Topics in Ship Structural Design
(Hull Buckling and Ultimate Strength)

Lecture 2 Column Buckling

Reference : Mechanics of Material Ch. 11
Columns

NAOE

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Design of Pillar

❖ **Pillars in Cruise ship**

Buckling Control for Pillar

❖ Axially compressed pillars

σ_f : minimum upper yield stress

σ_{el} : ideal compressive buckling stress

σ_{cl} : critical buckling stress

$$\sigma_c = \sigma_{el} \quad \text{when} \quad \sigma_{el} < \frac{\sigma_f}{2}$$

$$= \sigma_f \left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \text{when} \quad \sigma_{el} > \frac{\sigma_f}{2}$$

$$\sigma_{el} = 0.001\mathrm{E}\frac{I_A}{AL^2} \; (N/mm^2)$$

$$\sigma_{el} = \frac{P}{A} = \frac{\pi^2 \mathrm{E} I_A}{AL^2}$$

$$\because \frac{3.14^2 \mathrm{E}(10^{-4} I_A)}{(10^{-2}A)L^2} = 0.000985\frac{\mathrm{E}I_A}{AL^2}(N/mm^2)$$

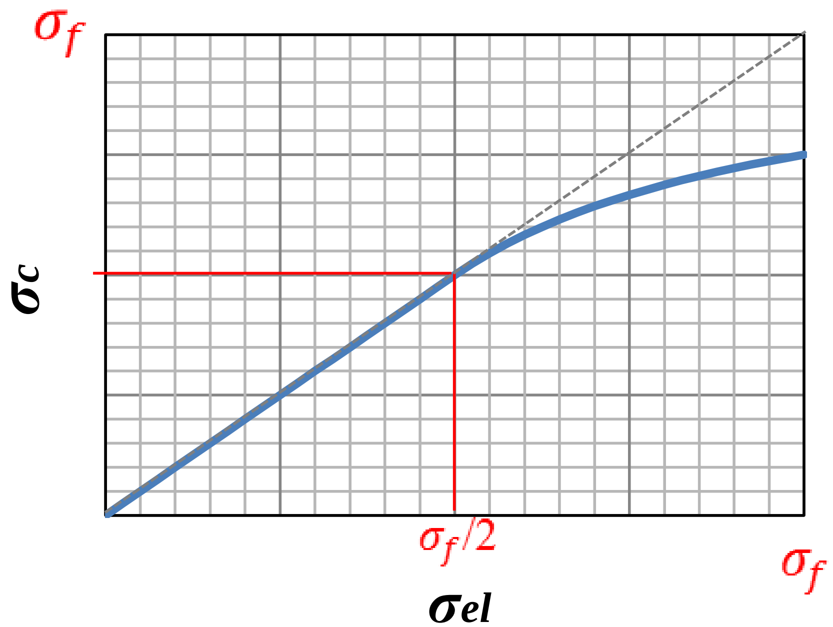
- I_A : moment of inertia in cm^4 about the axis perpendicular to the expected direction of buckling
- A: cross-sectional area in cm^2
- L: length of pillar in m

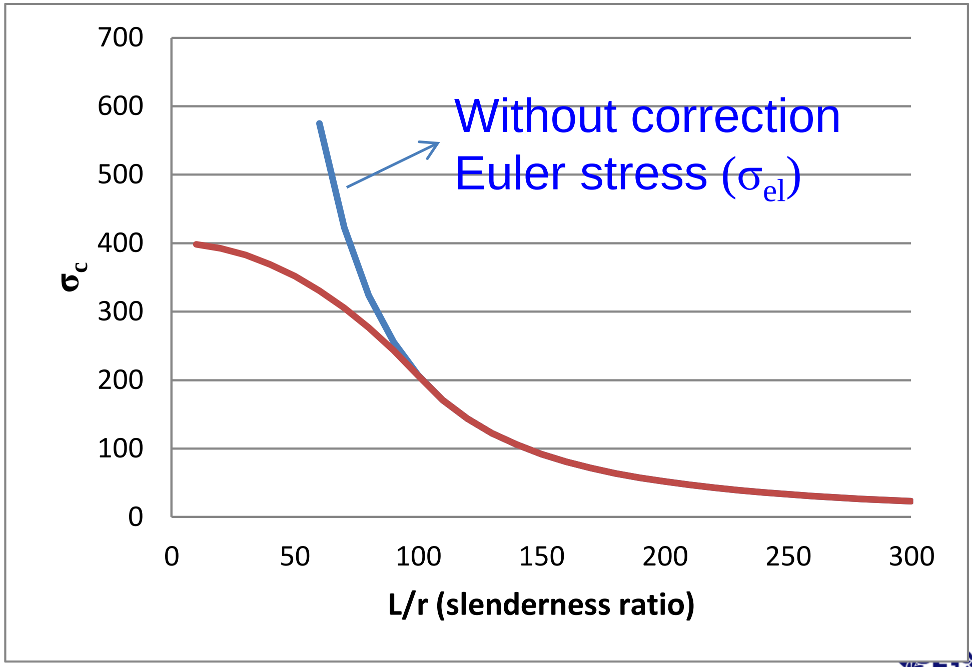
Buckling Control for Pillar

$$\sigma_{el} = \frac{\pi^2 E I_A}{A L^2} = \frac{\pi^2 E}{(L/r)^2}$$

$$\sigma_c = \sigma_{el} \quad \text{when} \quad \sigma_{el} < \frac{\sigma_f}{2}$$
$$= \sigma_f \left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \quad \text{when} \quad \sigma_{el} > \frac{\sigma_f}{2}$$

$$\sigma_c = \frac{\pi^2 E}{(L/r)^2} \quad \text{when} \quad \frac{\pi^2 E}{(L/r)^2} < \frac{\sigma_f}{2}$$
$$= \sigma_f \left(1 - \frac{(L/r)^2 \sigma_f}{4\pi^2 E}\right) \quad \text{when} \quad \frac{\pi^2 E}{(L/r)^2} > \frac{\sigma_f}{2}$$





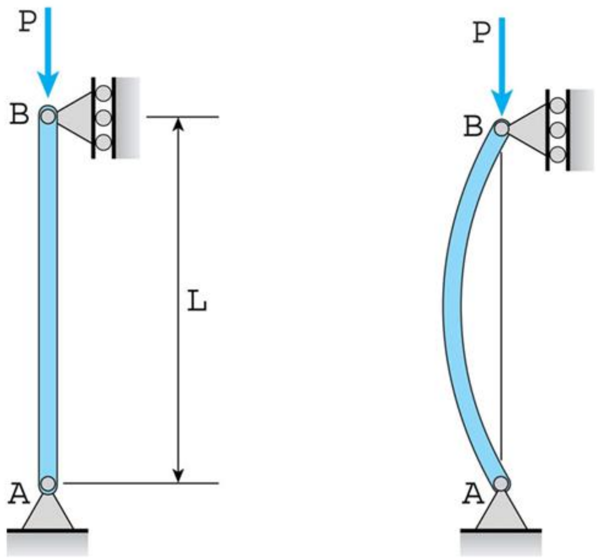
Q: Why correction ?

11.1 Introduction

❖ **Column : long, slender structural members**

- When a column is loaded axially in compression
- It may deflect laterally.
- If fail by bending rather than direct compression

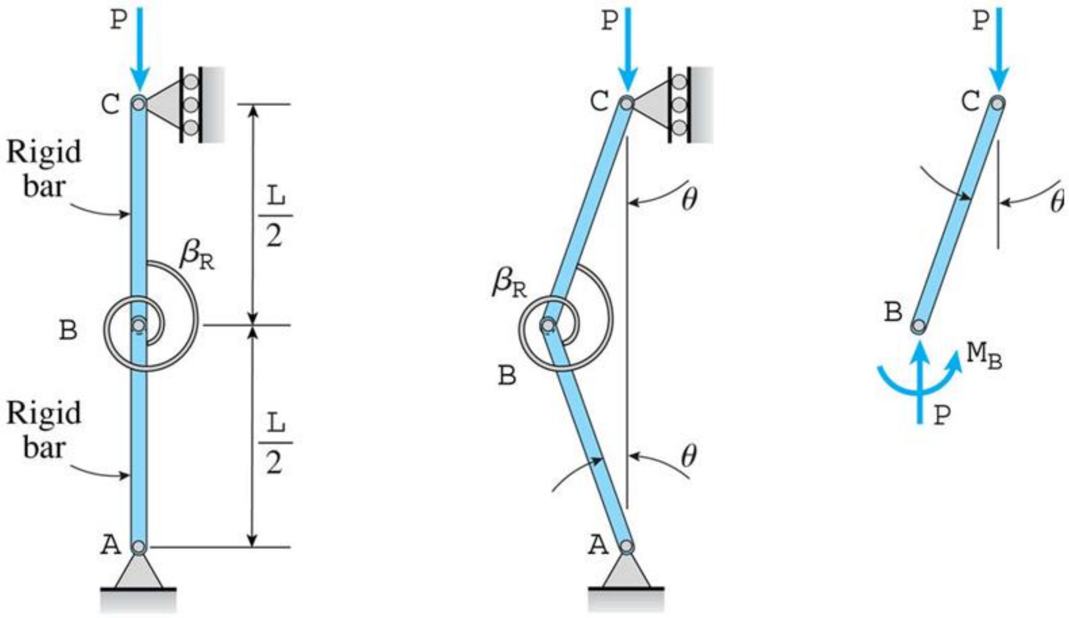
  → buckling



- Other phenomenon of buckling
  - ✓ When stepping on the top of an empty aluminum can, steel plate wrinkles under compressive stress.
  - ✓ One of the major causes of failures in structures

11.2 Buckling and Stability

❖ Stability in column buckling model

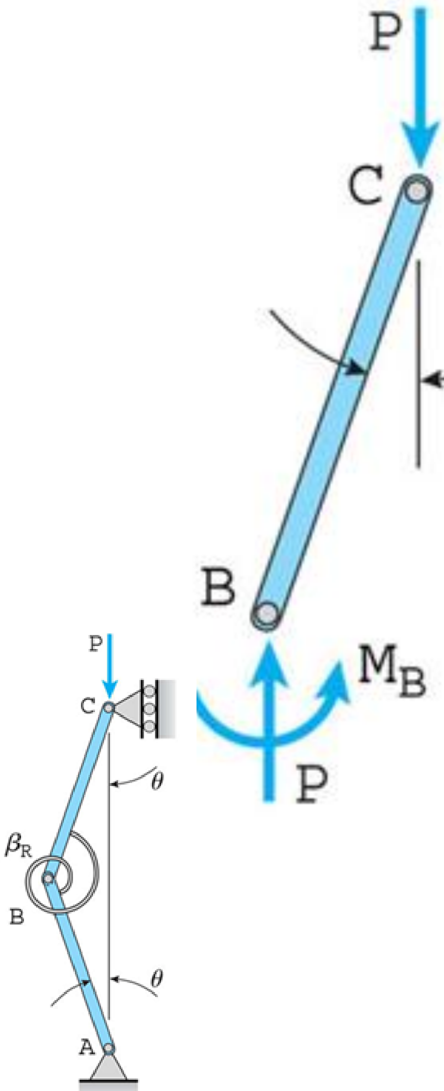


when disturbed by some external force

- **M_B** (Restoring moment by rotation spring having stiffness β_R) decreases lateral displacement.
- Axial compressive load, **P**, increases lateral displacement.
- Action of **M_B** > Action of **P** ➔ stable
- Action of **M_B** < Action of **P** ➔ unstable

11.2 Buckling and Stability

❖ **Critical load, P_{cr}**



Restoring moment by rotational spring is

$$M_B = 2\beta_R\theta$$

Considering moment equilibrium about Point B

$$M_B - P(\frac{L}{2}\theta) = 0$$

$$(2\beta_R - \frac{PL}{2})\theta = 0$$

θ=0 (trivial solution when it is perfectly straight)

$$\therefore P_{cr} = \frac{4\beta_R}{L}$$

11.2 Buckling and Stability

❖ Critical load and stability

1) $0<P<P_{cr}$

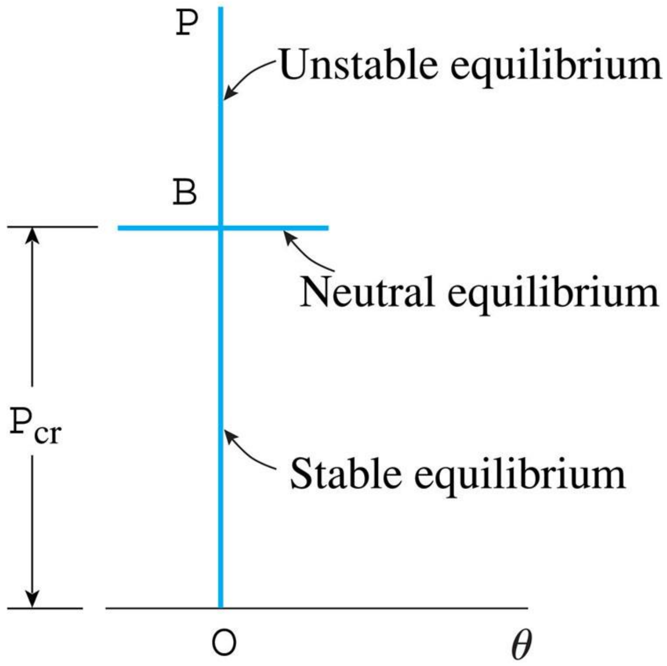
Stable. Structure returns to its initial position after being disturbed

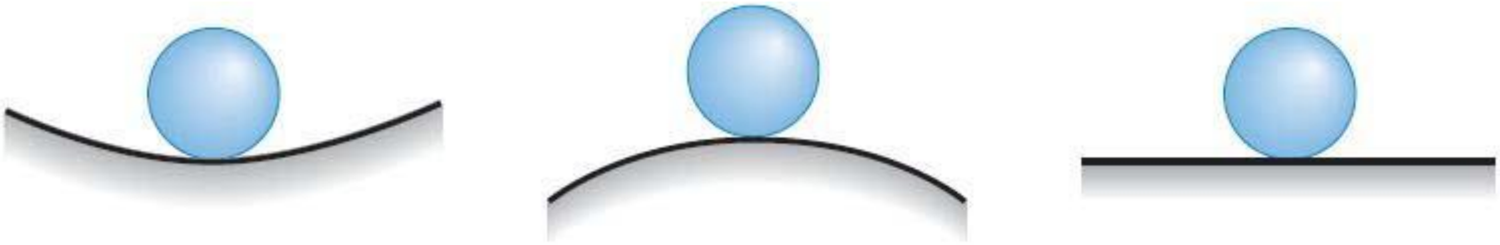
2) $P_{cr}<P$

Unstable. The slightest disturbance will cause the structure to buckle.

3) $P_{cr}=P$

Neutral equilibrium.at the boundary between stability and instability.

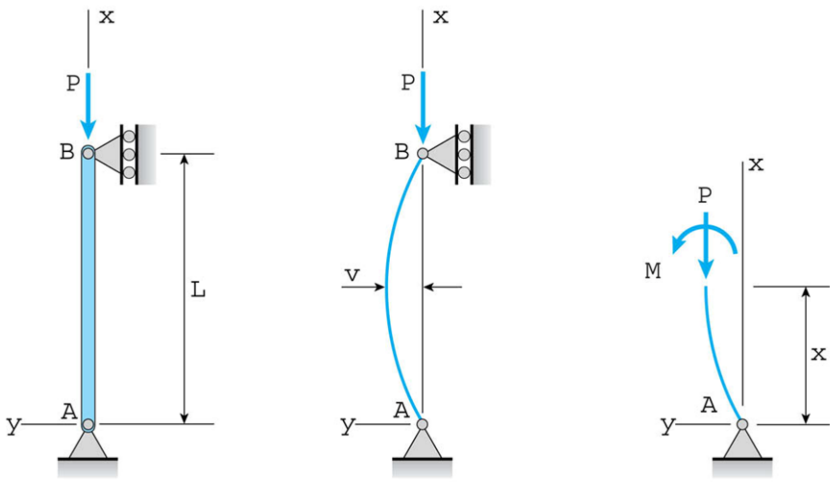




11.3 Columns with pinned ends

❖ Ideal column

- Perfectly straight
- no imperfection
- P is applied through the centroid of the end cross section



Ideal Column

Buckled shape

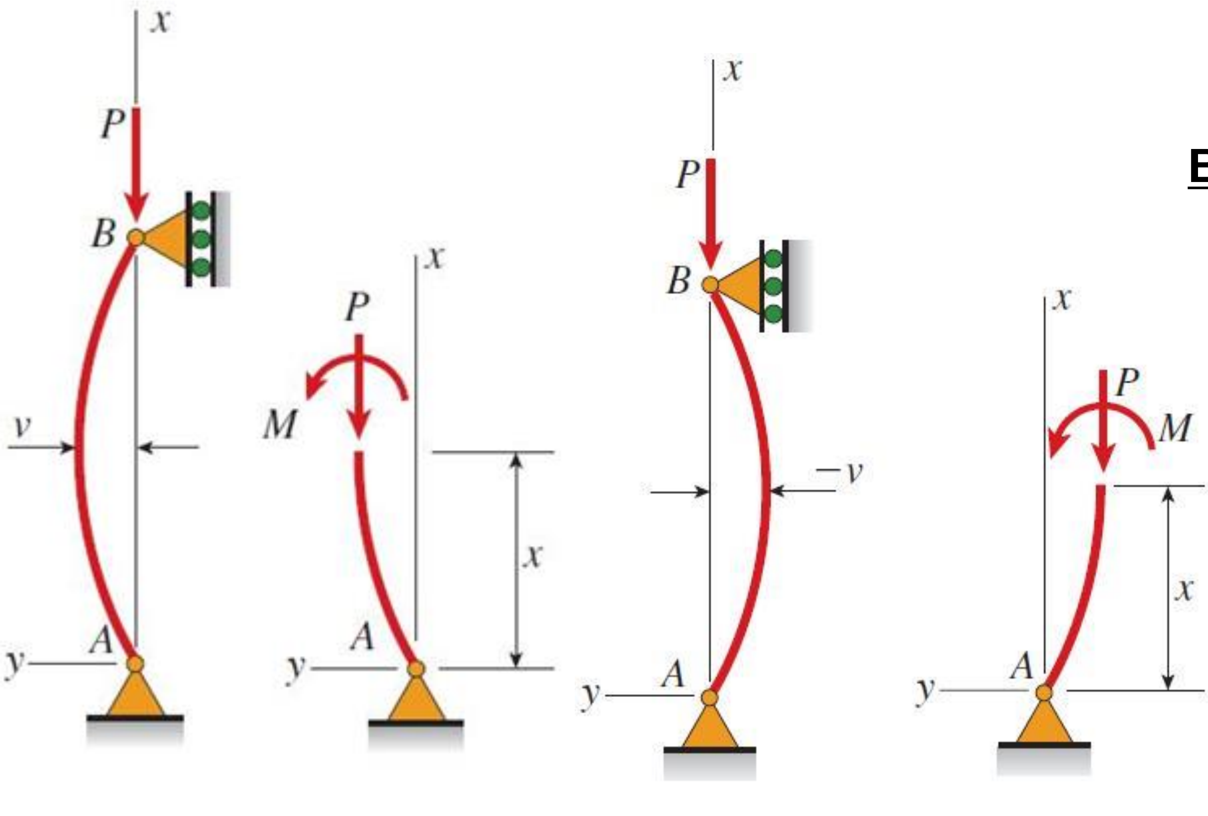
Free body diagram

- $0<P<P_{cr}$; Stabile equilibrium in straight position
- $P=P_{cr}$: neutral equilibrium in either the straight or a slightly bent position
- $P>P_{cr}$: unstable equilibrium and will buckle under the slightest disturbance

11.3 Columns with pinned ends

❖ Differential Equation for column buckling

- Applicable to a buckled column because the column bends like a beam
- Differential equations of the deflection curve of a beam can be used.
- Use the second-order equation since M is a function of lateral deflection, v.



(b) (c) (a) (b)

Bending moment equation

$$EI\,v'' = M$$

$$M + Pv = 0$$

$$M = -Pv$$

$$\therefore\ EI\,v'' + Pv = 0$$

11.3 Columns with pinned ends

❖ **Solution of the differential equation (Euler Load)**

$$EI\,v'' + P\,v = 0$$

put $k^2 = \dfrac{P}{EI}$

$$v'' + k^2 v = 0$$

General solution of this equation is

$$v = C_1 \sin kx + C_2 \cos kx$$

Deflection is zero when x=0 and L.

$$v(0) = 0 \Rightarrow C_2 = 0$$

$$v = C_1 \sin kx$$

$$v(L) = 0 \Rightarrow \quad C_1 \sin kL = 0$$

$$\sin kL = 0$$

$$kL = n\pi \qquad (n = 1,2,3,\ldots)$$

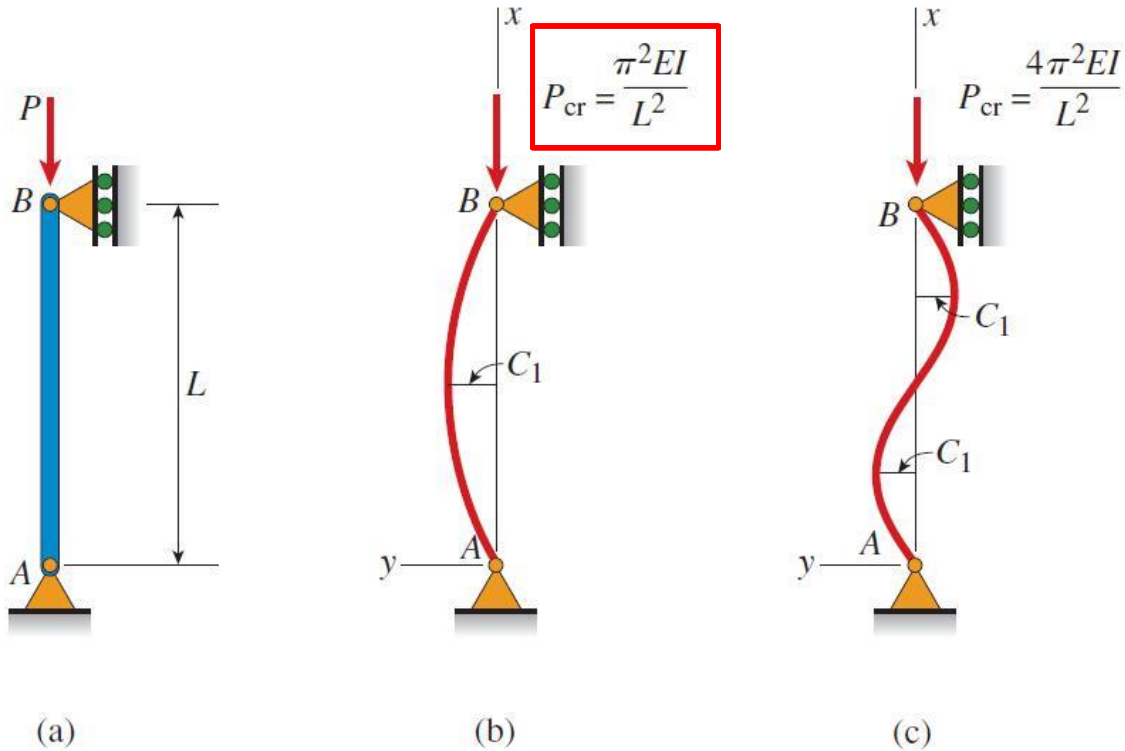
$$\boxed{v = C_1 \sin kx = C_1 \sin \frac{n\pi x}{L}}$$

$$\boxed{\therefore\ P_{cr} = \frac{n^2 \pi^2 EI}{L^2}}$$

11.3 Columns with pinned ends

❖ Buckling mode

- Higher values of the index n ➔ Higher modes, the critical load is proportional to the square of n.
- The lowest critical load occurs when n=1



(n=1) **(n=2)**

11.3 Columns with pinned ends

❖ **Euler Load**

$$P_{cr} = \frac{n^2 \pi^2 EI}{L^2}$$

❖ **Critical load is**

- proportional to the flexural rigidity EI.
- inversely proportional to the square of length.
- Independent of the strength of the material itself (e.g. proportional limit or yield strength).

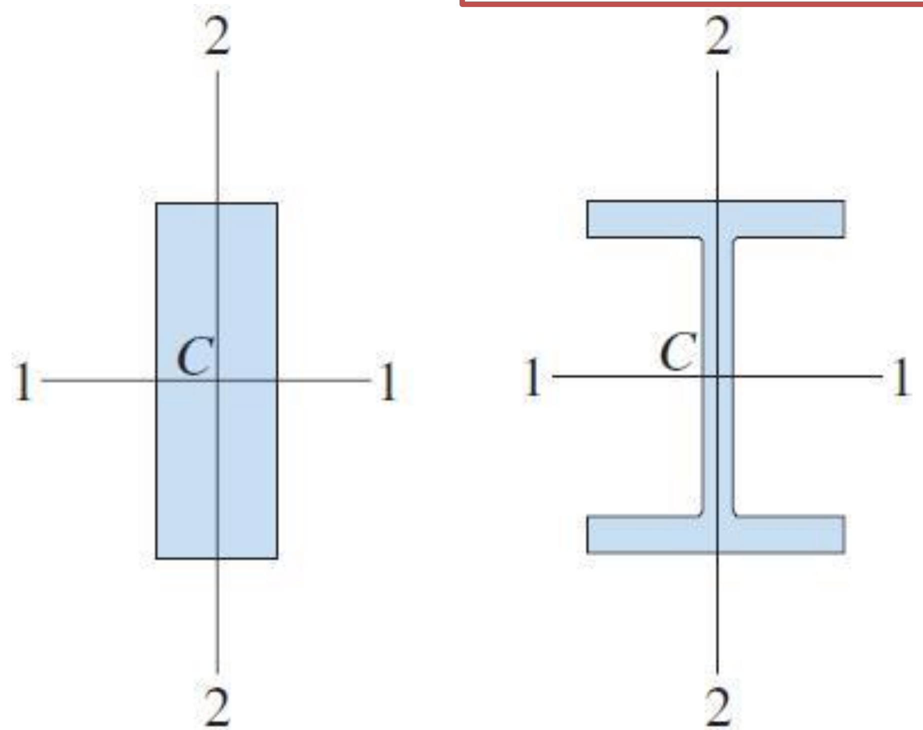
❖ **In order to increase the critical load**

- increase the flexural rigidity EI.
- reduce length.
- add additional restraint to prevent low buckling mode.

11.3 Columns with pinned ends

❖ **Buckling plane**

Q: In which plane buckling is likely to occur ? (or along which axis?)



- if $I_1 > I_2$, the column will buckle in the plane of 1-1
- the smaller moment of inertia I_2 should be used for the critical load

11.3 Columns with pinned ends

❖ Critical stress

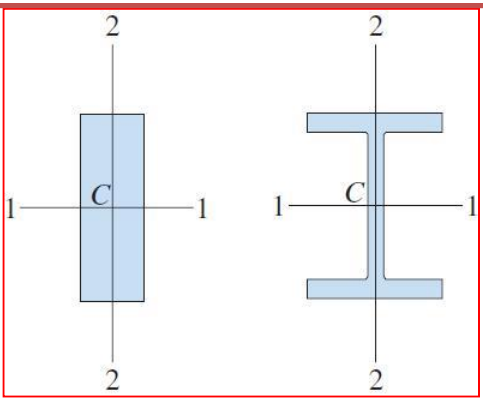
- Average compressive stress when load reaches the critical load

$$\boxed{P_{cr} = \frac{n^2 \pi^2 EI}{L^2}}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{AL^2}$$

$$\boxed{\sigma_{cr} = \frac{\pi^2 E}{(L/r)^2}}$$

Q: If areas are the same, which one a has large slenderness ratio?



where,

$$r = \sqrt{\frac{I}{A}}$$

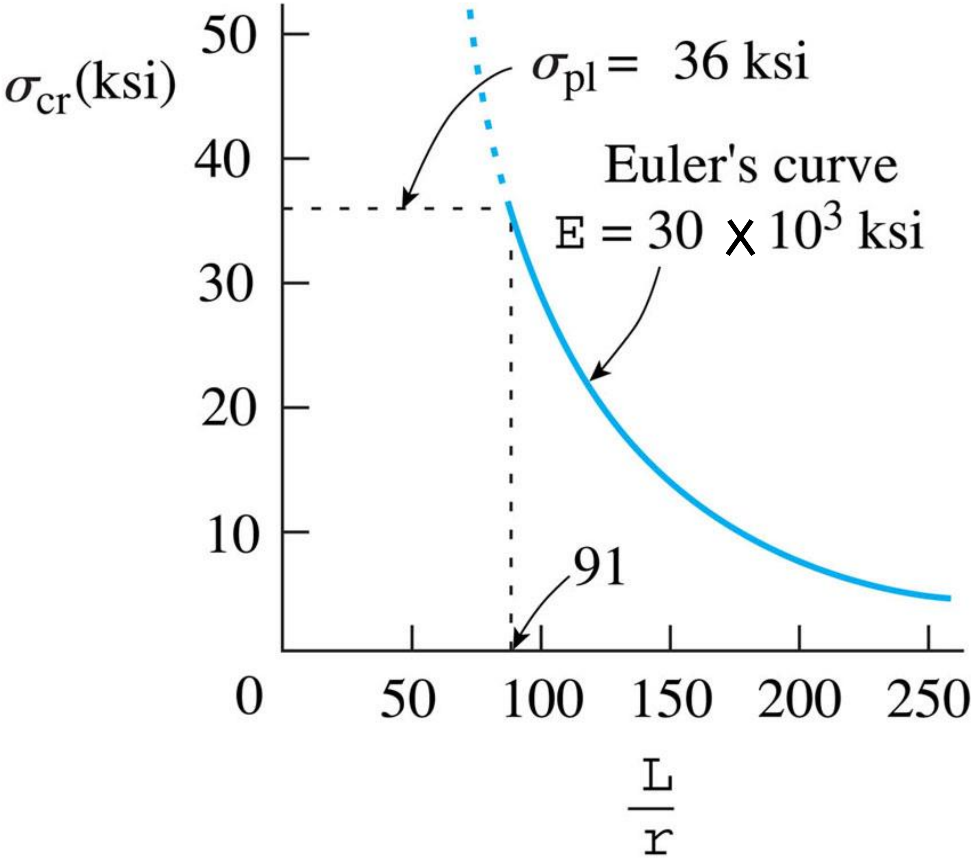
: radius of gyration

where L/r : slenderness ratio

11.3 Columns with pinned ends

❖ **Euler curve**

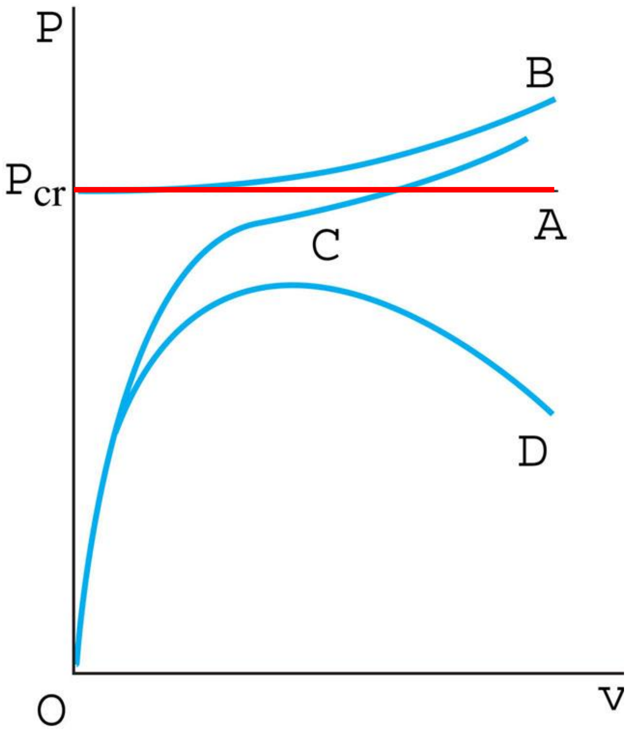
- A graph of the critical stress as a function of the slenderness ratio



σ_{pl} : *proportional limit*

11.3 Columns with pinned ends

❖ **Load-deflection diagram**



Line A:
ideal elastic column with small deflections

Curve B:
ideal elastic column with large deflections

Curve C:
elastic column with imperfections

Curve D:
inelastic column with imperfections

※ Ideal elastic column

: the loads are precisely applied without eccentricity

the construction is perfect

the material follows Hooke's law

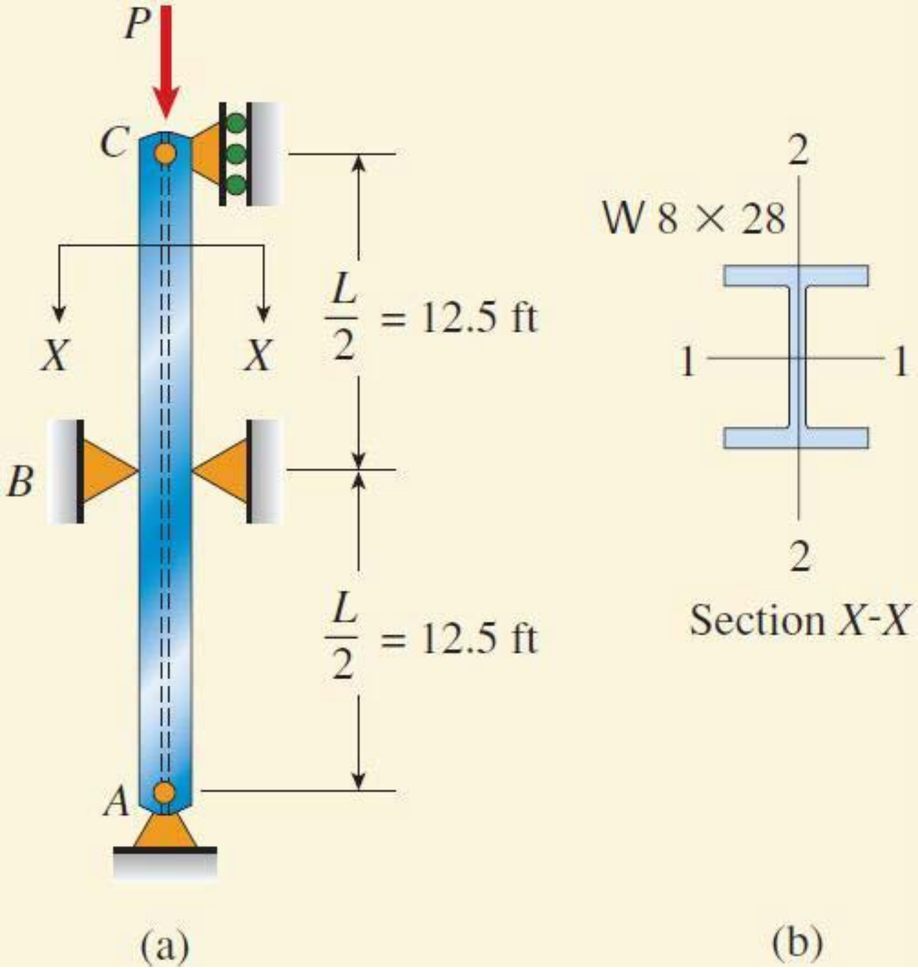
Example 11-1

❖ **Pin-supported at the ends compressed by an axial load P, Lateral support at the midpoint B**

$E = 200$ GPa, $\sigma_{pl} = 300$ MPa

WF (IPN220) steel, L=8 m

Safety factor n=2.5, P_{allow} =?.



Solution

Appendix E-2, the column property of IPN220 are

$I_1 = 3060\ cm^4$, $I_2 = 162\ cm^4$

$A = 39.5\ cm^2$

Example 11-1

❖ Critical load

- **If column buckles in the plane of the figure**

$$P_{cr} = \frac{\pi^2 EI_2}{(L/2)^2} = \frac{4\pi^2 EI_2}{L^2} = \frac{4\pi^2 (200\,\mathrm{GPa})(162\,\mathrm{cm}^4)}{(8\,\mathrm{m})^2} = 200\,\mathrm{kN}$$

- **If column buckles perpendicular to the plane of the figure**

$$P_{cr} = \frac{\pi^2 EI_1}{L^2} = \frac{\pi^2 (200\,\mathrm{GPa})(3060\,\mathrm{cm}^4)}{(8\,\mathrm{m})^2} = 943.8\,\mathrm{kN}$$

- **The critical load for the column**

$$P_{cr} = 200\,\mathrm{kN}$$

- **The critical stress do not exceed the proportional limit in the case of larger critical load ➔ satisfactory calculations**

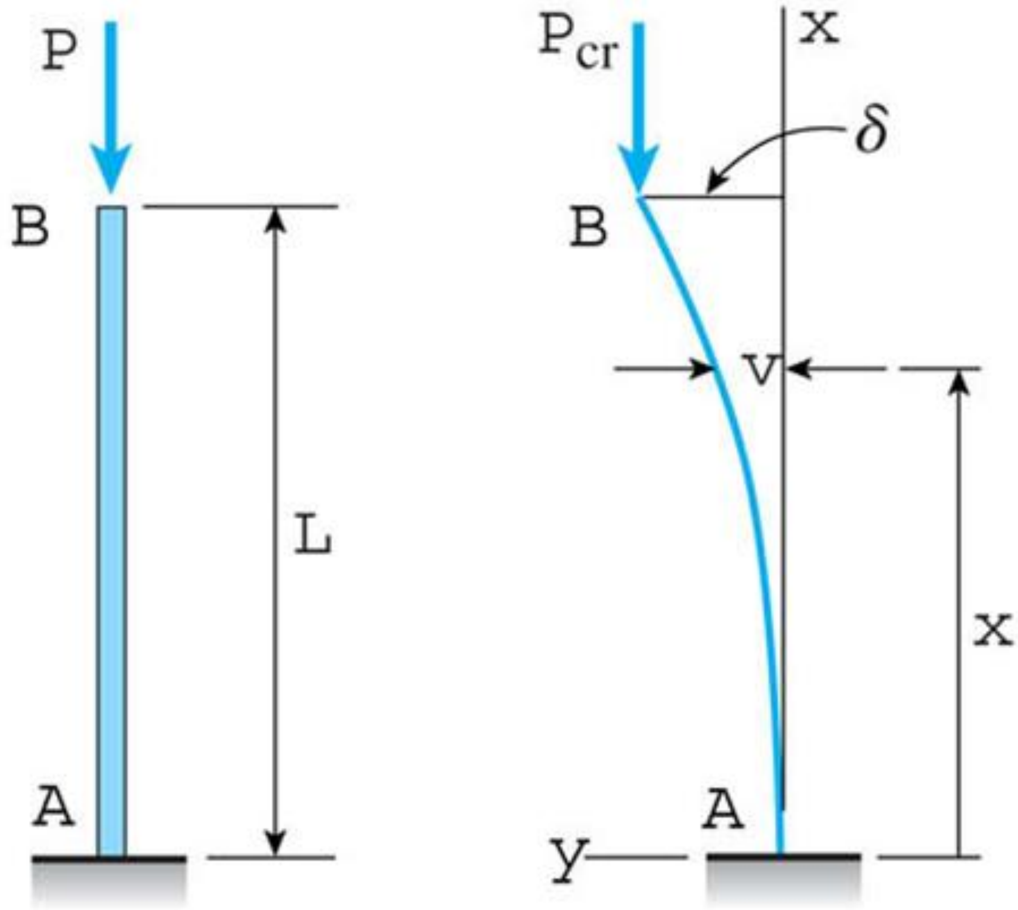
$$\sigma_{cr} = \frac{P_1}{A} = \frac{943.8\ \mathrm{kN}}{39.5\,\mathrm{cm}^2} = 238.9\,\mathrm{MPa} < 300\,\mathrm{MPa}$$

- **Allowable load**

$$P_{allow} = \frac{P_{cr}}{n} = \frac{200\ \mathrm{kN}}{2.5} = 79.9\,\mathrm{kN}$$

11.4 Columns with other support conditions

❖ **Column fixed at the base and free at the top**



11.4 Columns with other support conditions

❖ **Column fixed at the base and free at the top**

$$M = P(\delta - v)$$

$$EI\,v'' = M = P(\delta - v)$$

$$v'' + k^2 v = k^2 \delta \qquad (k^2 = \frac{P}{EI})$$

$$v_H = C_1 \sin kx + C_2 \cos kx$$

$$v_P = \delta$$

$$v = C_1 \sin kx + C_2 \cos kx + \delta$$

$$v' = C_1 k \cos kx - C_2 k \sin kx$$

$$v(0) = 0 \qquad C_2 = -\delta$$

$$v'(0) = 0 \qquad C_1 = 0$$

$$v = \delta(1 - \cos kx)$$

$$v(L) = \delta \qquad \delta \cos kL = 0$$

$$\cos kL = 0$$

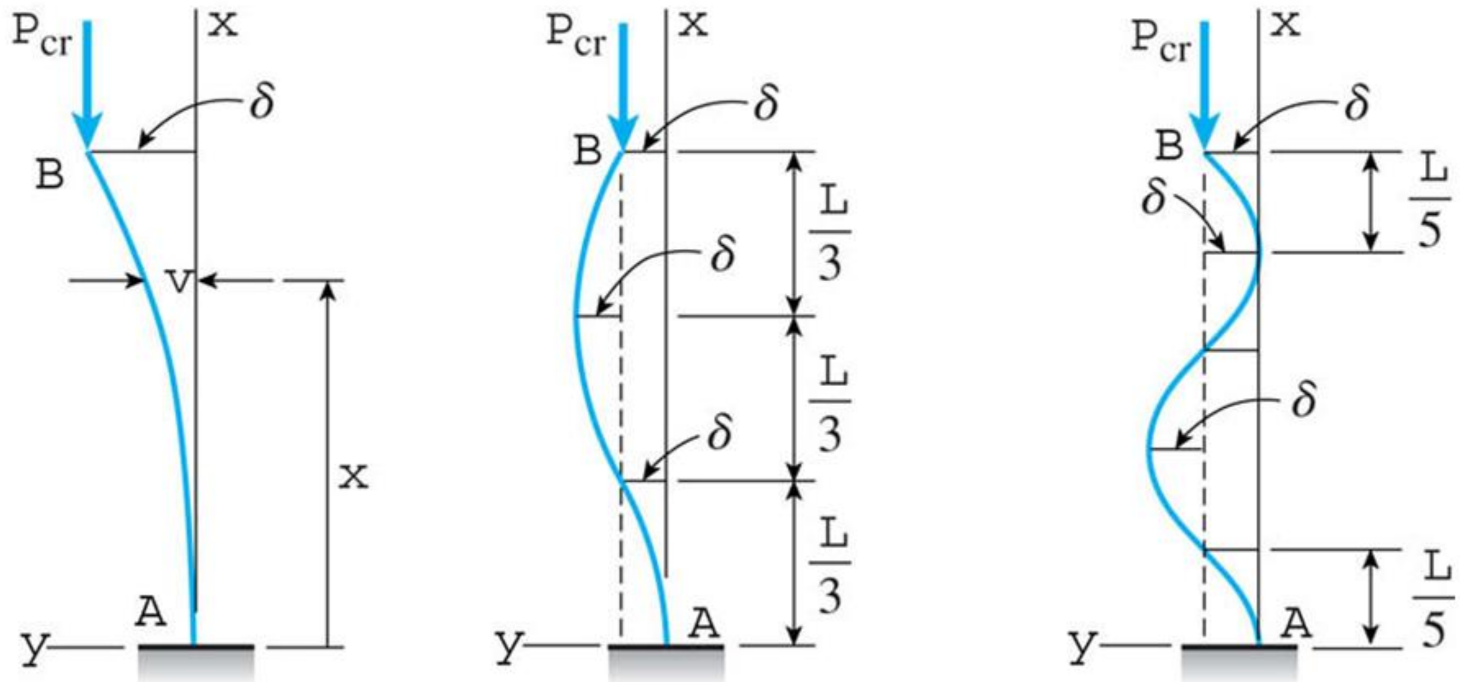
$$kL = \frac{n\pi}{2} \qquad (n = 1,3,5,\ldots)$$

$$v = \delta(1 - \cos\frac{n\pi x}{2L}) \qquad (n = 1,3,5,\ldots)$$

$$\therefore P_{cr} = \frac{n^2 \pi^2 EI}{4L^2} \qquad (n = 1,3,5,\ldots)$$

11.4 Columns with other support conditions

❖ **Buckling mode**

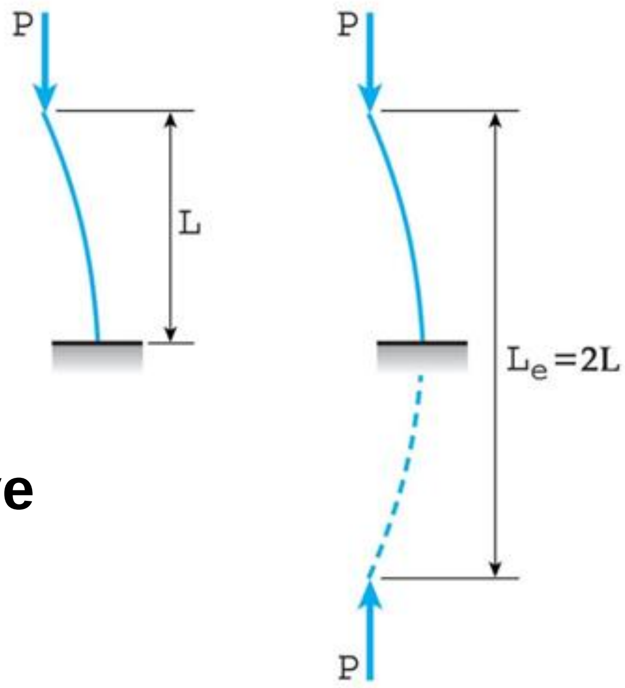


$$P_{cr} = \frac{\pi^2 EI}{4L^2} \qquad P_{cr} = \frac{9\pi^2 EI}{4L^2} \qquad P_{cr} = \frac{25\pi^2 EI}{4L^2}$$

(n=1) **(n=3)** **(n=5)**

11.4 Columns with other support conditions

- **Effective length**
  - The length of the equivalent pinned-end column
  - $L_e = KL$ (K : effective length factor)
  - A fixed-free column : $K=2$, $L_e=2L$



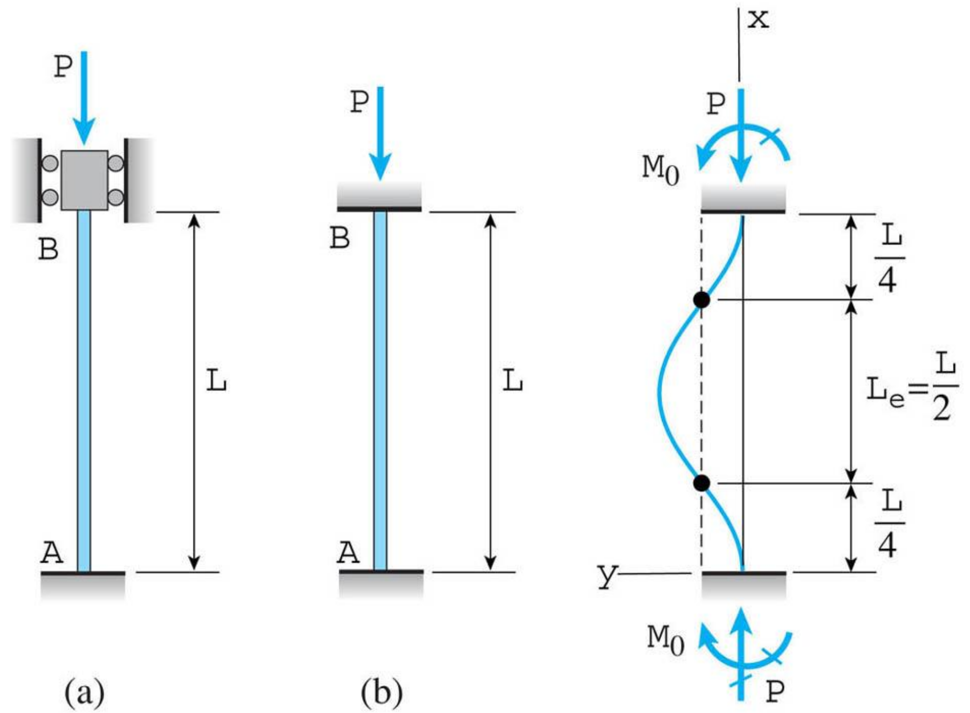
- **The critical load in terms of an effective length**

$$\boxed{P_{cr} = \frac{\pi^2 EI}{L_e^{\,2}}}$$

11.4 Columns with other support conditions

❖ Column with both ends fixed against rotation

- $K=1/2$
- $L_e=KL=L/2$

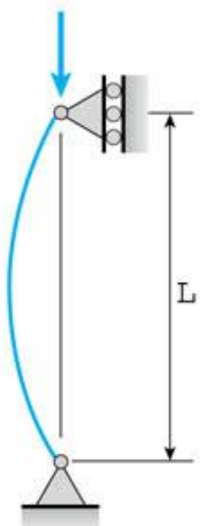
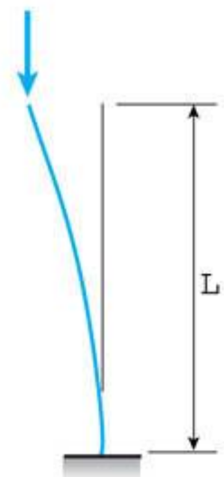
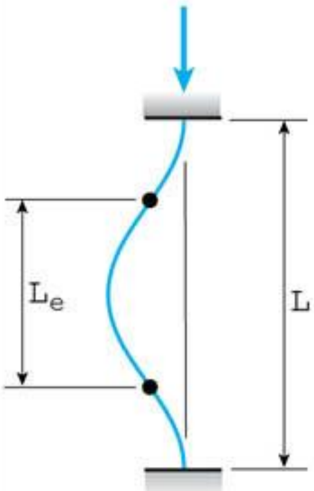
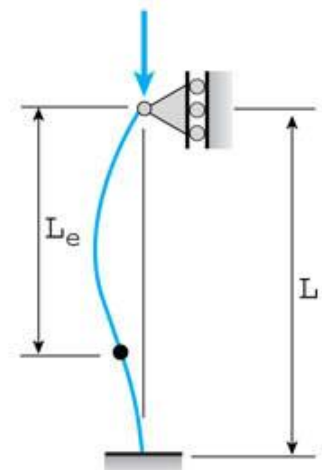


(a) (b) (c)

- Critical Load

$$P_{cr}=\frac{\pi^2 EI}{L_e{}^2}=\frac{4\pi^2 EI}{L^2}$$

11.4 Columns with other support conditions

(a) Pinned-pinned column	(b) Fixed-free column	(c) Fixed-fixed column	(d) Fixed-pinned column
$P_{cr} = \frac{\pi^2 EI}{L^2}$	$P_{cr} = \frac{\pi^2 EI}{4L^2}$	$P_{cr} = \frac{4\pi^2 EI}{L^2}$	$P_{cr} = \frac{2.046\ \pi^2 EI}{L^2}$
			
$L_e = L$	$L_e = 2L$	$L_e = 0.5L$	$L_e = 0.699L$
$K = 1$	$K = 2$	$K = 0.5$	$K = 0.699$

11.5 Columns with eccentric axial loads

- **Ideal column**
  - remains straight until the critical loads are reached,
  - after which bending may occur
- **If a small eccentricity, e**
  - Column begins to deflect at the onset of loading
  - The deflection then becomes steadily larger as the load increases.
  - Differential equation

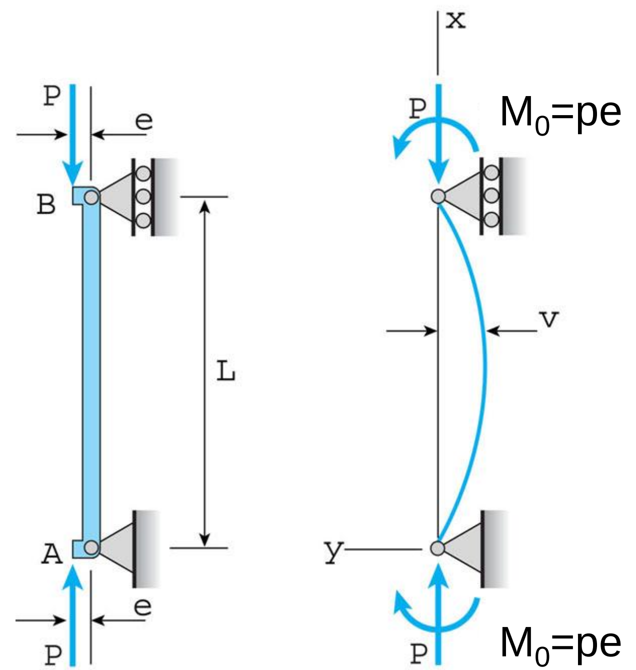
$$EIv'' = M = M_0 + P(-v) = Pe - Pv$$

$$v'' + k^2 v = k^2 e \qquad k = P/EI$$

  - Boundary condition

$$v = C_1 \sin kx + C_2 \cos kx + e$$

$$v(0) = 0, \ v(L) = 0$$



(a) (b)

11.5 Columns with eccentric axial loads

$$C_2 = -e \qquad C_1 = -\frac{e(1-\cos kL)}{\sin kL} = -e\tan\frac{kL}{2}$$

$$v = -e(\tan\frac{kL}{2}\sin kx + \cos kx - 1)$$

❖ **Maximum deflection**

$$\delta = -v(\frac{L}{2}) = e(\tan\frac{kL}{2}\sin kx + \cos kx - 1)$$

$$\delta = e(\sec\frac{kL}{2} - 1)$$

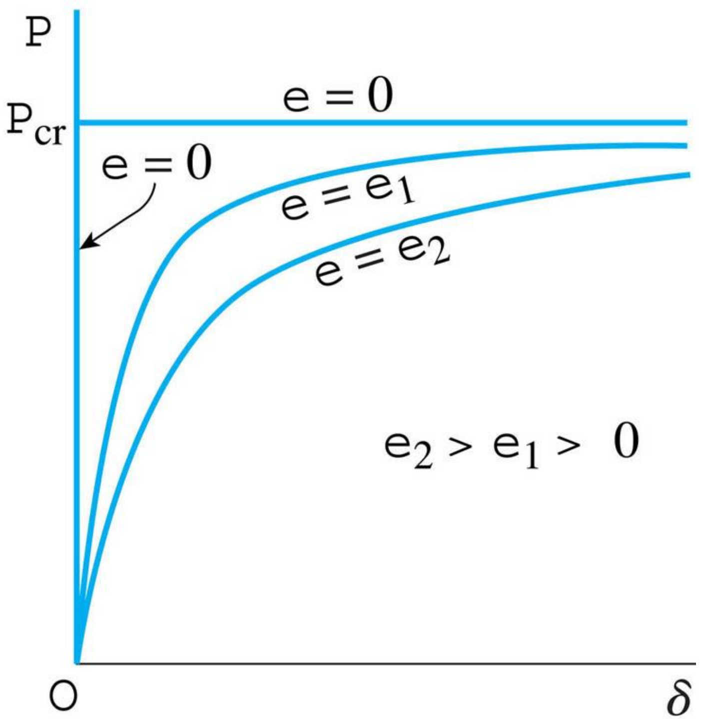
$$k = \sqrt{\frac{P}{EI}} = \sqrt{\frac{P\pi^2}{P_{cr}L^2}} = \frac{\pi}{L}\sqrt{\frac{P}{P_{cr}}} \qquad kL = \pi\sqrt{\frac{P}{P_{cr}}}$$

$$\delta = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right]$$

1) $\delta = 0$ when $e = 0$

2) $\delta = 0$ when $P = 0$

3) $\delta = \infty$ when $P \to P_{cr}$



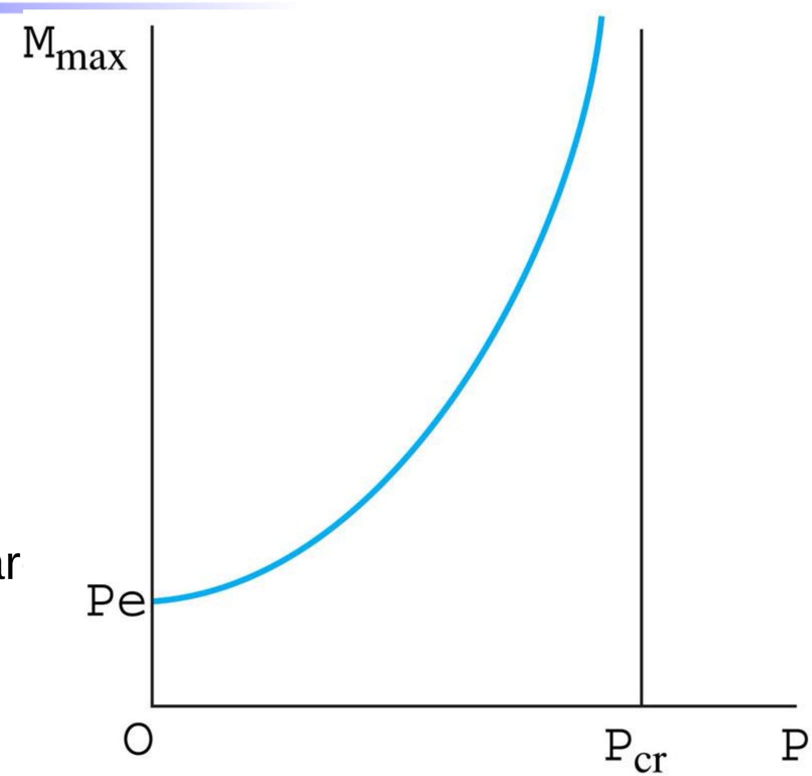
Load-deflection diagram for a column with eccentric axial loads

11.5 Columns with eccentric axial loads

❖ Maximum bending moment

$$M_{\max} = P(e+\delta)$$

$$M_{\max} = Pe\sec\frac{kL}{2} = Pe\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)$$



Maximum bending moment in a column with eccentric axial loads

❖ Other End Conditions

- Fixed-free condition : previous equations ar available with $L_e=2L$
- Fixed-pinned condition :

  Not available even with $L_e=0.699L$.

  A new set of equations to be derived.
- Fixed ends : the concept of an eccentric axial load has no meaning

  → Any moment applied at the end is resisted directly by the supports and no bending of the column itself.

11.6 The Secant Formula for Columns

❖ **The maximum stresses in a column with eccentric axial loads occur at the midpoint. (Compressive force + bending moment)**

$$\sigma_{\max} = \frac{P}{A} + \frac{M_{\max} c}{I}$$

$$M_{\max} = Pe \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right)$$

$$P_{cr} = \pi^2 EI / L^2, \; I = Ar^2$$

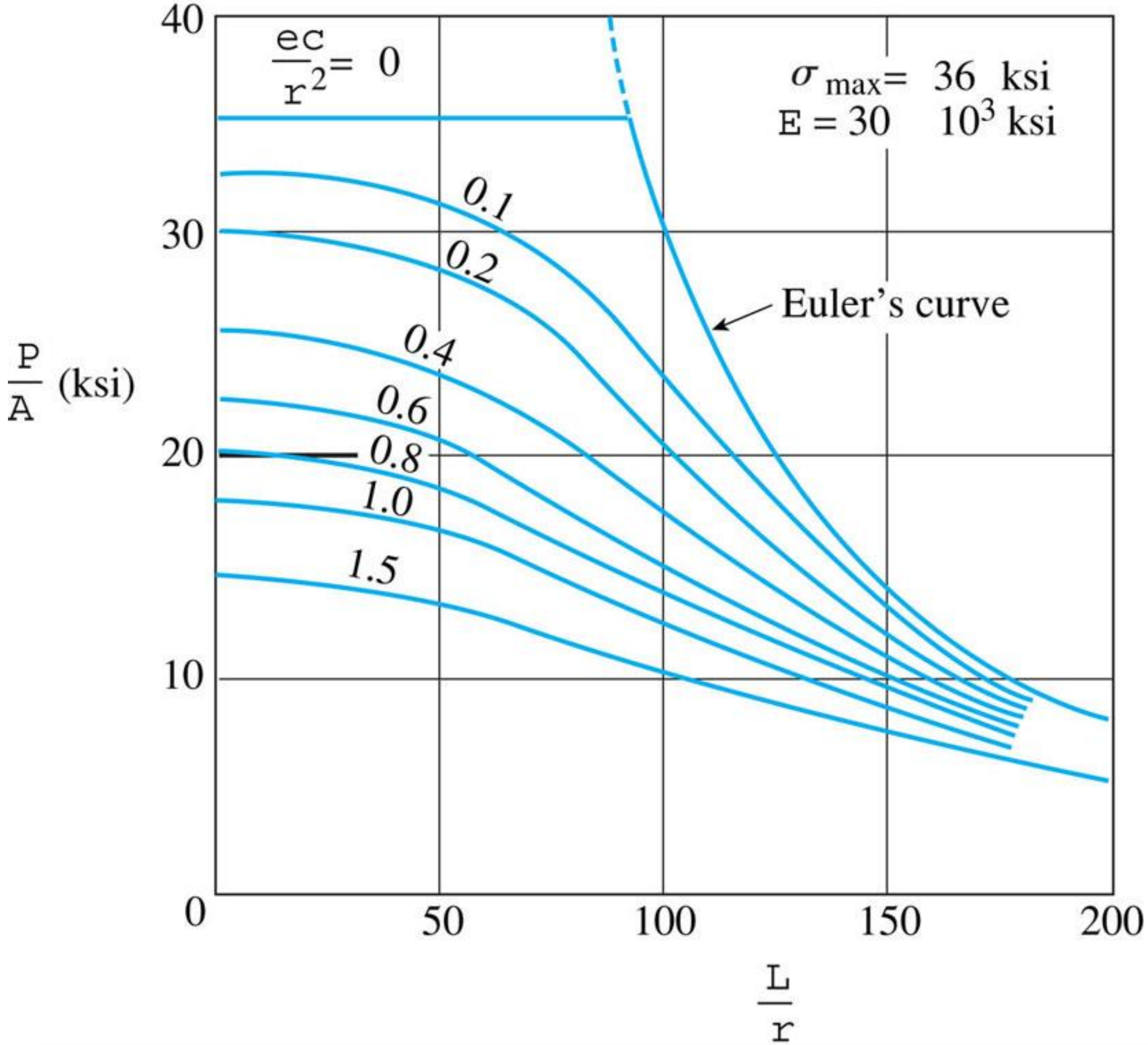
$$\sigma_{\max} = \frac{P}{A} + \frac{Pec}{I} \sec\left(\frac{L}{2r} \sqrt{\frac{P}{EA}} \right) = \frac{P}{A}\left(1 + \frac{ec}{r^2} \sec\left(\frac{L}{2r} \sqrt{\frac{P}{EA}} \right) \right)$$

➔ **Secant formula**

$$\text{Eccentricity ratio} = \frac{ec}{r^2}, \quad \text{the most common values} < 1$$

- if e=0, then $\sigma_{cr} = \dfrac{P_{cr}}{A} = \dfrac{\pi^2 EI}{AL^2} = \dfrac{\pi^2 E}{(L/r)^2}$

11.6 The Secant Formula for Columns



Graph of the secant formula for
σ_{max} = 36 ksi (σ_{yield}) and E = 30 X 10^3 ksi

11.6 The Secant Formula for Columns

❖ Discussion of Secant formula

- The load-carrying capacity decreases significantly as the slenderness ratio L/r increases.

  ➔ Long slender columns are much less stable than stocky columns.

- The load-carrying capacity decreases with increasing eccentricity e.

  ➔ The effect is relatively greater for short columns than for long ones.

- Applicable to fixed-free condition with $L_e=2L$

  but not to other end conditions

- An actual column has imperfections such as initial curvature, imperfect supports, non-homogeneity of the material. Assume eccentricity ratio (ec/r^2)of 0.25 for structural steel design instead of a safety factor.

Example 11-4

❖ **A steel of wide-flange column of HE 320A shape,**

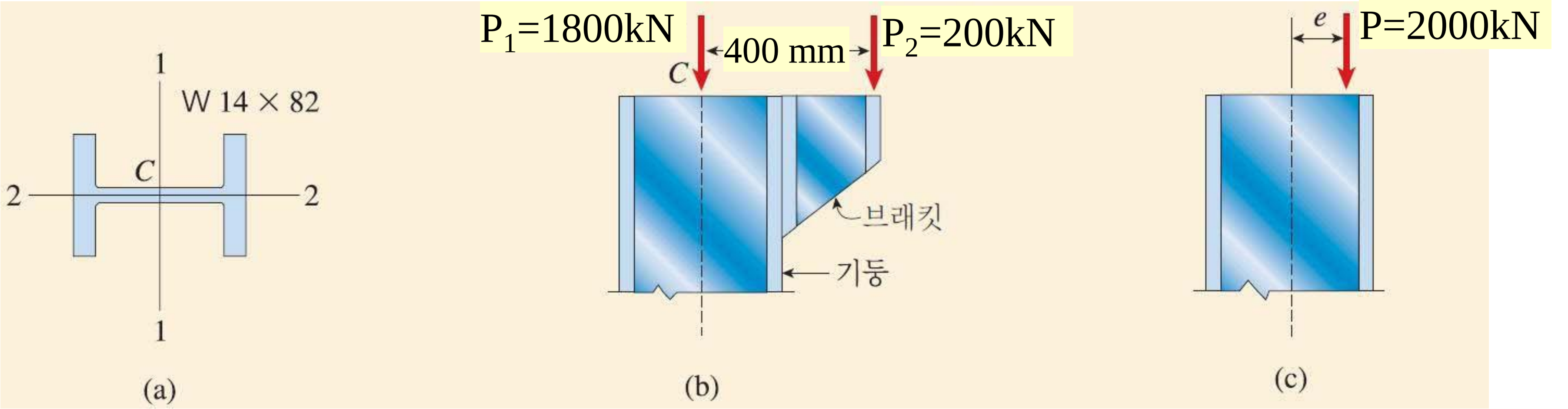
pin-supported at the ends, a length of 7.5 m,

a centrally applied load P_1= 1800 kN

an eccentrically applied load P_2= 200kN

(a) E=210 GPa, calculate the maximum compressive stress

(b) If yield stress = 300MPa, what is the factor of safety with respect to yielding?



Equivalent to a single load of P=2000 KN with e=40 mm

Example 11-4

(a) Maximum compressive stress

- The required properties of the HE 320A wide-flange shape are from Table E-1 in Appendix E

$$A = 124.4\ \text{cm}^2 \quad r = 13.58\ \text{cm} \quad c = \frac{310\ \text{mm}}{2} = 155\text{mm}$$

- Equivalent to a single load P=2000 KN with e=40 mm
- The compressive stress occurs at midpoint on the concave side

$$\sigma_{\max} = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right) = 235.6\,\text{MPa}$$

Example 11-4

(b) Factor of safety with respect to yielding

- Determine the value of the load P acting at the eccentricity e that will produce a maximum stress equal to the yield stress, σ_Y
- Nonlinear relationship between load and stress => solve numerically

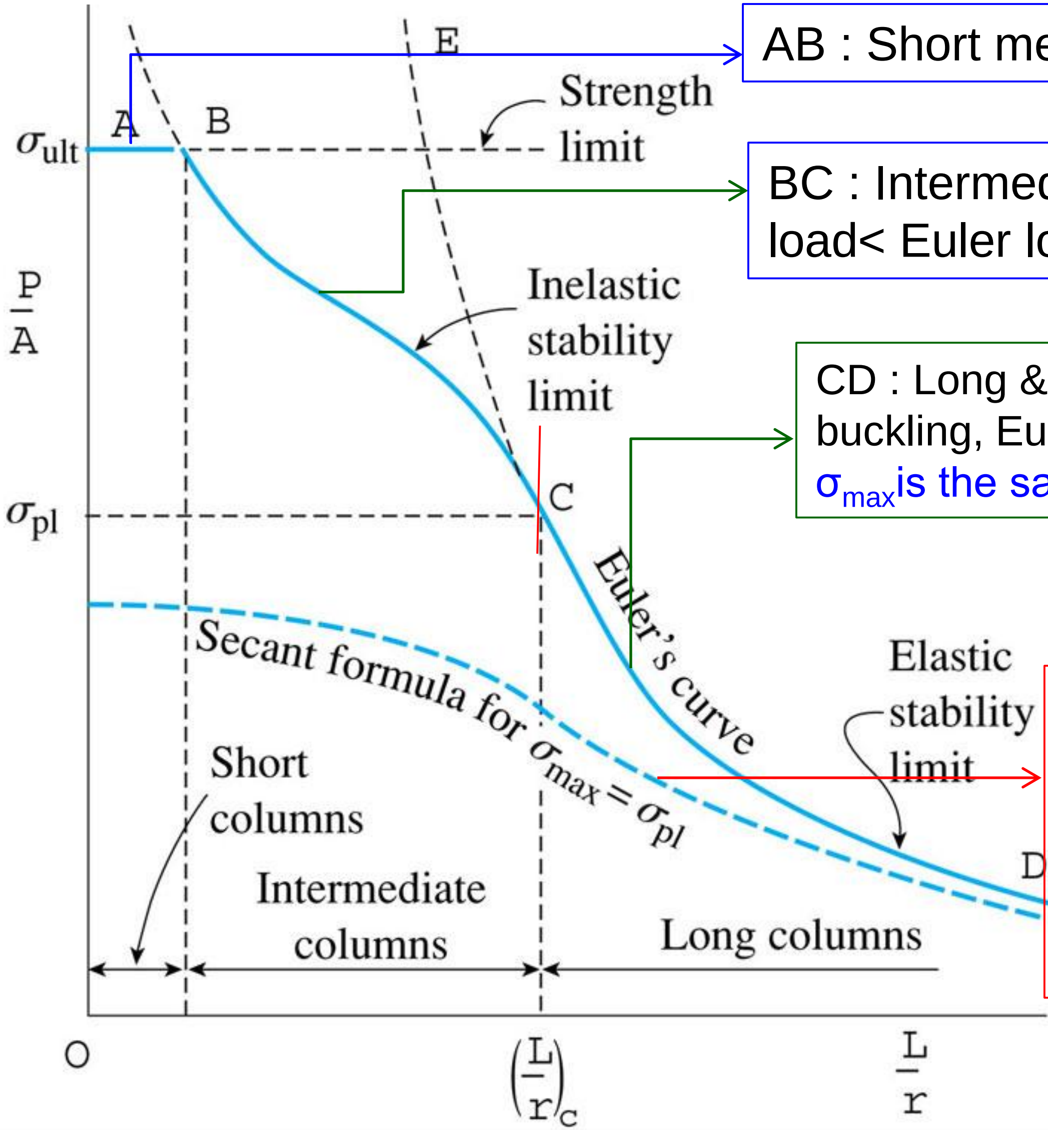
$$\sigma_Y = \frac{P_Y}{A}\left(1+\frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P_Y}{EA}}\right)\right)$$

- $P_Y = 2473\,\mathrm{kN}$ will produce yielding at the cross section of max. bending moment

$$300\,\mathrm{MPa} = \frac{P_Y}{124.4\,\mathrm{cm}^2}\left(1+0.336\sec\left(\frac{55.23}{2}\sqrt{\frac{P_Y}{(210\,\mathrm{GPa})(124.4\mathrm{cm}^2)}}\right)\right)$$

$$n = \frac{P_Y}{P} = \frac{2473\,\mathrm{kN}}{2000\,\mathrm{kN}} = 1.236$$

11.7 Elastic and Inelastic column behavior



AB : Short member, fails by yielding and crushing

BC : Intermediate, inelastic buckling, critical load< Euler load

CD : Long & slender member, fails by elastic buckling, Euler buckling,
σ_{max} is the same as P/A and decreases as C→D.

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}c}{I}$$

P/A decreases as C→D but
σ_{max} remains constant because
$\sigma_{max} = \sigma_{pl}$

Diagram of average compressive stress *P*/*A* versus slender-ness ration *L*/*r*

11.7 Elastic and Inelastic column behavior

- Inelastic buckling : the buckling of columns when the proportional limit is exceeded

- Euler buckling is valid only when axial stress (P/A) < proportional limit

- Slenderness ratio above which Euler's curve is valid by setting critical stress.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{AL^2} = \frac{\pi^2 E}{(L/r)^2} = \sigma_{pl}$$

- Critical slenderness ratio

$$\left(\frac{L}{r}\right)_c = \sqrt{\frac{\pi^2 E}{\sigma_{pl}}}$$

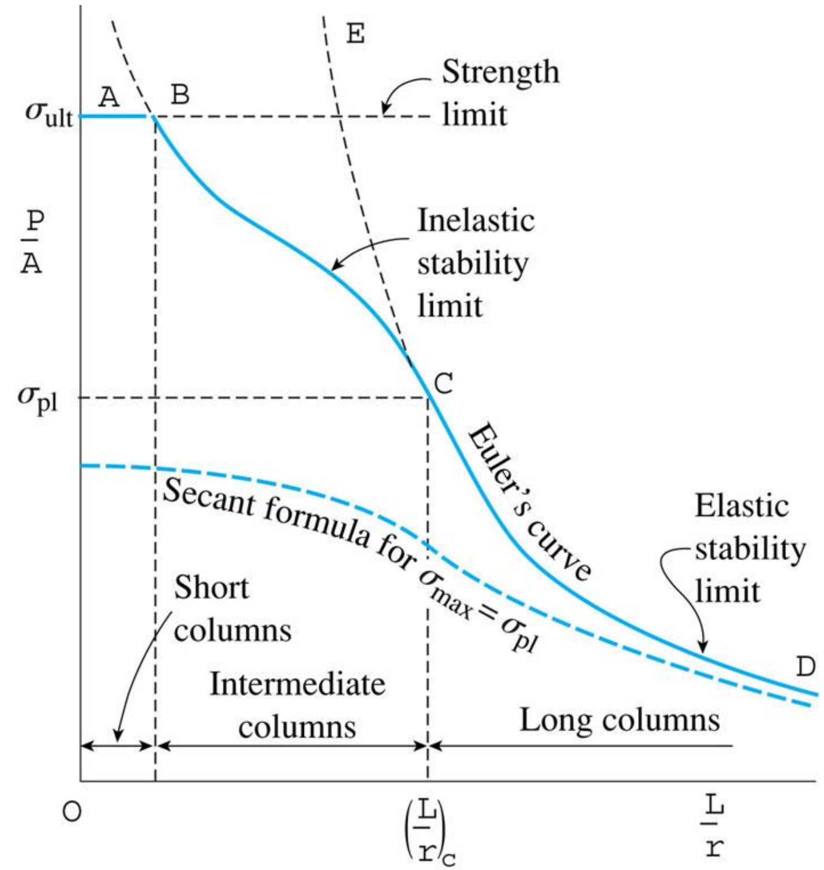
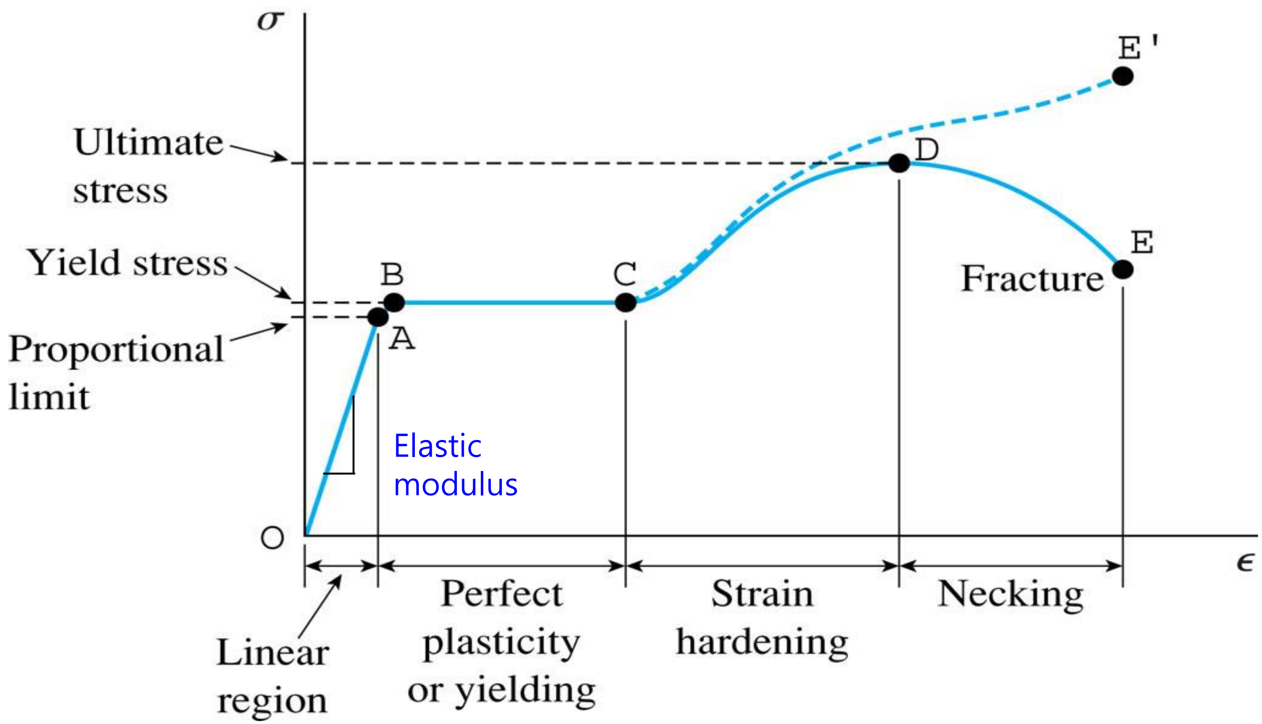


Diagram of average compressive stress *P*/*A* versus slenderness ratio *L*/*r*

11.8 Inelastic buckling

- Euler buckling strength of long column cannot be improved by higher-strength material since it is due to the instability of the column as a whole.
- For a column of intermediate length, the stress in the column will reach the proportional limit before buckling begins.
- The slope of the stress-stain curve is less than the the modulus of elasticity

  → the critical load is always less than the Euler load
- A theory of inelastic buckling is needed



11.8 Inelastic buckling

❖ Tangent-Modulus Theory

- Tangent modulus : the slope of the stress-stain diagram at point A.

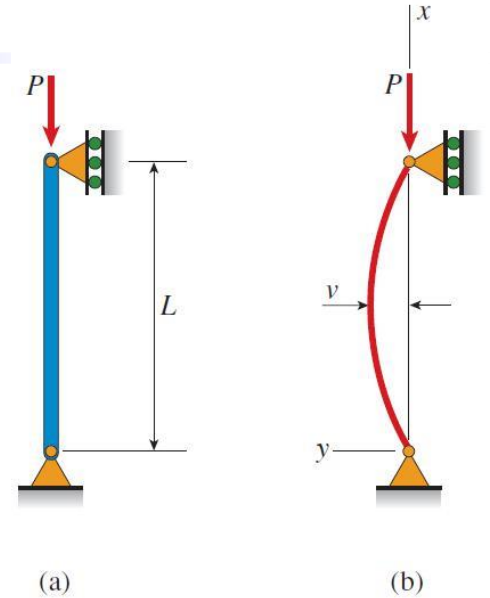
$$E_t = \frac{d\sigma}{d\varepsilon}$$

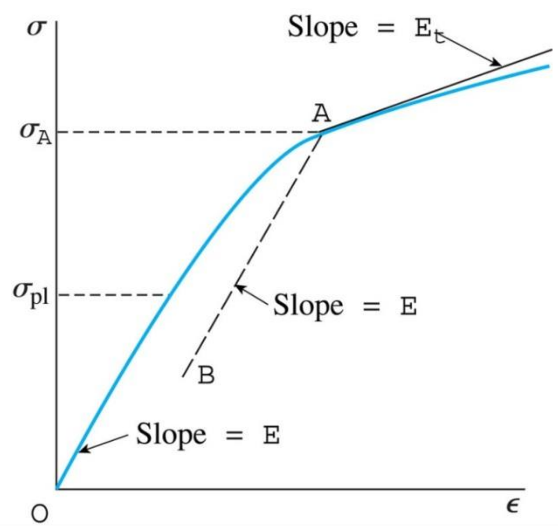
below σ_{pl} , $E = E_t$
beyond σ_{pl}, E_t decrease.

- Column remains straight until the inelastic critical load is reached. After, the column starts bending and the bending stress and is superimposed upon the axial compressive stress.
- The relationship between the resulting bending stress and the strain is given by tangent modulus.
- Expressions for curvature at point A are the same as those for linearly elastic bending with E_t

$$\kappa = \frac{1}{\rho} = \frac{d^2 v}{dx^2} = \frac{M}{E_t I} \qquad M = -Pv$$

➡ $E_t I v'' + Pv = 0$





Compression stress-strain diagram for the material of the column

11.8 Inelastic buckling

- Tangent-modulus load $P_t = \dfrac{\pi^2 E_t}{L^2}$
- Critical stress $\sigma_t = \dfrac{P_t}{A} = \dfrac{\pi^2 E_t}{(L/r)^2}$
- E_t varies with the compressive stress
- Calculate Tangent-modulus load in iterative way

1) Estimation of P_t (call it P_1)
2) $\sigma_1 ,= P_1 /A$
3) Determine E_t from the stress-strain diagram
4) Calculate P_t (call it P_2)
5) Iterate until P_2 is very close to P_1

Q: The reason of $\sigma_r > \sigma_t$?

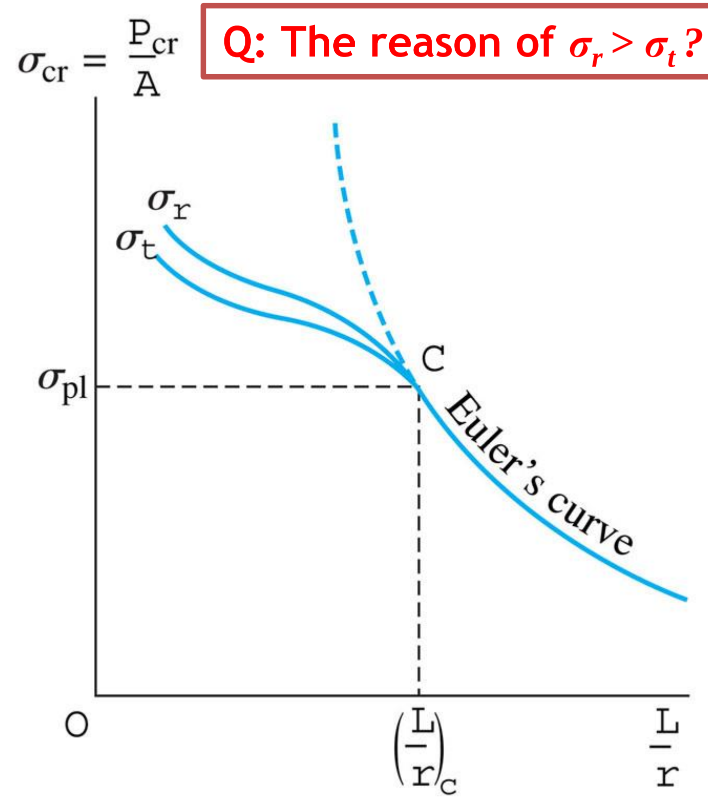


Diagram of critical stress versus slenderness ratio

Q: What is a defect of tangent-modulus theory?

11.8 Inelastic buckling

❖ Reduced-Modulus Theory

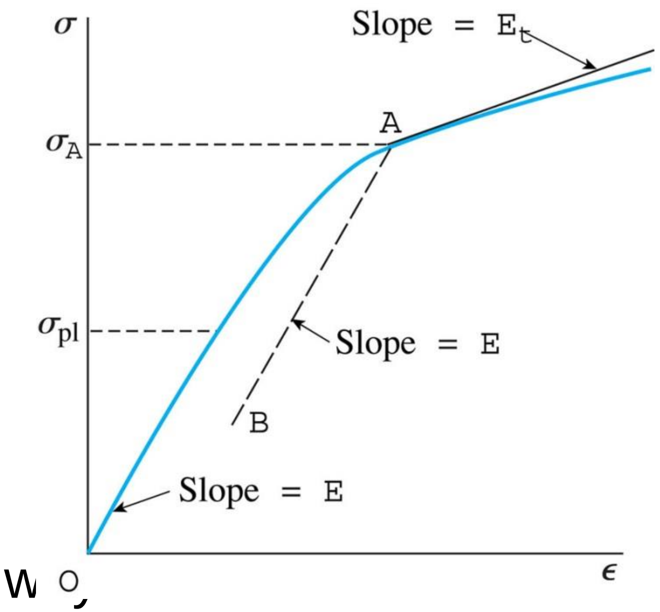
- when the column first departs from the straight position, bending stresses are added.
- the concave side : additional compressive stress, material E_t
- the convex side : tensile stress, material E
- **Reduced modulus E_r**

ex1) rectangular cross section

$$E_r = \frac{4EE_t}{(\sqrt{E}+\sqrt{E_t})^2}$$

ex2) wide-flange beam

$$E_r = \frac{2EE_t}{(E+E_t)^2}$$



- Calculate critical load and stress in iterative w

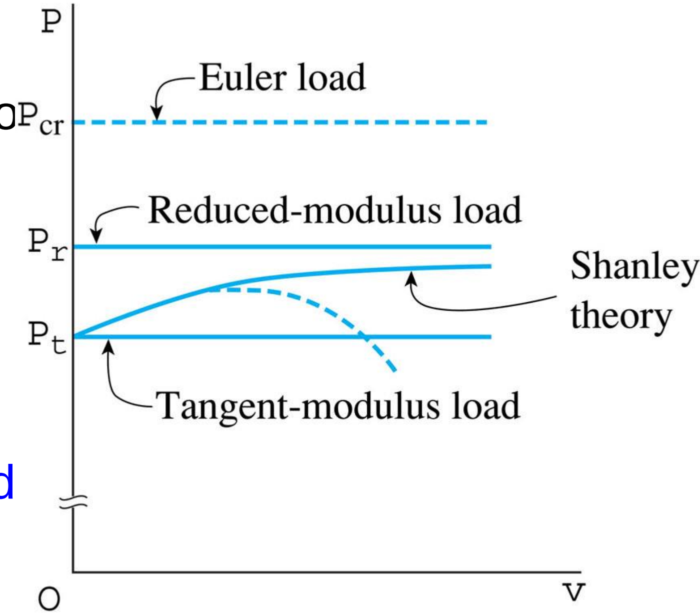
$$P_r = \frac{\pi^2 E_r}{L^2} \qquad \sigma_r = \frac{P_r}{A} = \frac{\pi^2 E_r}{(L/r)^2}$$

Q: What is a conceptual defect of this theory?

11.8 Inelastic buckling

❖ **Shanley Theory (verified by tests)**

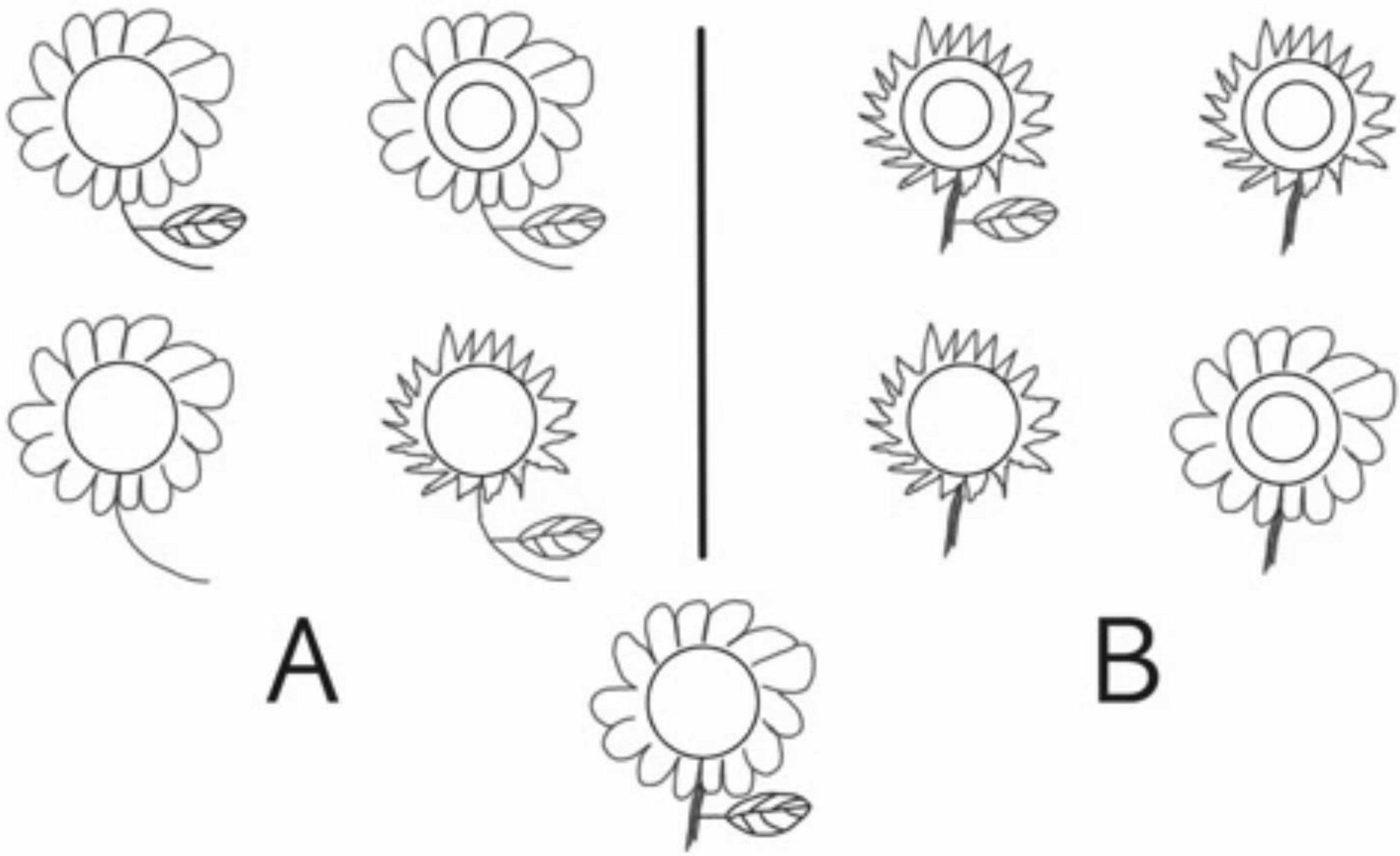
- Since P cannot reach P_t and P_r, it is not possible to buckle inelastically analogous to Euler buckling
  ➔ contradictory
- Instead of neutral equilibrium, a deflected shape, ever-increasing axial load. Bending increases with in load.
- When the load reaches the tangent-modulus load (P_t), bending can begin only if the load continues to increase.
- Decrease in strain on the convex side.
  → The effective modulus becomes lager than E_t.

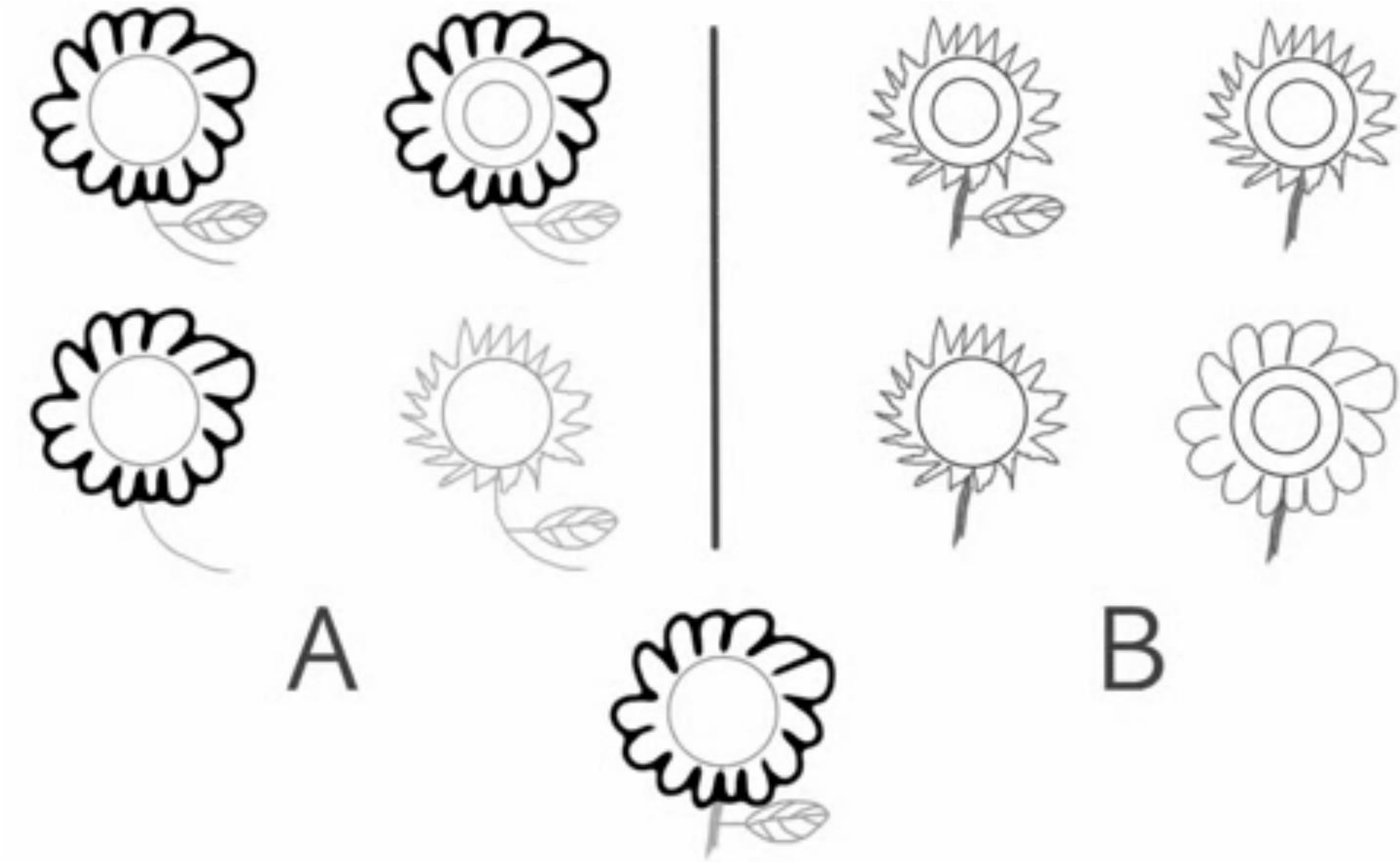


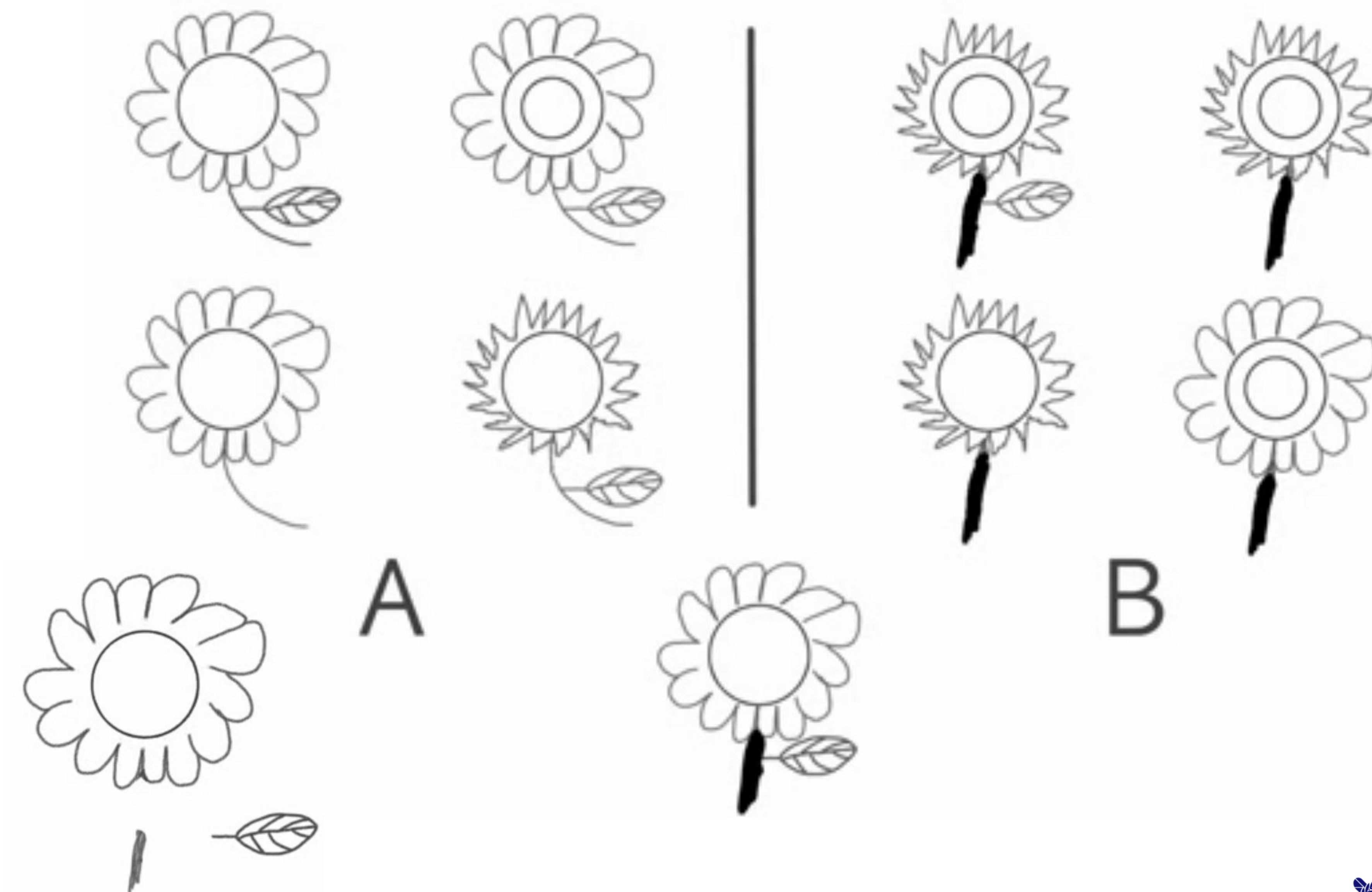
Load-deflection diagram for elastic and inelastic buckling

· but not as great as E_r since it is based on P is constant.

- Load increases but the load doesn't reach reduced modulus load (P_r) until infinitely large deflection.
- In reality the curve eventually goes downward due to other initial defects. For practical purpose, tangent-modulus load (P_t) is adopted.


A
B


A
B


A
B

Difference of Westerner and Asian

- Analytic thinking starts with the objects we pay attention their properties and categorize the object on the basis of the category , we bring rule about the category to somewhat abstract level gove apply the category
- 서양 : 개체를 배경과 분리해서 본다. 분리된 개체를 해석하는 작용도 나옴, 분리하고 해석하는 것 분석 (Analysis, Analyein 분리하다란 어원에 두고 있음)
- 독립된 개체들의 집합, 세상은 무수한 개체로 이루어졌다. 이를 정리할 필요가 생기게 되었다. 모든 속성을 관찰한 뒤 같은 유형들끼리 분리하기 시작했다.
- 분석적 사고 : 어떤 사물이나 인간을 관찰하여 그 속성을 발견하는 것에서부터 시작
- 알아낸 속성을 중심으로 사물을 분류. 분류에 대한 원칙을 정하고 그 원칙을 중심으로 분류가 일어나는 것입니다.
- 큰 장점, 지식 축적이 용이, 과학발달 (science의 어원 sceadan (to divide, separate) 가능


이 중에 두개를 묶는다면?

동서양의 차이 – 동양의 관점

- ❖ 음양 사상 : 음양의 조화,
- ❖ 주역 대대성 (對待性) : 상대가 없으면 존재할 수 없다.

이것이 있으므로 저것이 있고
이것이 생기므로 저것이 생겨난다
이것이 없으므로 저것이 없고
이것이 사라지므로 저것이 사라진다.
-잡아함경

아낌없이 나누어라

- 내가 어떻게 쌓은 노하운인데…
- 줄때는 가장 아까운 것을 확실하게 주어라.
- 신세 지고 나면 마음이 편할까?
- 어렵지 않게 도움을 요청할 수 있다.
- 구한자료는 저자를 명기해서 나누어주라.
- 정을 쌓아라. 도와주고 도와준 티를 내지 마라.
- 배려와 칭찬에 인색하지마라. 칭찬의 말로 자신을 속여라.

선입견을 버려라

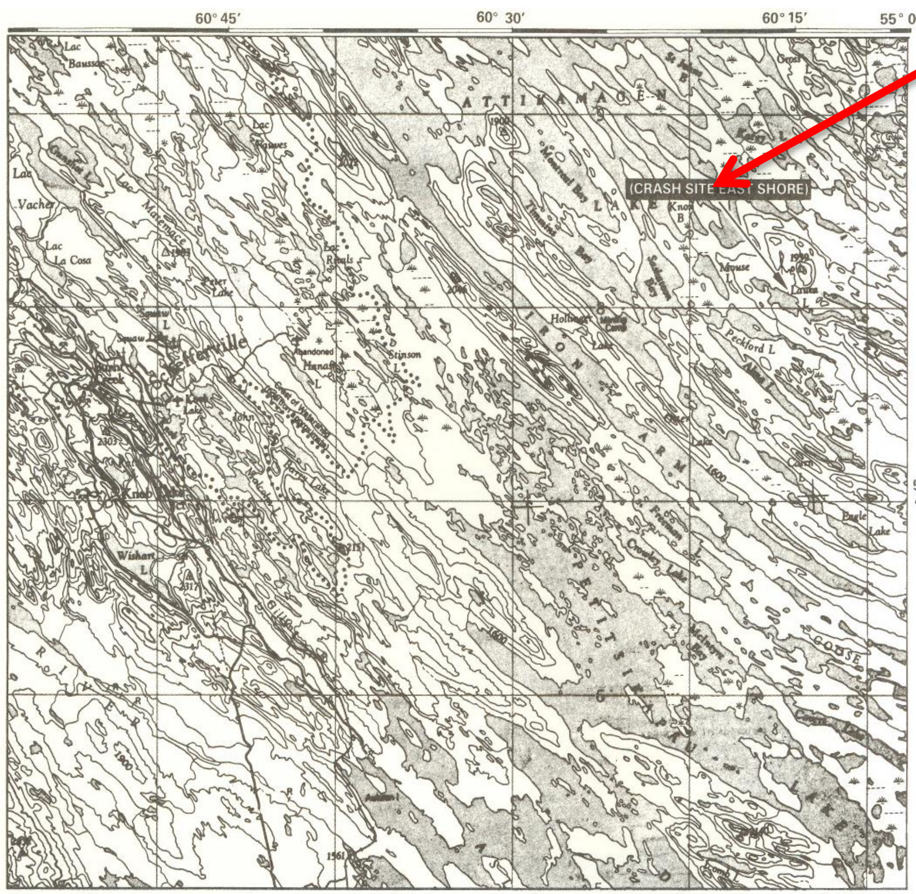
- 하버드 대학 심리학자, 교수와 학생의 키?
- Made in China (미녀들의 수다)
- 신속한, 불확실한 상황에서의 행동의 원칙 수립
- 원래 다 그렇게 해왔어 ☞ 안돼~ 그냥 하던대로 해~
- 부서간의 벽 v.s. 조직 전체적으로의 최적화

  ☞ 깨인 인재, 열린 인재, 통한다.
- 선입견 데이타베이스의 꾸준한 업데이트
- 사람마다 최적의 필터와 렌즈를 껴서 진심 이해 노력
- 미신을 타파하라.

기초 삶 역학 : 협업의 희열

❖ 하버드 교육과정 : 북극 부근에서 살아남기

- 비행기 타고 북근 근처에서 추락, 조난, 조종사 사망
- 조난지 환경에 대한 설명



Temperature Chart for Area

	Mean Daily Temp. °F	Mean Daily Temp. °C	Mean Daily Max. Temp. °F	Mean Daily Max. Temp. °C	Mean Daily Low Temp. °F	Mean Daily Low Temp. °C	Min. Temp. Expected °F	Min. Temp. Expected °C
Oct.	30.3	-0.9	35.8	2.1	24.8	-4.0	0	-17.8
Nov.	15.6	-9.1	22.4	-5.3	9.3	-12.6	-33.0	-36.1
Dec.	-0.3	-17.9	7.5	-13.6	-8.1	-22.3	-42.0	-41.1
Jan.	-9.8	-23.2	-1.5	-18.6	-18.0	-27.8	-53.0	-47.2

Mean Snowfall

Oct.	(Avg. 11 days of snowfall)	7.5 inches (19 cm)
Nov.	(Avg. 16 days of snowfall)	14.5 inches (37 cm)

Sunrise 6:15 a.m. **Sunset** 5:45 p.m.

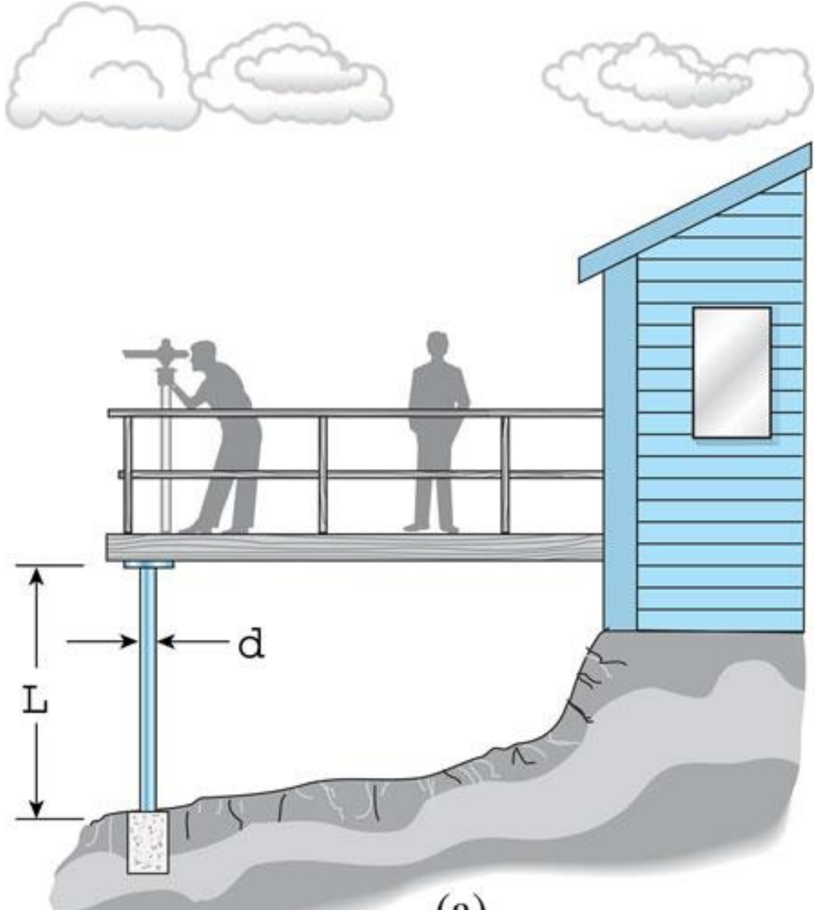
기초 삶 역학 : 협업의 희열

10명씩 총 4팀

Items	STEP 1 개인 순위	STEP 2 팀순위	STEP 3 전문가 순위	STEP 4 STEP1과 STEP 3 차	STEP 5 STEP3과 STEP 3 차이
나침반	1	2	13	12	11
침낭	8	11	5	3	6
간이 정수기	5	4	4	1	0
천막	4	10	15	11	5
성냥	6	8	3	3	5
나이론 로프	2	1	1	1	0
휴대용 랜턴	9	6	6	3	0
스노우 슈즈	11	12	10	1	2
술(럼주)	7	3	7	0	4
거울와 면도 세트	10	10	11	1	1
알람 시계	14	7	9	5	2
손 도끼	15	15	12	3	3
북극성에 의한 행해법	12	5	2	10	3
				54	42

기초 삶 역학 : 협업의 희열

- 사오정? 주파수를 만들어 맞장구를 치며 공감대를 형성하라
- 나만 잘하면 되지? 기계 계열의 업무의 특성, 혼자서는 못한다.
- 업무의 동지를 만들라, 직장 생활의 최대의 기쁨
- 소통과 대화는 상호간의 주고받는 것
- 로미오와 줄리엣 효과
- 설득에만 목을 매면 설득은 멀리 도망간다.
- 상대가 얻고자 하는 것, 중요하게 여기는 것이 무엇인가?
- 오늘 당장 성과가 없어도 좋다.



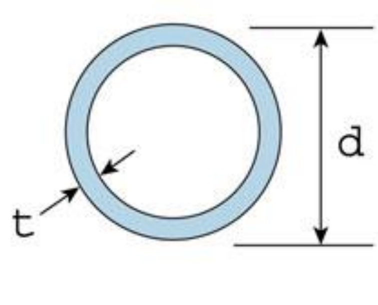


FIG. 11-20
Example 11-2. Aluminum pipe column

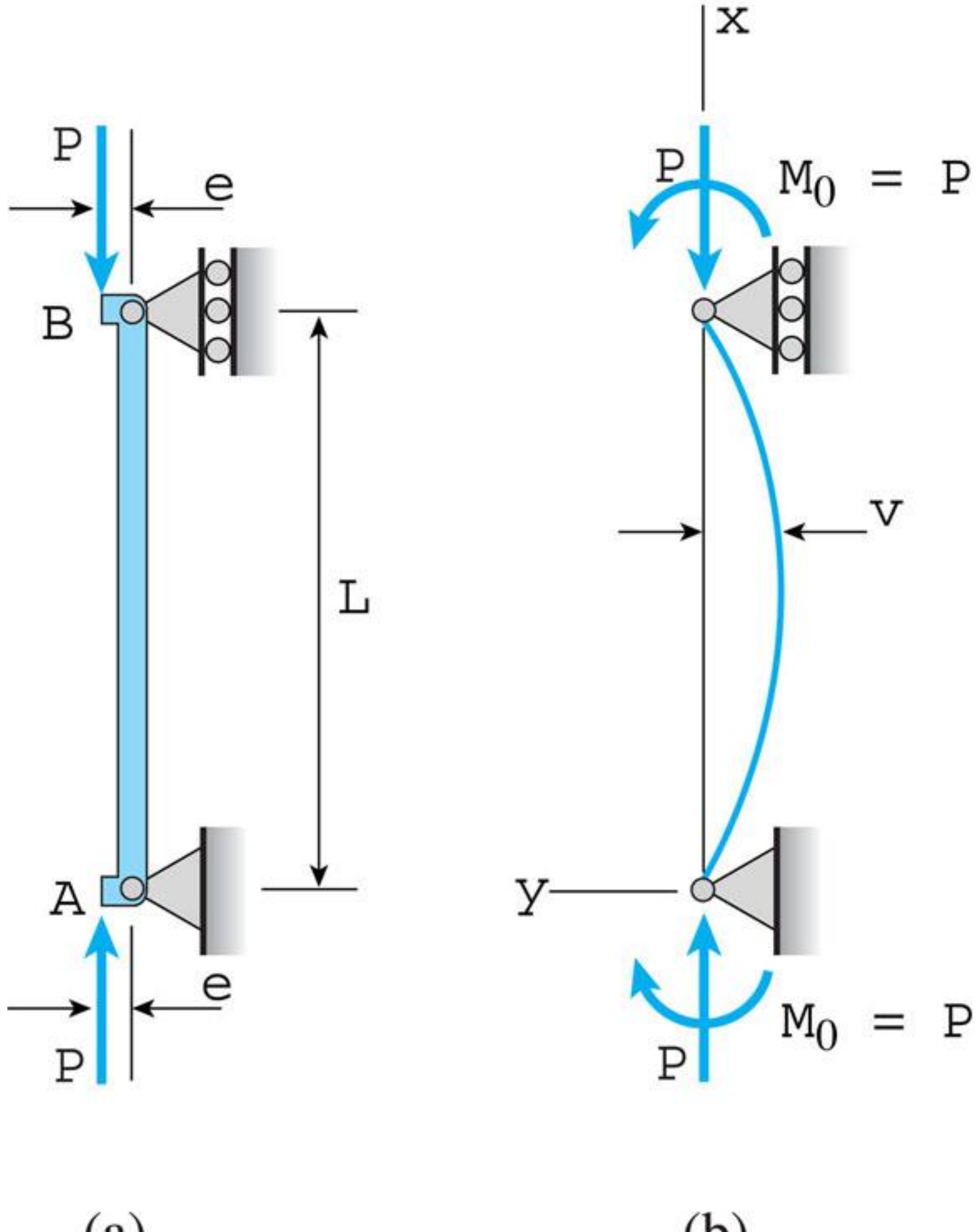


FIG. 11-21
Column with eccentric axial loads

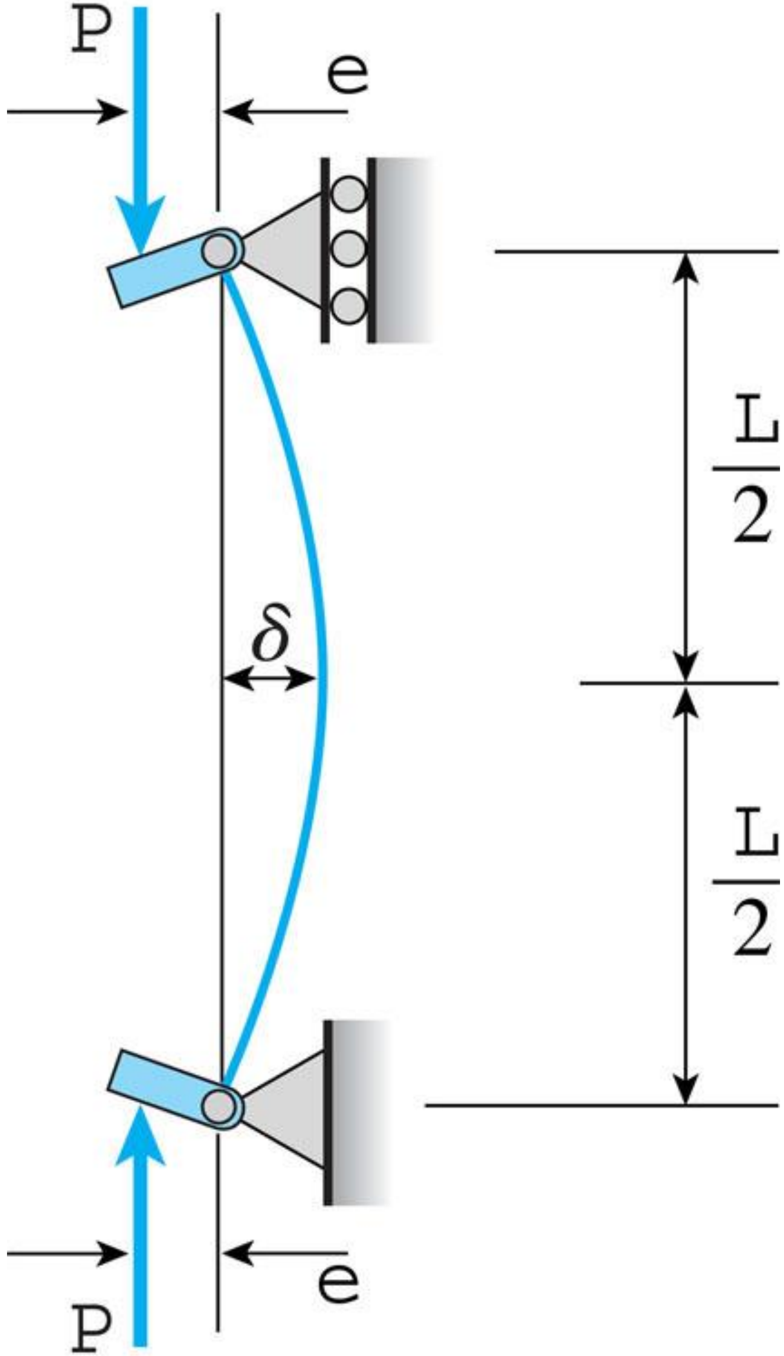


FIG. 11-22
Maximum deflection δ of a column with eccentric axial loads

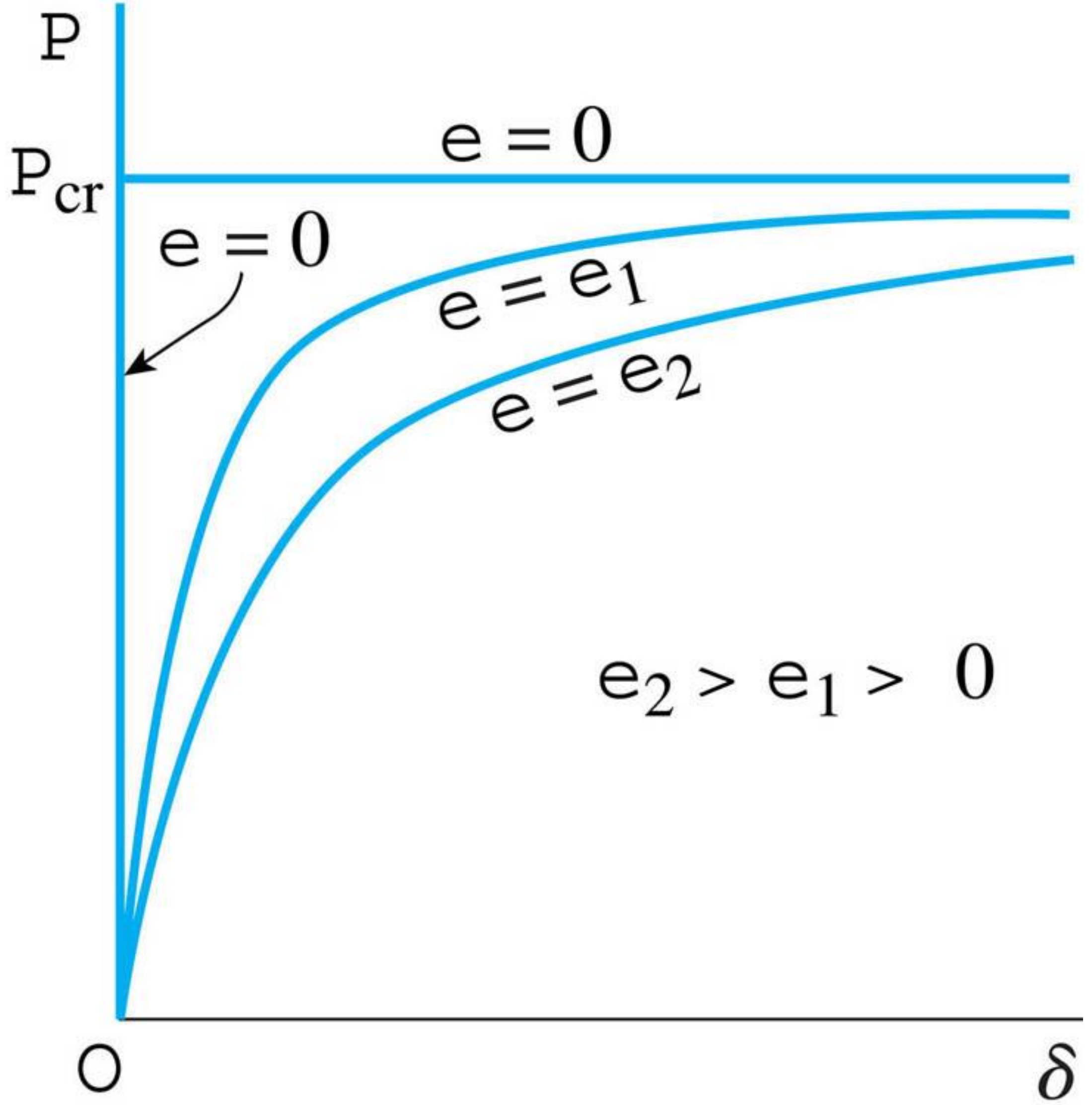


FIG. 11-23 **Load-deflection diagram for a column with eccentric axial loads (see Fig. 11-22 and Eq. 11-54)**

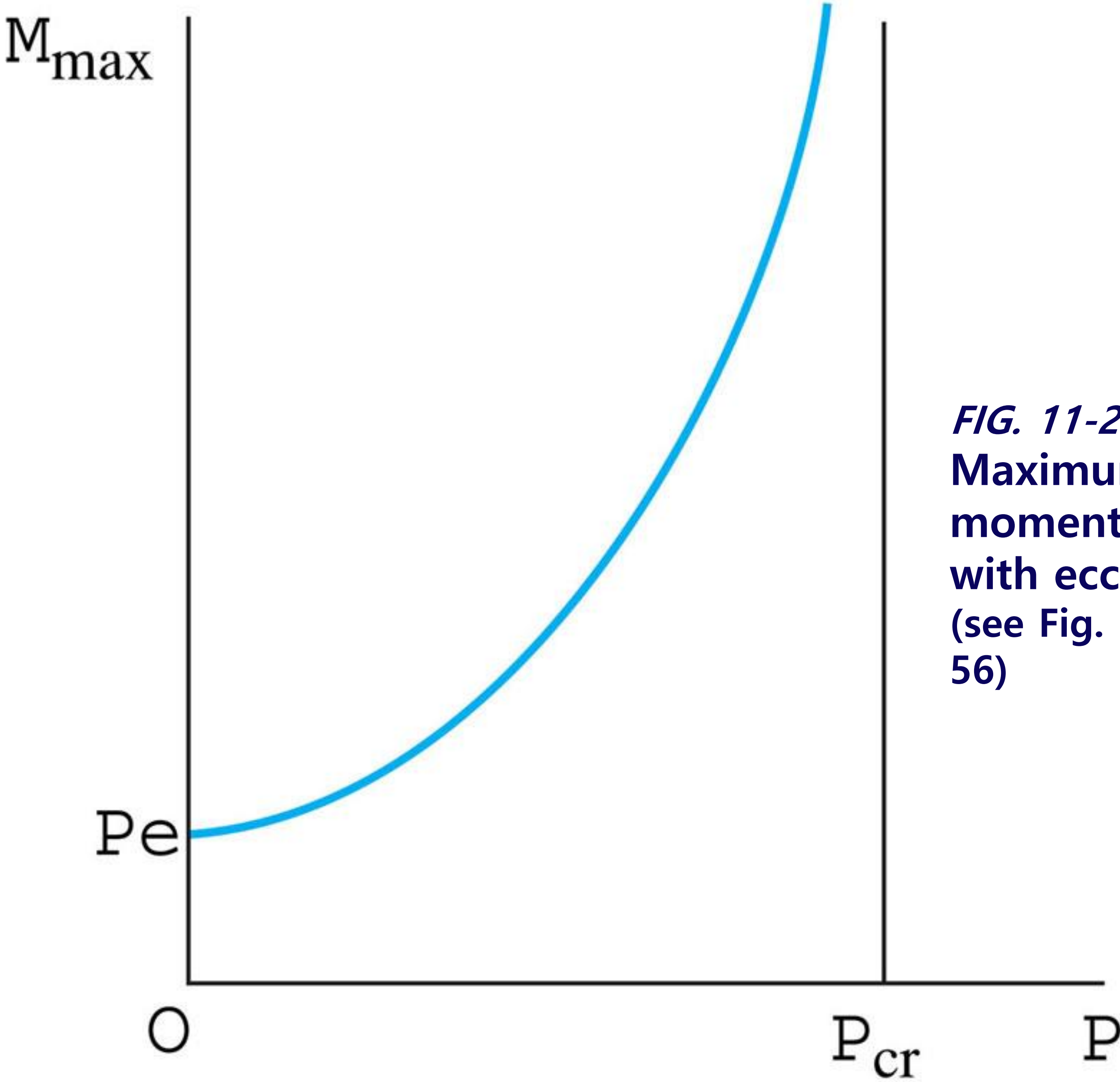


FIG. 11-24
Maximum bending moment in a column with eccentric axial loads (see Fig. 11-22 and Eq. 11-56)

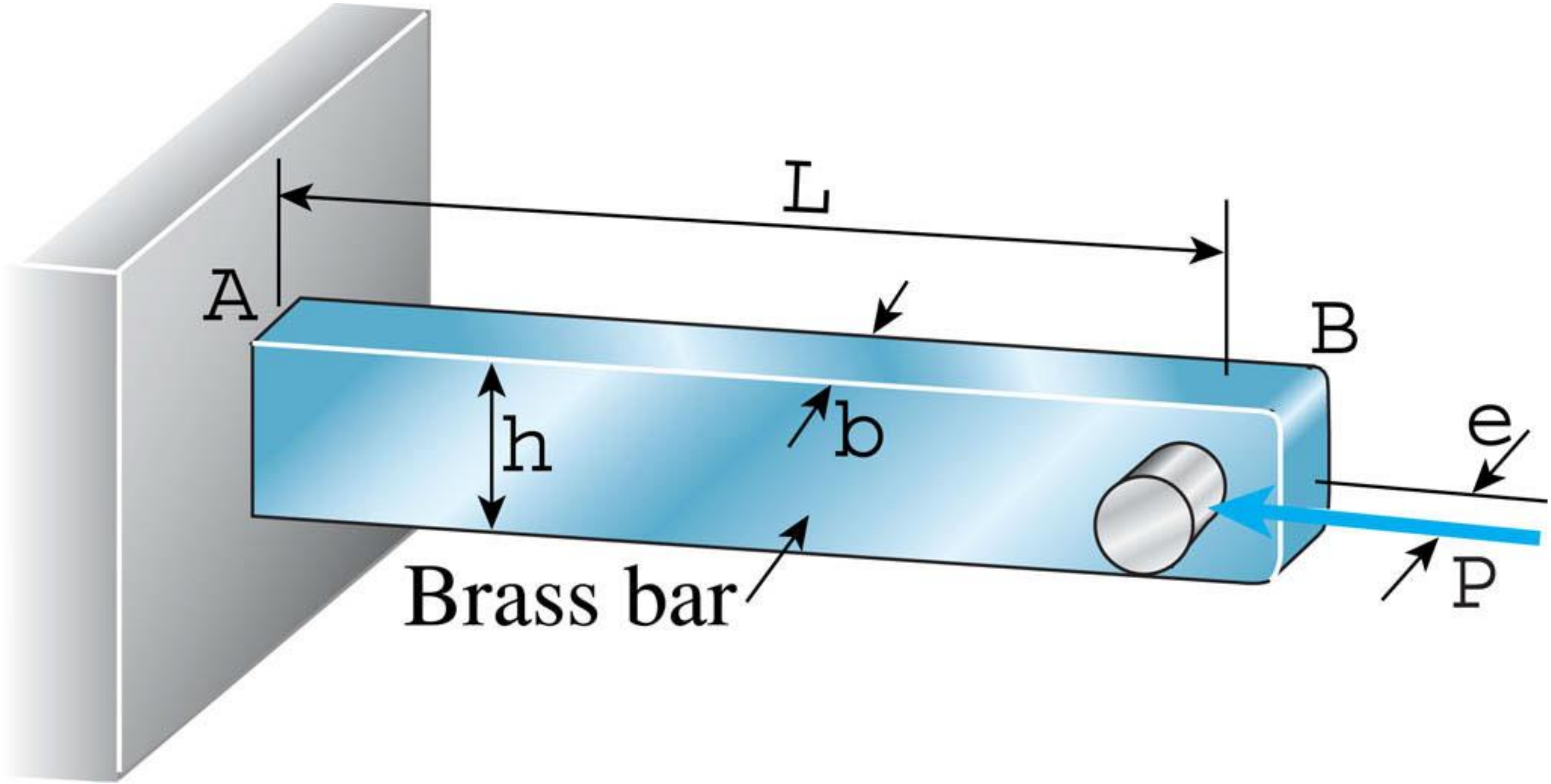
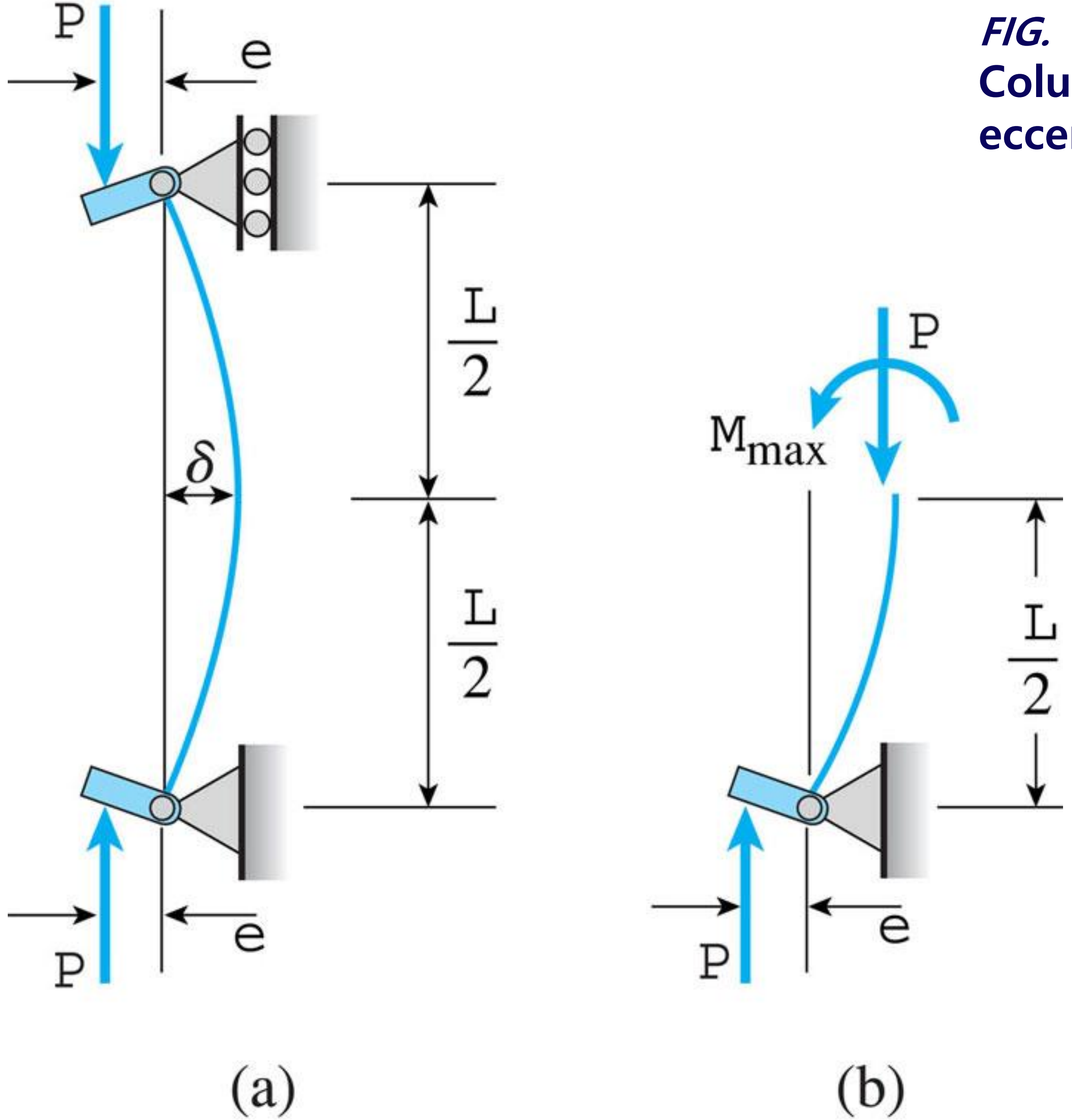


FIG. 11-25 **Example 11-3. Brass bar with an eccentric axial load**

FIG. 11-26
Column with eccentric axial loads



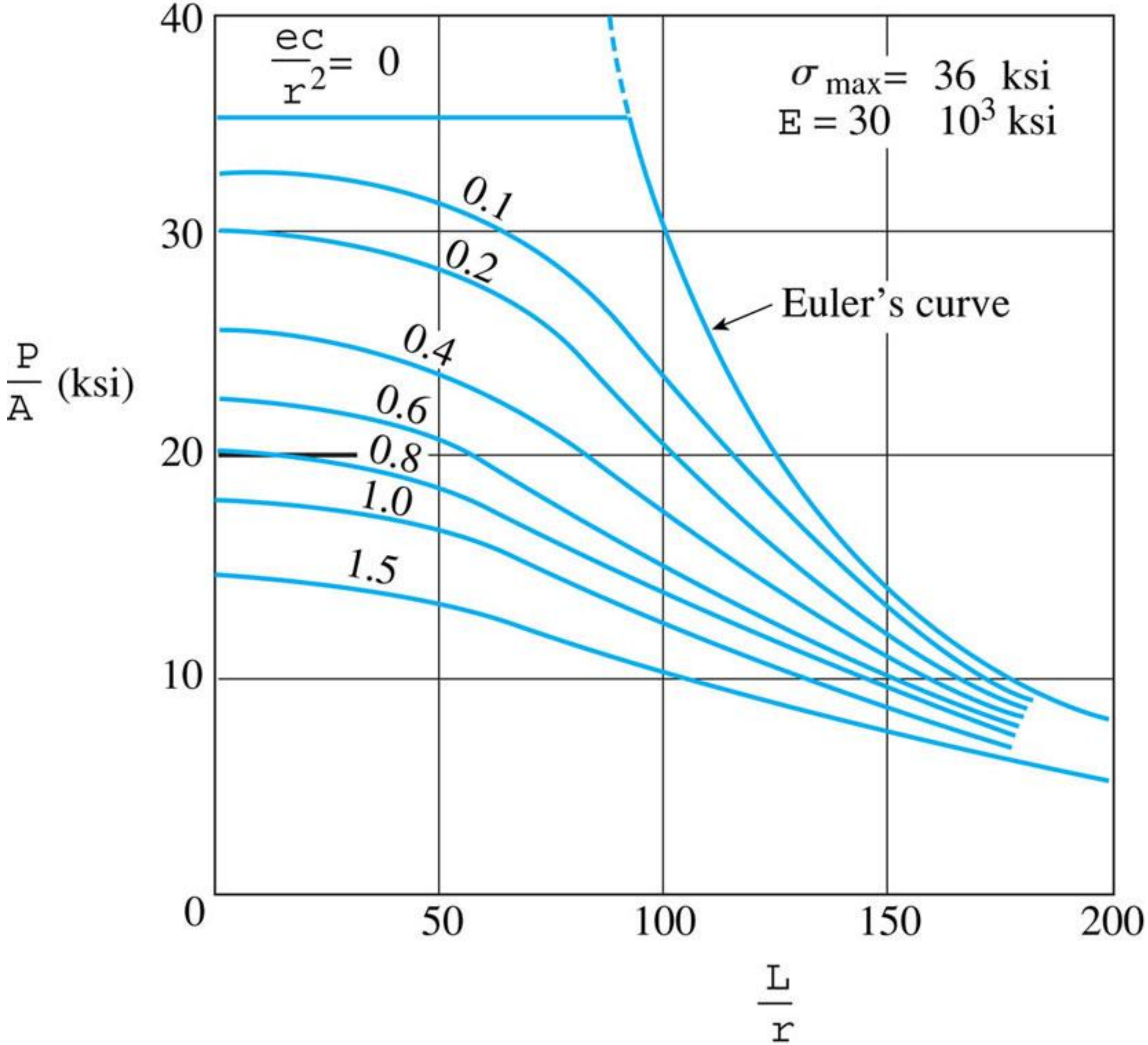


FIG. 11-27 **Graph of the secant formula (Eq. 11-59) for σ_{max} = 36 ksi and E = 30 X 10^3 ksi**

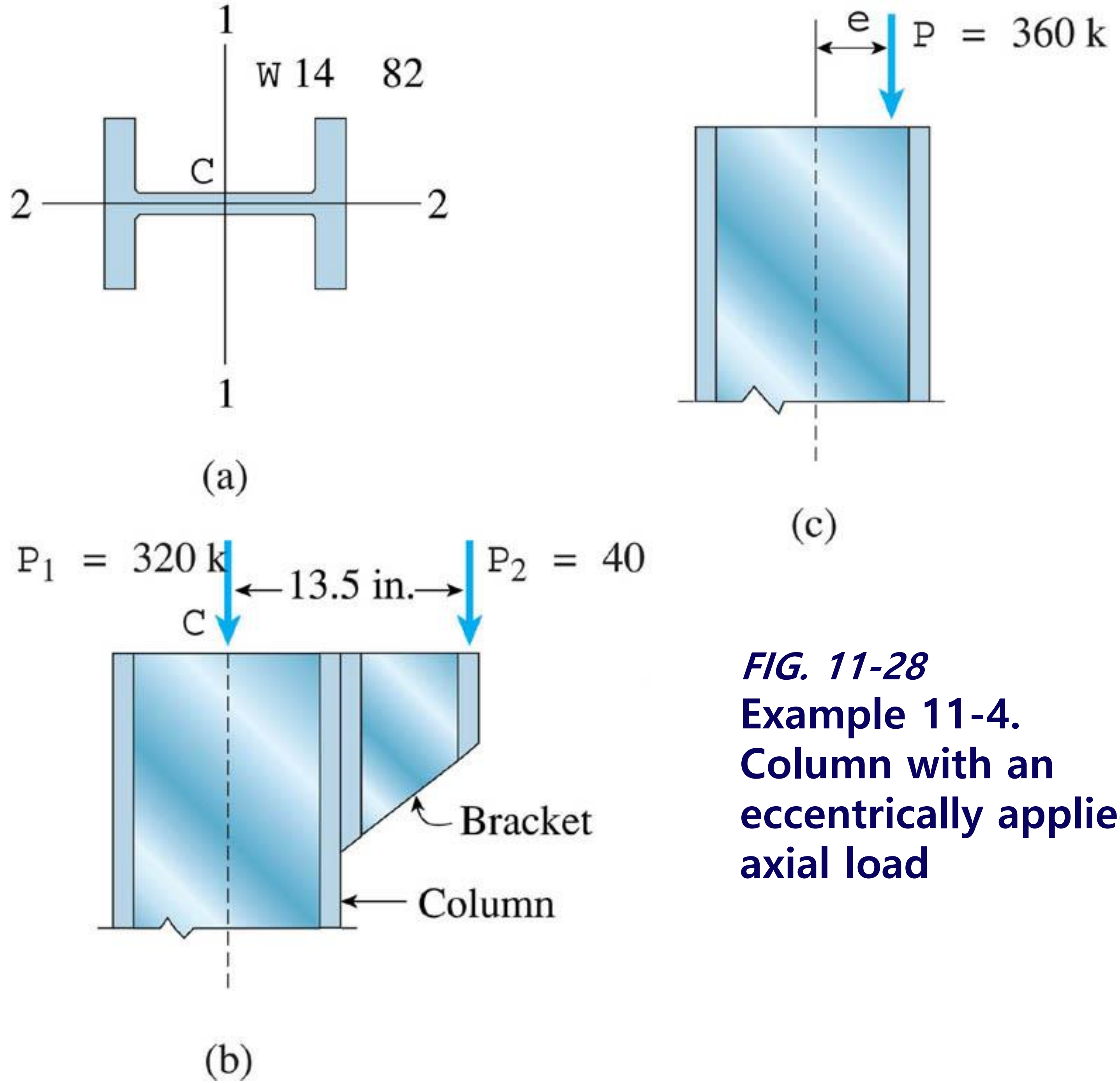


FIG. 11-28
Example 11-4. Column with an eccentrically applied axial load

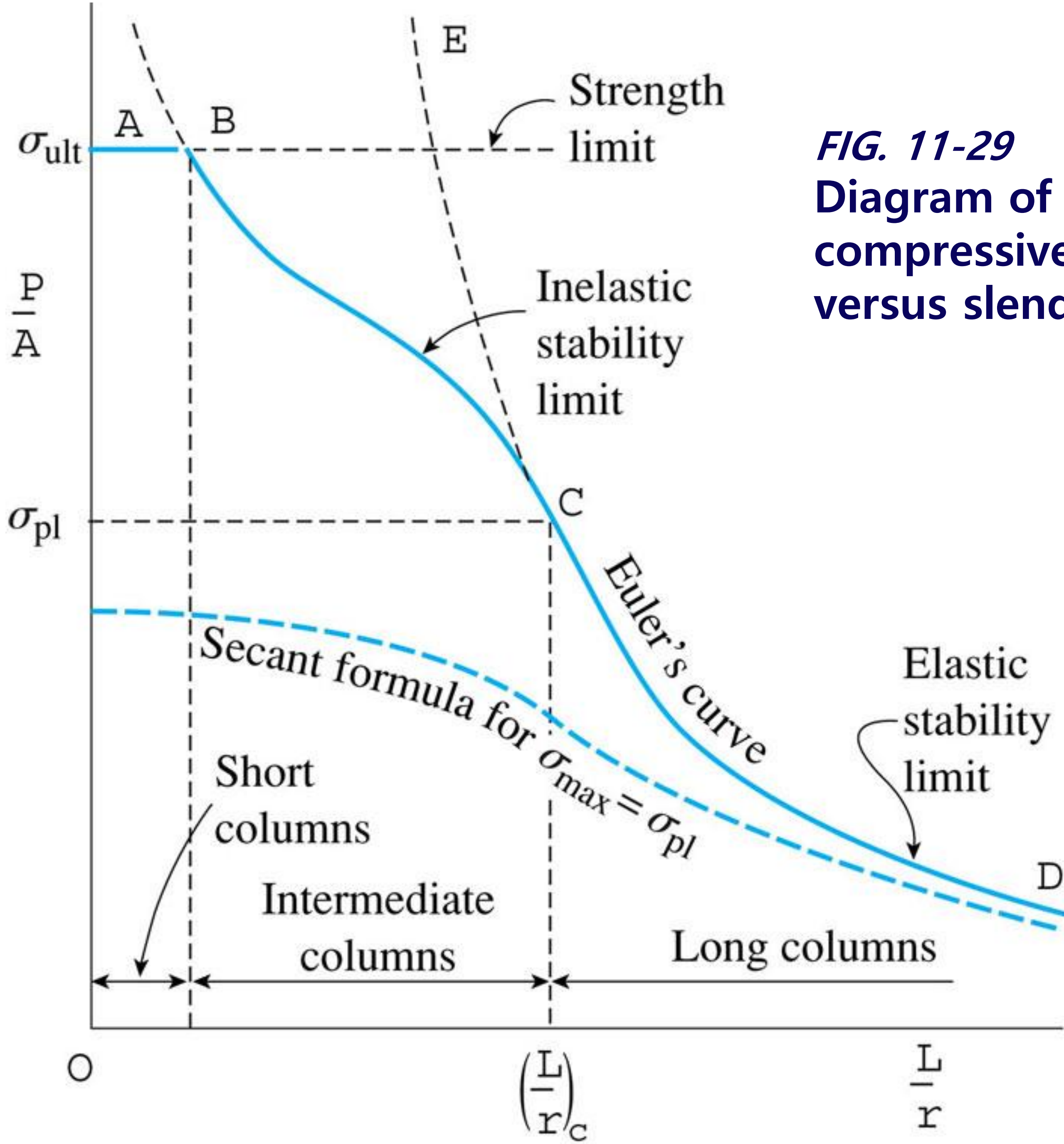


FIG. 11-29
Diagram of average compressive stress *P/A* versus slender-ness ration *L/r*

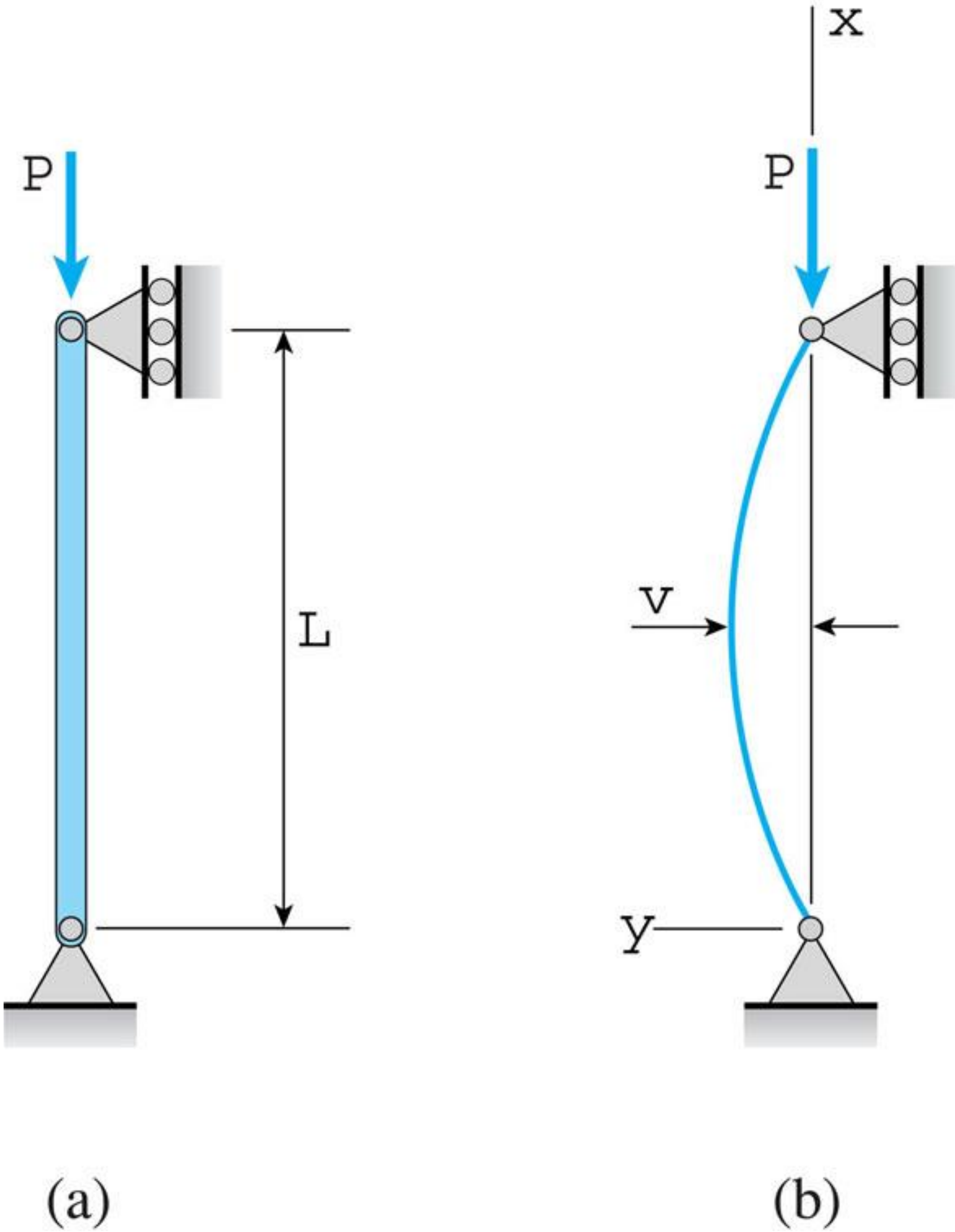


FIG. 11-30
Ideal column of intermediate length that buckles inelastically

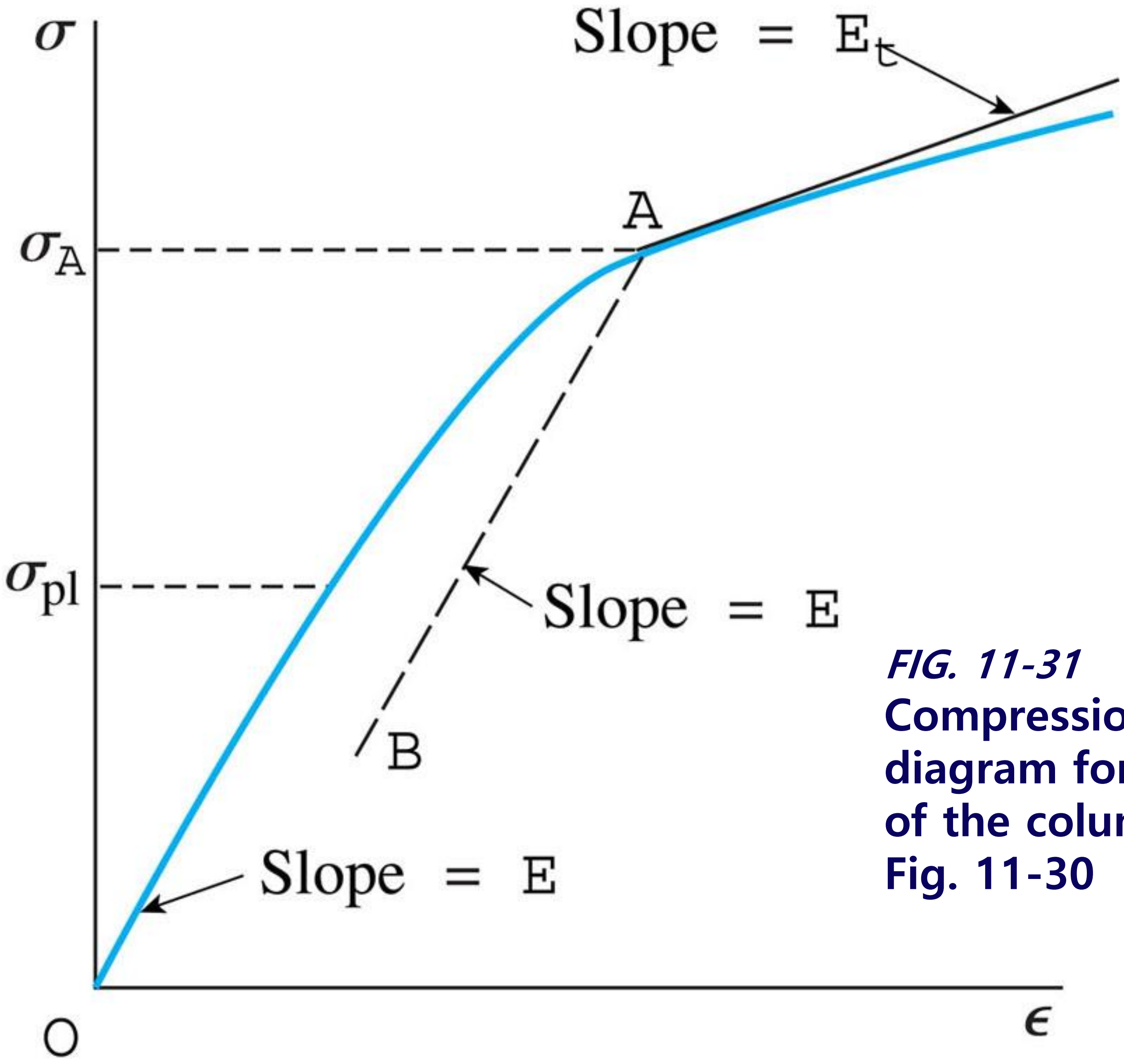


FIG. 11-31

Compression stress-strain diagram for the material of the column shown in Fig. 11-30

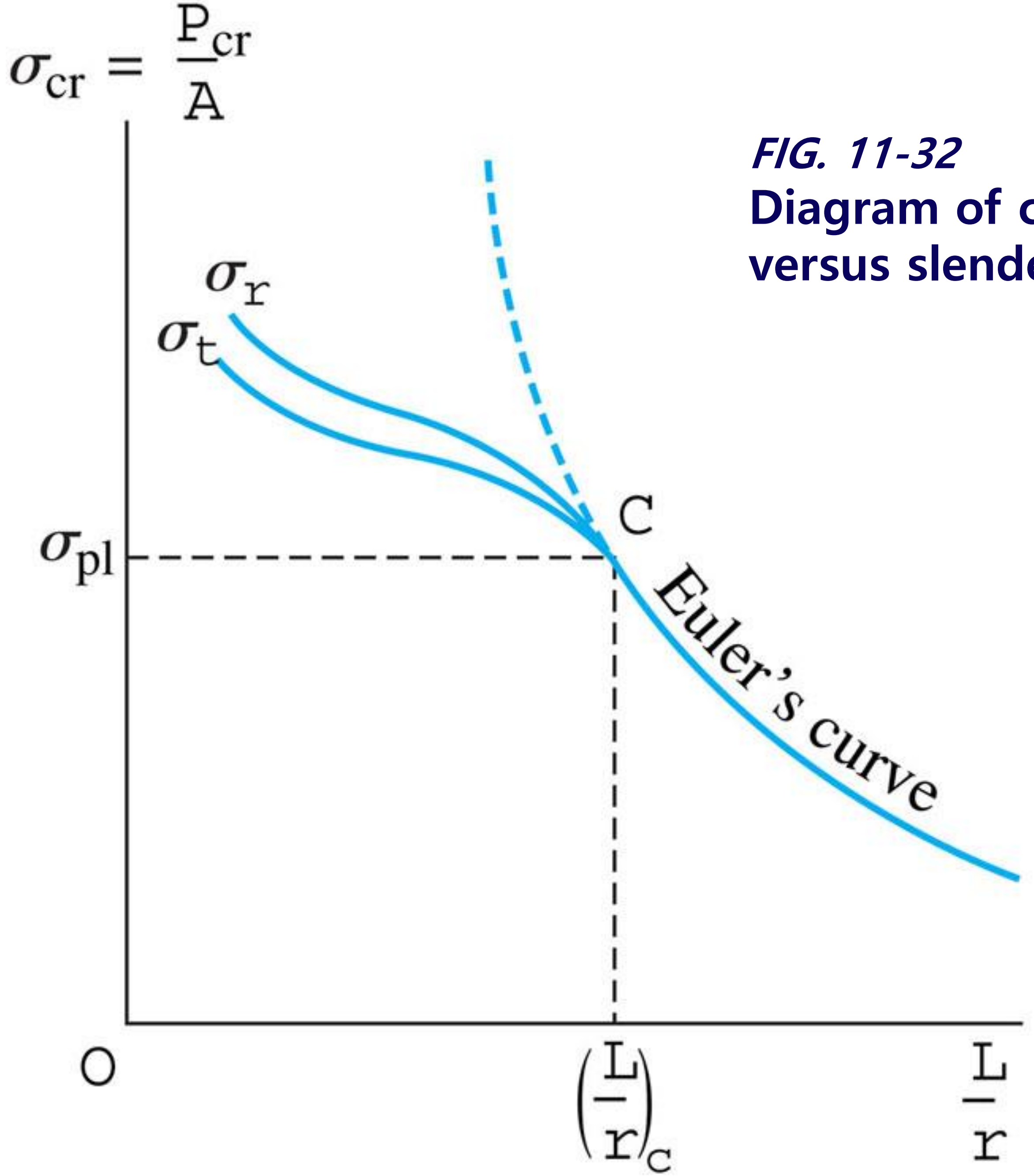


FIG. 11-32
Diagram of critical stress versus slenderness ratio

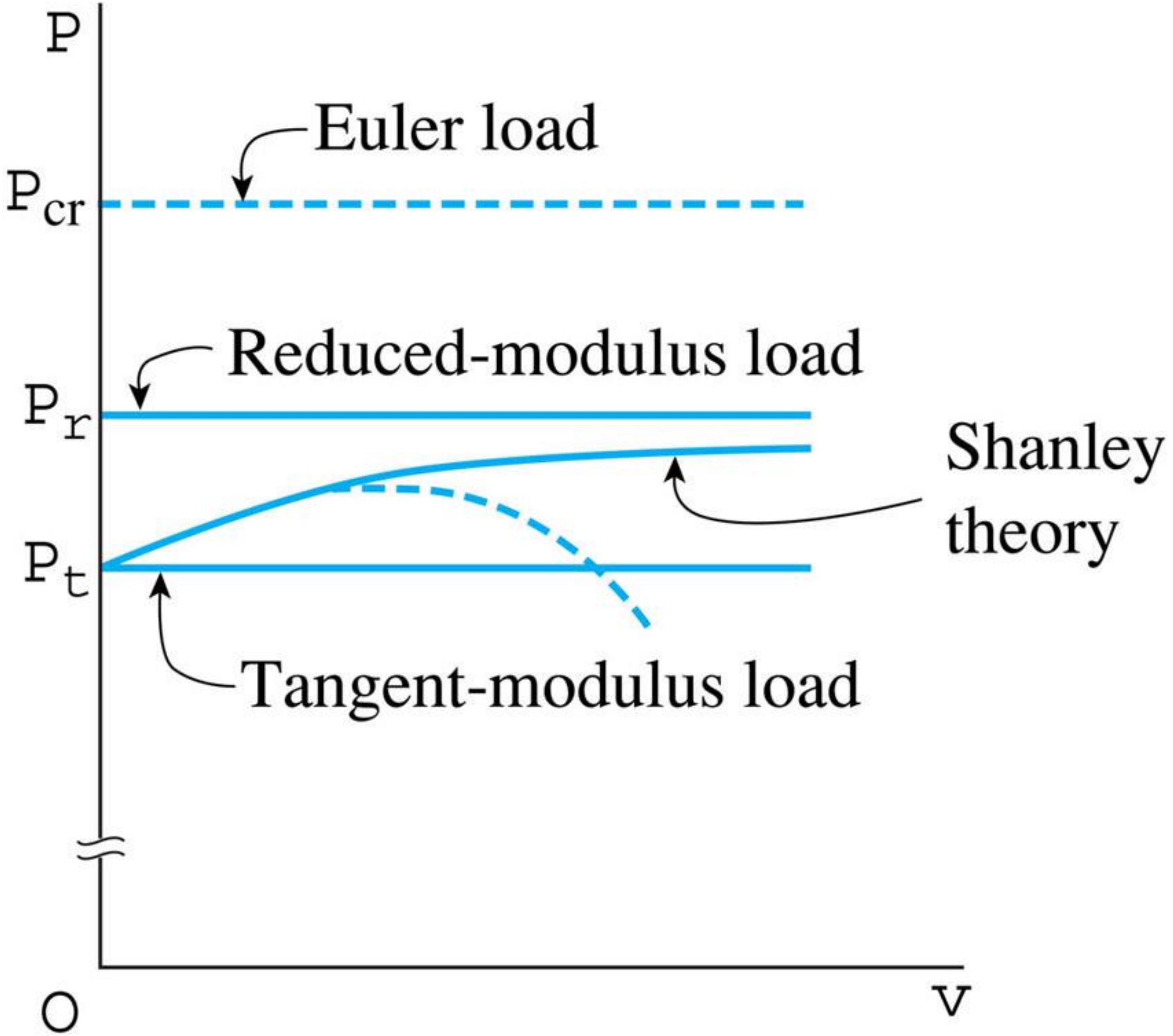


FIG. 11-33 **Load-deflection diagram for elastic and inelastic buckling**

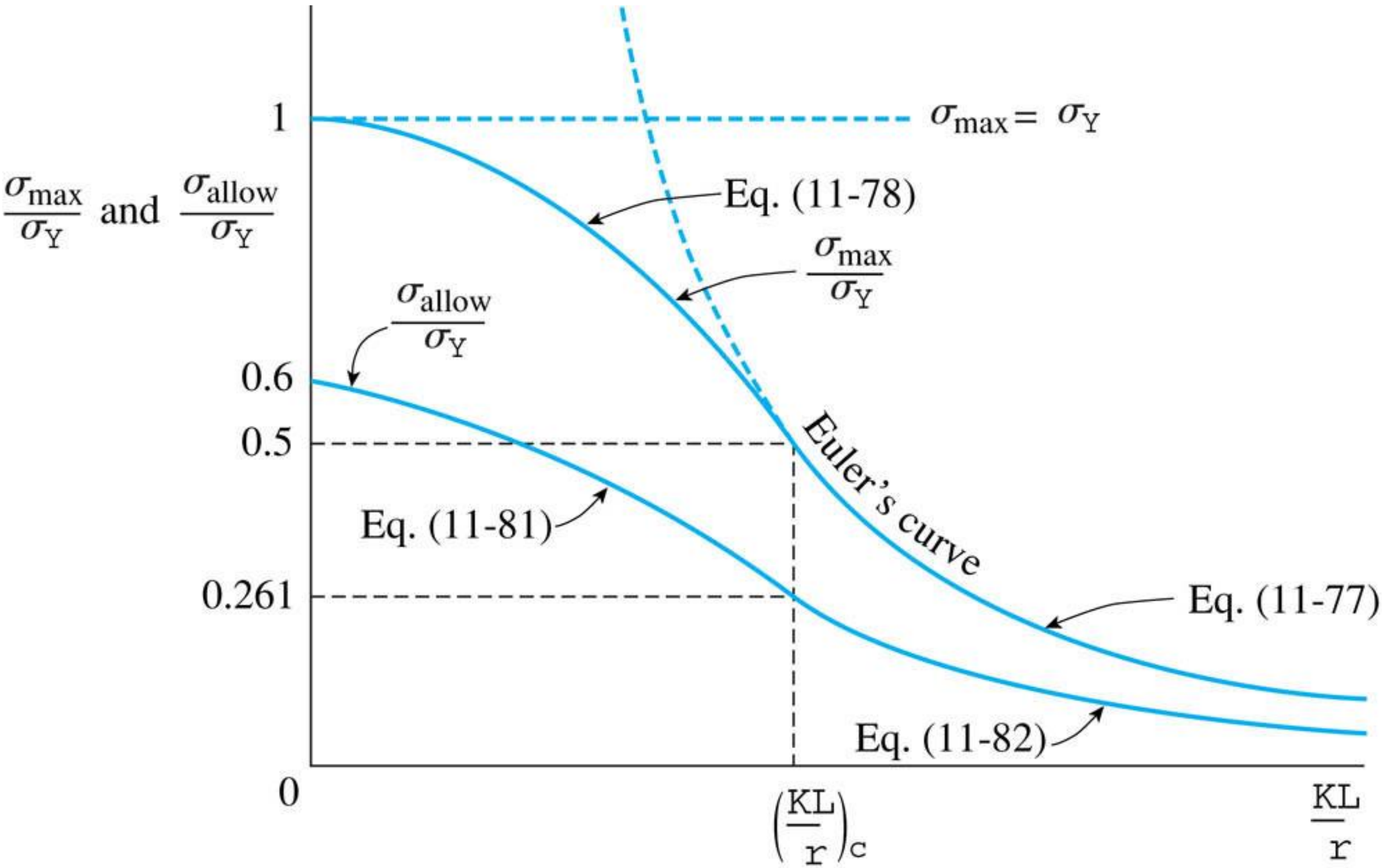
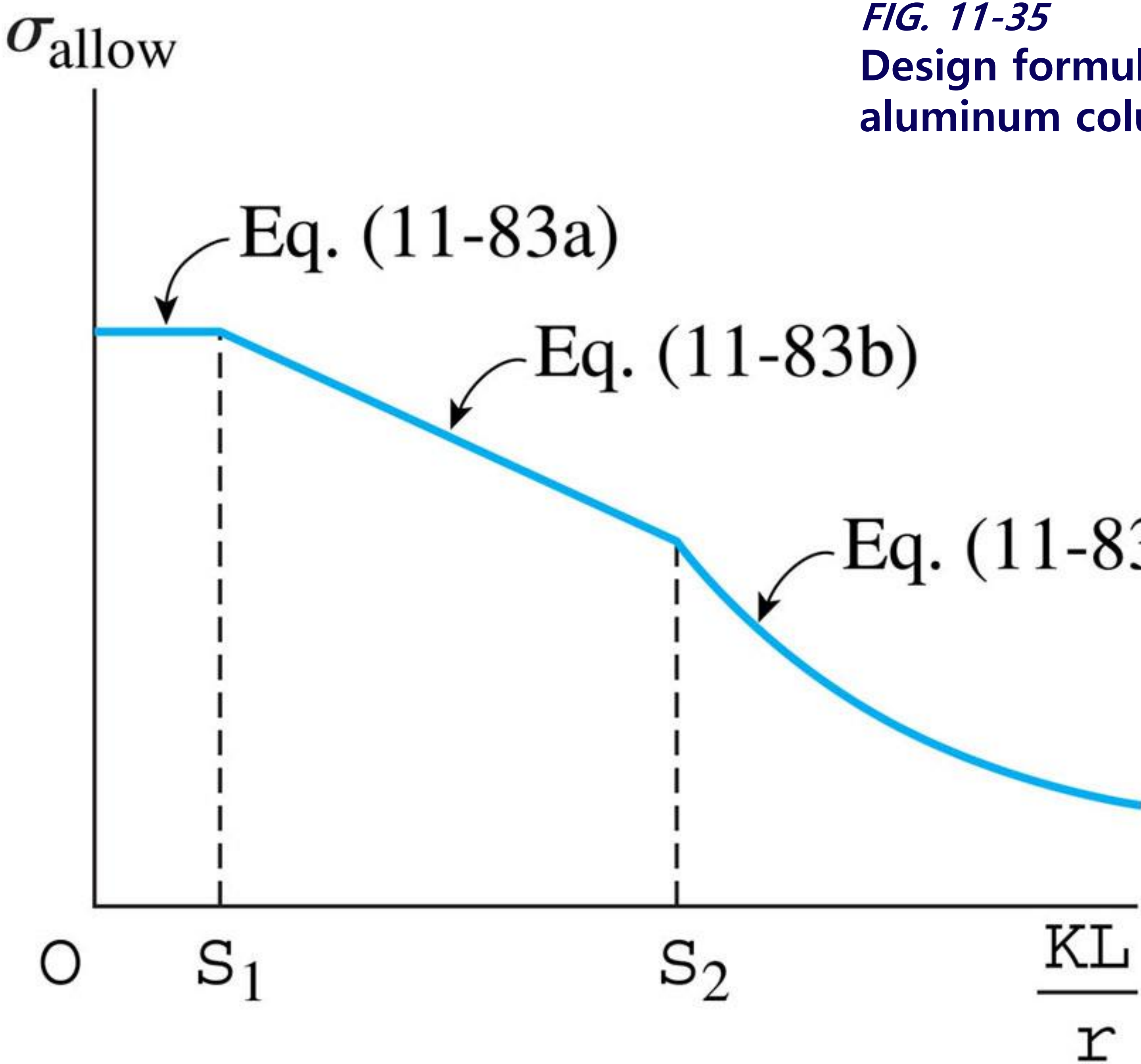


FIG. 11-34 **Design formulas for structural-steel columns**

FIG. 11-35
Design formulas for aluminum columns



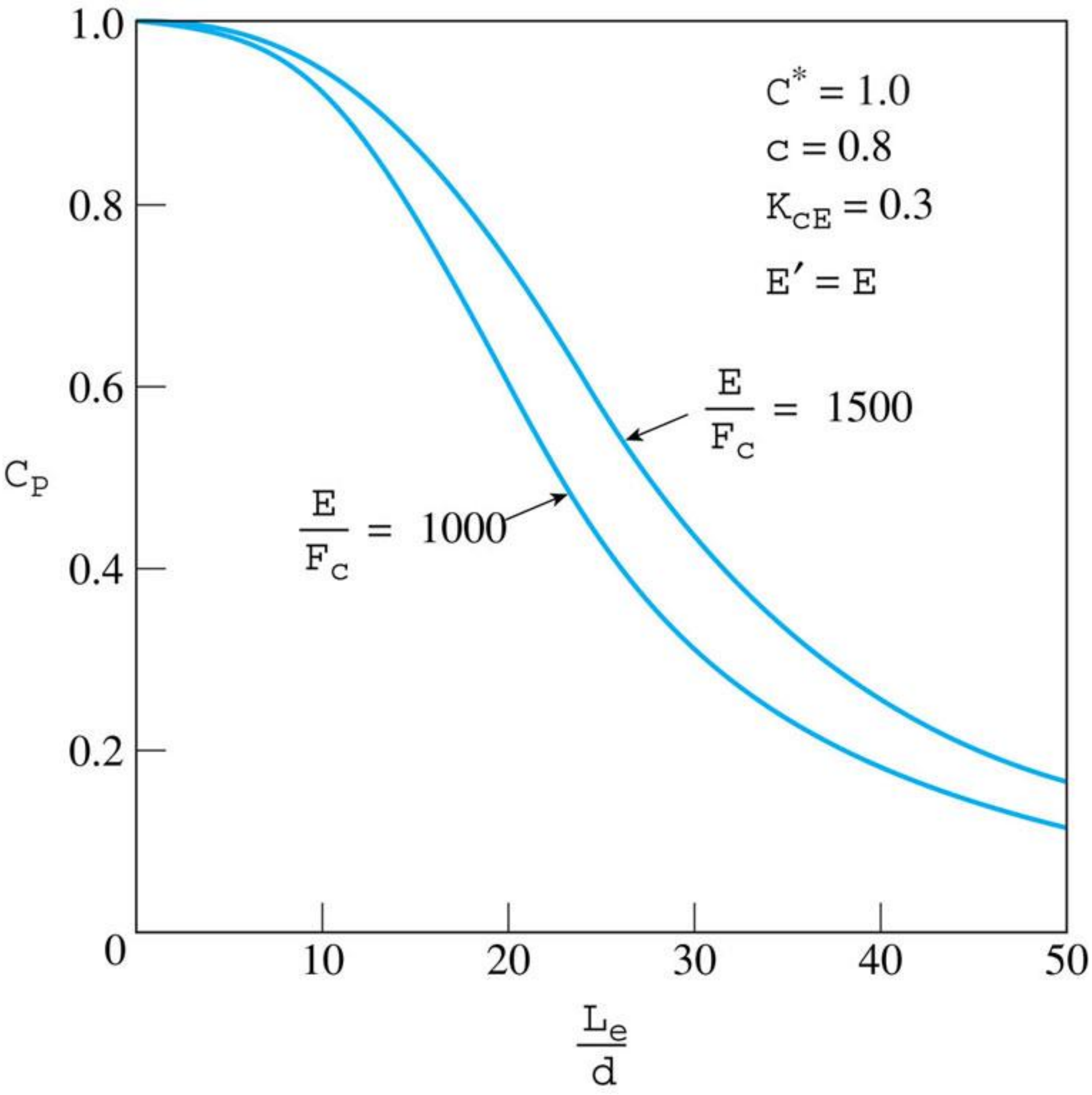


FIG. 11-36
Typical curves for the column stability factor C_P (rectangular wood columns)

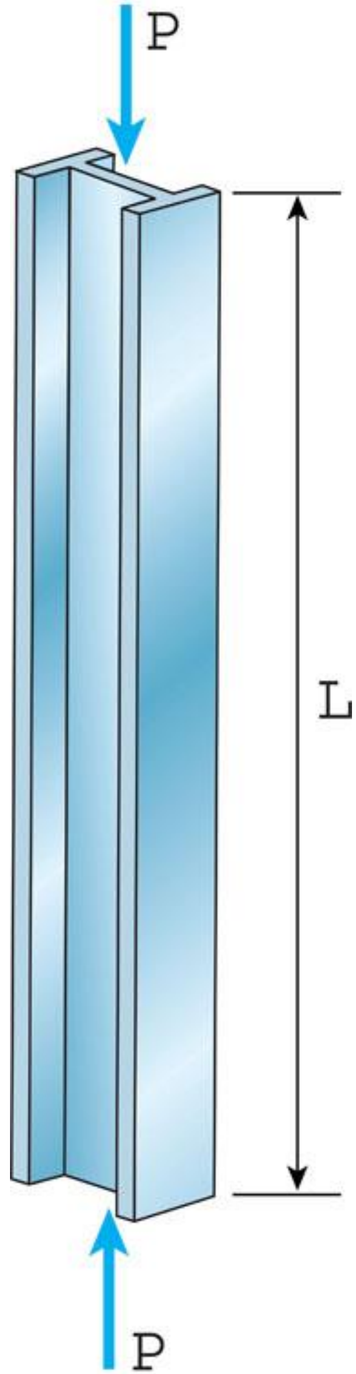


FIG. 11-37
Example 11-5.
Steel wide-flange column

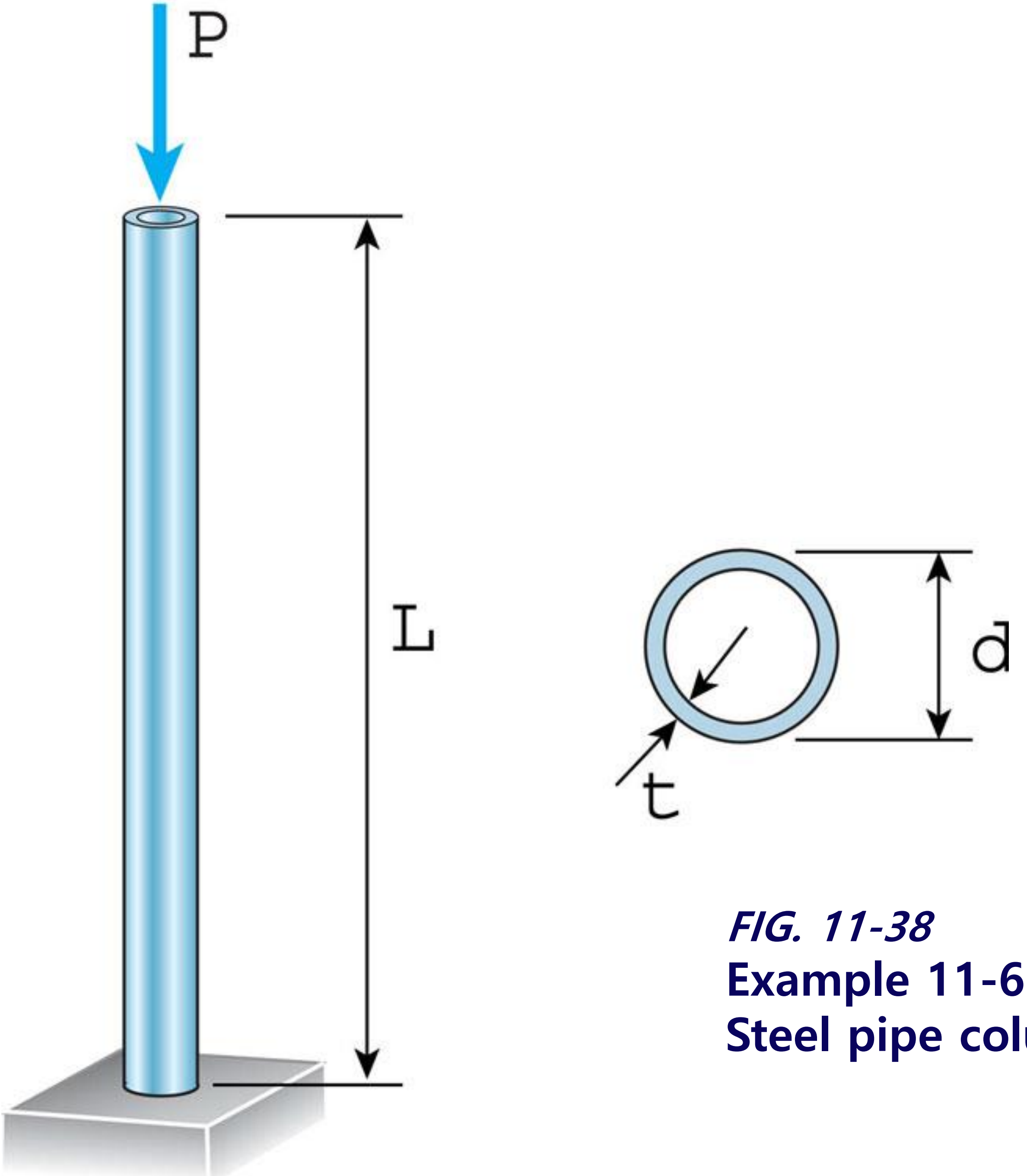


FIG. 11-38
Example 11-6.
Steel pipe column

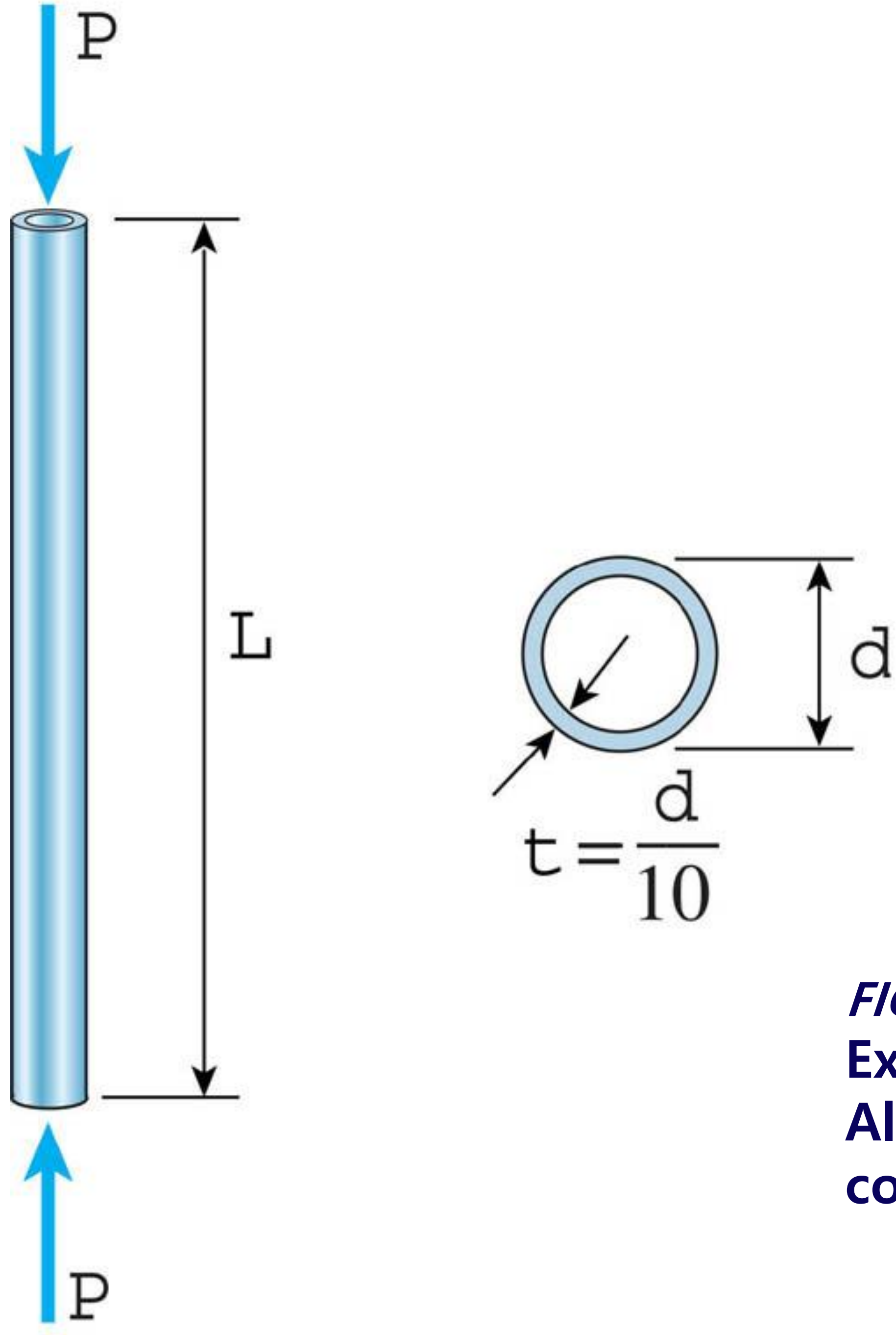


FIG. 11-39
Example 11-7. Aluminum tube in compression

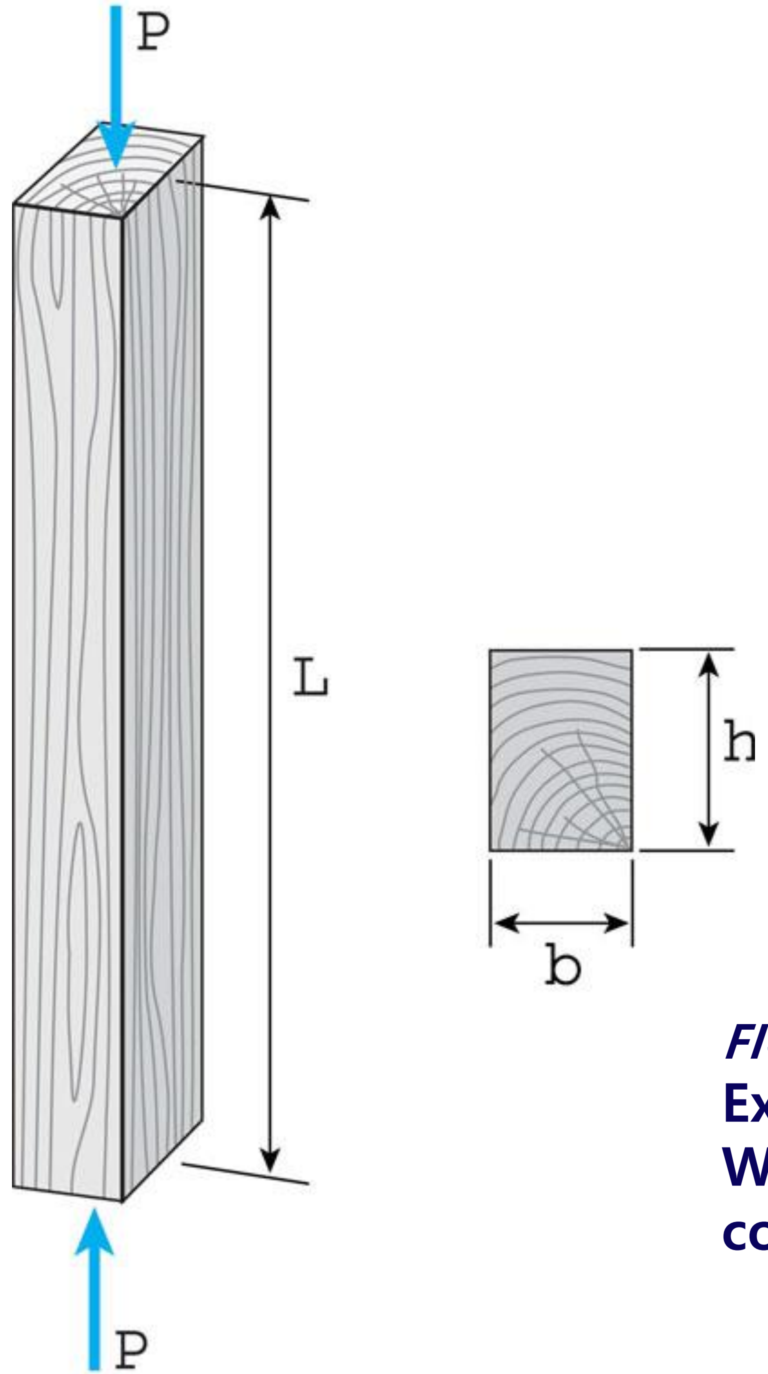


FIG. 11-40
Example 11-8. Wood post in compression

Buckling Control for Plating

- Local plate panels between stiffeners may be subject to uni-axial or bi-axial compressive stresses
- σ_f : minimum upper yield stress
- σ_{el} : ideal compressive buckling stress
- σ_{cl} : critical buckling stress

$$\sigma_c = \sigma_{el} \quad \text{when} \quad \sigma_{el} < \frac{\sigma_f}{2}$$

$$= \sigma_f \left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \quad \text{when} \quad \sigma_{el} > \frac{\sigma_f}{2}$$

$$\sigma_{el} = 0.9\mathrm{kE}\left(\frac{t - t_k}{1000s}\right)^2 \ (N/mm^2)$$

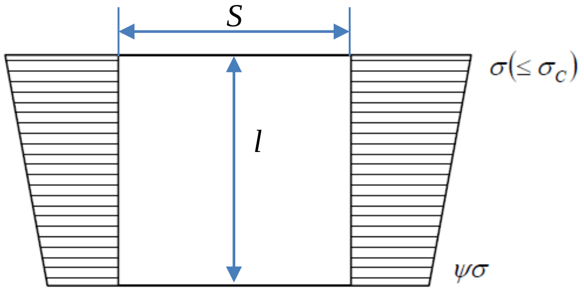
t_k : corrosion addition

Buckling Control for Plating

❖ For plating with longitudinal stiffeners (in direction of compression stress):

$$k = k_l = \frac{8.4}{\psi + 1.1} \quad \text{for} \quad (0 \le \psi \le 1)$$

ψ : the ratio between the smaller and the larger compressive stress assuming linear variation



❖ For plating with transverse stiffeners (perpendicular to compression stress):

$$k = k_s = c\left[1 + \left(\frac{s}{l}\right)^2\right]^2 \frac{2.1}{\psi + 1.1} \quad \text{for} \quad (0 \le \psi \le 1)$$

c = 1.21 when stiffeners are angles or T-sections
= 1.10 when stiffeners are bulb flats
= 1.05 when stiffeners are flat bars
c = 1.3 when the plating is supported by floors or deep girders.