(3) Ferromagnetism

▶ Below T_c, the magnetic moments within domains are aligned parallel.
Above T_c, the randomizing effect of thermal energy overcomes the aligning effect of the interaction energy,
leading to the disordered magnetic state.

▶ Theory of Ferromagnetism

(a) Localized electron theory

Weiss theory of ferromagnetism

Weiss field H_e : a very strong internal field which allows the atomic moments to couple or act cooperatively despite the strong
disordering effects of temperature below T_c.

Two important assumptions for ferromagnetism by P. Weiss : Later, both assumptions were experimentally confirmed.
(i) Spontaneous magnetization M_s without applying field H_a
(ii) Existence of magnetic domains with M_s in the demagnetized state

Interatomic interaction field (or exchange field) H_e
Suppose that any magnetic moment $m_i$experiencing an effective field H_{eij} due to another moment m_j.
If we assume that this field is also in the direction of m_j

$$H_{ei} = \mathcal{J}_{ij} m_j$$

Total exchange interaction field at the moment m_i will be the vector sum of the interactions with other moments.

$$H_{ei} = \sum_{allj} \mathcal{J}_{ij} m_j$$

(3) Ferromagnetism

(a) Localized electron theory

i) **Mean-field approximation** : useful in the paramagnetic region

If the interactions between all moments are identical and hence independent of displacement between the moments, then all of the $\mathcal{J}_{ij}$ are equal and let $\mathcal{J}_{ij} = \alpha/v$

$$\boldsymbol{H}_{ei} = \sum_{allj} \mathcal{J}_{ij} \boldsymbol{m}_j = (\alpha/v) \sum_{allj} \boldsymbol{m}_j$$

Within a domain

$$\boldsymbol{H}_e \approx \alpha \boldsymbol{M}_s$$

The interaction energy of the moment under these conditions is

$$E_e = -\mu_o \boldsymbol{m}_i \cdot \boldsymbol{H}_e$$
$$= -\mu_o \alpha \boldsymbol{m}_i \cdot \boldsymbol{M}_s$$: original formulation of the Weiss theory

ii) **Nearest-neighbor interactions** : appropriate in the ferromagnetic regime

The atomic moments interact only with those of its z nearest neighbors (fcc, $z = 6$; bcc, $z = 8$; fcc, $z = 12$)

The exchange interaction field

$$\boldsymbol{H}_e = \sum_{\text{nearest neighbors}} \mathcal{J}_{ij} \boldsymbol{m}_j$$

Assuming an identical interaction, $\mathcal{J}_{ij} = \mathcal{J}$,

where $\mathcal{J} = 0$ corresponds to the noninteracting limit described by Langevin theory.

$$\boldsymbol{H}_e = \sum_{\text{nearest neighbors}} \mathcal{J} \boldsymbol{m}_j = \mathcal{J} \sum_{\text{nearest neighbors}} \boldsymbol{m}_j$$

If $\mathcal{J} > 0$, ferromagnetic alignment

If $\mathcal{J} < 0$, antiferromagnetic alignment

(3) Ferromagnetism

(a) Localized electron theory (continued)

▶ **Classical Weiss theory** (derived from the Lagevin theory of paramagnetism)

With the external field $\boldsymbol{H}$

$E = -\mu_o \boldsymbol{m_i} \cdot (\boldsymbol{H} + \boldsymbol{H_e}) = -\mu_o \boldsymbol{m_i} \cdot (\boldsymbol{H} + \alpha \boldsymbol{M_s})$

$M_s/M_o = \coth a - \frac{1}{a}$ where $a = \mu_o m(H + \alpha M_s)/kT$

Without the external field,

$\boldsymbol{H}_{\mathbf{tot}} = \boldsymbol{H}_{\mathbf{e}} = \alpha \boldsymbol{M}_{\mathbf{s}}$, $M_s/M_o = \coth a - \frac{1}{a}$ where $a = \mu_o m \alpha M_s/kT$

The above equation leads to perfect alignment of magnetic moments within a domain as $T \rightarrow 0$ K.

As T increases, $\boldsymbol{M_s}$ decreases.

▶ **Quantum theory** (derived from the quantum theory of paramagnetism)

$E = -\mu_o \boldsymbol{m_i} \cdot (\boldsymbol{H} + \alpha \boldsymbol{M})$

$\alpha \boldsymbol{M}$ represents the interaction of the atomic moment with other moments due to the electrons on neighboring atoms interacting with one another.

Assuming $J = S = 1/2$,

$B_J(x) = \tanh x$ where $x = \mu_o \mu_B (H + \alpha M_s)/kT$

$M = N\mu_B B_J(x)$

For multielectron atoms

$M = NgJ\mu_B B_J(x)$, $x = \mu_o gJ\mu_B (H + \alpha M_s)/kT$

(3) Ferromagnetism

▶ Temperature dependence of the saturation magnetization within a domain

Ferromagnetic Brillouin function in the absence of a magnetic field

$M_s = NgJ\mu_B B_J(x),\ \ x = \mu_o gJ\mu_B \alpha M_s/kT$

In the case of $S = 1/2$ (e.g., Ni)

$M_s = N\mu_B \tanh(\mu_o \mu_B \alpha M_s/kT)$

- Graphical solution : Fig. 3.19 (O'Handley)
  For Ni, the intersection of the two theoretical curves in Fig. 3.19: Fig. 3.20 (O'Handley)
- M_s is only weakly dependent on T below $0.75T_c$, and above that it decreases rapidly towards zero at T_c

▶ The Curie-Weiss law

At high temperature ($T > T_c$), M will be uniform throughout the material

Thus, $M = N_v\mu_o g^2\mu_B{}^2 J(J+1)(H + \alpha M)/3kT$, $\chi = M/H = C/(T - T_c)$

where $C = N_v\mu_o g^2\mu_B{}^2 J(J+1)$, $\quad T_c = \alpha N\mu_o g^2\mu_{B2} J(J+1)/3k$

cf) $\chi \propto 1/(T - T_c)^{1.33}$: detailed calculation at temperatures very close to near T_c

- In ferromagnetic regime ($T < T_c$), the magnetization M will not be uniform throughout the solid. The atomic moments will couple to the M_s (spontaneous magnetization within the domain) rather than the bulk magnetization M in the ferromagnetic state, and hence a nearest-neighbor model, with possible extension to next and higher-order neighbors, is more appropriate.

- Exchange coupling in magnetic insulators: The electrons on the metallic ions can be coupled together via their interactions with electrons on the nonmetallic ion, which is known as *superexchange*.

(3) Ferromagnetism

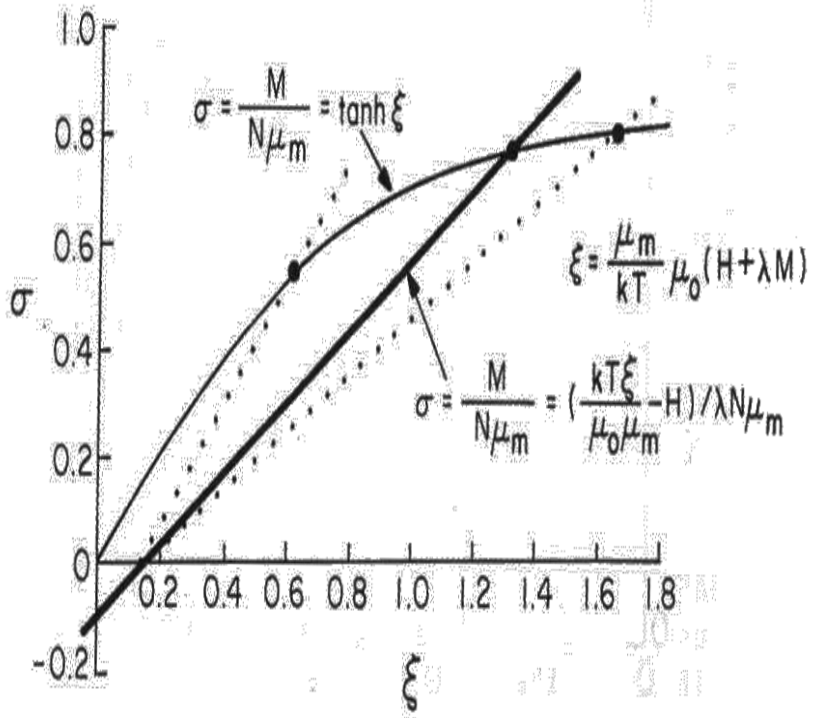

Figure 3.19 Construction for the solution, $\sigma(\xi)$, of the transcendental equation (4.25). The linear curve is shown for three different temperatures. The intersection of the linear and tanh curves is the solution for the temperature dependence of magnetization. Here the offset of the linear curve has been exaggerated to show the effect of an external field.

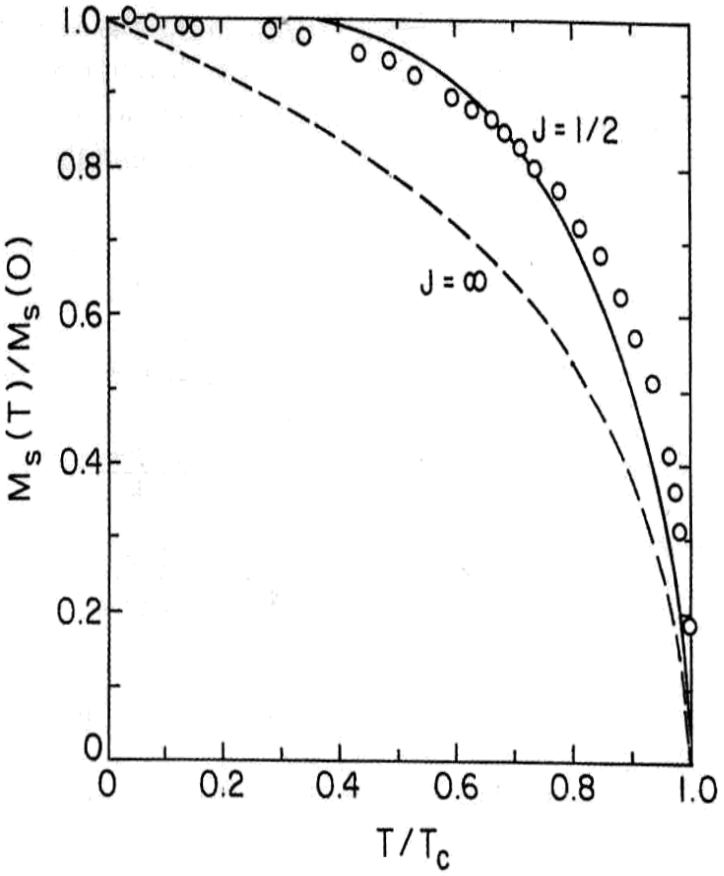

Figure 3.20 Reduced magnetization versus reduced temperature for nickel [open data points, from Weiss and Forrer (1926) and Brillouin function, $B_{1/2}(x)$ (solid line]. The dashed line is the classical solution for $J = \infty$.

(3) Ferromagnetism

(b) The Itinerant Electron Theory

▶Key points

- Local moment model: Quite realistic for the lanthanides with their closely bound 4f electrons which determine the magnetic properties of the solid. Not realistic for the ferromagnets of the 3*d* series of which magnetic electrons are outer electrons which are relatively free to move through the solid and thus *itinerant*.
- In metallic ferromagnets such as Fe, Co, Ni, the magnetic properties are due principally to the conduction electrons. *Itinerant exchange* between these electrons.

▶Band Theory of Ferromagnetism

- A simple extension of the band theory of paramagnetism by the introduction of an exchange coupling between the electrons.
- Source of magnetic moments : unpaired electrons (if paired, $L = S = 0$)
- In partially filled energy band, an imbalance of spins leads to a net magnetic moment per atom.

Magnetic properties of 3d band electrons : rigid band model

Let n = number of 3d + 4s electrons per atom

x = number of 4s electrons per atom

$n - x$ = the number of 3d electrons per atom

Then, $\mu_{tot} = [5 - (n - x - 5)]\mu_B = [10 - (n - x)]\mu_B$ since $(n - x)\uparrow = 5$ and $(n - x)\downarrow = (n - x - 5)$

Assuming $x = 0.6$, $\mu_{tot} = (10.6 - n)\mu_B$

(3) Ferromagnetism

(b) The Itinerant Electron Theory (continued)

The Slater-Pauling curve (see Fig. 5.1 in O'Handley)

Magnetic moments per atom of 3d metals and their alloys (from Mn to Cu) from the premises of the itinerant electron theory.

- Interpretation: in terms of the rigid band model.
- Max. moments : at a point between Fe and Co.
- 3d and 4s electrons are responsible for the magnetic properties of these metals and alloys.