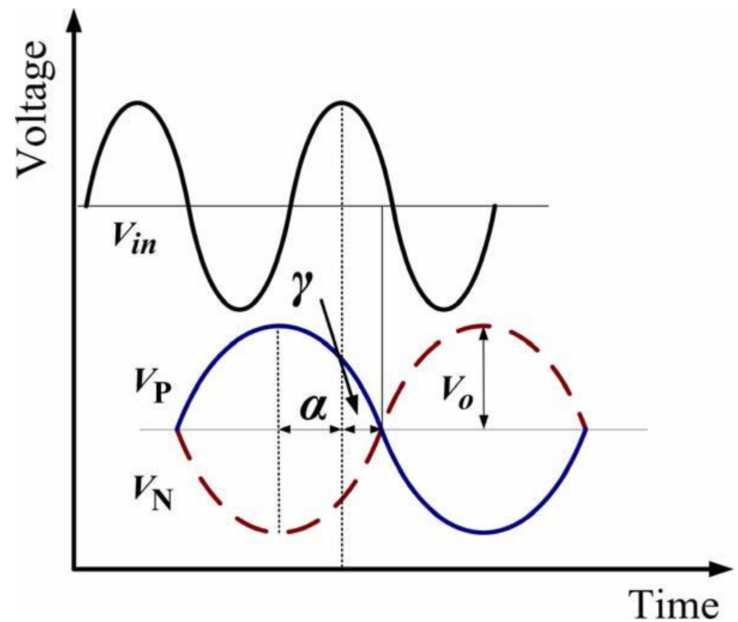
- **Direct Injection Locked Frequency Divider**
- **Regenerative IL FD**

Direct Injection-Locked FD

- **Divide-by-two Direct ILFD**

[TMTT2008 T. Luo et al.]

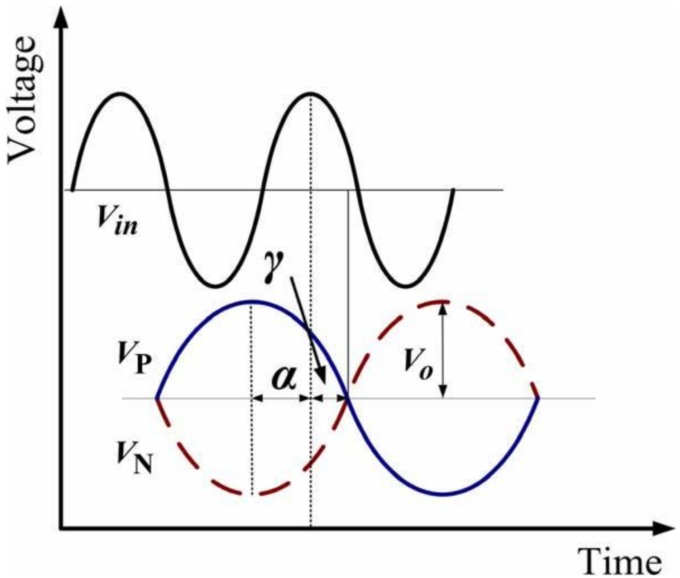




$$I_R = I_o + I_{\text{inj}}$$

Direct Injection-Locked FD

- **Divide-by-two Direct ILFD**



$$I_{\text{inj}} = \eta I_o \cdot \frac{2}{\pi}\left[1 - \frac{\sqrt{2}(V_{\text{gs}} - V_t)_{\text{sw}}}{\pi \cdot V_{\text{ins}}}\right]$$

$$\eta = \frac{2V_o \sin\gamma}{r_{\text{DS}} \cdot I_o}$$

$$r_{\text{DS}} = \frac{1}{\mu_{\text{sw}} C_{\text{ox}} \left(\frac{W}{L}\right)_{\text{sw}} (V_{\text{gs}} - V_t)_{\text{sw}}}$$

$$\gamma = \frac{\pi}{2} - \alpha$$

Output frequency

$$\boxed{f_{L\_o} = \frac{f_r}{Q} \cdot \frac{I_{\text{inj}}}{\sqrt{I_o^2 - I_{\text{inj}}^2}}}$$

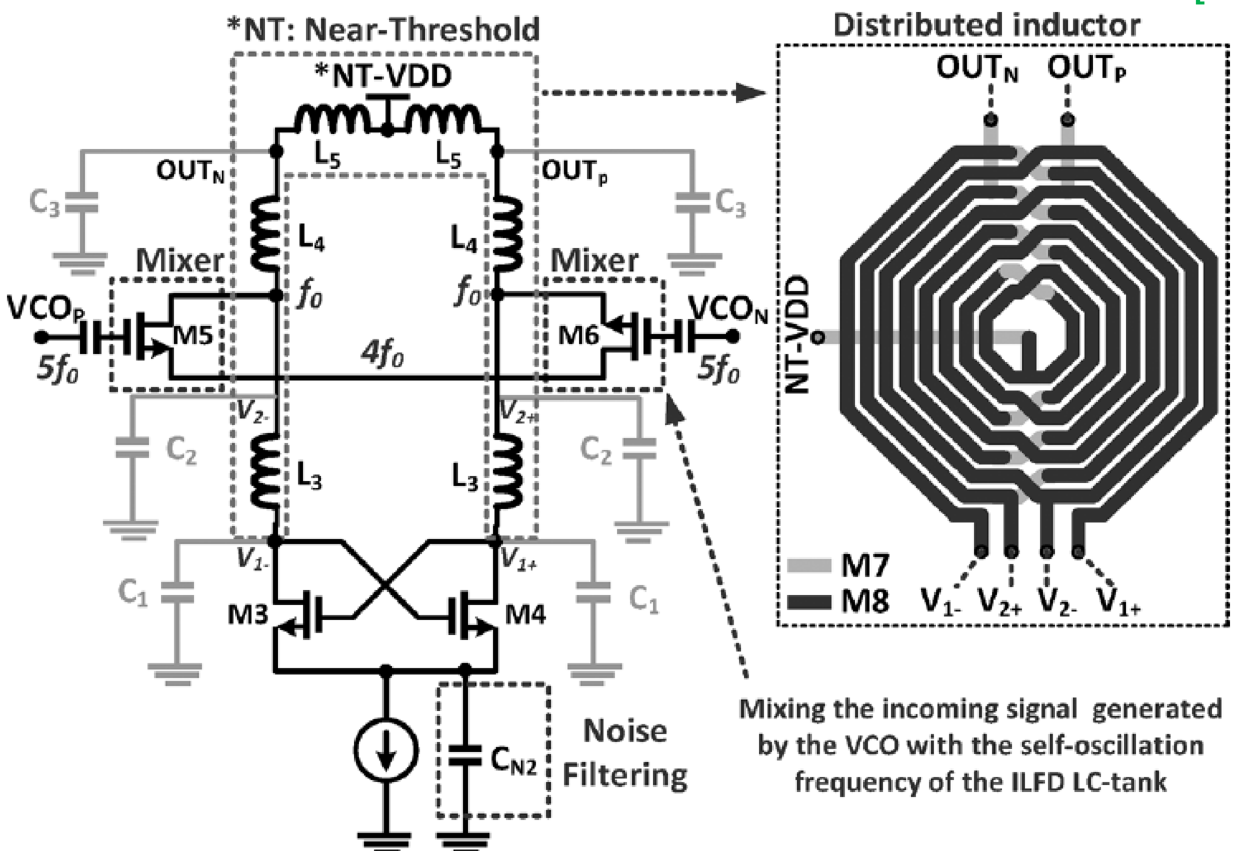
Input frequency

$$\boxed{f_L = \frac{2f_r}{Q} \cdot \frac{I_{\text{inj}}}{\sqrt{I_o^2 - I_{\text{inj}}^2}}}$$

Frequency Divider based on LC Osc

- **L3 and L4 reduces the effect of parasitic capacitances**
- **Divide-by-5 operation**

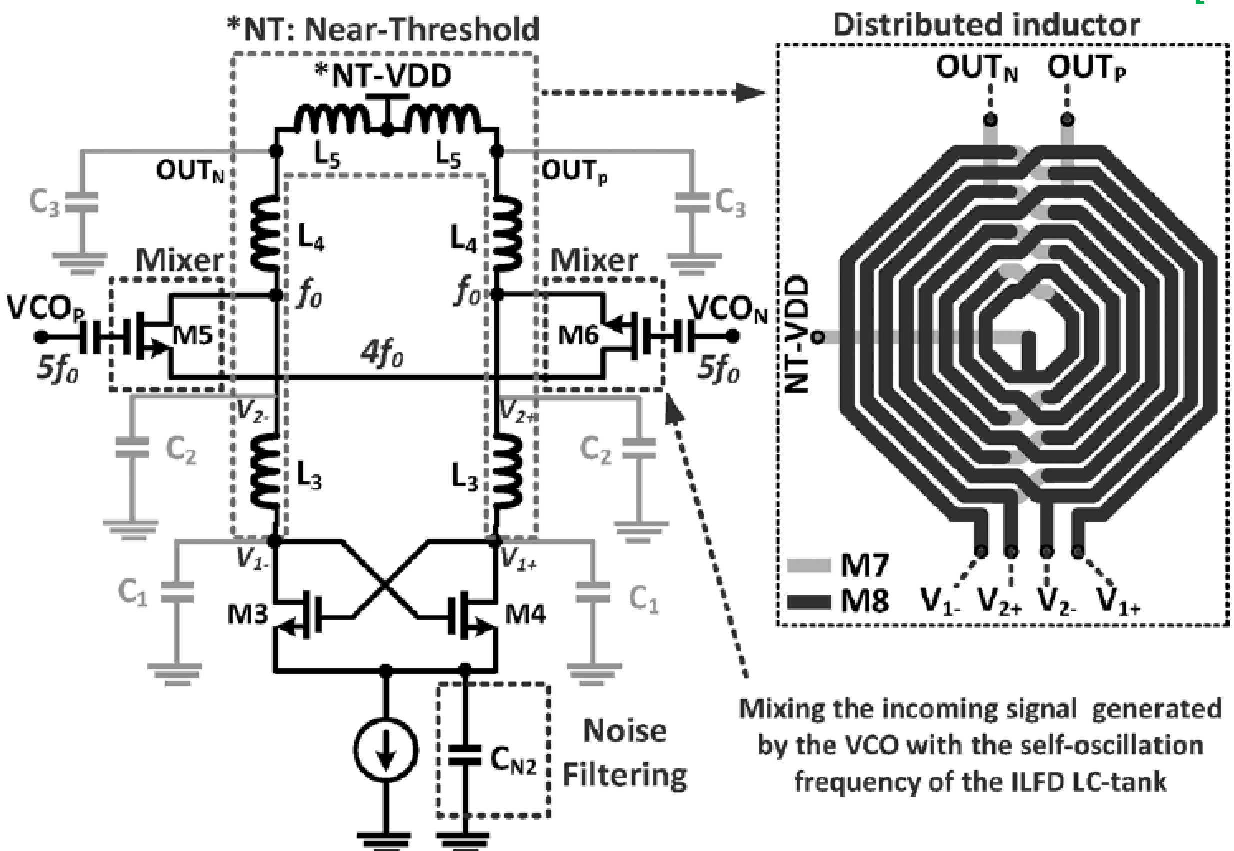
[MWC2014 M. Jalalifar et al.]



Frequency Divider based on LC Osc

- **M5 mixes f_0 and $5f_0$ to generate $4f_0$ and $6f_0$, which is mixed with $5f_0$ by M6 to generate f_0 and $9f_0$ and $11f_0$.**

[MWC2014 M. Jalalifar et al.]



Frequency Divider based on LC Osc

- **Resonance frequency**

$$f_0(ILFD) = \frac{1}{2\pi}\sqrt{\frac{C_1(L_3+L_4+L_5)+C_2(L_4+L_5)+C_3L_5-\sqrt{\Delta}}{2\left(C_1L_3L_5(C_3+C_2)+C_1C_2L_3L_4\right)}} \tag{4}$$

$$\Delta = \left(C_1\left(L_3+L_4+L_5\right)+C_2\left(L_4+L_5\right)+C_3L_5\right)^2 - 4\left(C_1L_3L_5(C_3+C_2)+C_1C_2L_3L_4\right). \tag{5}$$

- **If C_1=C_2=C_3=C and L_5 > L_3, L_4**

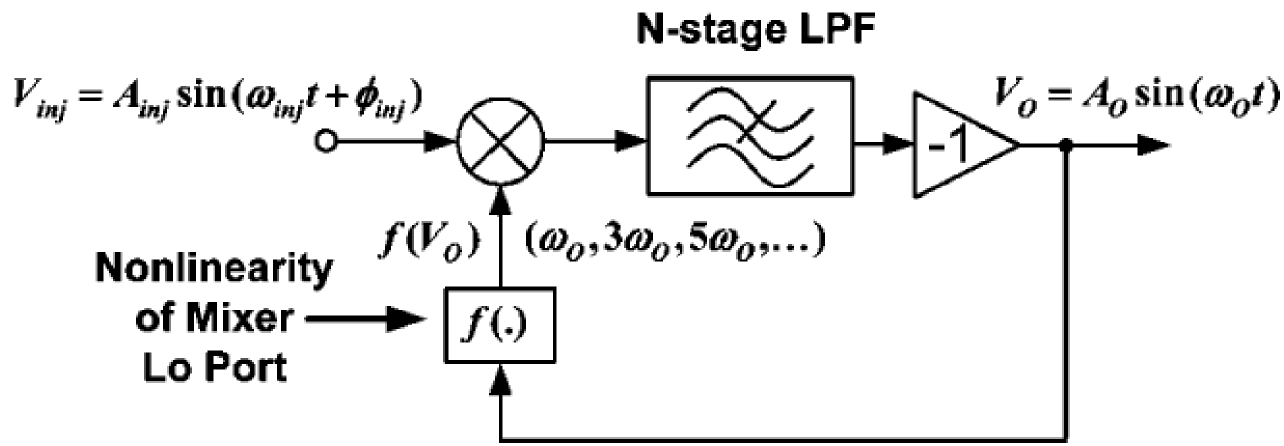
$$f_0(ILFD) = \frac{1}{2\pi}\sqrt{\frac{1-\sqrt{1-\frac{8}{9}\left(\frac{L_3}{L_5}\right)}}{\frac{4}{3}CL_3}}$$

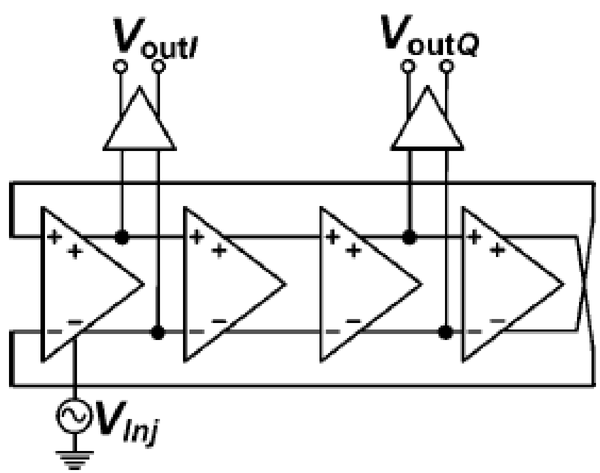
- **L_3 must be minimized to increase f_0**

RO-based FD

[MWC2009 M. Farazian et al.]

- **4-stage RO based ILFD**





$f(\cdot)$ ±1 square wave

$$f(V_O) = \sum_{k=-\infty}^{\infty} a_k \, e^{jk\omega_o t}$$

RO-based FD

- **4-stage RO based ILFD**

$$I_D = \sum_{k=1}^{\infty} (\pm) \frac{2 g_m A_{\text{inj}}}{(2k-1)\pi} \cos\left[\omega_{\text{inj}} t + \phi_{\text{inj}} \mp (2k-1)\omega_o t\right]$$

- **N-stage Lowpass filter removes high-frequency components and only the following term survives.**

$$\left|\omega_{\text{inj}} - (2k-1)\omega_o\right| = \omega_o$$

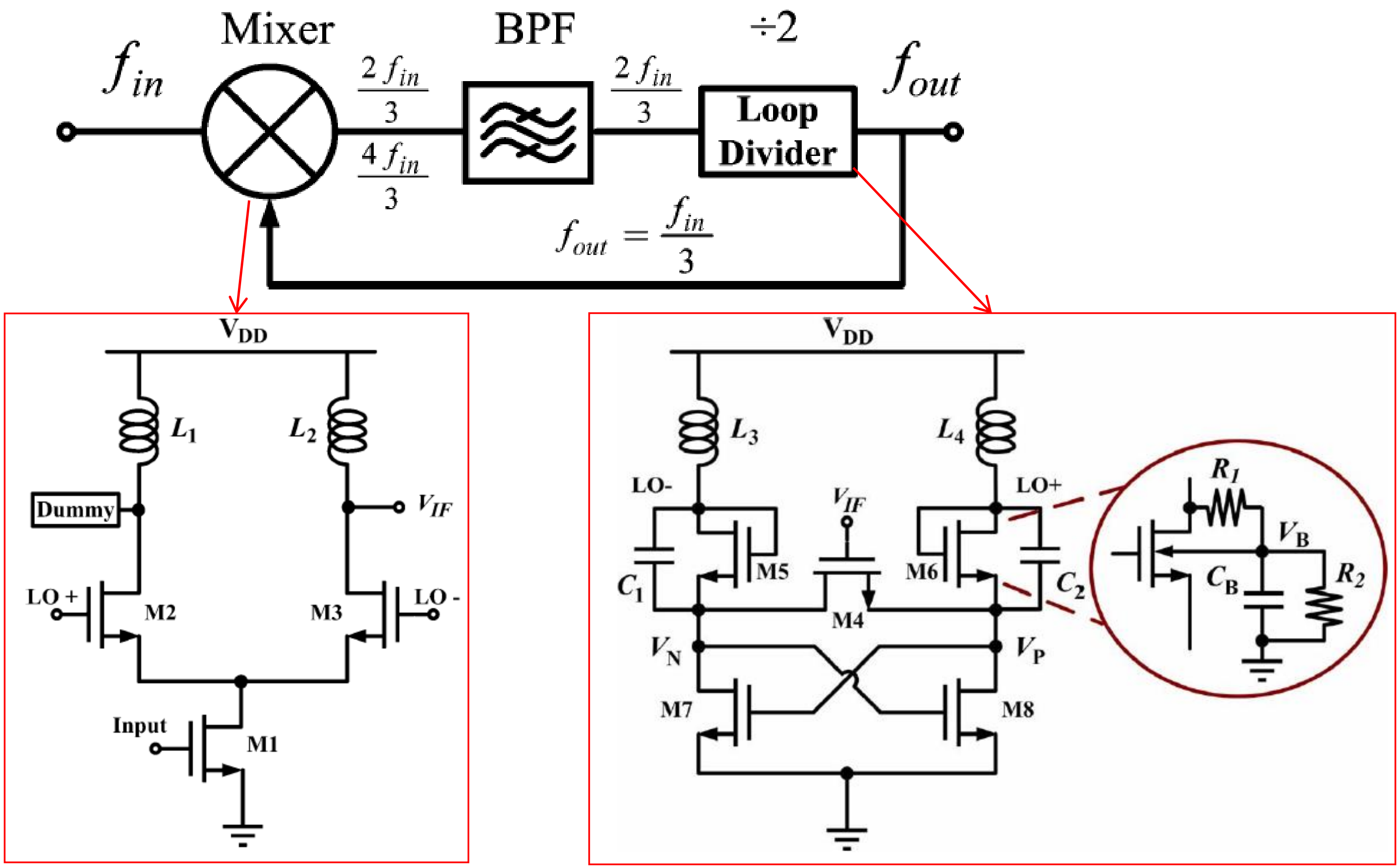
- **Therefore,**

$$\frac{\omega_o}{\omega_{\text{inj}}} = \frac{1}{2k}$$

Regenerative Injection-Locked FD

- **60GHz divide-by-three operation**

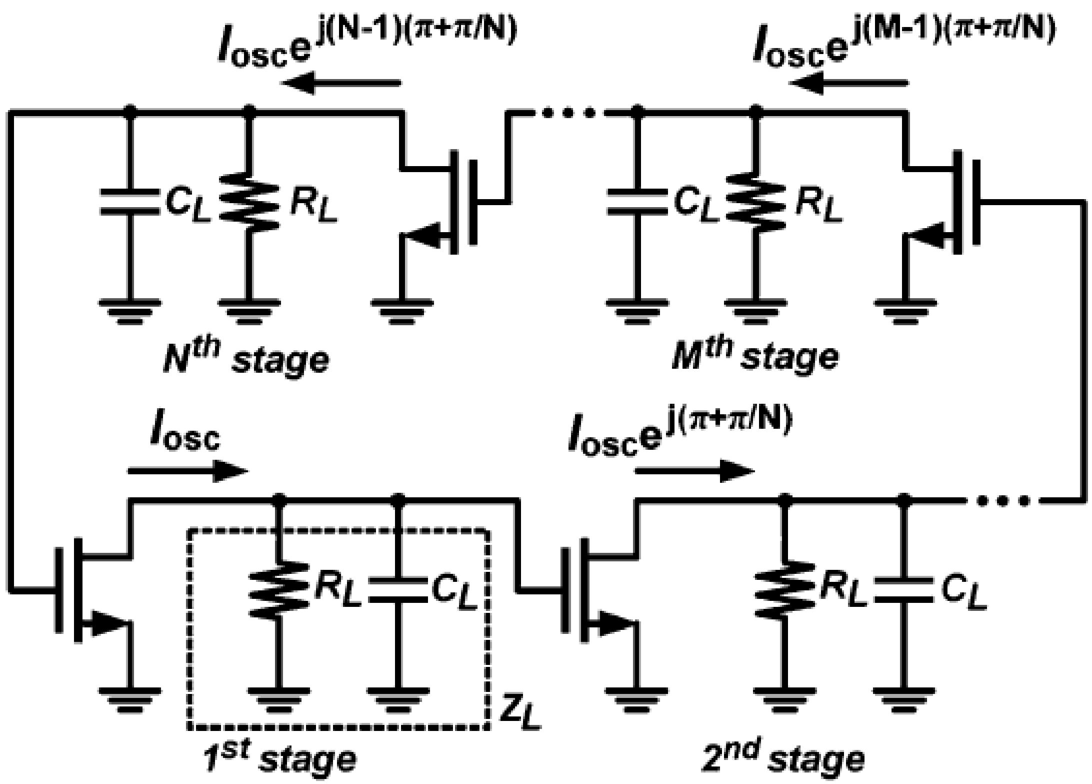
[TMTT2008 T. Luo et al.]

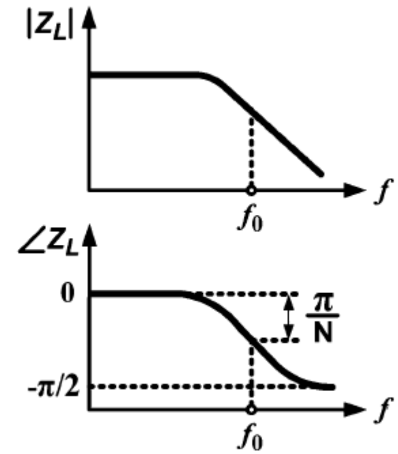


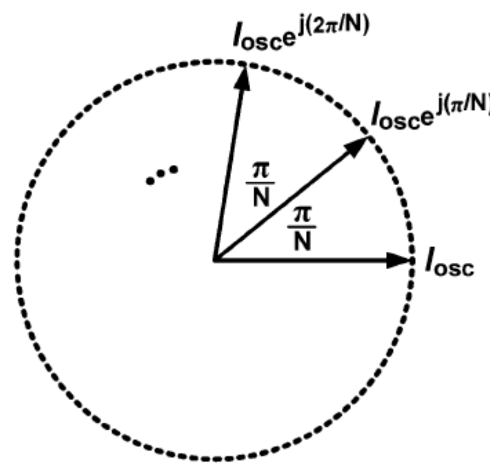
Injection Locked Ring Oscillators

- **N-stage resistive-load RO at free running**

[JSSC2007 J. Chien et al.]



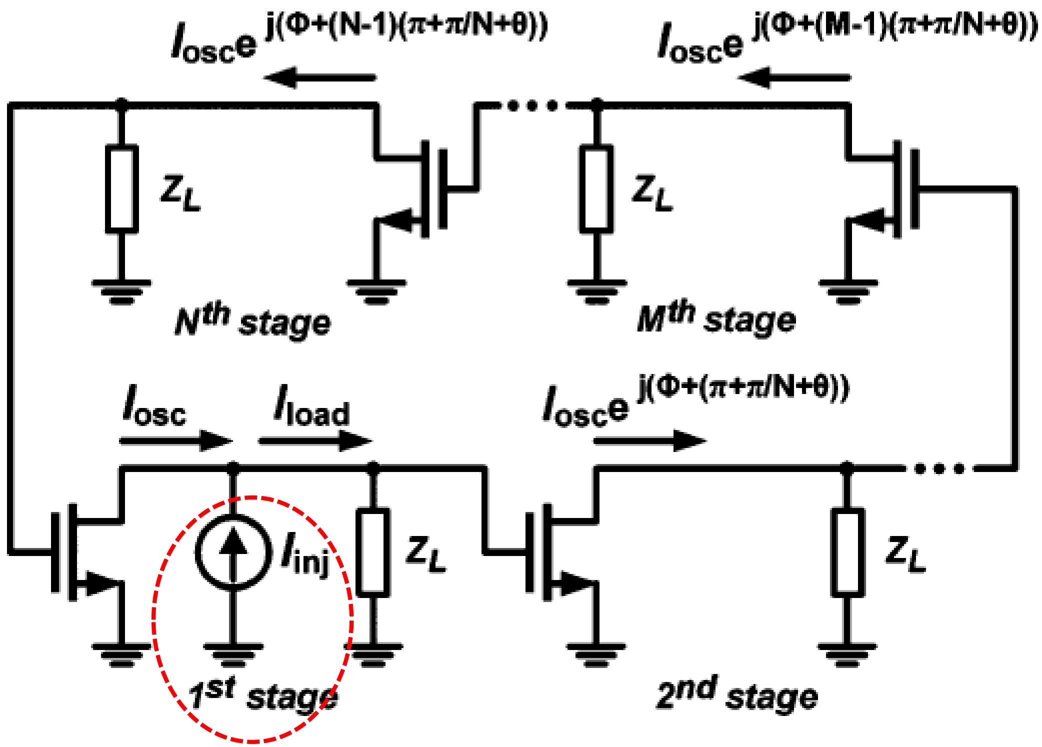


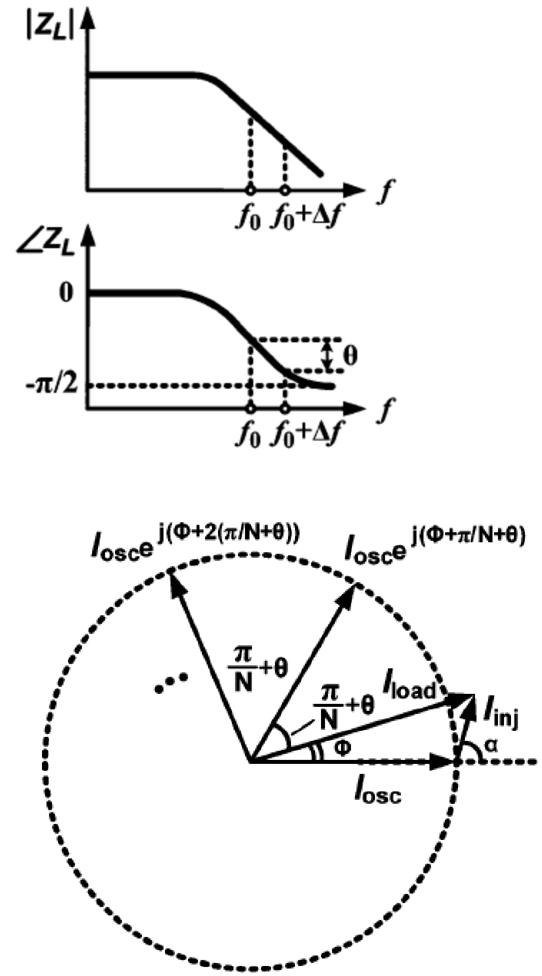


Injection Locked Ring Oscillators

- **Single-ended injection**

[JSSC2007 J. Chien et al.]

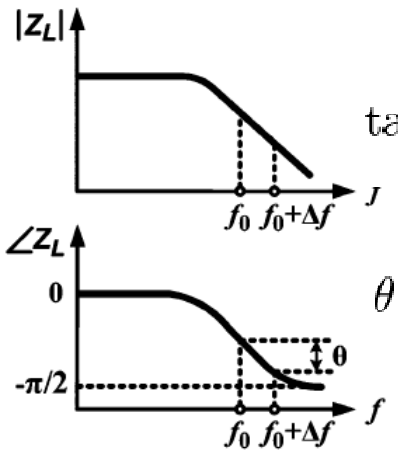




Injection Locked Ring Oscillators

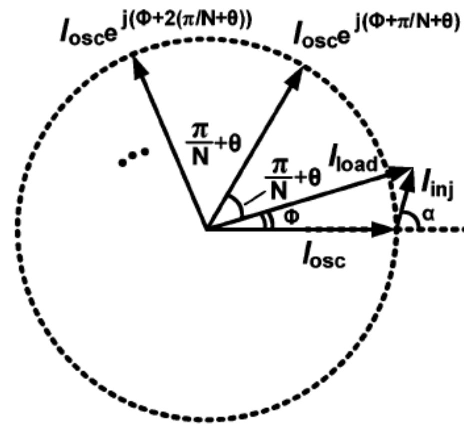
- **Single-ended injection**

[JSSC2007 J. Chien et al.]



$$\tan^{-1}\left(\frac{f}{f_0}\right)=\frac{\pi}{N}+\theta$$

$$\theta \approx \frac{\tan\frac{\pi}{N}}{1+\tan^2\frac{\pi}{N}}\cdot\frac{\Delta f}{f_0}$$



$$\tan\phi=\frac{|I_{\mathrm{inj}}|\sin\alpha}{|I_{\mathrm{inj}}|\cos\alpha+|I_{\mathrm{osc}}|}$$

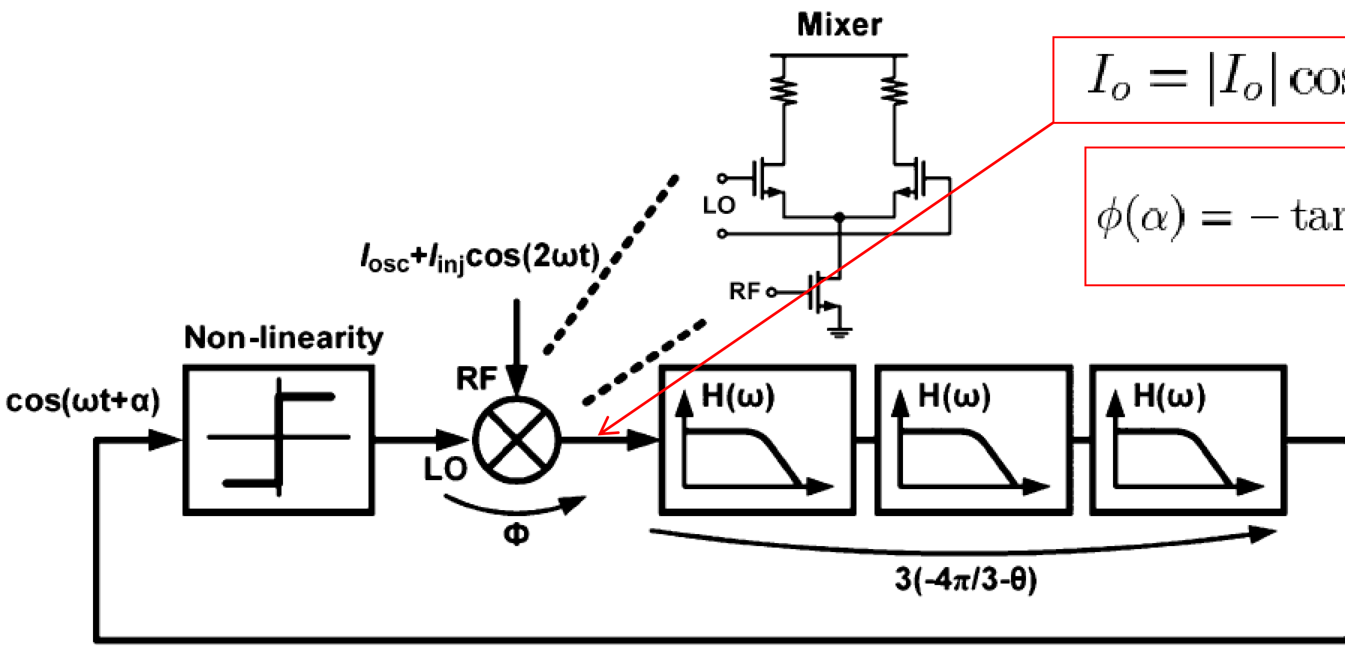
$$\phi_{\max}=\frac{|I_{\mathrm{inj}}|}{\sqrt{|I_{\mathrm{osc}}|^2-|I_{\mathrm{inj}}|^2}}$$

- **Since** $\theta=-\dfrac{1}{N}\phi$

$$\boxed{\frac{\Delta f}{f_0}\leq\frac{1}{N}\cdot\frac{1+\tan^2(\pi/N)}{\tan(\pi/N)}\cdot\left|\frac{I_{\mathrm{inj}}}{I_{\mathrm{osc}}}\right|\cdot\left(1-\left|\frac{I_{\mathrm{inj}}}{I_{\mathrm{osc}}}\right|^2\right)^{-\frac{1}{2}}}$$

3-Stage Ring Oscillator as 2:1 Divider

- **Superharmonic IL RO as 2:1 frequency divider** [JSSC2007 J. Chien et al.]
  - **Mixer acts as a phase shifter and a frequency translator**



$$I_o = |I_o|\cos\left(\omega t + \alpha + \phi(\alpha)\right)$$

$$\phi(\alpha) = -\tan^{-1}\left(\frac{2\eta\sin(2\alpha)}{3+\eta\cos(2\alpha)}\right)$$

$$\eta = I_{\mathrm{inj}}/I_{\mathrm{osc}}$$

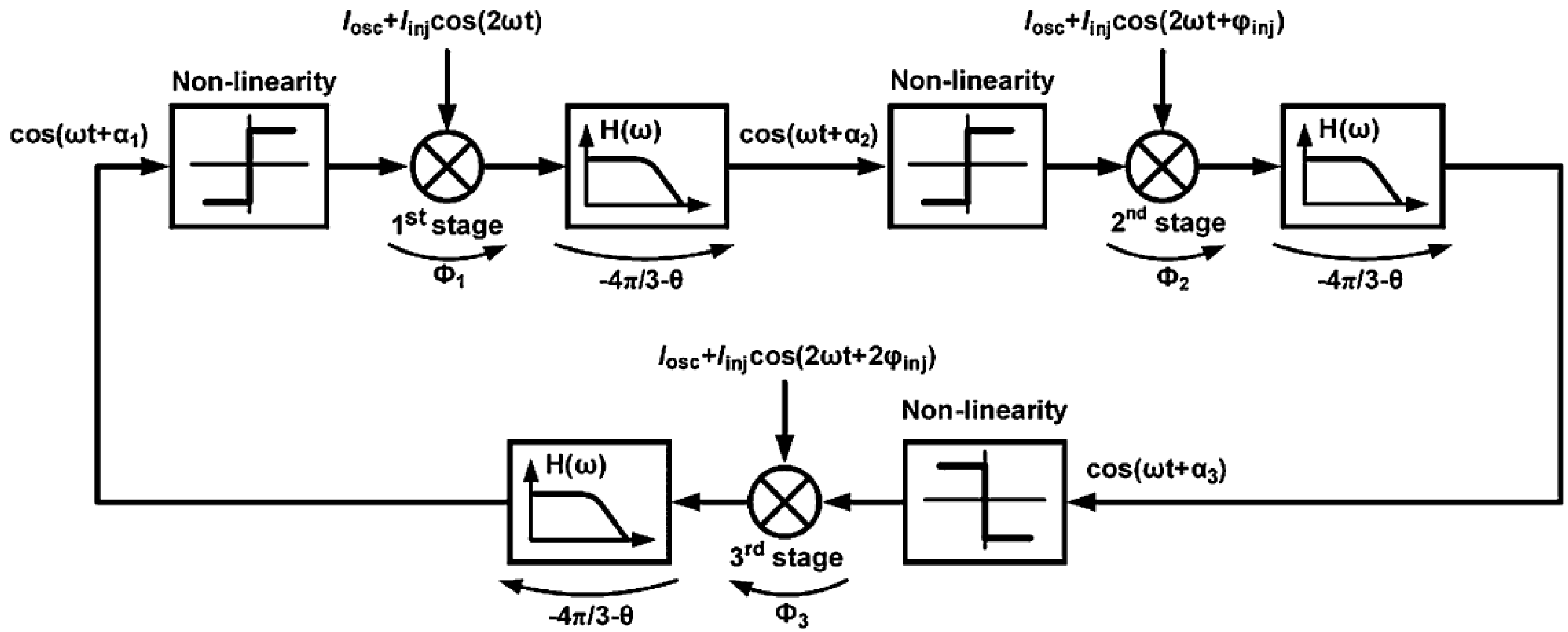
$$\theta = \frac{1}{3}\phi(\alpha) \leq \frac{1}{3}\phi_{\max} = \frac{1}{3}\tan^{-1}\left(\frac{2}{\sqrt{(3/\eta)^2-1}}\right)$$

$$\frac{\Delta f}{f_0} \leq \frac{4}{3\sqrt{3}}\tan^{-1}\left(\frac{2}{\sqrt{(3/\eta)^2-1}}\right)$$

Multiple-Input Injection

[JSSC2007 J. Chien et al.]

- **Multiple-input injection**
  - **Injection in three points with phase shift of φ_{inj}**

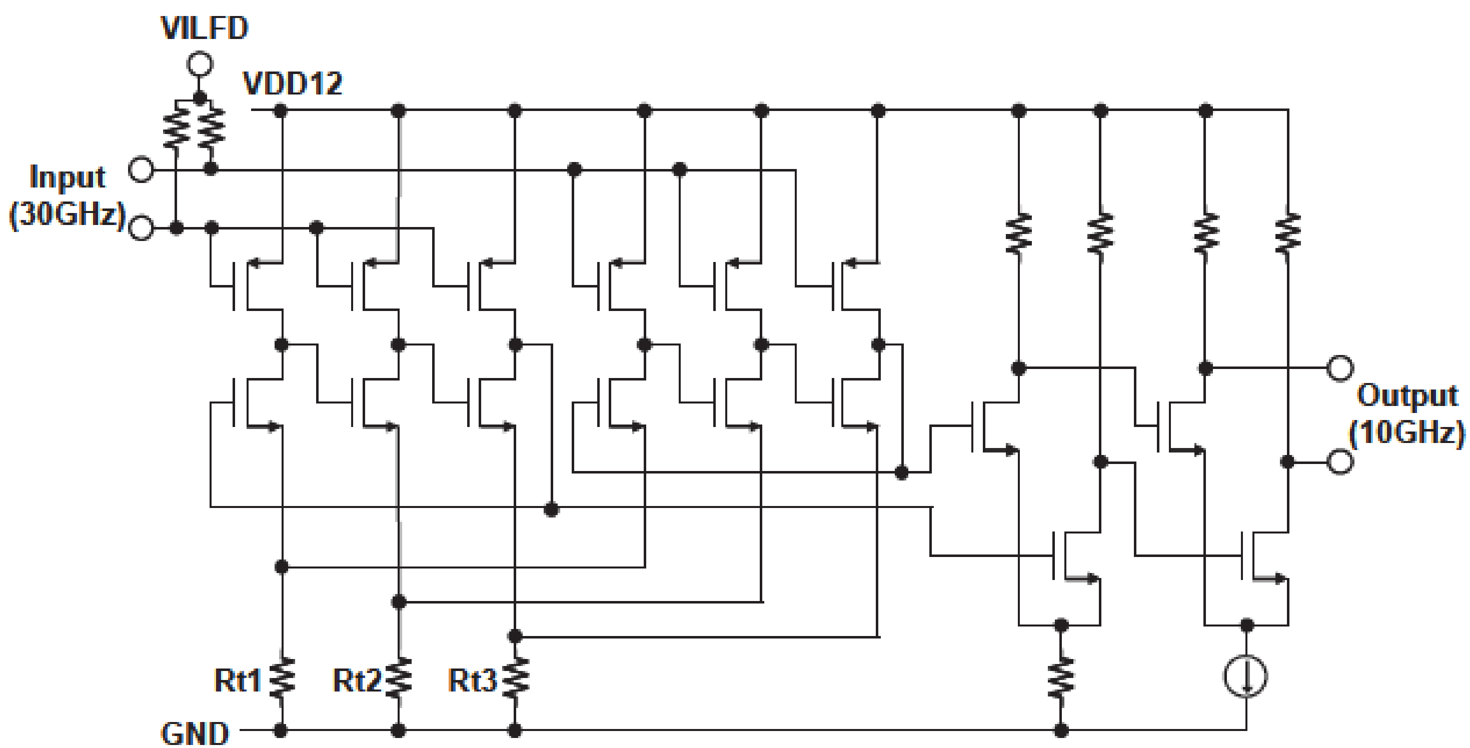


- **Lock range is enhanced by 3 times with $\varphi_{inj} = -2\pi/3$**

ILFD

[APMC2011 T. Shima et al.]

- RO locks to the 3rd harmonic of the input
  - Free-running frequency is set by VILFD
  - Each three-stage RO runs at 10GHz. Its 2nd harmonic is mixed with the 30GHz input and creates 10GHz output.



References

- [4.4.1] J. Chien et al., “Analysis and Design of Wideband Injection-Locked Ring Oscillators With Multiple-Input Injection”, JSSC Sept. 2007
- [4.4.2] E. Larson et al., “A CMOS Multi-Phase Injection-Locked Frequency Divider for V-Band Operation,” MWCL 2009
- [4.4.3] M. Jalalifar et al., “A K-Band Divide-by-Five Injection-Locked Frequency Divider Using a Near-Threshold VCO,” IEEE Microwave and Wireless Components Letters, vol 24, no. 12, 2014
- [4.4.4] T. Luo, “A 60-GHz 0.13-um CMOS Divide-by-Three Frequency Divider,” TMTT, vol 56, no. 11, 2008
- [4.4.5] T Shima et al., “A 60 GHz CMOS PLL Synthesizer using a Wideband Injection-Locked Frequency Divider with Fast Calibration Technique,” APMC 2011
- [4.4.6] C.-Y. Wu and C.-T. Yu, “Design and analysis of a millimeter-wave direct injection-locked frequency divider with large frequency locking range,” *TMTT*, vol. 55, no. 8, pp.1649–1658, 2007
- [4.4.7] Y. Lo et al., “A 1.8 V, sub-mW, over 100% locking range, divide-by-3 and 7 complementary-injection-locked 4 GHz frequency divider, “ CICC 2009

Topics in IC Design

4.5 Analysis of Pulsed Injection Locked Oscillators

Deog-Kyoon Jeong

dkjeong@snu.ac.kr

School of Electrical and Computer Engineering

Seoul National University

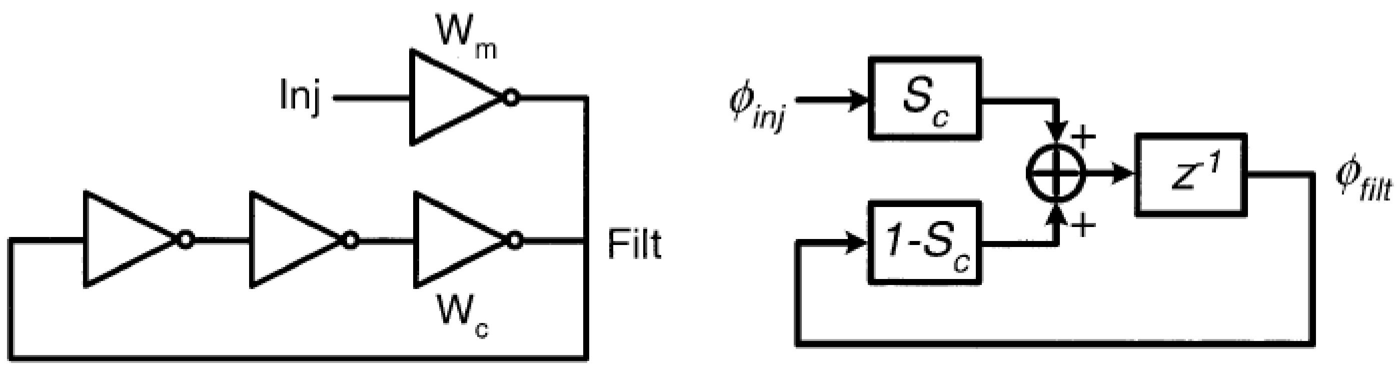
2020 Fall

Outline

- Superharmonic pulsed injection lock model
- Transfer function derivation
- Phase Domain Response (PDR)
- Tuning
- 2-path injection issue

Injection Lock Model

- **Signal Injection** $S_c = \dfrac{W_m}{W_m + W_c}$

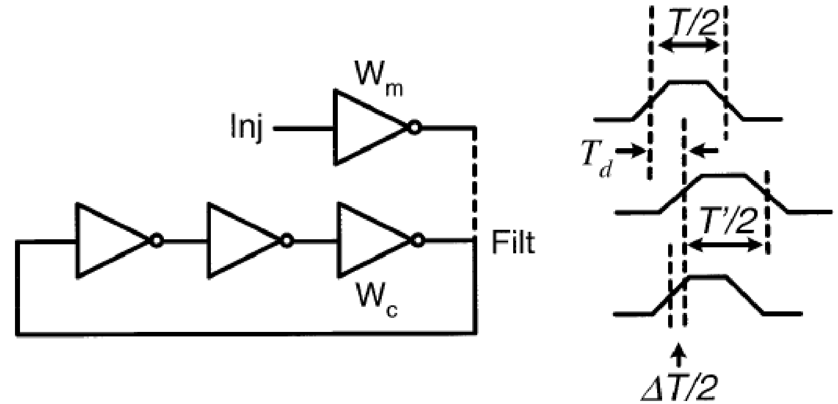


- Transfer function is $\dfrac{\phi_{\text{filt}}}{\phi_{\text{inj}}} = \dfrac{S_c}{z - (1 - S_c)}$

- The pole of this lowpass function is $p_0 = \dfrac{\ln(1 - S_c)}{T_i}$ (rad/s)

Injection Lock Model

- **Lock range calculation**
  - **Period of injected signal = *T***
  - **Free-running frequency = *T'***
  - **Phase shift is *ΔT=T'-T***



- **For phase lag of T_d, injection events per clock edge creates phase shift of**

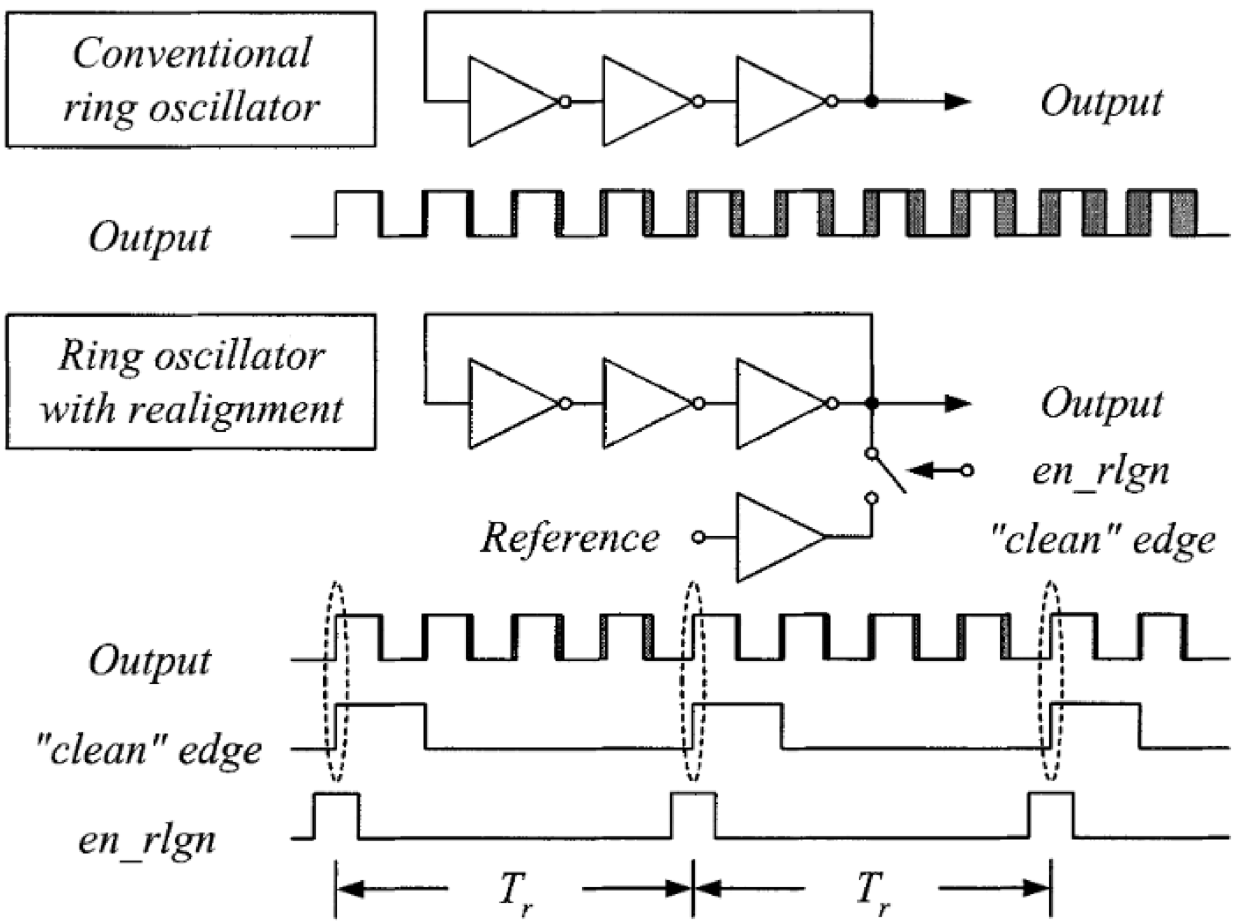
$$\frac{\Delta T}{2} = T_d S_c$$

- **Period difference must not be more than *T/4* or *T'/4***
- **Therefore, T_d=*T/4* and the lock range is**

$$\frac{\Delta T}{T} = \frac{S_c}{2}$$

Timing diagram of IL-VCO

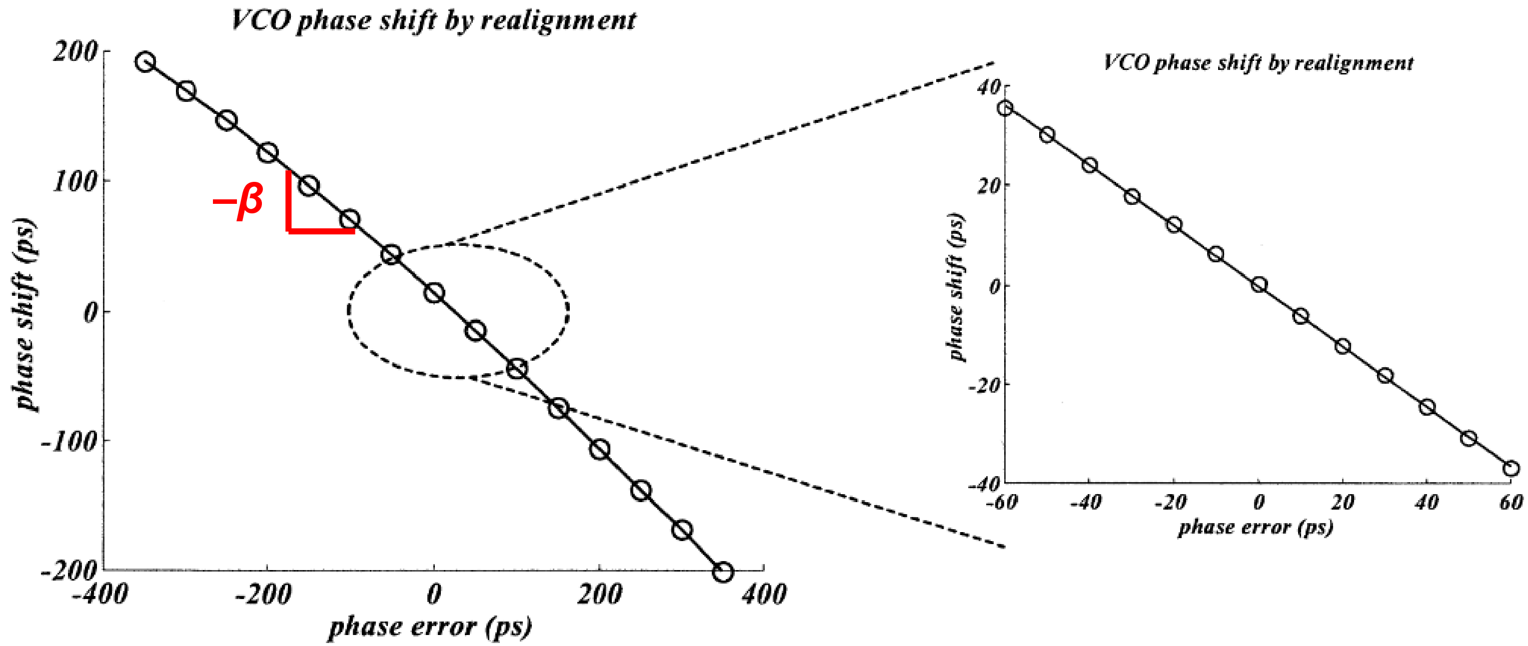
[JSSC'02 S. Ye *et al.*]



Assuming
$N \cdot f_{ref} = f_{vco}$ ($\Delta f = 0$),
"Tuned"

Phase Realignment

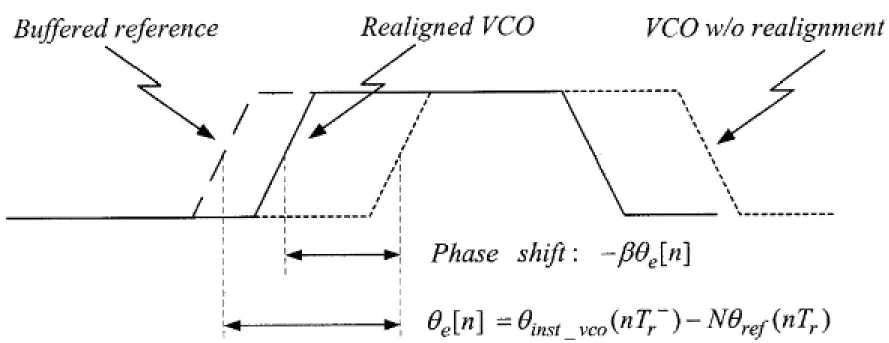
[JSSC'02 S. Ye *et al.*]



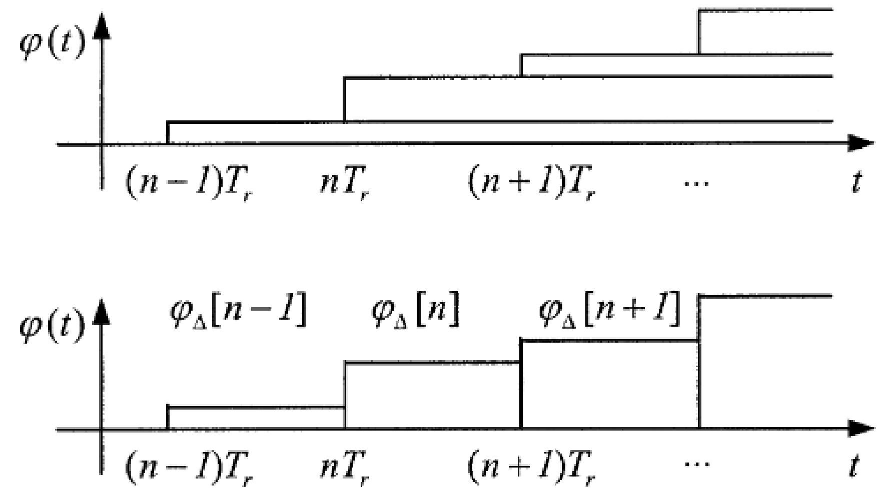
- **Assumptions**
  - **Linear phase shift (*β*)**
  - **Amplitude fluctuation is negligible**

Theoretical analysis (1)

[JSSC'02 S. Ye *et al.*]



[Time-domain waveform]



[Extra phase shift]

- $\theta_{inst\_vco}(t) = \theta_{vco}(t) + \varphi(t)$ —— *(1)*
  - $\theta_{inst\_vco}(t)$: instantaneous VCO phase noise
  - $\theta_{vco}(t)$: intrinsic VCO phase noise
  - $\varphi(t)$: extra phase shift due to injection (or realignment)

Theoretical analysis (2)

[JSSC'02 S. Ye *et al.*]

- $\theta_e[n] = \theta_{inst\_vco}(nT_r^-) - N \cdot \theta_{ref}(nT_r)$
  - nT_r^-: the time instant just before the nth reference edge
  - $\theta_{ref}(t)$: the reference phase noise at 1/N frequency
- $-\beta \cdot \theta_e[n]$
  - VCO phase shift after nth phase realignment
- $\varphi(t) = -\beta \cdot \sum_{-\infty}^{+\infty} \theta_e[n] \cdot u(t - nT_r)$
  - VCO can be modeled as a phase error integrator
- $\varphi(t) = \sum_{-\infty}^{+\infty} \varphi_\Delta[n] \cdot h_{hold}(t - nT_r)$
  - $\varphi_\Delta[n] - \varphi_\Delta[n-1] \equiv -\beta \cdot \theta_e[n]$ *(see the figure of previous slide)*
  - $h_{hold}(t) = u(t) - u(t - T_r)$ (zero-order hold with pulse width of T_r)

Theoretical analysis (3)

[JSSC'02 S. Ye *et al.*]

- $\varphi(jw) = T_r \cdot e^{-jwT_r/2} \cdot \frac{\sin(wTr/2)}{(wTr/2)} \cdot \varphi_\Delta(z)\,|_{z=e^{jwT_r}}$ —— (2)
  - Fourier transform of "$\varphi(t) = \sum_{-\infty}^{+\infty} \varphi_\Delta[n] \cdot h_{hold}(t-nT_r)$"
  - $\varphi_\Delta(z)$: z transform of $\varphi_\Delta[n]$
- $\varphi_\Delta[n] - \varphi_\Delta[n-1] = -\beta \cdot (\theta_{vco}[n] + \varphi_\Delta[n-1] - N \cdot \theta_{ref}[n])$

Z transform

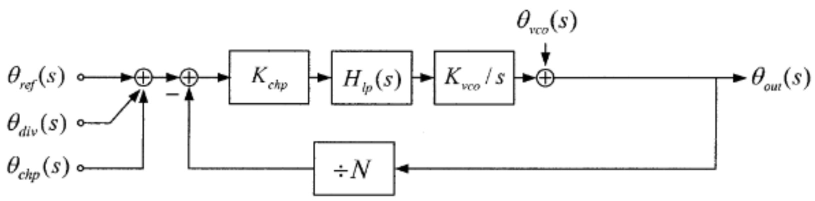
  - $\varphi_\Delta[n] - \varphi_\Delta[n-1] \equiv -\beta \cdot \theta_e[n]$
  - $\theta_e[n] = \theta_{inst\_vco}(nT_r^-) - N \cdot \theta_{ref}(nT_r)$

combining

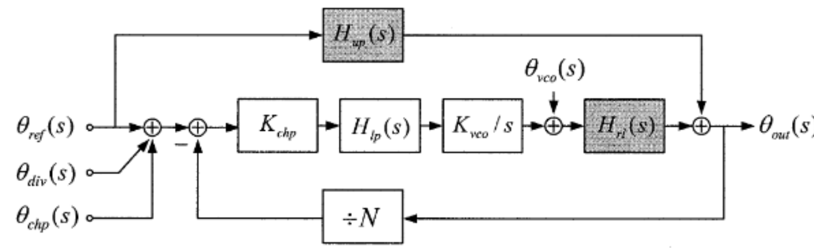
- $\varphi_\Delta(z) = \frac{-\beta}{1+(\beta-1)z^{-1}} \cdot \theta_{vco}(z) + \frac{N \cdot \beta}{1+(\beta-1)z^{-1}} \cdot \theta_{ref}(z)$ —— (3)
- $\theta_{inst\_vco}(jw) = \theta_{vco}(jw) \cdot H_{rl}(jw) + \theta_{ref}(jw) \cdot H_{up}(jw)$ – (1),(2),(3)
  - $H_{rl}(jw) = 1 - \frac{\beta}{1+(\beta-1)e^{-jwT_r}} \cdot e^{-jwT_r} \frac{\sin(wT_r/2)}{(wT_r/2)}$ (phase realignment, high pass)
  - $H_{up}(jw) = \frac{N \cdot \beta}{1+(\beta-1)e^{-jwT_r}} \cdot e^{-jwT_r} \frac{\sin(wT_r/2)}{(wT_r/2)}$ (reference noise upconversion, low pass)

Theoretical analysis (4)

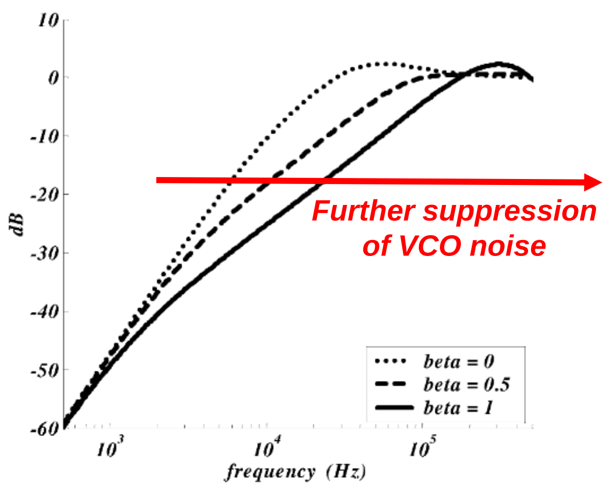
[JSSC'02 S. Ye *et al.*]



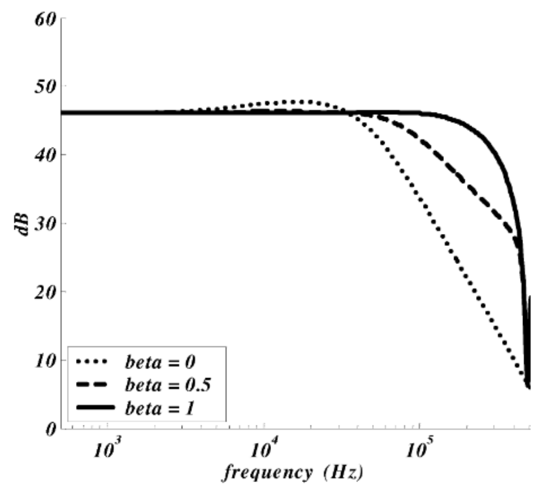
Conventional PLL



Realigned PLL



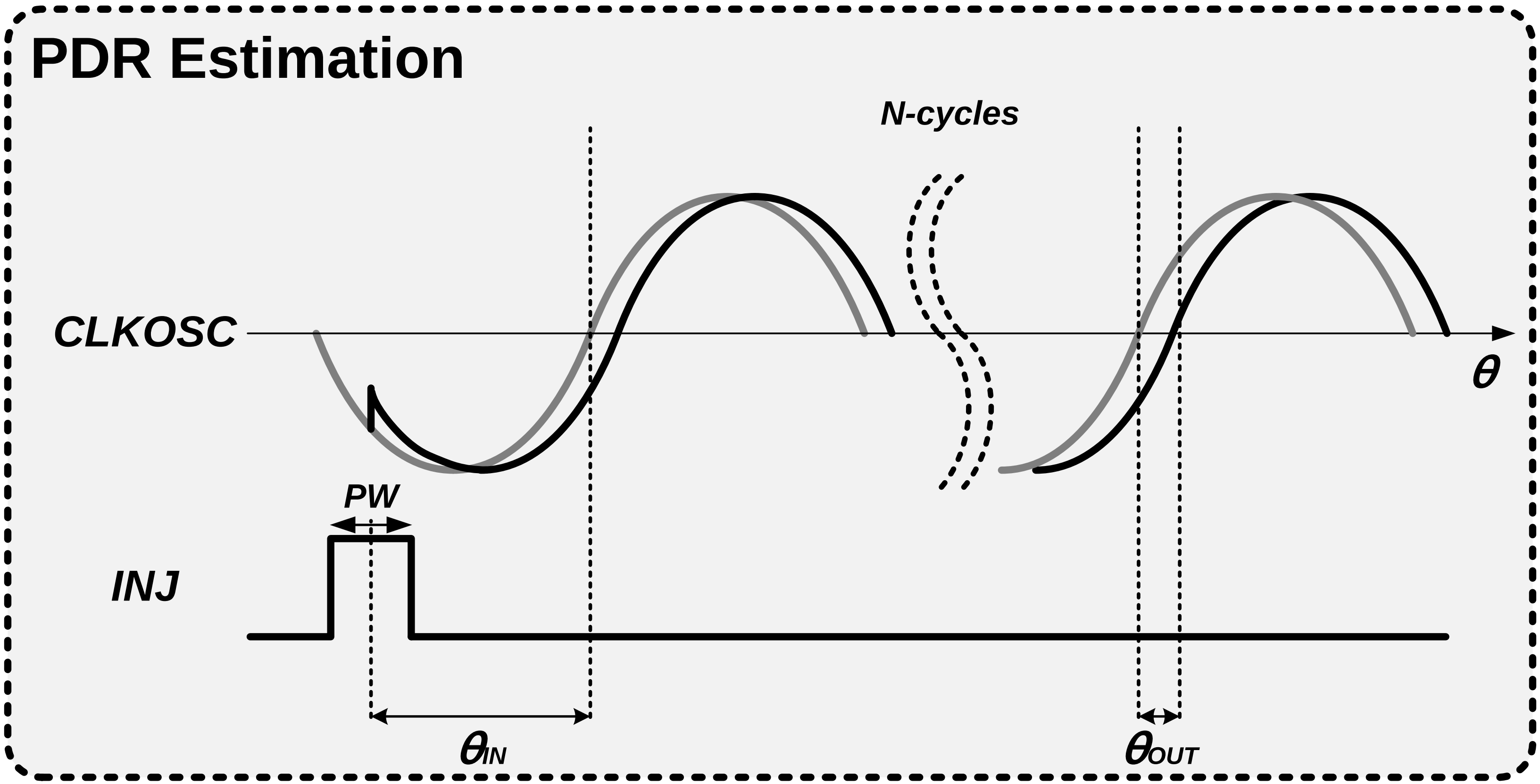
VCO phase noise transfer function of RPLL



Reference phase noise transfer function of RPLL

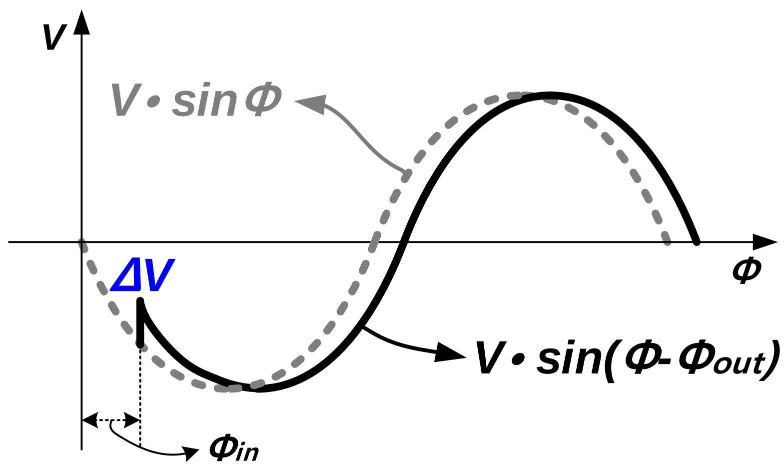
What is Phase Domain Response?

- ***Re-adjustment of edges by a single pulse***

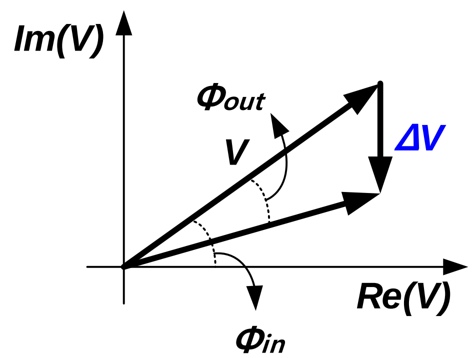


PDR Calculation (1)

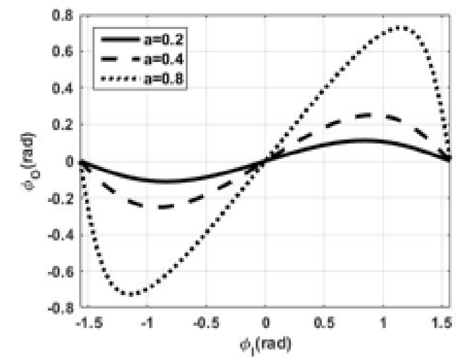
[JSSC'13 Y. C. Huang *et al.*]



(a) Time domain



(b) Phasor diagram

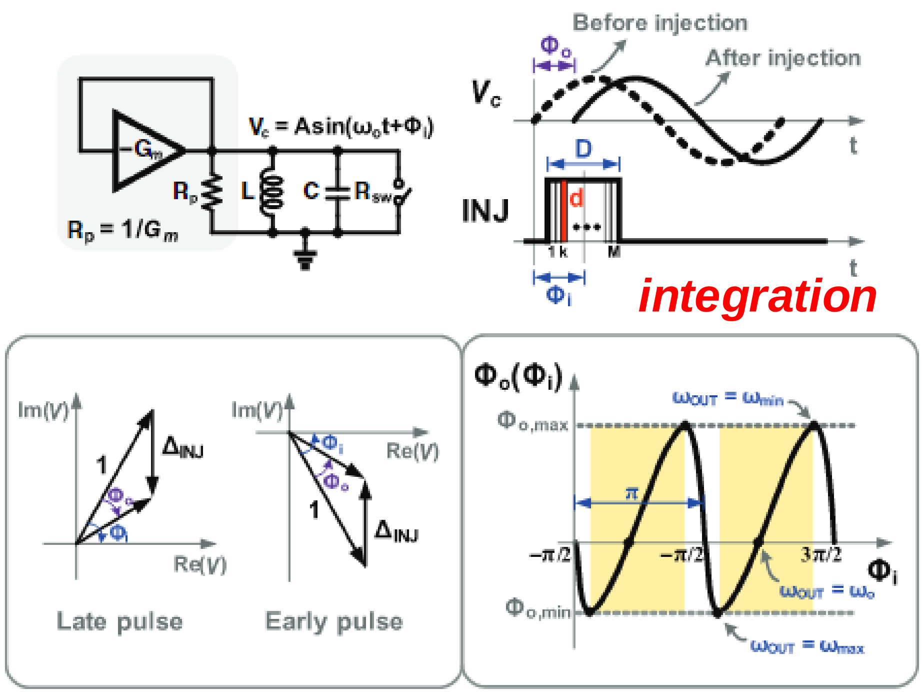


(c) PDR curves

- $\Delta V = V \cdot a \cdot \sin(\Phi_{in})$, $\left(a = 1 - e^{-\frac{d}{R_{SW}C}}\right)$
- $\Phi_{out} = \Phi_{in} - \mathrm{Arctan}[\tan(\Phi_{in}) - \Delta V / V\cos(\Phi_{in})]$
- **Injection pulse width: impulse (d~0)**
- **Symmetric PDR**

PDR Calculation (2)

[JSSC'13 A. Elkholy *et al.*]

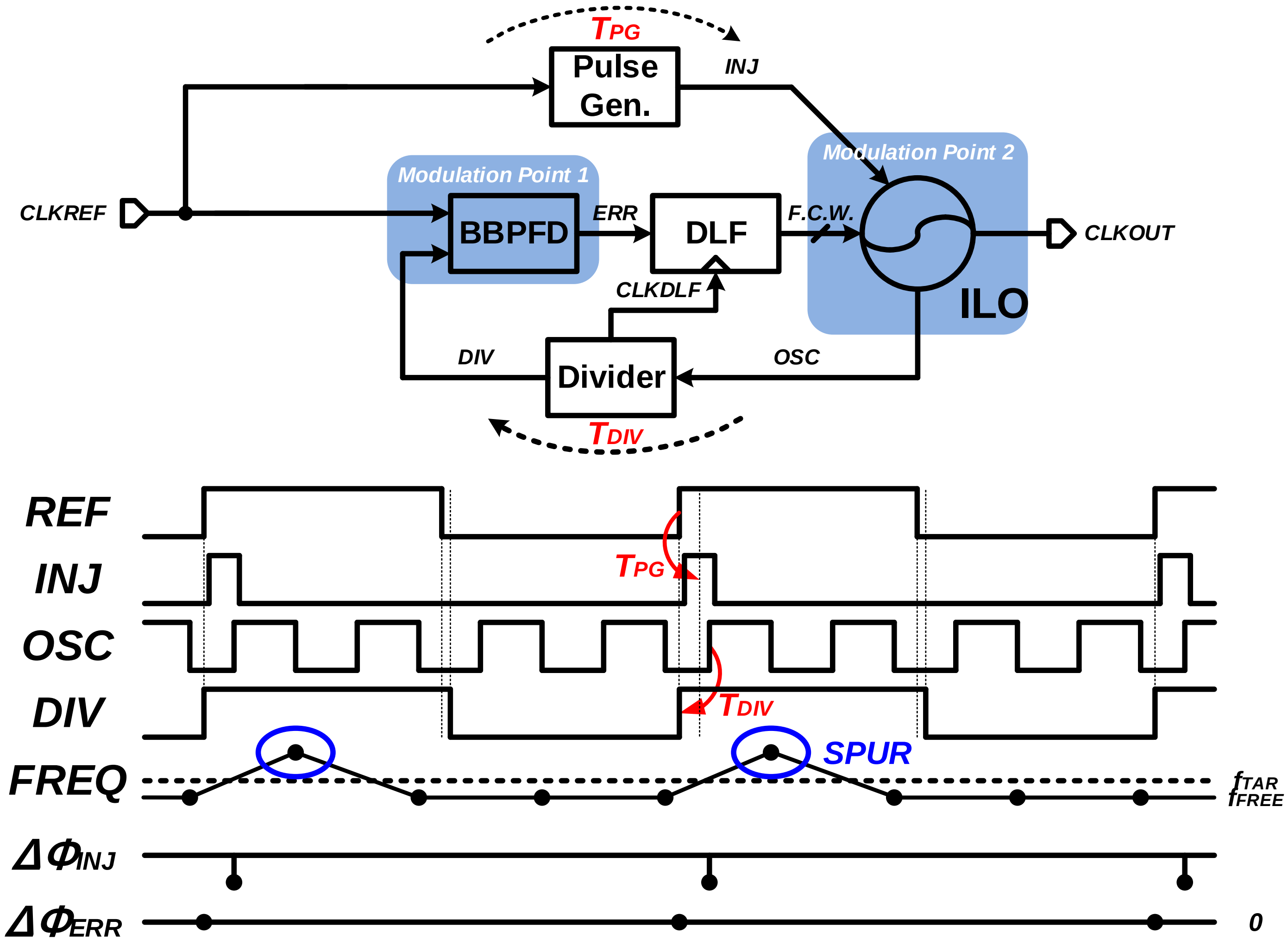


- ***Integration*** **of phase shifts**
- **Injection pulse width: large signal (D>0)**
- **Asymmetric PDR**

Tuning

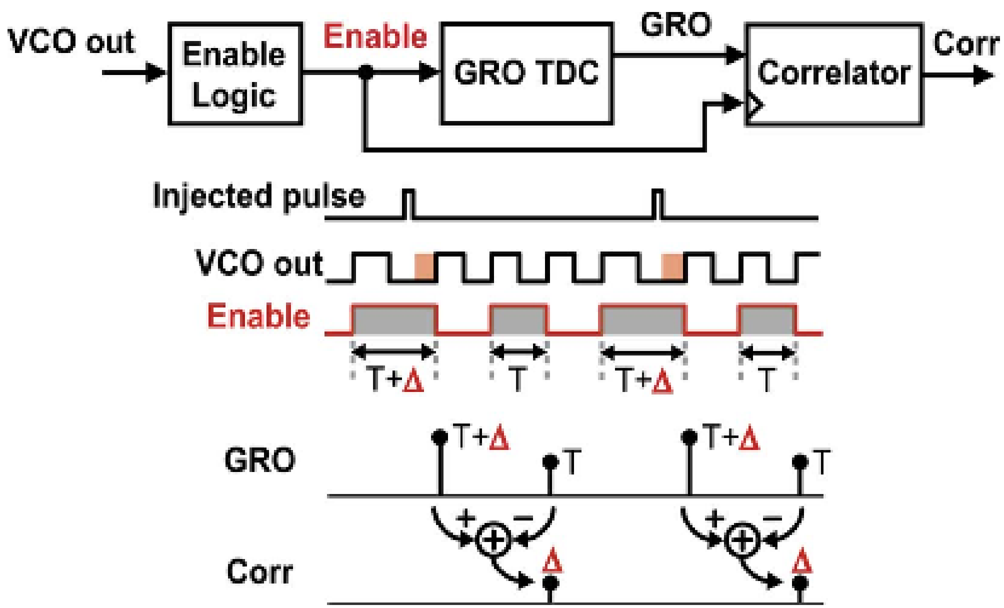
- **Tuning**
  - **Free-running frequency = injection frequency**
  - **Free-running frequency can drift**
  - **Continuous background calibration necessary**
- **Frequency Locked Loop (FLL) is necessary**
  - **Phase is determined by injected signal**
- **What if a PLL is used?**
  - **Phase is locked by two paths**
  - **Average free-running frequency with spur = injection frequency**
  - **But real free-running frequency ≠ injection frequency**

2-path phase adjustment

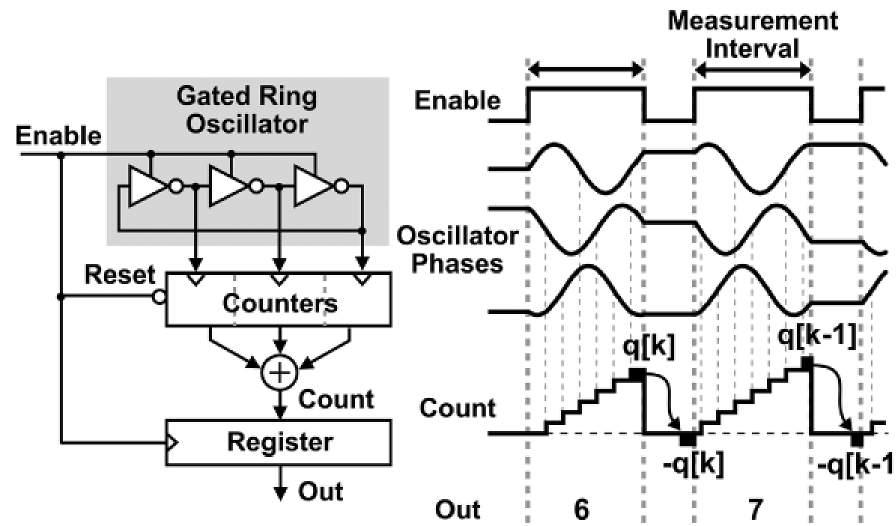


Tuning 1 – GRO TDC

[JSSC'09 B. M. Helal *et al.*]



[Overall architecture]

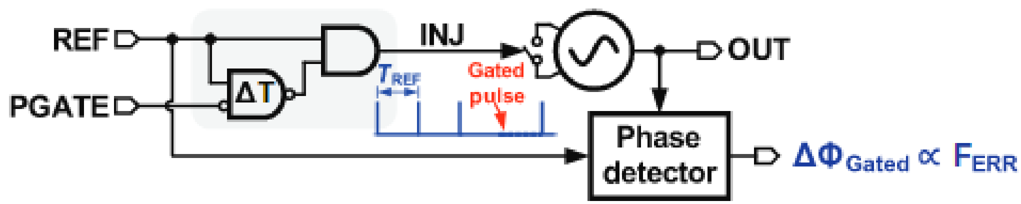


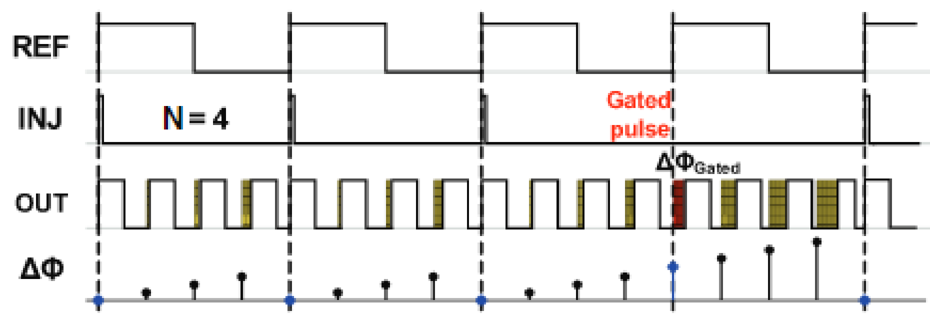
[GRO TDC]

- It can detects Δf using GRO TDC
- Only 1 point injection: no BBPD from reference

Tuning 2 – signal gating

[JSSC'13 A. Elkholy *et al.*]

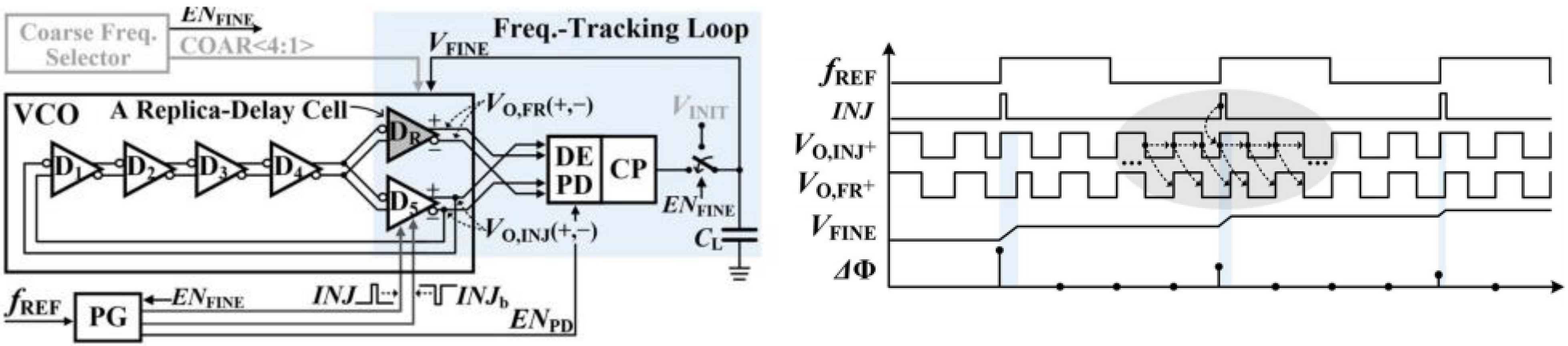




- Gating injection pulse
- When gating pulse, BBPD can detect Δf
- 2 point modulation: but calibrates two different paths
- But this method waste useful information every 1/N

Tuning 3 – replica (1)

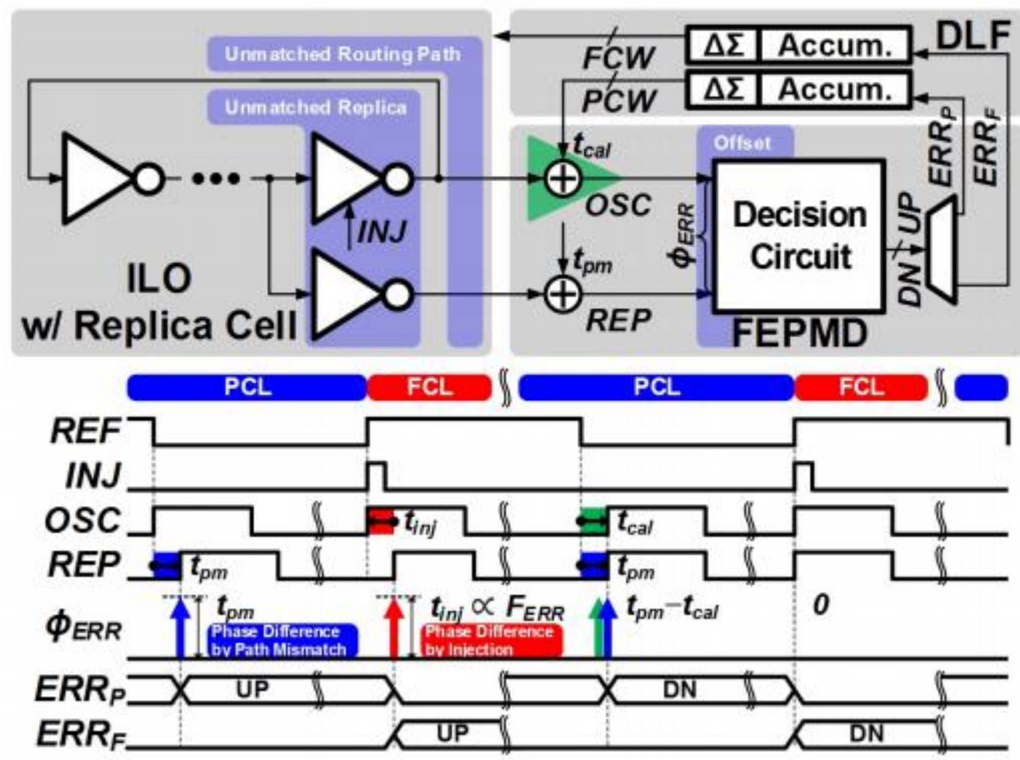
[JSSC'16 Choi *et al.*]



- **It can detects Δf using a replica delay cell**
- **Mismatch is reduced comparing replica oscillator**
- **DE-PD: comparing phase of VCO & REPLICA**
- **Only 1 point injection: no BBPD from reference**

Tuning 4 – 2-path delay matching (1)

[ISSSC'17 Kim *et al.*]

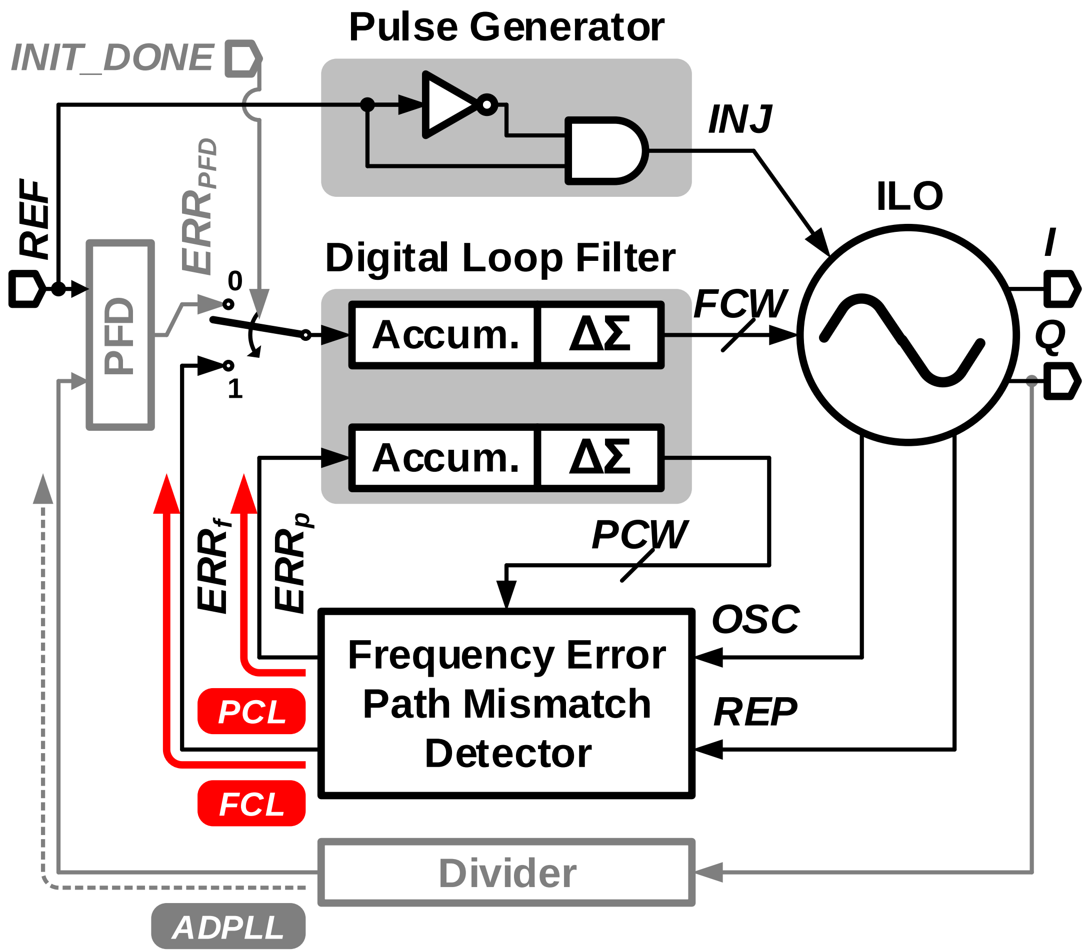


- **1 replica delay cell as in JSSC'16 Choi**
- **But this architecture calibrates mismatches using PCL**
- **Only 1 point injection: no BBPD from reference**

Tuning 4 – 2-path delay matching (2)

[ISSSC'17 Kim *et al.*]

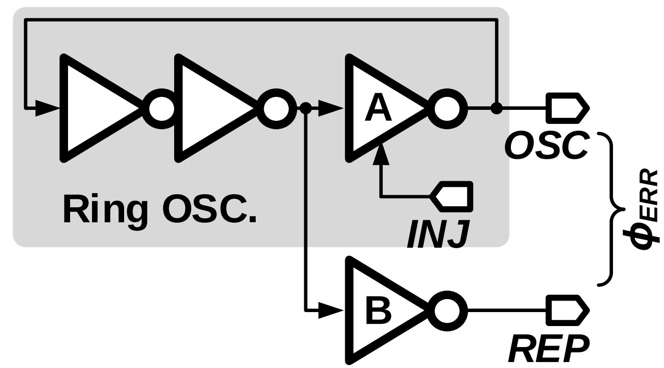
- **Architecture**

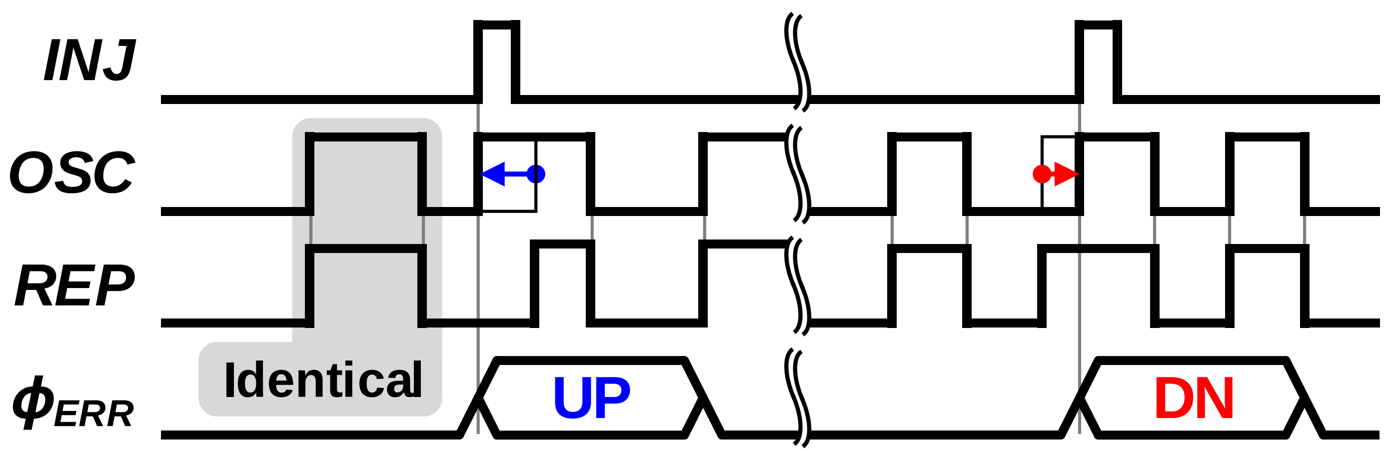


Tuning 4 – 2-path delay matching (3)

[ISSSC'17 Kim *et al.*]

- If 2 modulation paths are not matched, it is shown as
  - Large spur (PLL)
  - Increased Phase jitter

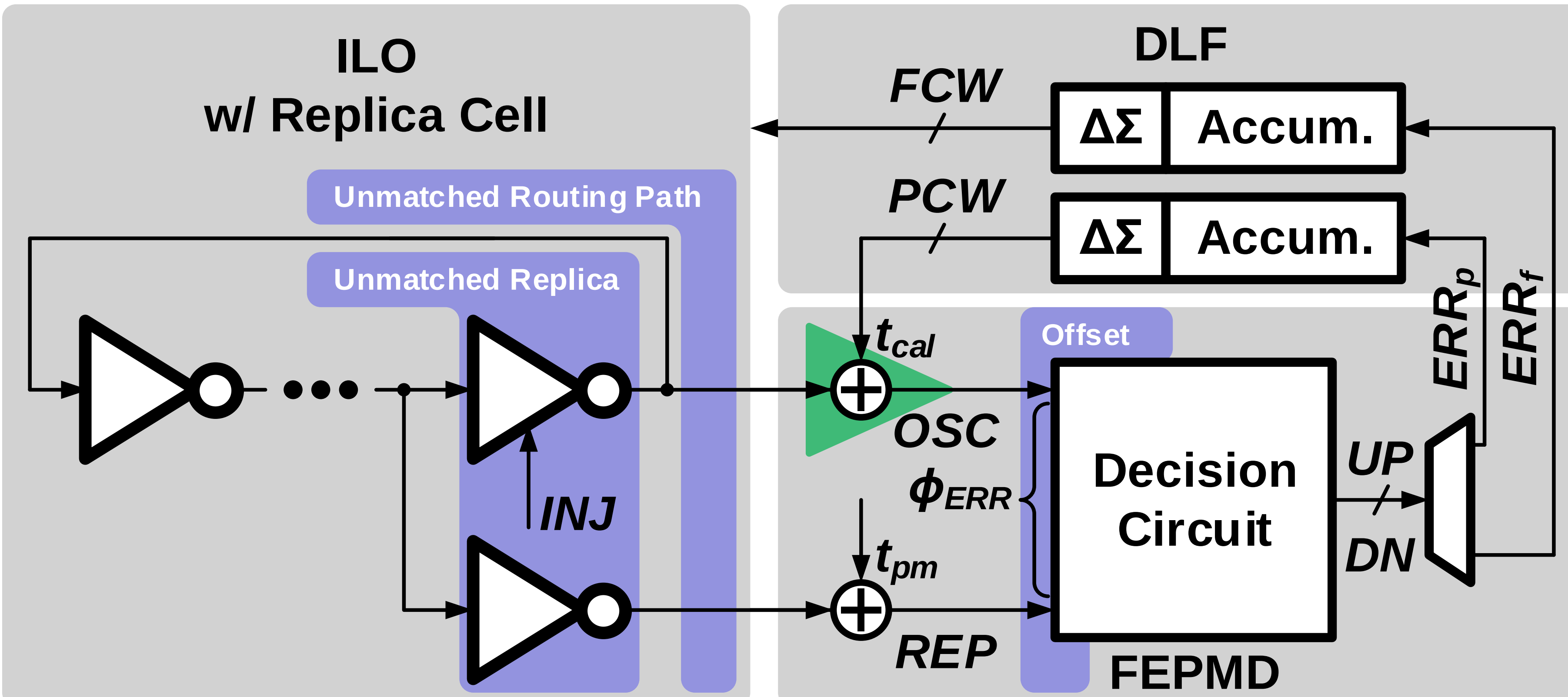




Tuning 4 – 2-path delay matching (4)

[ISSSC'17 Kim *et al.*]

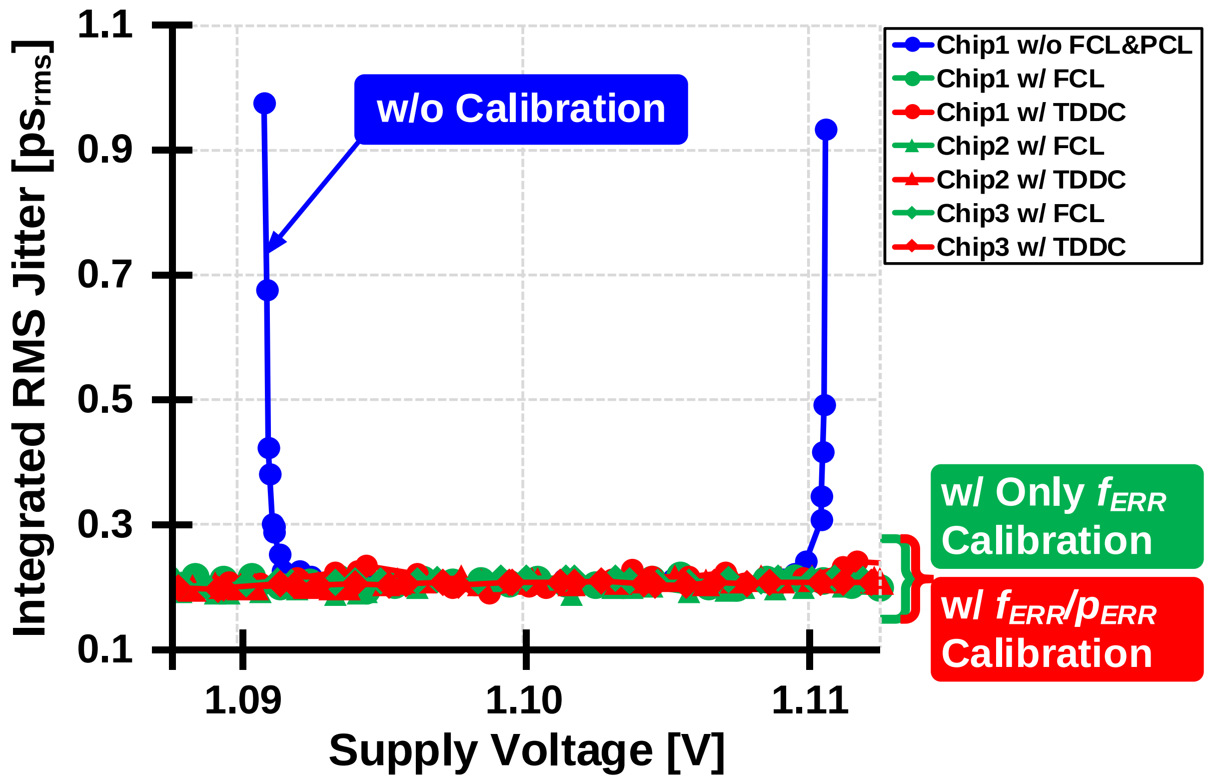
- t_{pm} represents total path mismatch error
- DLF controls t_{cal} to match t_{pm}



Tuning 4 – 2-path delay matching (5)

[ISSSC'17 Kim *et al.*]

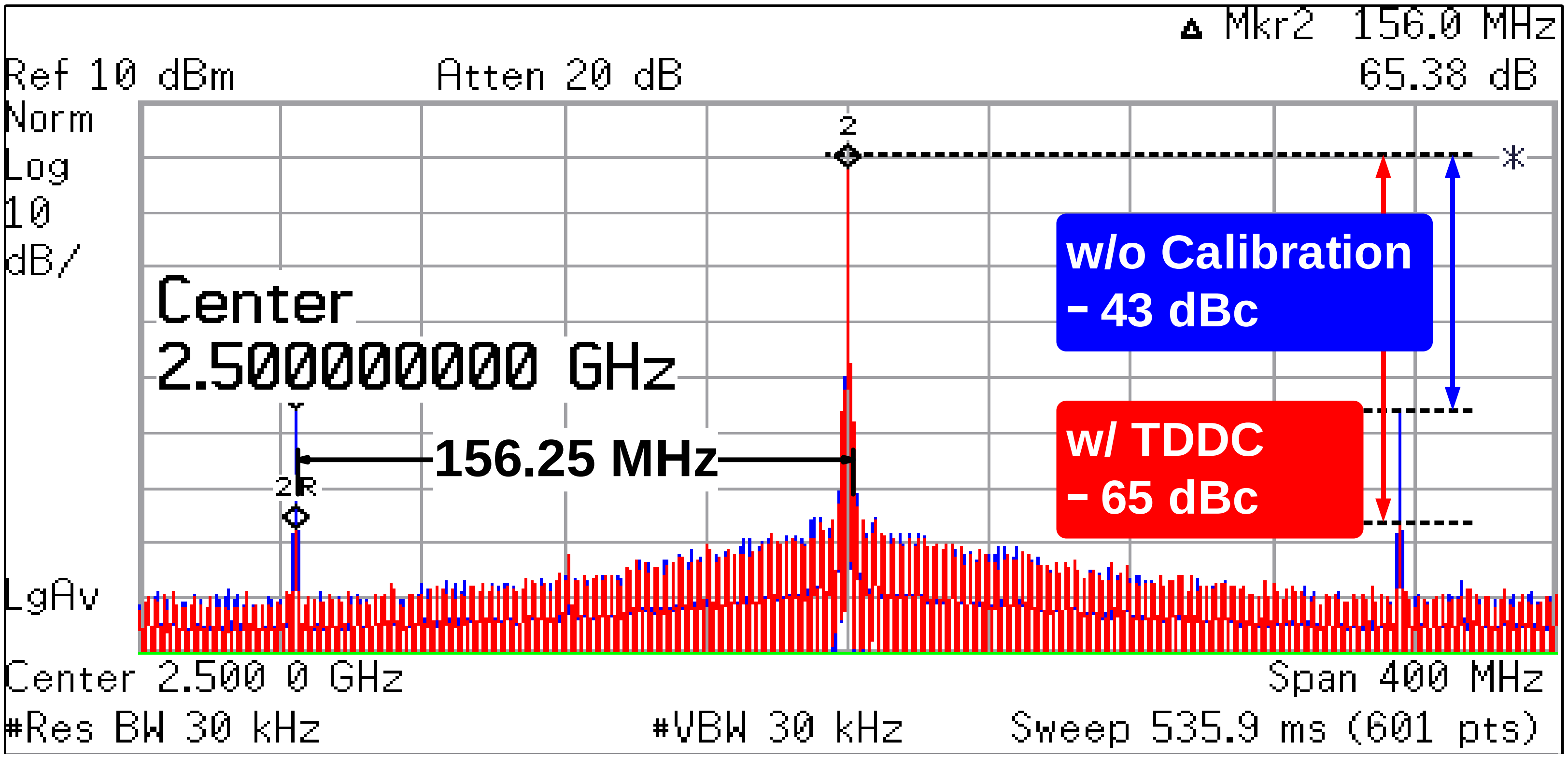
- **Measured results**



Tuning 4 – 2-path delay matching (6)

[ISSSC'17 Kim *et al.*]

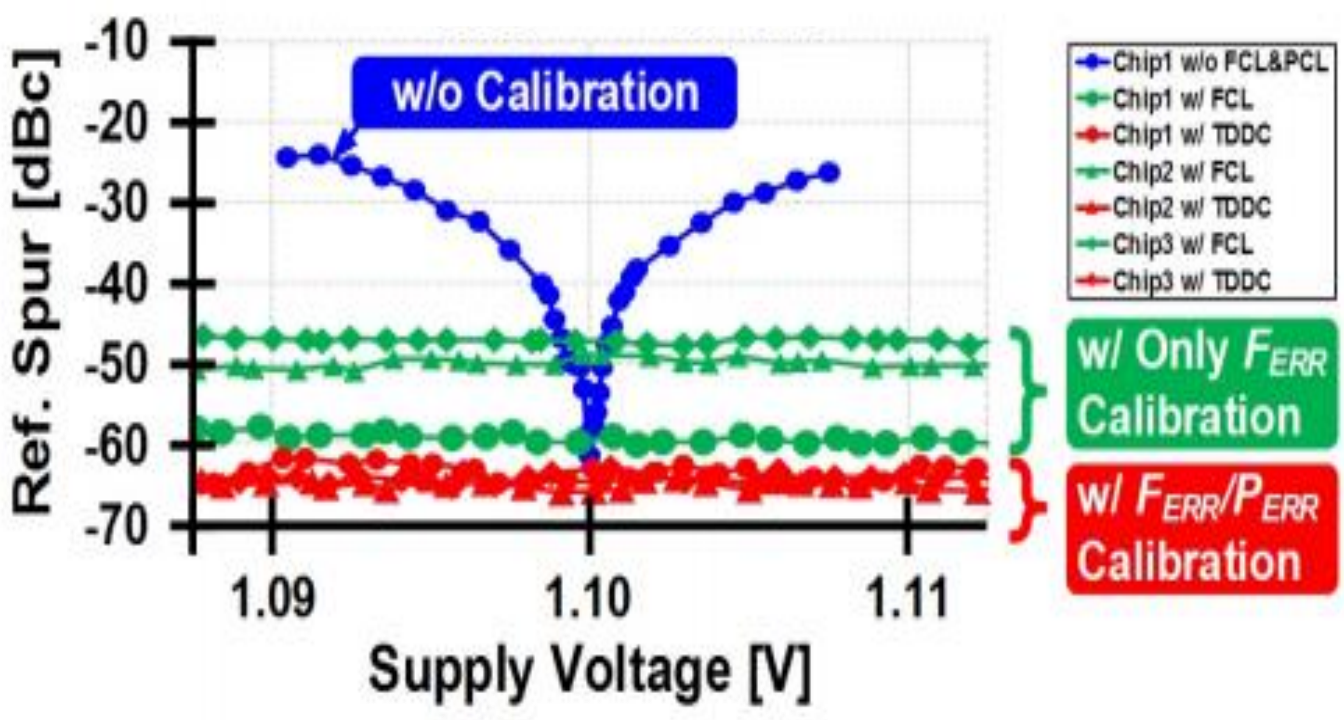
- **Measured results**



Tuning 4 – 2-path delay matching (7)

[ISSSC'17 Kim *et al.*]

- **Measured results**



References

- [4.5.1] **Ng et al.,** “A Second-Order Semidigital Clock Recovery Circuit Based on Injection Locking,” JSSC no. 12, 2003
- [4.5.2] **S. Ye et al.,** “A multiple-crystal interface PLL with VCO realignment to reduce phase noise,” *JSSC,* 2002.
- [4.5.3] **D. Dunwell et al.,** “Modeling Oscillator Injection Locking Using the Phase Domain Response,” *TCAS I,* 2013.
- [4.5.4] **Y. C. Huang et al.,** “A 2.4-GHz Subharmonically Injection-Locked PLL With Self-Calibrated Injection Timing,” *JSSC,* 2013.
- [4.5.5] **A. Elkholy et al.,** “Design and Analysis of Low-Power High-Frequency Robust Sub-Harmonic Injection-Locked Clock Multipliers,” *JSSC,* 2015.
- [4.5.6] **B. M. Helal et al.,** “A Low Jitter Programmable Clock Multiplier Based on a Pulse Injection-Locked Oscillator With a Highly-Digital Tuning Loop,” *JSSC,* 2009.
- [4.5.7] **S. Choi et al.,** “A PVT-Robust and Low-Jitter Ring-VCO-Based Injection-Locked Clock Multiplier With a Continuous Frequency-Tracking Loop Using a Replica-Delay Cell and a Dual-Edge Phase Detector,” *JSSC,* 2016.
- [4.5.8] **S. Kim *et al.*,** “A 2.5GHz injection-locked ADPLL with 197fs_{rms} integrated jitter and -65dBc reference spur using time-division dual calibration,” *ISSCC*, 2017.