(2) Magnetoelastic Effects

❑ **The dimension of a ferromagnetic material changes when it is magnetized. The resulting strain is called, the magnetostriction *λ***

❑ Observations

- Isotropic effect : ***volume magnetostriction***

- When the magnetic ordering is produced by an applied field, they are called ***forced magnetostriction***.

- On a smaller scale, the volume expansion can show an anisotropy for $T < T_c$, that is, the linear strain is different in different directions relative to the direction of magnetization.

- The magnetization vector ***M*** is associated with a stress which causes a mechanical deformation of the material.

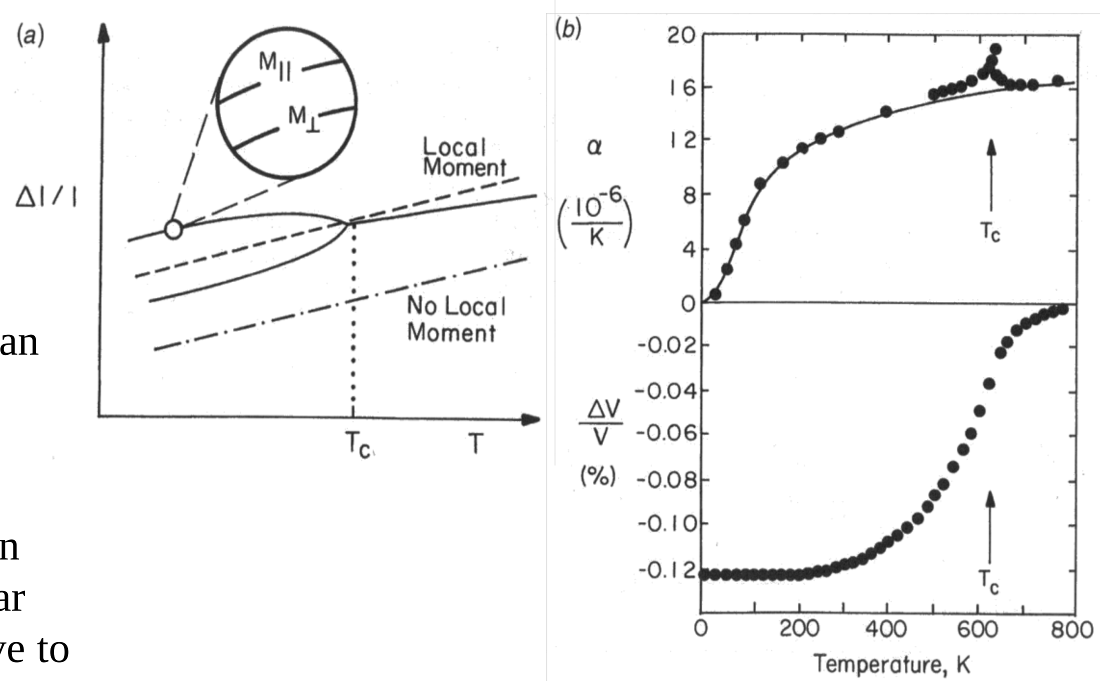

Figure 7.1 (*a*) Schematic of the thermal expansion of a magnetic material as a function of temperature illustrating the increased volume due the presence of a local magnetic moment and the onset of magnetic anomalies below the Curie temperature; (*b*) volume expansion and thermal expansion coefficient of Ni (Kollie 1977). A small anisotropic strain, depending on the direction of magnetization (circled inset, left), is also observed below T_C. The latter is usually referred to as anisotropic magnetostriction.

(2) Magnetoelastic Effects (continued)

▶ **Joule (or Anisotropic) Magnetostriction,** $\lambda = \triangle l/l$

The anisotropic strain associated with the direction of magnetization was first observed in iron by Joule on 1842 yr.

- Field dependence of anisotropic strain (see Fig. 7.2 in O'Handley) for strain measured
parallel to the field, $e_{//} = (\triangle l/l)_{//}$
perpendicular to the field, $e_{\perp} = (\triangle l/l)_{\perp}$

- λ **ranging from zero (<** 10^{-7}**)**
to nearly ± 10^{-4} **in 3*d* metals and alloys**
to over 10^{-3} **in some 4*f* metals, intermetallic compounds, and alloys.**

▶ **Two Ways in describing Anisotropic Magnetostriction**

- Saturation magnetostriction, λ_s **:**
the strain produced at magnetic saturation

- Magnetoelastic coupling coefficient, B_{ij} **:**
the magnetic stress causing λ_s

▶ **The Magnetic Stress Tensor, called Magnetoelastic Coupling Coefficient** B_{ij}

- The components B_{ij}**, can be related to its magnetostrictive strains by a analogy with Hooke's law:**

$$B_{ij} \propto -c_{ijkl}\lambda_{kl}$$

For Ni, $B_1 = 6.2$**MPa, Young's modulus,** $E = 200$**GPA,**
$\lambda = 30\times10^{-6}$

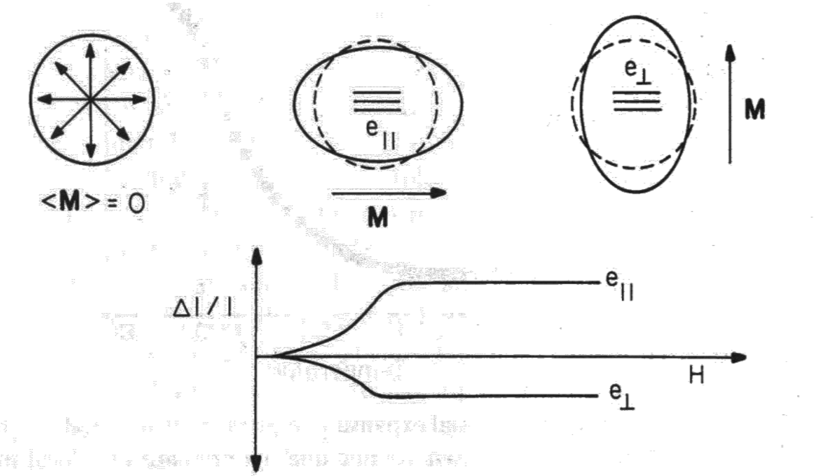

Figure 7.2 When a demagnetized sample has its magnetization aligned by an external field, the sample strains anisotropically. The direction in which the strain is measured in the samples above is indicated by the three parallel lines. The strain in the direction of magnetization will be opposite in sign to the strain perpendicular to the direction of magnetization. Notice that $e_{||}$ and $e_{\perp}$ need not be related by Poisson's ratio because of ;the arbitrariness of the zero-field magnetization configuration, which defines the zero-field strain. The strains depicted above are those for a material with a positive magnetostriction constant: $(\Delta l/l)_{||} > 0$.

(2) Magnetoelastic Effects (continued)

▶ The Inverse Effect

- Stressing or straining a magnetic material
→ a change in its preferred magnetization direction can be produced. (see Fig. 7.3 in O'Handley): inverse Joule effects, Villari effects, piezomagnetism, or stress-induced anisotropy.
- If $\lambda_s > 0$, it is easier to magnetize a material in the tensile stress ($\sigma > 0$) direction.
- It is harder to magnetize a material in a direction for which $\lambda_s < 0$ and $\sigma > 0$ or for which $\lambda_s > 0$ and $\sigma < 0$.

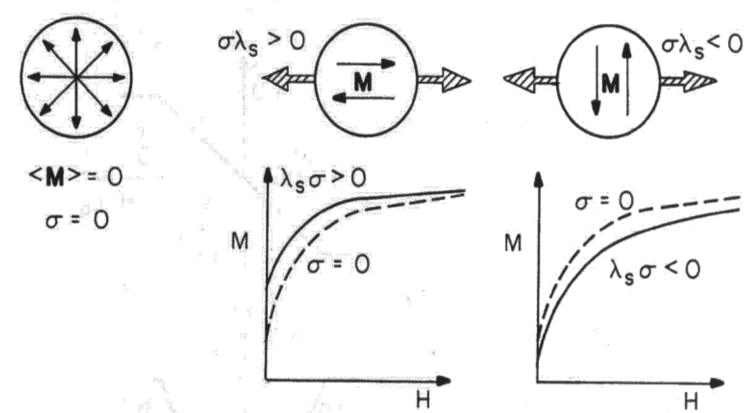

Figure 7.3 Imposing a strain on a magnetic material by a mechanical stress alters the preferred direction of magnetization (magnetic anisotropy) and thus changes the shape of the M–B_0 curve below saturation. The cases above illustrate the changes observed for an isotropic material. When the product $\lambda_s\sigma$ is positive, the magnetization is favored along that stress axis (shaded arrows).

▶ Torsional Effects

- A current passing through a magnetic material in the direction of ***M*** causes a twisting of the magnetization around the current axis.
- If $\lambda_s \neq 0$, a torsional motion of the sample occurs: ***Wiedemann effect***.
- ***Inverse Wiedemann effect***, named after Matteucci: a mechanical twisting of the sample causes a voltage to appear along the sample length, consistent with Faraday's law and the strain-induced magnetization change.
- The existence of anisotropic magnetoelastic(ME) effects → the existence of a coupling between the magnetization direction and mechanical strains.

(2) Magnetoelastic Effects (continued)

❑ Two Main Types of Magnetostriction

- Spontaneous Magnetostriction λ_s : arising from the ordering of magnetic moments into domains below T_c
- Field-induced Magnetostriction λ : arising from the reorientation of λ_s under the action of a magnetic field

In both cases, $\lambda = \triangle l/l$

▶ Spontaneous magnetostriction λ_o in isotropic materials (see Fig. 5.6 in Jiles)

Consider spherical volumes of unstrained solid above T_c (in the disordered phase) for a isotropic material.

- Below T_c, spontaneous magnetization appears within the domains and associated spontaneous strain e (or magnetostriction λ_o)

$$e(\theta) = e\cos^2\theta$$

where θ is the angle between the strain measuring direction and the direction of spontaneous magnetization

The average deformation throughout the solid, assuming randomly oriented domains

$$\lambda_o = \pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e\cos^2\theta \sin\theta d\theta = \frac{e}{3}$$

→ No change in shape although the sample changes in dimensions.

▶ Saturation Magnetostriction λ_s

The maximum fractional change in length from a demagnetized to a saturated magnetization state along the magnetic field direction,

$$\lambda_s = e - \lambda_o = 2e/3$$

λ_s can be increased even after the magnetization has reached technical saturation by a forced magnetization.

(2) Magnetoelastic Effects (continued)

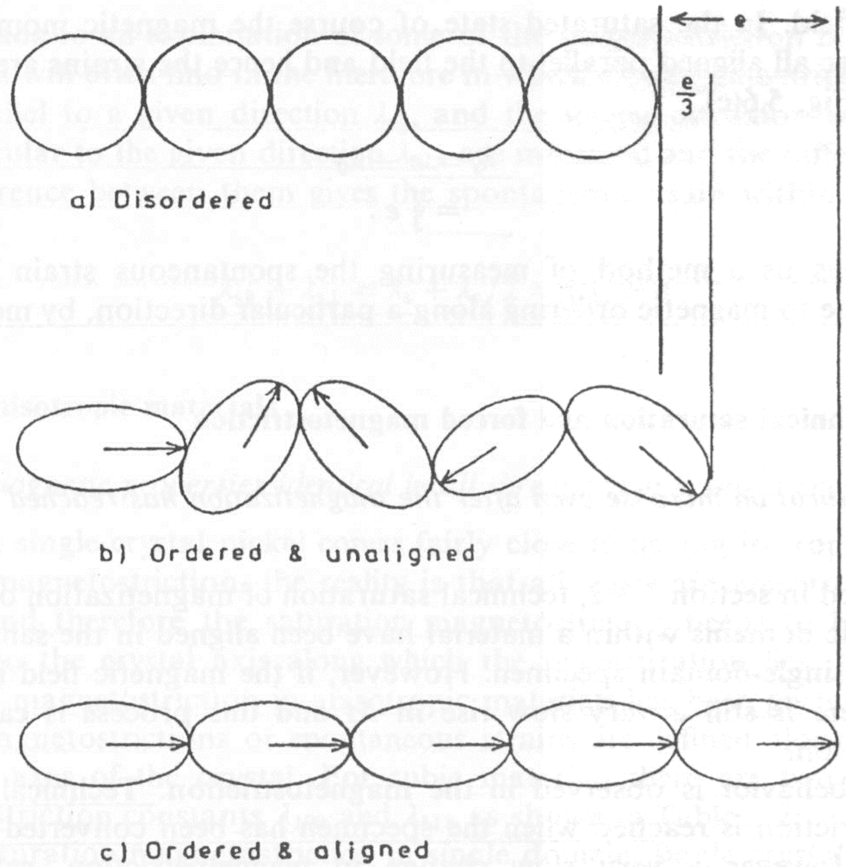

Fig. 5.6 Schematic diagram illustrating the magnetostriction in: (a) the disordered (paramagnetic) regime; (b) the ferromagnetic regime demagnetized; (c) the ferromagnetic regime, magnetized to saturation.

(2) Magnetoelastic Effects (continued)

Field Dependence of Joule Magnetostriction

The *anisotropic magnetostrictive strain e* (sometimes called, *magnetostriction* λ_p) relative to the direction of magnetization (see Fig 7.2 in O'Handley)

▶ For an isotropic material

(see Fig 7.4 in O'Handley)

$$e = \lambda_0 = (3/2)\lambda_s(\cos^2\theta - 1/3)$$

where $e = \triangle l/l$ is the strain measured at an angle θ relative to the saturation magnetization direction, and the saturation magnetostriction coefficient λ_s is a measure of the strain on changing the direction of magnetization in the material.

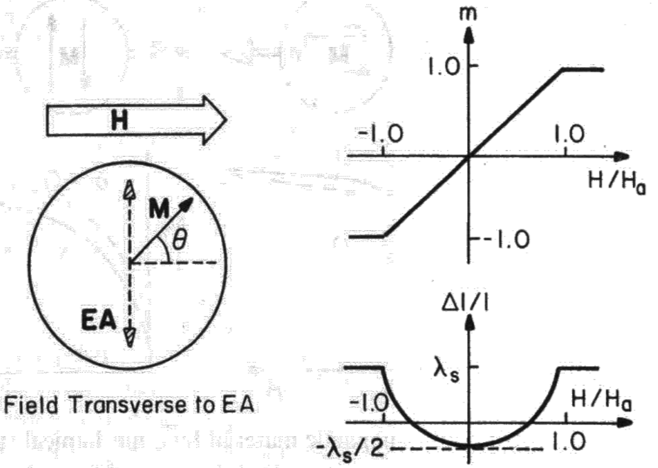

Figure 7.4 Application of a magnetic field perpendicular to the easy axis (EA) of a material causes a rotation of the magnetization direction and results in a linear M–H characteristic and a quadratic dependence of strain on field.

- For the hard-axis magnetization process in a first-order uniaxial material,

$$M = M_s H/H_a \quad \text{or} \quad m(= M/M_s) = h(= H/H_a).$$

$$e = \lambda_s(m^2 - 1/3) \rightarrow e \propto H^2$$

Above saturation,

$$e_{//} = (3/2)\lambda_s(1 - 1/3) = \lambda_s, \qquad e_{\perp} = \lambda_s(0 - 1/3) = -1/2\lambda_s$$

(longitudinal magnetostriction) (transverse magnetostriction)

Hence, for isotropic materials, $e_{//} - e_{\perp} = 3/2\lambda_s$ or $\lambda_s = 2/3(e_{//} - e_{\perp})$

(2) Magnetoelastic Effects (continued)

- The magnetostrictive strain of a material with a 180° domain wall (see Fig. 7.5 in O'Handley)

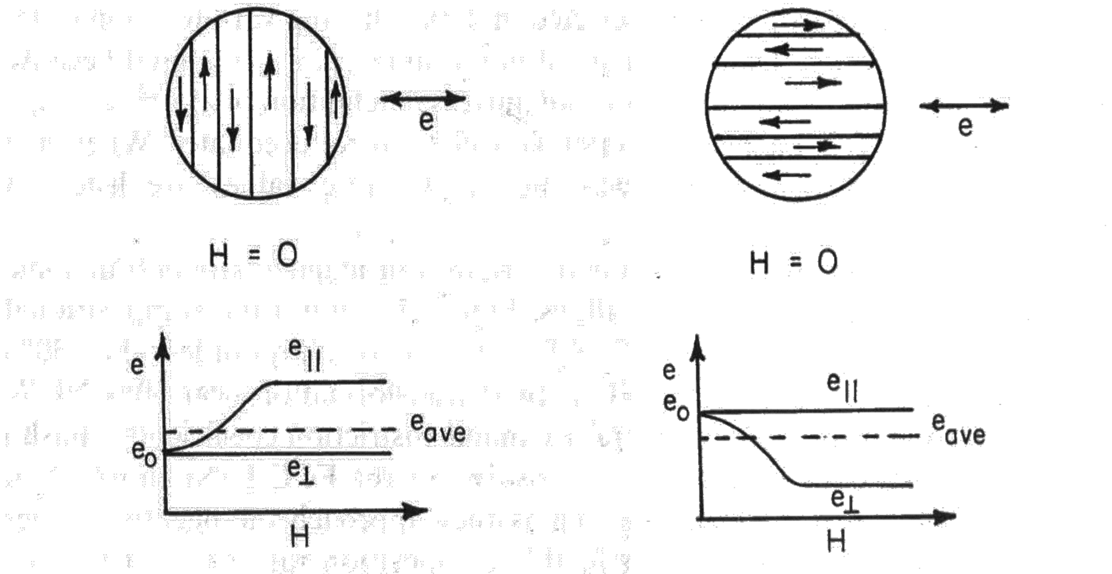

Figure 7.5 Above, schematic illustration of two demagnetized states that have different orientations relative to the strain sensing direction (indicated by *e*); below, field dependence of strain for each case in the presence of fields applied either parallel ($e_{||}$) or perpendicular ($e_\perp$) to the strain-sensing direction.

A 180° domain wall motion does not produce any magnetostrictive change in dimensions.

- Some data (see Table 7.1, Fig. 7.6-9 in O'Handley)

- Surface Magnetostriction (see Chap 16 in O'Handley)

(2) Magnetoelastic Effects (continued)

▶ **For anisotropic materials**

A general equation for the saturation magnetostriction in a single domain, single cubic crystal in a direction defined by the cosines β_1, β_2, β_3, relative to the crystal axes, when it is magnetized from the demagnetized sate to saturation in a direction defined by α_1, α_2, α_3:

$$\lambda_s = (3/2)\lambda_{100}(\alpha_1^2\beta_1^2 + \alpha_2^2\beta_2^2 + \alpha_3^2\beta_3^2 - 1/3)$$
$$+ 3\lambda_{111}(\alpha_1\alpha_2\beta_1\beta_2 + \alpha_2\alpha_3\beta_2\beta_3 + \alpha_3\alpha_1\beta_3\beta_1)$$

TABLE 7.1 Magnetostriction Constants λ_{100} and λ_{111} ($\times 10^6$) at 4.2 K and Room Temperature for Several Materials[a]

	$T = 4.2\,K$		Room Temperature		
	$\lambda_{100}(\lambda^{\gamma,2})$	$\lambda_{111}(\lambda^{\varepsilon,2})$	$\lambda_{100}(\lambda^{\gamma,2})$	$\lambda_{111}(\lambda^{\varepsilon,2})$	Polycrystal λ_s
		3d Metals			
BCC-Fe	26	−30	21	−21	−7
HCP-Co[a]	(−150)	(45)	(−140)	(50)	(−62)
FCC-Ni	−60	−35	−46	−24	−34
BCC-FeCo	—	—	140	30	—
a-$Fe_{80}B_{20}$	48 (isotropic)	—	—	—	+32
a-$Fe_{40}Ni_{40}B_{20}$	+20	—	—	—	+14
a-$Cos_{80}B_{20}$	−4	—	—	—	−4
		4f Metals/Alloys			
Gd[a]	(−175)	(105)	(−10)	0	—
Tb[a]	—	(8700)	—	(30)	—
$TbFe_2$	—	4400	—	2600	1753
$Tb_{0.3}Dy_{0.7}Fe_2$	—	—	—	1600	1200
		Spinel Ferrites			
Fe_3O_4	0	50	−15	56	+40
$MnFe_2O_4$[a]	—	—	(−54)	(10)	—
$CoFe_2O_4$	—	—	−670	120	−110
		Garnets			
YIG	−0.6	−2.5	−1.4	−1.6	−2
		Hard Magnets			
$Fe_{14}Nd_2B$[a]	—	—	—	—	—
$BaO \cdot 6Fe_3O_4$[a]	—	—	(13)	—	—

(2) Magnetoelastic Effects (continued)

- The spontaneous strains along crystal axes for $T < T_c$:

$$e_{111} = 3/2\lambda_{111} \text{ and } e_{100} = 3/2\lambda_{100}$$

From the above equation,

(a) it is possible to calculate the dimensional change of a single domain due to a rotation of its $\boldsymbol{M}_s$ vector out of the easy axis.

(b) it is possible to circumvent, in magnetostriction measurements, the uncertainty about the demagnetized state.

Ex) (see Fig. 8.3-4 in Cullity)

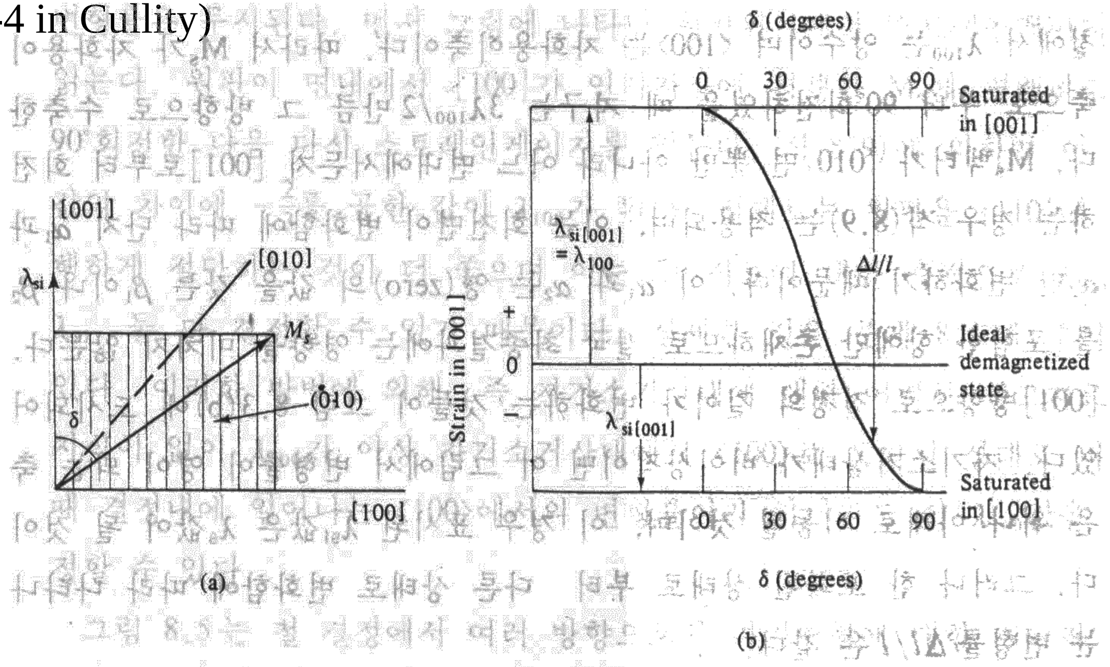

Fig. 8.3 Magnetostrictive strains in a cubic crystal for which λ_{100} is positive

(2) Magnetoelastic Effects (continued)

$$\lambda_s = (3/2)\lambda_{100}(\alpha_1^2\beta_1^2 + \alpha_2^2\beta_2^2 + \alpha_3^2\beta_3^2 - 1/3) + 3\lambda_{111}(\alpha_1\alpha_2\beta_1\beta_2 + \alpha_2\alpha_3\beta_2\beta_3 + \alpha_3\alpha_1\beta_3\beta_1)$$

- The magnetostriction of an isotropic material, $\lambda_\theta = (3/2)\,\lambda_s(\cos^2\theta - 1/3)$ can be derived from the above equation by putting $\lambda_{100} = \lambda_{111} = \lambda_s$

- The saturation magnetization in the same direction as the field (*i.e.*, saturation magnetization), since $\alpha_1 = \beta_1$, $\alpha_2 = \beta_2$, and $\alpha_3 = \beta_3$,

$$\lambda_s = \lambda_{100} + 3(\lambda_{111} - \lambda_{100})(\alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^2 + \alpha_3^2\alpha_1^2)$$

- Experimental curves for magnetostriction in various directions in a Fe crystal(see Fig 8.5 in Cullity) and in a Ni crystal (see Fig. 8.6 in Cullity)

- For a polycrystalline sample, $\lambda_s = 2/5\lambda_{100} + 3/5\lambda_{111}$ (proof: homework)
  The magnetostriction at an angle θ to the magnetization with no preferred orientation (or magnetically isotropic),

$$\lambda_\theta = 3/2\lambda_s(\cos^2\theta - 1/3)$$

- Experimental curves for polycrystalline samples (see Fig. 8.13 in Cullity or Fig. 5.7 in Jiles)

(2) Magnetoelastic Effects (continued)

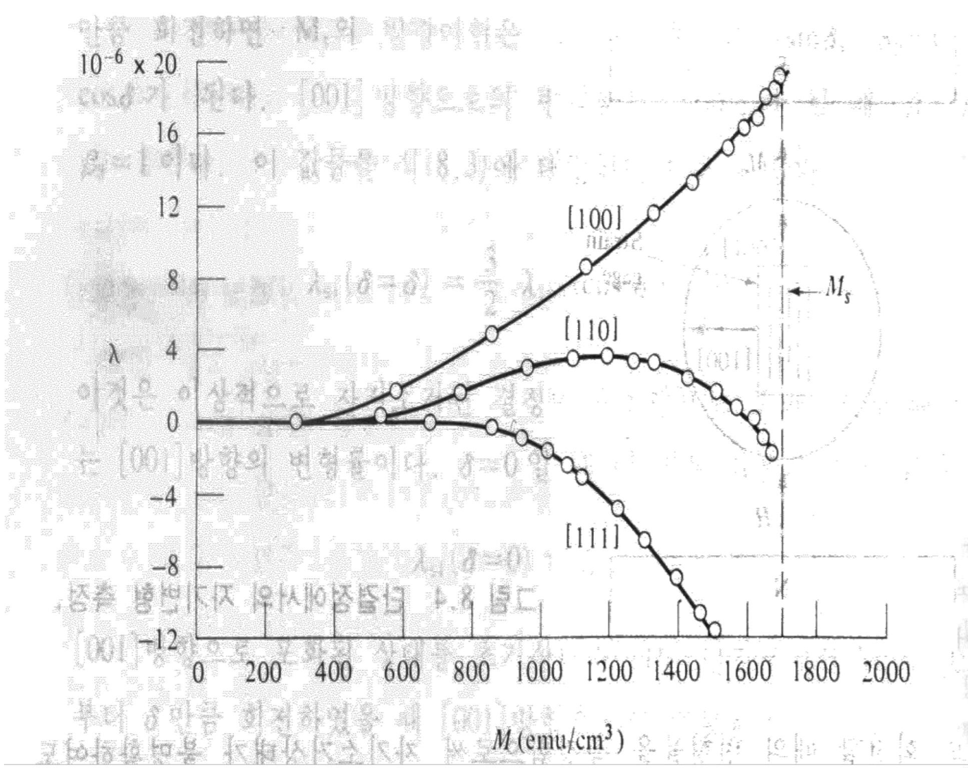

Fig. 8.5 Magnetostriction as a function of magnetization of iron single crystal in the form of rods cut parallel to the principle crystal directions. Webster [8.14].

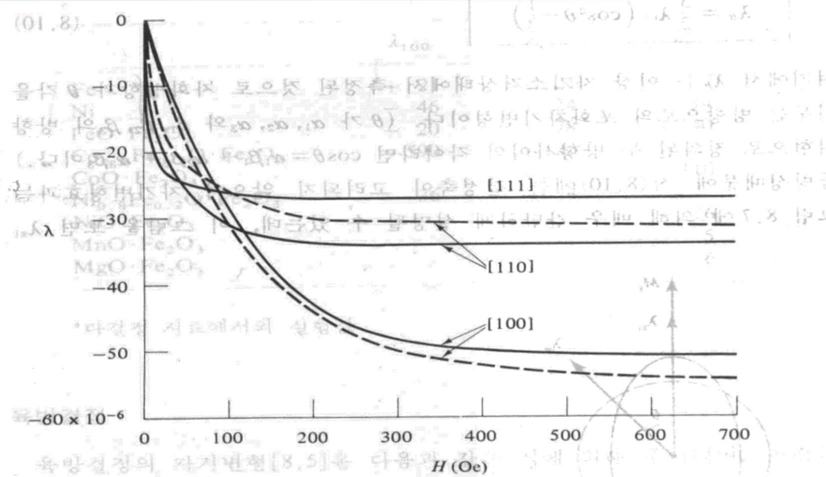

Fig. 8.6 Magnetostriction as a function of field and crystal direction for nickel single crystals in the form of oblate spheroids (disks) cut parallel to the (110) plane (solid curves) and the (100) plane (dashed curves). Masiyama [8.15].

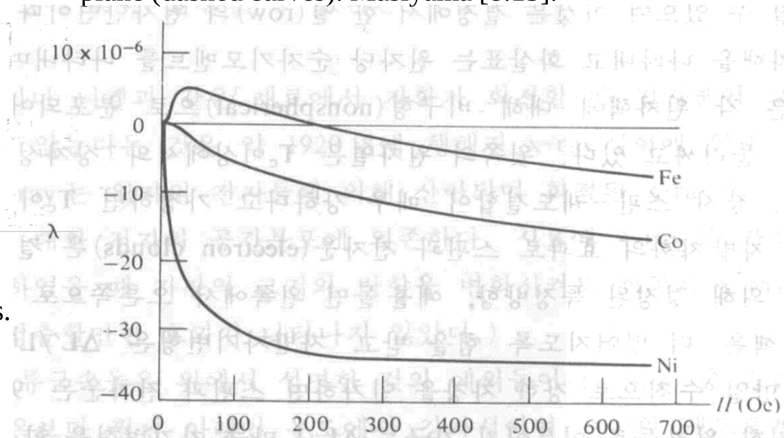

Fig. 8.13 Magnetostriction of polycrystalline iron, cobalt, and nickel. After Lee [8.1].

(2) Magnetoelastic Effects (continued)

❑ Phenomenology

▶ The magnetization energy u_m, expanded in a Maclaurin series

- A coupling between the magnetization direction and mechanical deformations in a material, which depends on the direction of magnetization relative to the crystal axes.

$$u_m = U_m/V_o$$
$$= f_o + K_1\alpha_i^2\alpha_j^2 + K_2\alpha_i^2\alpha_j^2\alpha_k^2 + \dots$$

(dependent only on the direction of magnetization)

$$+ c_{ijkl}e_{ij}e_{kl} + H_{ijkmnl}e_{ij}e_{kl}e_{mn} + \dots$$

(pure elastic energy and the strain dependence of the c terms)

$$+ B_{ij}e_{ij}\alpha_i\alpha_j + \dots + D_{ijkl}e_{ij}e_{kl}\alpha_i\alpha_j\alpha_k\alpha_l + \dots$$

(dependent on the direction cosines of the magnetization)

- For a cubic material, the first-order terms,

$$u = u_m + u_{me} + u_{el}$$
$$= U_o/V_o + K_1(\alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^2 + \alpha_3^2\alpha_1^2) + K_2(\alpha_1^2\alpha_2^2\alpha_3^2) + \dots$$
$$+ (1/2)c_{11}(e^2_{xx} + e^2_{yy} + e^2_{zz}) + c_{12}(e_{xx}e_{yy} + e_{yy}e_{zz} + e_{zz}e_{xx})$$
$$+ (1/2)c_{44}(e_{xx}e_{yy} + e_{yy}e_{zz} + e_{zz}e_{xx}) + \text{const} \times e_{ij}$$
$$+ B_1(\alpha_1^2e_{xx} + \alpha_2^2e_{yy} + \alpha_3^2e_{zz}) + B_2(\alpha_1\alpha_2e_{xy} + \alpha_2\alpha_3e_{yz} + \alpha_3\alpha_1e_{zx})$$ (see Table 7.2 in O'Handley)

(2) Magnetoelastic Effects (continued)

❑ Physical Origin

Mainly due to the spin-orbit coupling

A crude picture (see Fig. 8.14 in Cullity)

❑ Magnetoelastic Contribution to Anisotropy

There is a close physical connection between crystal anisotropy and magnetostriction.

- First-Order Anisotropy Due to an External Strain

    Effects of Imposed Strain

- Second-Order Anisotropy Due to an Magnetostriction

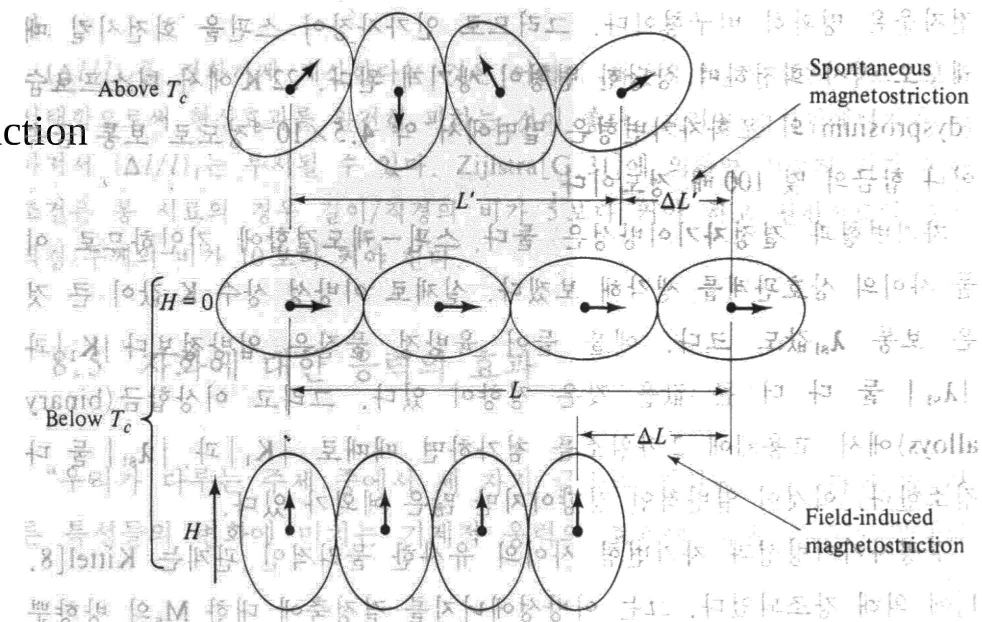

Fig. 8.14 Mechanism of magnetostriction [8.17].

(2) Magnetoelastic Effects (continued)

❑ ΔE Effect and Thermodynamics of Magnetomechanical Coupling

▶ △*E* Effect

- Effect of added magnetic strain due to the magnetostrictive strain, which is important for acoustic waves, vibrations, and damping.

-The total strain e_{tot} of a ferromagnetic sample under stress σ

$$e_{tot} = \sigma/E_M + (3/2)\lambda_s[\cos^2\theta - 1/3]$$

where E_M is Young's modulus for fixed ***M*** (no magnetic contribution) and θ is the angle between ***M*** and the strain measuring direction. (see Fig. 7.13 in O'Handley)

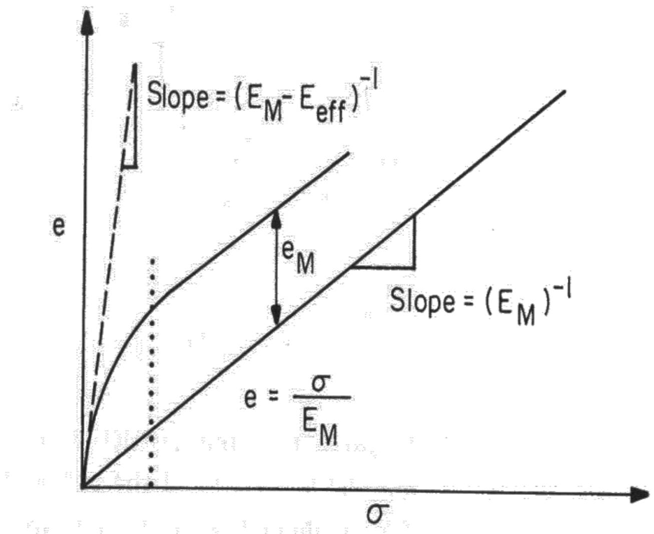

Figure 7.13 Strain versus stress in a magnetic material in the elastic regime. When the magnetization is fixed, the small strain behavior is purely linear. When the magnetization is free to respond to the applied stress, the material appears softer because of the additional magnetostrictive strain e_M. [Adapted from Lacheisserie (1994)].

(2) Magnetoelastic Effects (continued)

-For a hard-axis magnetization process, since $\cos\theta = m = \frac{H}{H_a^{eff}}$

$$1/E_{eff} = 1/E_H = \partial e_{tot}/\partial\sigma = 1/E_M - \{3\lambda_s H^2/(H_a^{eff})^3\}(\partial H_a^{eff}/\partial\sigma)$$

Since $H_a^{eff} = H_a - 3\lambda_s\sigma/M_s$, $\frac{E_M - E_{eff}}{E_M} \equiv \frac{\Delta E}{E} = \frac{9\lambda_s^2 H^2}{M_s H_a^3} E_{eff}$

▶ **Thermodynamics of Magnetomechanical Cupling**

The magnetoelastic contribution to the internal energy of a magnetic material : - $V\sigma_M\lambda$

❑ Symmetry-Invariant Notation

-The expansion coefficients in a set of orthogonal harmonic functions are chosen so that the anisotropy energy is
invariant under all point operations of the crystal symmetry.

- Symmetry-based

cubic magnetostriction coefficients :

$\lambda_{100} \approx 2/3\lambda^{\gamma,2}$ or $\lambda^{\gamma,2} \approx 3/2\lambda_{100}$, $\lambda_{111} \approx 2/3\lambda^{\varepsilon,2}$ or $\lambda^{\varepsilon,2} \approx 3/2\lambda_{111}$

hexagonal magnetostriction coefficients: (see eqn. (7.37) in O'Handley)

$$\lambda^{hex} = \frac{1}{3}\lambda_1^{\alpha,0} + \lambda_2^{\alpha,0}(\beta_3^2 - \frac{1}{3}) + [\frac{1}{3}\lambda_1^{\alpha,2} + \lambda_2^{\alpha,2}(\beta_3^2 - \frac{\beta_1^2 + \beta_2^2}{3})](\alpha_3^2 - \frac{1}{3})$$

$$+ \lambda^{\varepsilon,2}[\frac{1}{2}(\beta_2^1 - \beta_2^2)(\alpha_1^2 - \alpha_2^2) + 2\beta_1\beta_2\alpha_1\alpha_2] + 2\lambda^{\zeta,2}(\beta_2\beta_3\alpha_2\alpha_3 + \beta_1\beta_3\alpha_1\alpha_3) + ...$$

(2) Magnetoelastic Effects (continued)

❑ Temperature Dependence

- Case studies
  - Ni single crystals (see Fig. 7.17 in O'Handley)
  - Rare earth metals (see Fig. 7.18-19 in O'Handley)
  - Yttrium iron garnet(YIG) (see Fig. 7.20 in O'Handley)
  - Amorphous magnetic alloys (see Fig. 7.21 in O'Handley)
- $\lambda(T)$ drops much more sharply with increasing temperature than does $M(T)$.
- Theoretical interpretations: C. Zener, *Phys. Rev.* **96**, 1335 (1954); E.R. Callen and H.B. Callen, *Phys. Rev.* **129**, 578(1963), and **139A**, 255(1965)

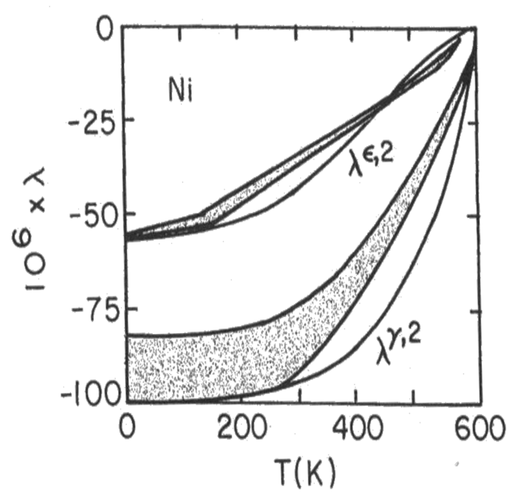

Figure 7.17 Shaded areas show range of experimental magnetostriction of Ni single crystals (Franse 1970, Bower 1971, Lee 1971), and solid lines show calculated temperature dependence (Lacheisserie 1972).

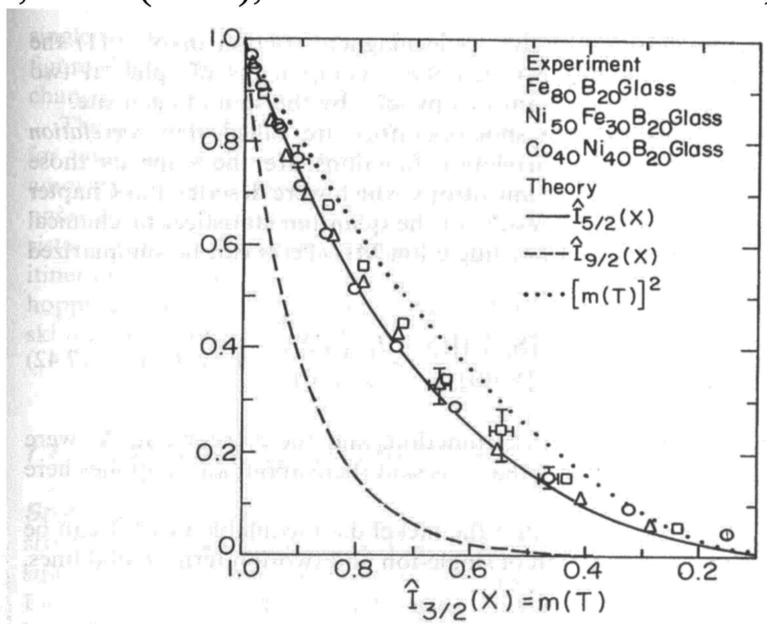

Figure 7.21 Temperature dependence of magnetostriction in three amorphous alloys after O'Handley (1978).

(2) Magnetoelastic Effects (continued)

❑ Measurement Techniques of Magnetostriction

- Strain Gauges :
  Measurement of resistance variation due to a dimensional change in a metal foil (e.g., Ni-Cr alloys) and conversion into measured strain.
  Capacitance bridges; measurement of the relative change in resonant frequency
- Small-Angle Magnetization Rotation :
  Effective for ribbon-shaped samples such as metallic glass strips
- Strain-Modulated FMR
- Thin-Film Techniques

❑ Magnetostrictive Materials and Applications

(ref. Chap 8.7 in Cullity)

- Magnetostrictive transducer: electrical energy $\leftrightarrow$ mechanical energy
- Acoustic delay line