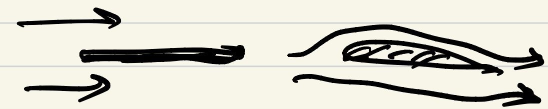
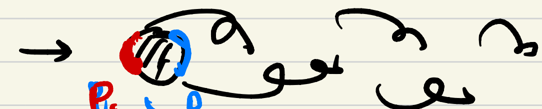
⊙ Reynolds number and geometry effects

two types ; ① streamlined body



② bluff body →

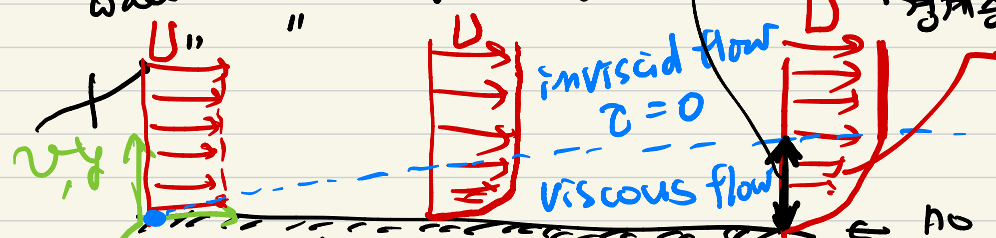


• **Flat plate**

wall-normal direction velocity

$\delta(x)$: boundary layer thickness 경계층 두께

$\tau = \mu \boxed{\frac{\partial u}{\partial y}}$?



$u(x, y)$

$v(x, y)$

$w = 0$ (laminar)

$\neq 0$ (turbulent)

U free-stream velocity 자유흐름속도

← no slip b.c.

$u = v = w = 0$

x : streamwise direction 유선방향

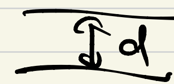
u : " velocity " " 속도

z, w spanwise direction

" velocity

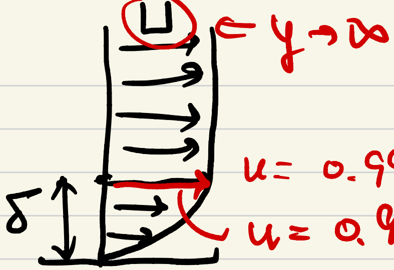
$Re_x = \frac{Ux}{\nu}$, $Re_\delta = \frac{U\delta}{\nu}$

$\sim f(x)$



$Re_d = \frac{Ud}{\nu}$

const

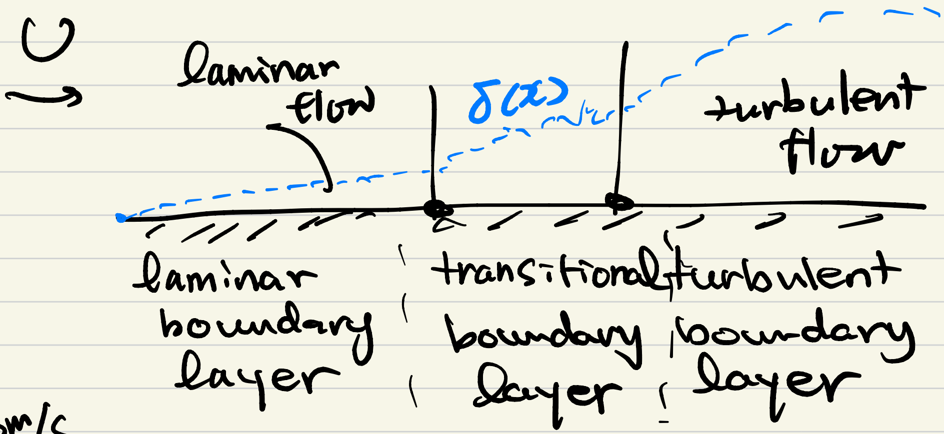


$u = 0.999U \rightarrow \delta_{0.999U}$ ✓

$u = 0.99U \rightarrow \delta_{0.99U}$ ✓

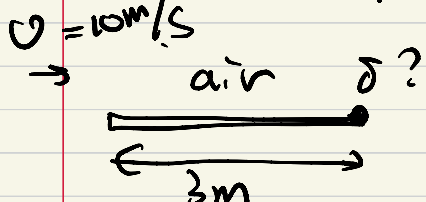
$u = 0.95U \rightarrow \delta_{0.95U}$

δ ?



Blasius (1908)

$$\frac{\delta}{x} = \begin{cases} \dfrac{5.0}{\sqrt{Re_x}} & \text{laminar} \quad \rightarrow \delta \sim \sqrt{x} \\[2ex] \dfrac{0.16}{Re_x^{1/7}} & \text{turbulent} \quad \rightarrow \delta \sim x^{6/7} \end{cases}$$

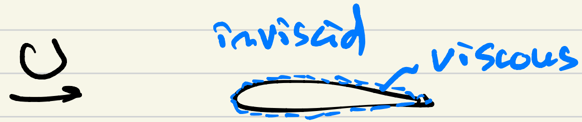


$$Re_x = \frac{Ux}{\nu} = \frac{10 \times 3}{1.5 \times 10^{-5}} = 2 \times 10^6 \rightarrow \delta_{laminar} = \frac{5.0 \times 3}{\sqrt{2 \times 10^6}} \doteqdot 1\,cm$$

$$\delta_{turb} = \frac{0.16 \times 3}{(2 \times 10^6)^{1/7}} \doteqdot 6\,cm$$

"thin"!

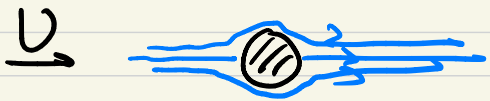
- flow around slender bodies



Plates
airfoils) thin boundary layer ← theory

- flow around bluff bodies

flow separation 유동박리



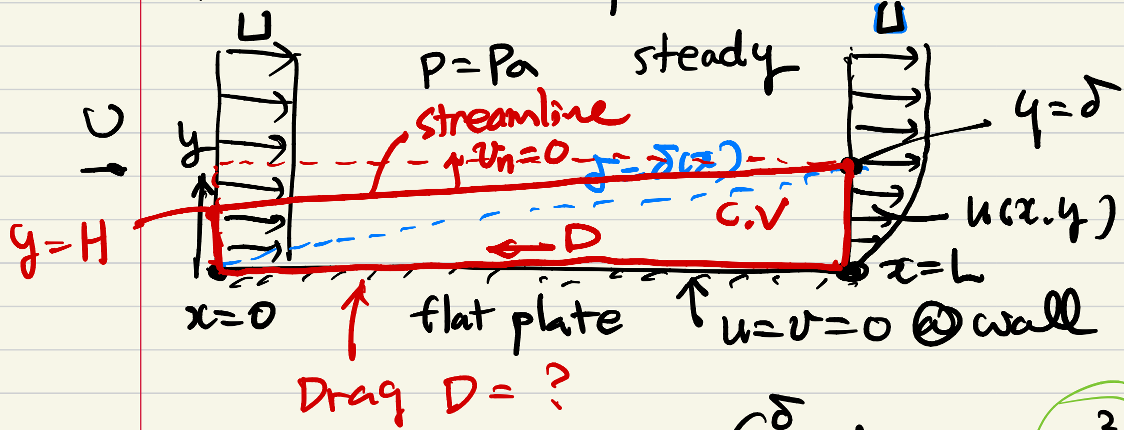
↑ theory: Stokes flow

$U \uparrow$ $\quad Re_d = \frac{Ud}{\nu} \uparrow$ $\quad$ theory fails.

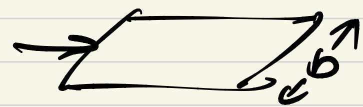
↓

experiment
computational fluid dynamics (CFD)

7.2 Momentum-integral estimates



b: width

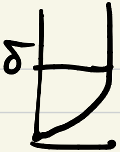


continuity : $UH = \int_0^\delta u\,dy \rightarrow U^2H = \int_0^\delta uU\,dy$

x-mtm : $\sum F_x = -D = b\int_{c.s} \rho u(\underline{V}\cdot\underline{n})\,dA = b\left(\int_0^\delta \rho u^2 dy - \underbrace{\int_0^H \rho U^2 dy}_{\rho U^2 H}\right)$

$$= \rho b\left(\int_0^\delta u^2 dy - \int_0^\delta uU\,dy\right)$$

$$\therefore D = \rho b\int_0^\delta (uU - u^2)\,dy = \rho b U^2 \underbrace{\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)dy}_{\equiv\, \theta(x)}$$



$$\boxed{\theta = \int_0^{\delta} \frac{u}{U}\left(1-\frac{u}{U}\right)dy}$$: momentum thickness 운동량두께

$$\boxed{D(x) = \rho b U^2 \theta(x)} \rightarrow \frac{dD}{dx} = \rho b U^2 \frac{d\theta}{dx}$$

$$\parallel$$

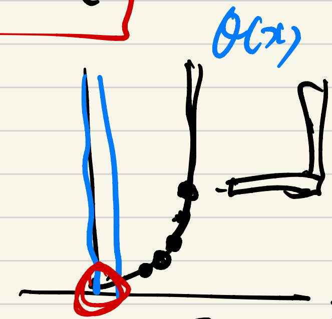
$$\int_0^x \tau_w \, dx \cdot b \Rightarrow \frac{dD}{dx} = \tau_w \cdot b \;\; /\!/ \rightarrow \boxed{\tau_w = \rho U^2 \frac{d\theta}{dx}}$$

wall shear stress
벽 전단응력

$$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_w$$

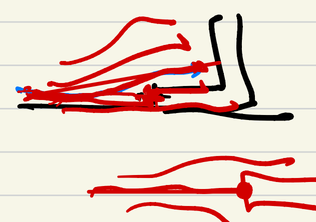
$$c_f \equiv \frac{\tau_w}{\frac{1}{2}\rho U^2} = 2\frac{d\theta}{dx}$$

skin-friction coefficient
마찰계수

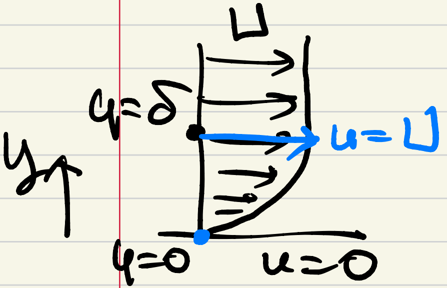


$$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_w$$

valid for laminar & turbulent flows $\Rightarrow$ $$\boxed{\frac{c_f}{2} = \frac{d\theta}{dx}}$$ momentum-integral relation for flat-plate boundary layer flow



- von Karman's assumption on velocity profile



$$\left.\begin{aligned} u&=0 \ @\ y=0 \\ u&=U \ @\ y=\delta \\ \frac{\partial u}{\partial y}&=0 \ @\ y=\delta \end{aligned}\right\} \rightarrow u = a+by+cy^2$$

$$\Downarrow$$

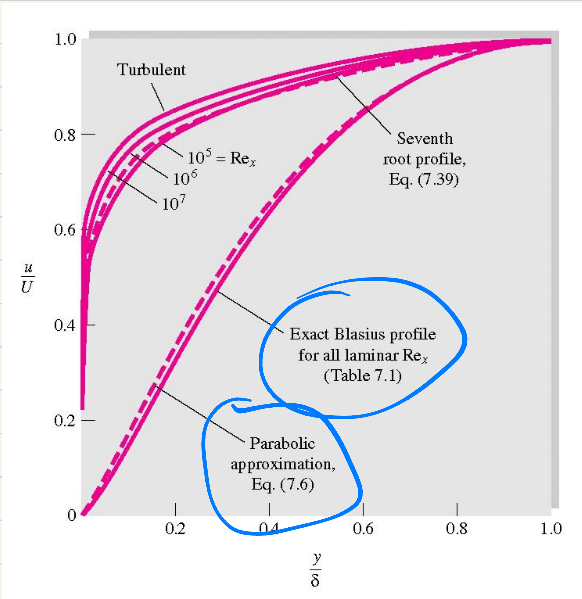
$$\boxed{u(x,y) = U\left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)} \quad 0 \le y \le \delta$$

$$\theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy = \frac{2}{15}\delta$$

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{\mu \cdot 2U}{\delta}$$

$$\rho U^2 \frac{d\theta}{dx} = \rho U^2 \cdot \frac{2}{15}\frac{d\delta}{dx}$$

$$\rightarrow \frac{\delta^2}{2} = 15\frac{\nu x}{U}$$



$$\boxed{\frac{\delta}{x} \approx 5.5\sqrt{\frac{\nu}{Ux}} = \frac{5.5}{\sqrt{Re_x}}}$$

$$\frac{\delta}{x} = \frac{5.0}{\sqrt{Re_x}}$$

Blasius

10% error

$$\rightarrow \delta \sim \sqrt{x}$$

$$\frac{\theta}{x} = \frac{\theta}{\delta}\cdot\frac{\delta}{x} = \frac{2}{15}\cdot\frac{5.5}{\sqrt{Re_x}} = \frac{0.733}{\sqrt{Re_x}} \rightarrow \theta \sim \sqrt{x}$$

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{2\mu U/\delta}{\frac{1}{2}\rho U^2} = \frac{4\mu}{\rho U\delta} = \frac{x}{\delta}\cdot\frac{4\mu}{\rho U x} = \frac{0.73}{\sqrt{Re_x}} \rightarrow 10\% \text{ error}$$

$$\rightarrow c_f \sim x^{-\frac{1}{2}}$$

$$c_f = \frac{0.664}{\sqrt{Re_x}} \text{ Blasius}$$