Topics in Ship Structural Design (Hull Buckling and Ultimate Strength)

Lecture 4 Buckling and Ultimate Strength of Columns

Reference : Ship Structural Design Ch.09

NAOE

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Goal

❖ **In Lecture 02 :**

- Euler buckling load
- Load eccentricity → Secant formula
- Elastic and Inelastic column behavior
  - ✓ Tangent-Modulus Theory
  - ✓ Reduced-Modulus Theory
  - ✓ Shanley Theory

❖ **In this Lecture:**

- Residual stress effect
- Load eccentricity + column geometric eccentricity (initial deflection)
  → Perry-Robertson formula
- Effect of lateral load

Ideal Columns

- The Euler buckling load P_E is

$$P_E = \frac{\pi^2 EI}{L_e^2} \quad (L_e : Effective\ Length)$$

- The Euler buckling load is the load for which an ideal column will first have an equilibrium deflected shape.
- Mathematically, it is the eigenvalue in the solution to Euler's differential equation

$$\frac{d^2 w}{dx^2} + \frac{Pw}{EI} = 0$$

- The "ultimate load" P_{ult} is the maximum load that a column can carry.
- It depends on
  - ✓ initial eccentricity of the column, eccentricity of the load
  - ✓ transverse load, end condition,
  - ✓ in-elastic action, residual stress.
- P_{ult} of a practical column is less than P_E
- Buckling - the sudden transition to a deflected shape - only occurs in the case of "ideal" columns.

Ideal Columns

- If the compressive stress exceeds proportional limit even in ideal column
  → Actual buckling load < Euler load
  due to the diminished slope of the stress-strain curve.
- For an ideal column (no eccentricity or residual stress) buckling will occur at the tangent modulus load, given by

$$P_t = \frac{\pi^2 E_t I}{L_e^2}$$ (E_t : *tangent modulus, the slope of the stress-stain curve corresponding to* P_t/A)

- Since E_t depends on P_t modulusthe calculation of P_t is generally an iterative process.
- It is more convenient to deal in terms of stress rather than load
  → The effects of yielding and residual stress to be included.

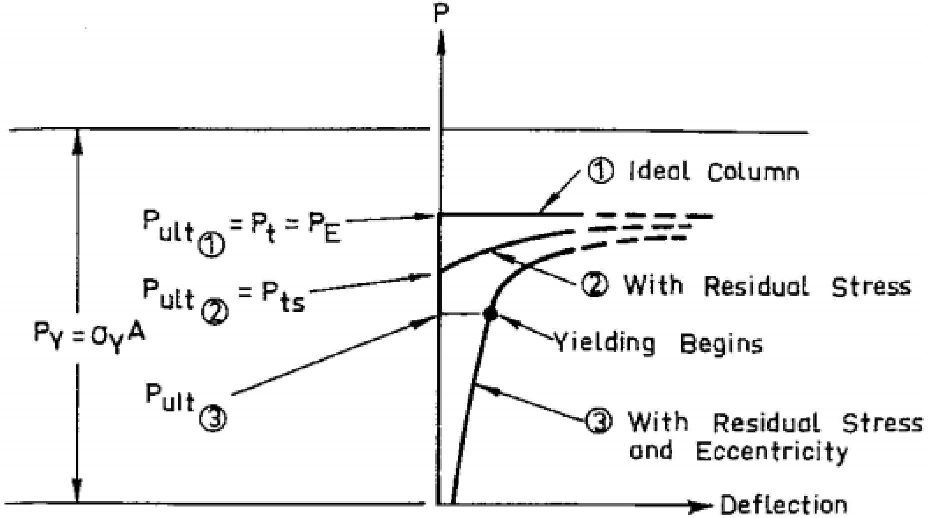
$$\sigma_E = \frac{P_E}{A} = \frac{\pi^2 E}{(L_e/\rho)^2}$$

ρ : *the radius of gyration* , $I = \rho^2 A$
L_e/ρ : *slenderness ratio*

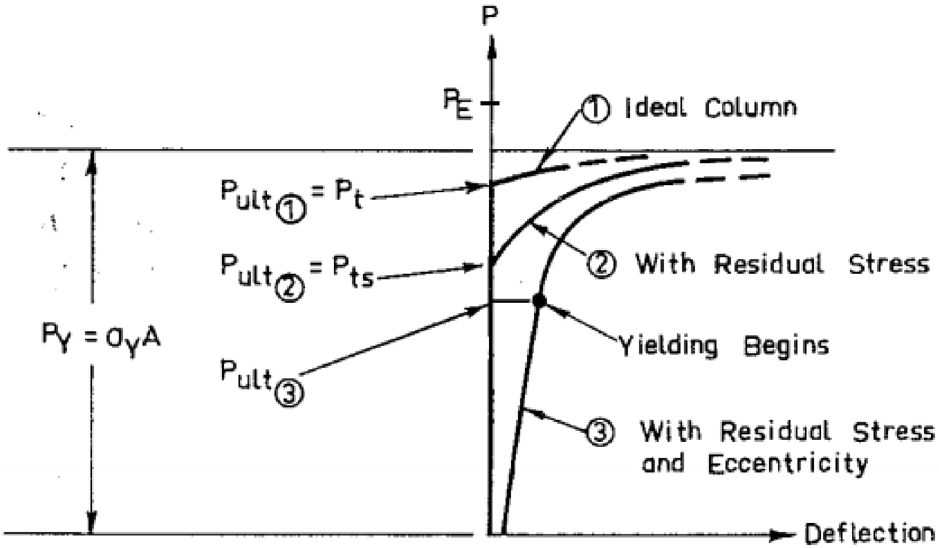
Ideal Columns

❖ **Load-deflection behavior for practical columns containing residual stress and eccentricity**

Q: Explain P_t?



Slender Column ($P_E<P_Y$)



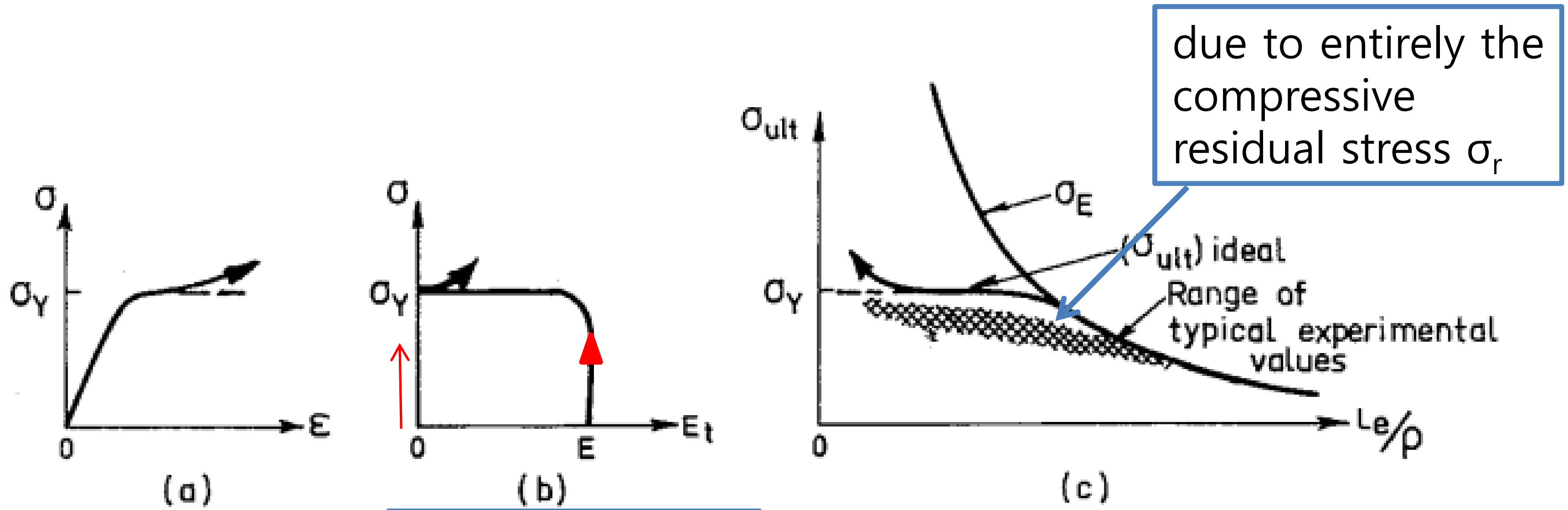
Stocky Column ($P_E>P_Y$)

$$P_t = \frac{\pi^2 E_t I}{L_e^2}$$

Ideal Columns

- The ultimate strength of a column is defined as the average applied stress at collapse

$$\sigma_{\text{ult}} = \frac{P_{\text{ult}}}{A} \qquad (\sigma_{\text{ult}})_{\text{ideal}} = \frac{\pi^2 E_t}{\left(\frac{L_e}{\rho}\right)^2}$$



Stress-strain curve

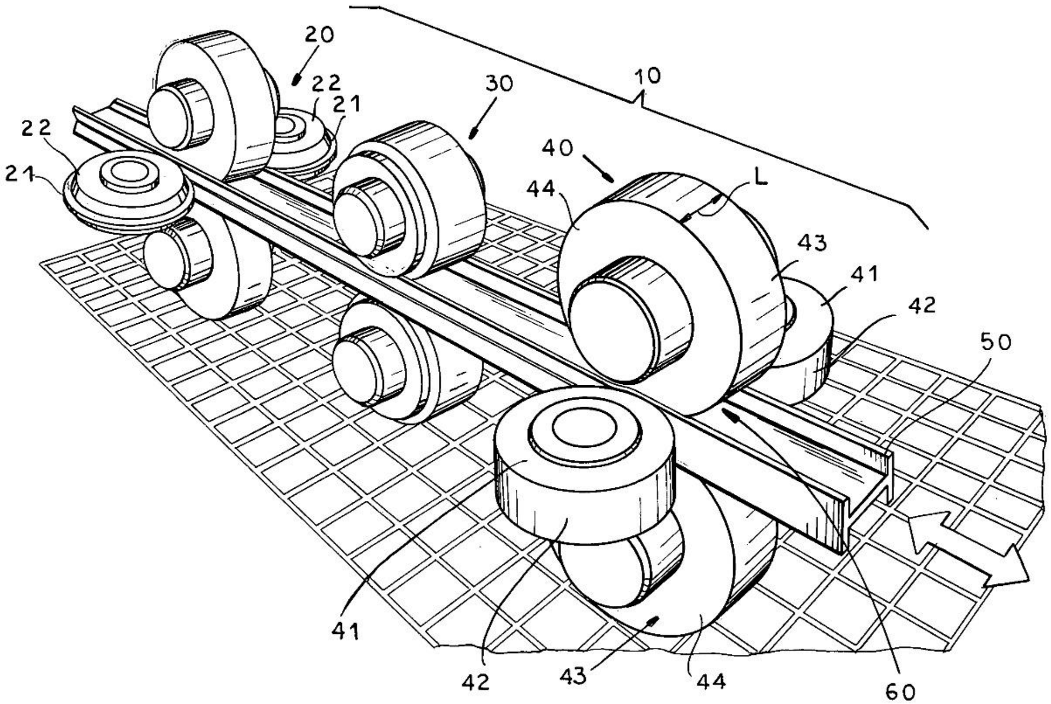
Tangent modulus curve

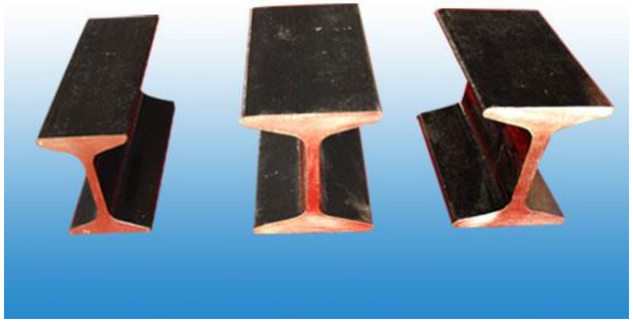
Typical experiment values for straight rolled sections and welded sections

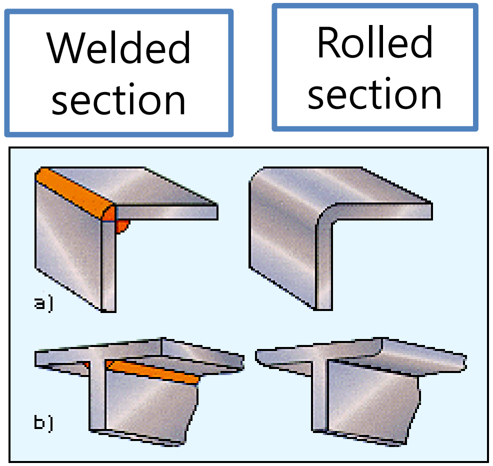
Residual Stress-Rolled Sections

❖ What is rolled section?

- a metal part produced by rolling and having any one of a number of cross-sectional shapes.







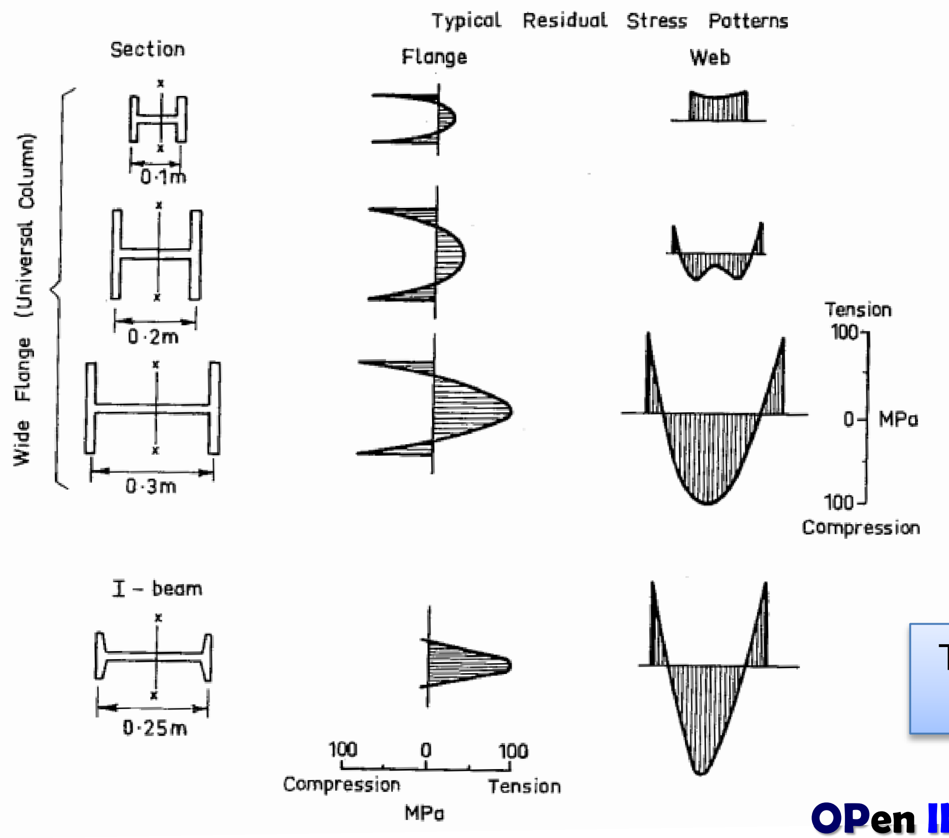
Residual Stress-Rolled Sections

❖ The uneven cooling between Residual stresses will result after the cooling because of the non-uniform temperature distribution through the cross section during the cooling process at ambient temperature.

flange root : **tesile** residual stresses

flange tip : **compressive** residual stresses,

about 80 MPa for mild steel



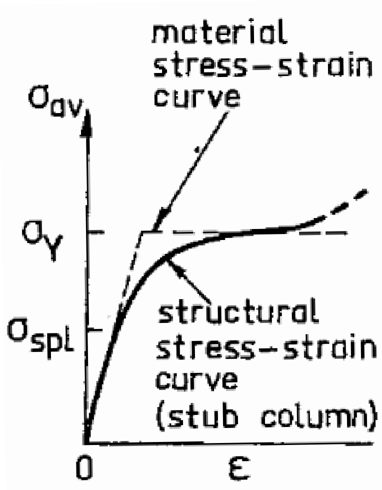
Typical residual stresses in rolled section

Residual Stress-Rolled Sections

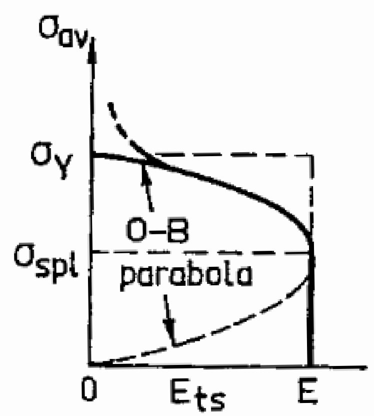
- The parts of the cross section that have **a compressive residual stress** may commence yielding when applied stress = $\sigma_Y - \sigma_r$

  → particularly detrimental because it occurs in the flange tips.
- The material is no longer homogeneous, and the simple tangent modulus approach is no longer valid.
- A comparatively simple solution can be achieved for the buckling strength in the primary direction by assuming an elastic-perfectly plastic stress-strain relationship.
- The progressive loss of bending stiffens is linearly proportional to the extent of the yielded zone → the use of an average value of tangent modulus.

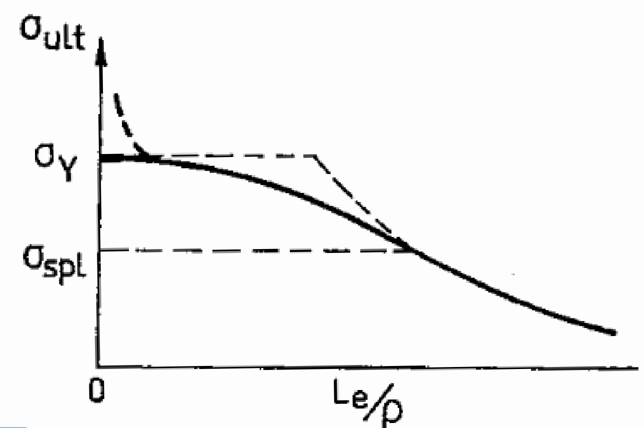
Q: What is σ_{spl} : Structural proportional limit?



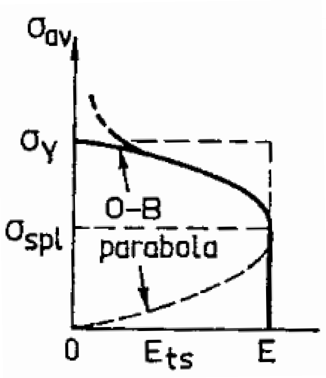
Typical stub column stress-strain curve



Structural tangent modulus diagram



Residual Stress-Rolled Sections



- Structural tangent modulus E_{ts} by Ostenfeld-Bleich Parabola

$$\frac{E_{ts}}{E}=\frac{\sigma_{av}(\sigma_Y-\sigma_{av})}{\sigma_{spl}(\sigma_Y-\sigma_{spl})} \text{ for } (\sigma_{spl}<\sigma_{av}<\sigma_Y)$$

σ_{spl} : Structural proportional limit

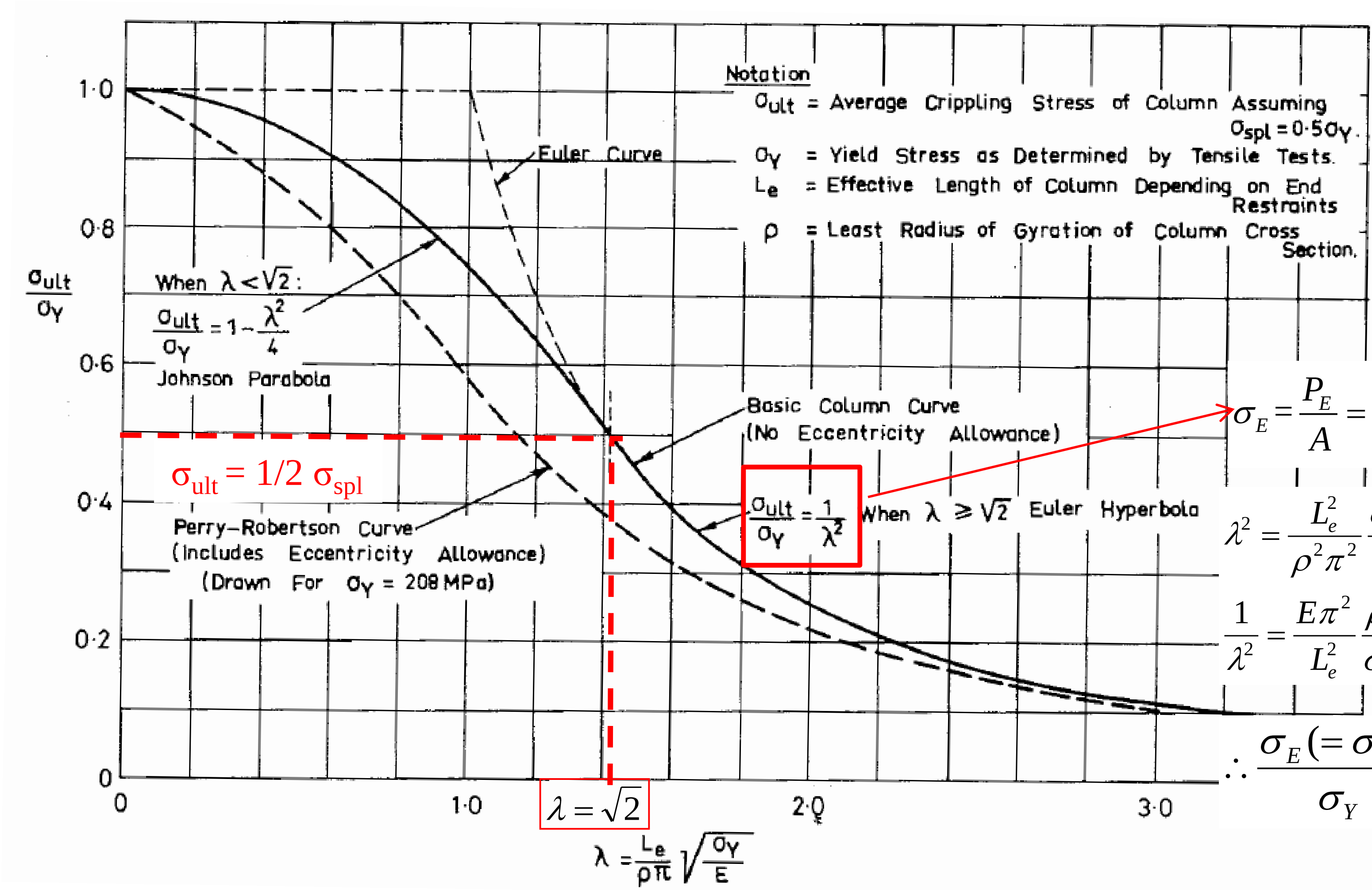
- Substituting E_{ts} in place of E_t

$$(\sigma_{ult})_{ideal}=\frac{\pi^2 E_t}{\left(\frac{L_e}{\rho}\right)^2} \Rightarrow \frac{\sigma_{ult}}{\sigma_Y}=1-\frac{\sigma_{spl}}{\sigma_Y}\left[1-\frac{\sigma_{spl}}{\sigma_Y}\right]\lambda^2,\quad \lambda=\frac{L_e}{\rho\pi}\sqrt{\frac{\sigma_Y}{E}}$$

- For rolled wide flange section, typical value of $\sigma_{spl} = 1/2\ \sigma_Y$.
  Johnson parabola for rolled section (essentially straight and pinned end)

$$\frac{\sigma_{ult}}{\sigma_Y}=1-\frac{\lambda^2}{4},\ \text{for } \lambda>\sqrt{2} \Leftarrow \text{when } \sigma_{ult}=1/2\ \sigma_{spl} \quad \frac{1}{2}=1-\frac{\lambda^2}{4},\quad \frac{1}{2}=\frac{\lambda^2}{4},\quad \lambda^2=2$$

Residual Stress-Rolled Sections



$$\sigma_E = \frac{P_E}{A} = \frac{\pi^2 E}{(L_e/\rho)^2}$$

$$\lambda^2 = \frac{L_e^2}{\rho^2 \pi^2}\frac{\sigma_Y}{E}$$

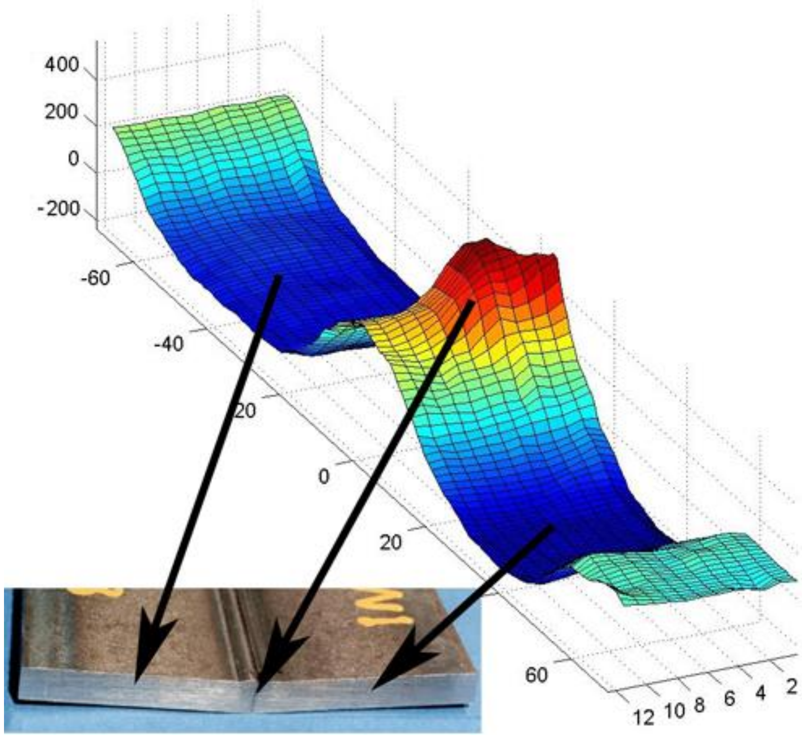
$$\frac{1}{\lambda^2} = \frac{E\pi^2}{L_e^2}\frac{\rho^2}{\sigma_Y} = \frac{E\pi^2}{(L_e/\rho)^2 \sigma_Y}$$

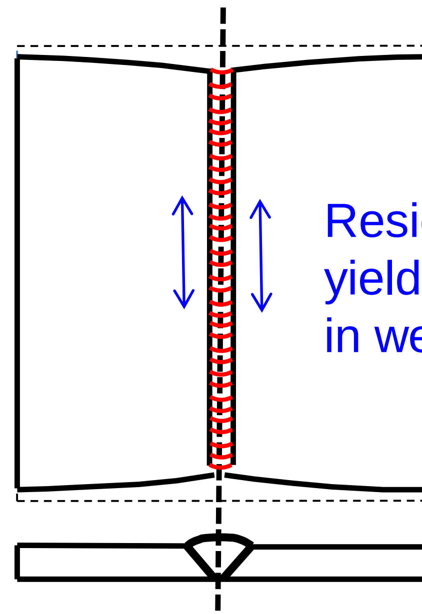
$$\therefore \frac{\sigma_E (= \sigma_{ult})}{\sigma_Y} = \frac{1}{\lambda^2}$$

Basic column curve (Johnson parabola and Perry-Robertson curve

Residual Stress-Welded Sections

❖ Residual thermal stress is induced by uneven internal temperature distribution

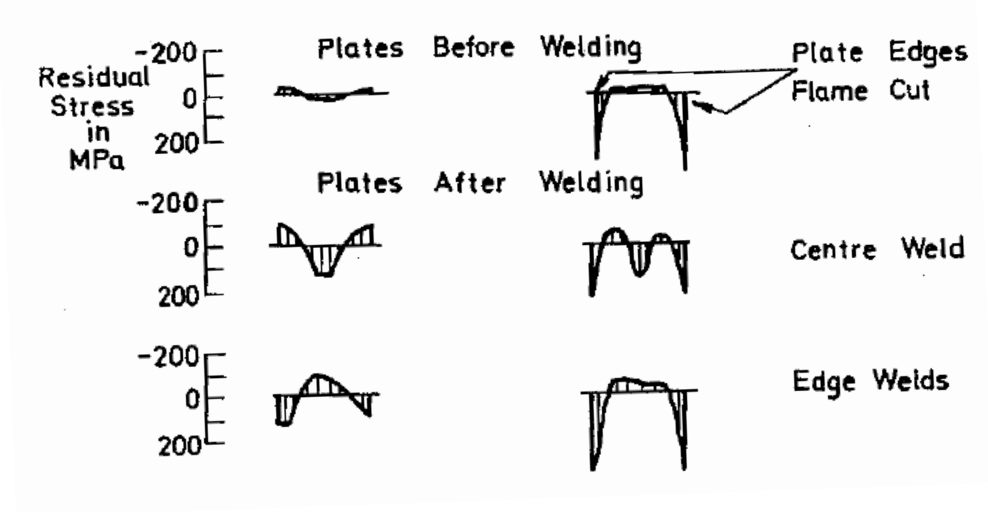


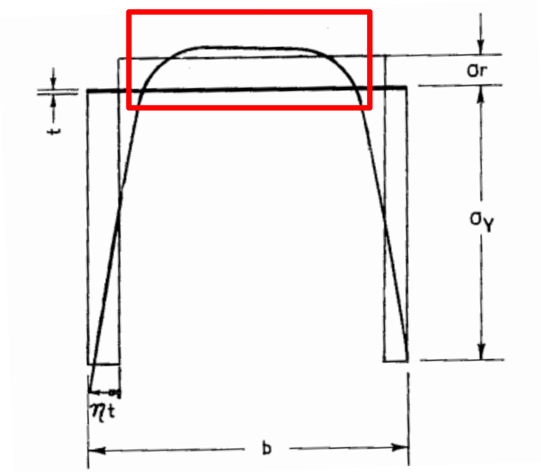


Residual stress close to yield stress
in welding direction

Residual Stress-Welded Sections

- The interaction between the different fibers results in a locked-in tensile stress in and near the weld which is equal to approximately equal to the yield stress.

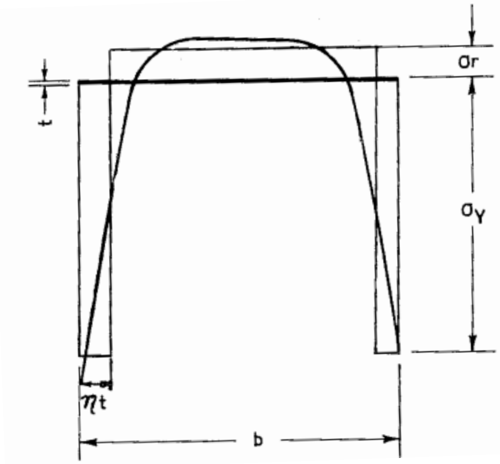




Typical residual stresses in welded section

- The extent of the tension yield zone (3~6 times thickness) depends mainly on the total heat input, the cross-sectional area of the weld deposit, the type of welding and the welding sequence.

Residual Stress-Welded Sections



- From the equilibrium

$$\sigma_r(b-2\eta t)=2\eta t\sigma_Y \quad \Rightarrow \quad \frac{\sigma_r}{\sigma_Y}=\frac{2\eta}{\frac{b}{t}-2\eta}$$

ηt : width of the tension yield zone, b : total flange width

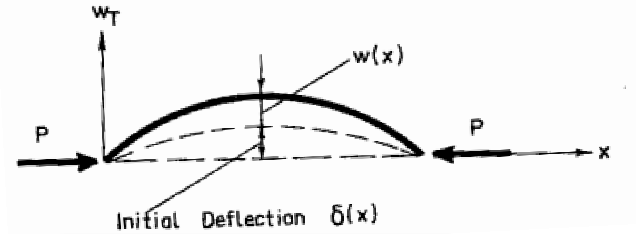
- For narrow thick sections residual stress will be high, and this will seriously diminish the strength of the section.

- It should be noted that
  - ✓ the effect will be much worse for open sections than for box sections due to the tensile stress at corners of box sections.
  - ✓ flame-cutting creates conditions similar to welding → high tensile residual stress is beneficial.
  - ✓ the effect of residual stress is somewhat diminished for higher yield steels due to narrower tension zone.

Eccentricity: Magnification Factor

- Pinned column with an initial deflection $\delta(x)$ and addition deflection (x) due to axial load. Bending moment is $P(\delta+w)$.

$$\frac{d^2w}{dx^2}+\frac{P}{EI}(\delta+\mathrm{w})=0$$



Eccentric columns

- Deflection continues to grow and magnified as long as P increases.
  → no static equilibrium configuration and no sudden buckling.

- Let δ be represented by a Fourier series

$$\delta=\sum_{n=1}^{\infty}\delta_n \sin\frac{n\pi x}{L}$$

- Assume that the additional deflection due to the bending is also a Fourier series

$$w=w_n \sin\frac{n\pi x}{L}$$

Eccentricity: Magnification Factor

- $$\frac{d^2w}{dx^2}=\sum_{n=1}^{\infty}-\frac{n^2\pi^2}{L^2}w_n\sin\frac{n\pi x}{L}$$

- $$\sum_{n=1}^{\infty}\left[-\frac{n^2\pi^2}{L^2}w_n+\frac{P}{EI}(w_n+\delta_n)\right]\sin\frac{n\pi x}{L}=0 \Rightarrow -\frac{n^2\pi^2}{L^2}w_n+\frac{P}{EI}(w_n+\delta_n)=0$$

$$\left(P-\frac{n^2EI\pi^2}{L^2}\right)w_n+\delta_n=(P-n^2P_E)w_n+\delta_n=0 \Rightarrow w_n=\frac{\delta_n}{\frac{n^2P_E}{P}-1},\ where\ n=1,2,3,\ldots$$

- The dominant term is $n=1$. When $P_E/P\rightarrow 1$, $w_n/\delta_n\rightarrow\infty$

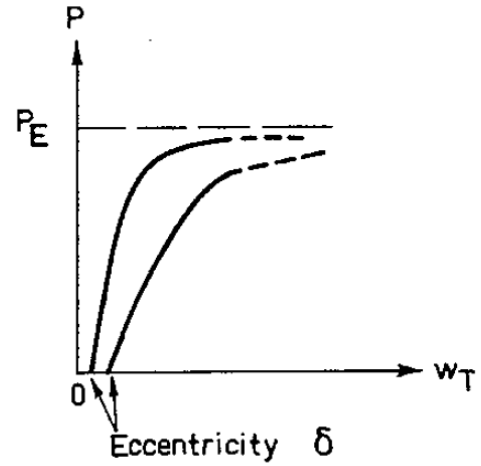
$$w=\frac{\delta}{\frac{P_E}{P}-1},$$

and the total deflection is

$$w_T=\delta+w=\delta+\frac{P}{P_E-P}\delta=\frac{P_E}{P_E-P}\delta=\phi\delta$$

where ϕ is called the magnification factor.

$$\boxed{\phi=\frac{P_E}{P_E-P}}$$



Load-deflection for eccentric columns

Eccentricity: Magnification Factor

- Eccentricity effects may also arise due to eccentricity of load. In this case the magnification factor is given by

$$\phi = \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_E}}\right)$$

$$\boxed{M_{\max} = Pe\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)}$$

Secant Formula in Lecture 2

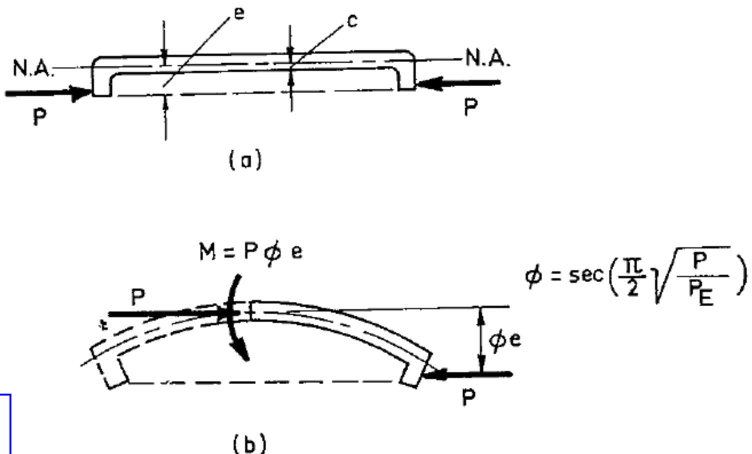
- The two types of eccentricity (load application and column geometry) can be combined linearly $\Delta = \delta + e$.

$$\sigma_{\max} = \frac{P}{A} + \frac{P\phi\Delta}{Z} = \frac{P}{A}\left(1 + \phi\frac{\Delta A}{Z}\right) = \frac{P}{A}\left(1 + \phi\frac{\Delta}{r_c}\right),$$

$$r_c = \text{core radius} = \frac{Z}{A} = \frac{\rho^2}{c}$$

$$\frac{\Delta}{r_c} = \text{eccentricity ratio} = \frac{\Delta c}{\rho^2}$$

$$\boxed{\frac{ec}{r^2} = \text{eccentricity ratio}}$$



Column with eccentric load

Secant Formula in Lecture 2

- Eccentricity can be related to the slenderness ratio

$$\frac{\Delta}{r_c} = \alpha\frac{L}{\rho}, \quad \alpha \approx 0.003 \text{ for lower bound for test results}$$

Perry-Robertson Formula

- It adopts Robertson`s value for α and assumes that the column will collapse when the maximum compressive stress reaches the yield stress.

$$\sigma_Y = \frac{P_{\text{ult}}}{A}\left(1+\frac{\alpha(L/\rho)P_E}{P_E-P_{\text{ult}}}\right) \qquad \sigma_Y = \sigma_{\text{ult}}\left(1+\frac{\alpha L/\rho}{1-\sigma_{\text{ult}}/\sigma_E}\right)$$

- It is a quadratic, and may be rewritten in a nondimensional, factored form

where R is the strength ratio of the column $R=\dfrac{\sigma_{\text{ult}}}{\sigma_Y}$

η is the eccentricity ratio $\eta=\dfrac{\alpha L}{\rho}$

λ is the column slenderness parameter $\lambda=\sqrt{\dfrac{\sigma_Y}{\sigma_E}}=\dfrac{L}{\pi\rho}\sqrt{\dfrac{\sigma_Y}{E}}$

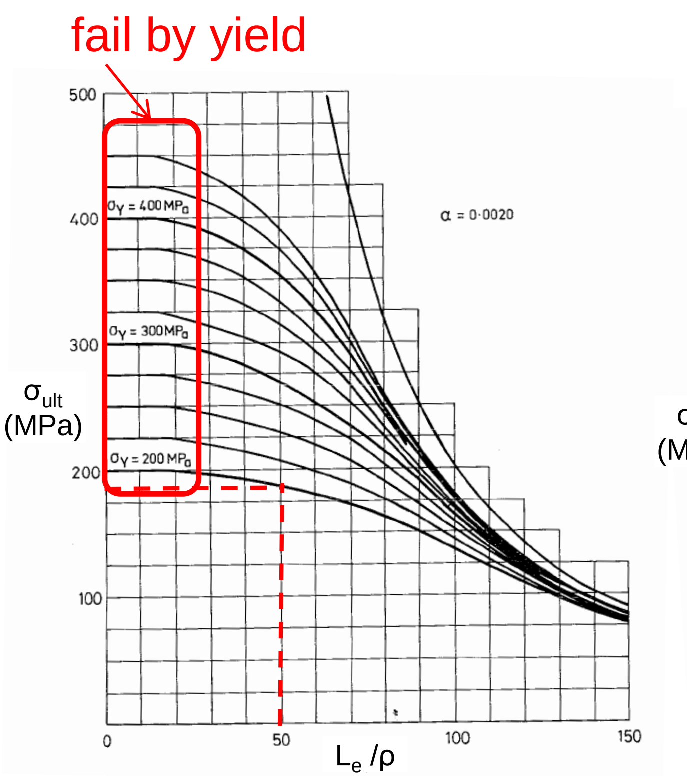
$$\frac{\sigma_Y}{\sigma_{ult}}-1=\left(\frac{\eta}{1-\lambda^2\sigma_Y/\sigma_{ult}}\right), \Rightarrow \frac{1}{R}-1=\frac{\eta}{1-\lambda^2 R} \qquad \boxed{(1-R)(1-\lambda^2R)=\eta R}$$

$$\Rightarrow R=\frac{1}{2}\left(1+\frac{1+\eta}{\lambda^2}\right)-\sqrt{\frac{1}{4}\left(1+\frac{1+\eta}{\lambda^2}\right)^2-\frac{1}{\lambda^2}}$$

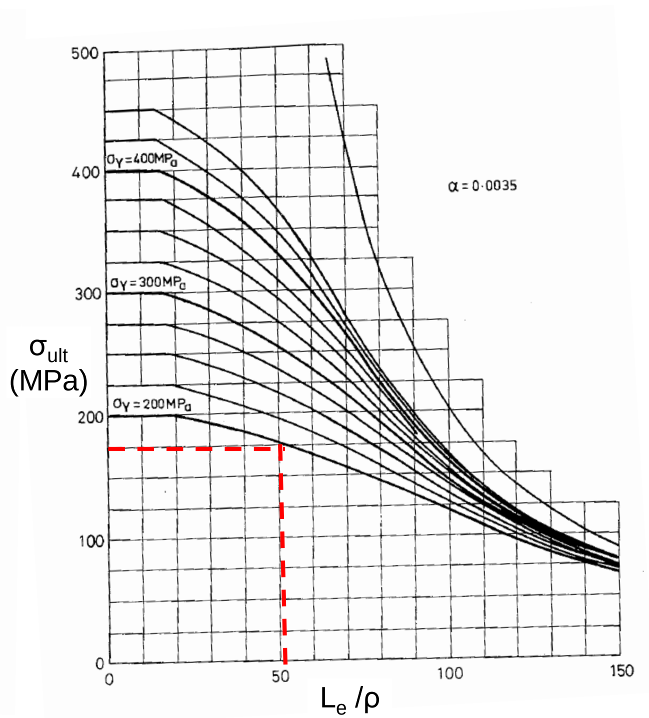
- For a perfectly straight column, having $\eta=0$, reduces to the Euler curve, , providing that $\lambda>1$. $R=\dfrac{1}{\lambda^2}$

Accurate Design Curves

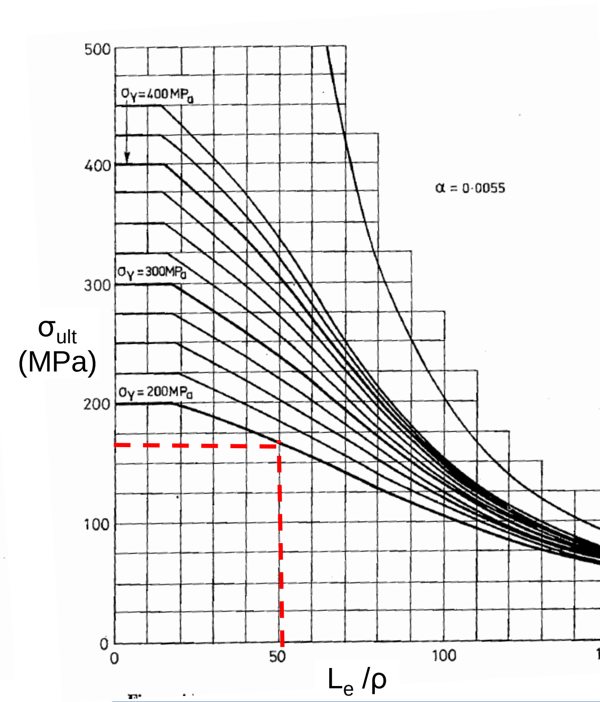
- Perry-Robertson formula with different values of α being used for different types of sections



Column design curve for α=0.002



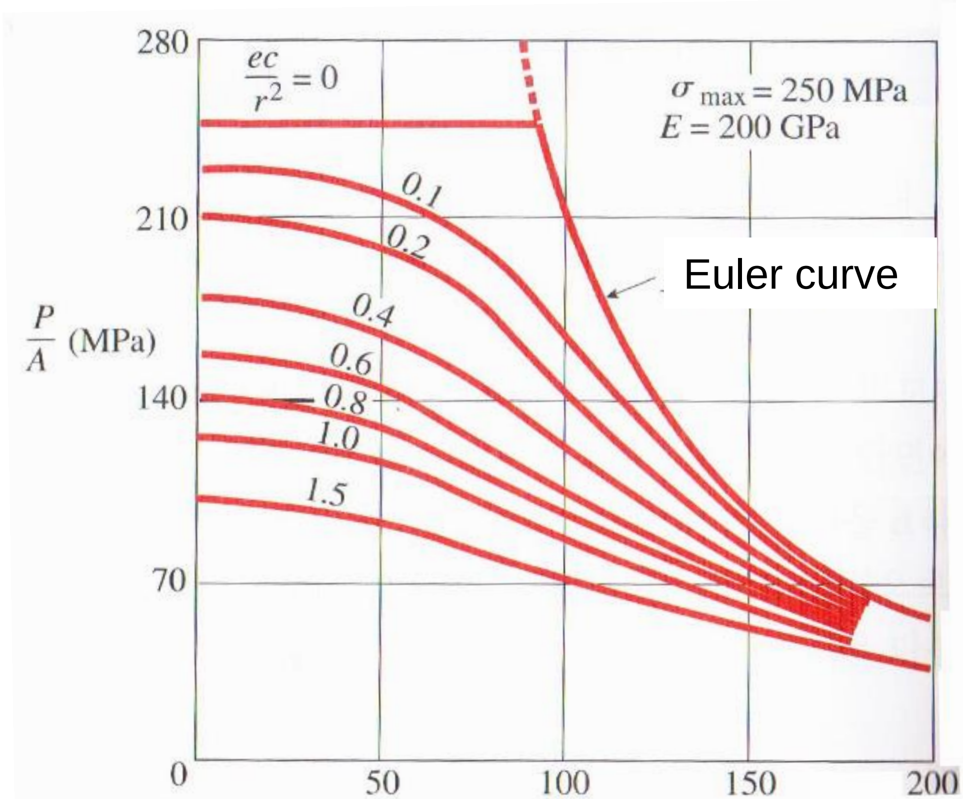
α=0.0035



α=0.0055

Homework #1

❖ **Plot Secant curve shown below and Perry-Robertson Formula in a graph and discuss the differences of two approach associate with the plots.**



Graph of the secant formula for σ_{max} = 250 MPa (σ_{yield}) and E = 200 GPa

Accurate Design Curves

- The curves have a horizontal plateau at values of slenderness ratio less than some threshold value $(L/\rho)_0$. To achieve this the value of the eccentricity ratio is defined

for $\dfrac{L}{\rho} < \left(\dfrac{l}{\rho}\right)_{0'}$ $\qquad \dfrac{\Delta}{r_c} = 0$

for $\dfrac{L}{\rho} > \left(\dfrac{l}{\rho}\right)_{0'}$ $\qquad \dfrac{\Delta}{r_c} = \alpha\left[\dfrac{L}{\rho} - \left(\dfrac{L}{\rho}\right)_0\right]$

where $(L/\rho)_0$ is given by $\left(\dfrac{L}{\rho}\right)_0 = 0.2\pi\sqrt{\dfrac{E}{\sigma_Y}}$

- For welded I-and box sections, the yield stress should be reduced by 5% to allow for welding residual stress.

Section	Axis of Buckling	α
Universal column	xx	0.0035
Universal column	yy	0.0055
Universal beam	xx	0.0020
Universal beam	yy	0.0035
UC or UB with cover-plates	xx	0.0035
UC or UB with cover-plates	yy	0.0020
Channel	xx	0.0055
Channel	yy	0.0055
Tee	xx	0.0055
Tee	yy	0.0055
Angle	any	0.0055
Round tube	any	0.0020
Rectangular hollow	xx	0.0020
Rectangular hollow	yy	0.0020
Welded I-sections	xx	0.0035
Welded I-sections	yy	0.0055
Welded box-sections		0.0035

Curve selection(α) table for rolled and welded sections

Use of Magnification Factor

- For a pinned column subjected to lateral pressure q and P, the w_{max} is

$$w_{\max} = \underbrace{\frac{5qL^4}{384EI}}_{\text{Central deflection of a laterally loaded member}} \underbrace{\left[\frac{24}{5\xi^4}\left(\sec\xi - 1 - \frac{\xi^2}{2}\right)\right]}_{\text{The effect of axial load}} \qquad \xi = \left(\frac{L}{2}\right)\sqrt{\left(\frac{P}{EI}\right)}$$

Homework #2 Derive w_{max} and M_{max}.

- The maximum bending moment

$$M_{\max} = \frac{qL^2}{8}\left[\frac{2(1-\sec\xi)}{\xi^2}\right]$$

$\rightarrow$ different for deflection, end condition, load

- The simpler magnification factor which was derived for sinusoidal initial eccentricity is used.

$$\phi = \frac{P_E}{P_E - P}$$

- Max. bending moment =due to lateral load(M_0)+ due to eccentricity(δ_0)

$$M_{\max} = M_0 + P\phi(\delta_0 + \Delta) \qquad \sigma_{\max} = \frac{P}{A} + \frac{M_{\max}}{Z}$$

Use of Magnification Factor

- To demonstrate the accuracy of this approach, if $P=0.5P_E$, $\Delta=0$

$$M_{\max} = \frac{qL^2}{8}\left[\frac{2(1-\sec\xi)}{\xi^2}\right] \quad \xi = \left(\frac{L}{2}\right)\sqrt{\left(\frac{P}{EI}\right)} \quad \Rightarrow \quad M_{\max} = 2.030M_0$$

$$M_{\max} = M_0 + 0.5\left(\frac{\pi^2 EI}{L^2}\right)2\frac{5qL^4}{384EI} = M_0\left(1+\frac{5\pi^2}{48}\right) = 2.028M_0$$

- We assume that the column will collapse when the maximum compressive stress reaches the yield stress. Therefore

$$\sigma_Y = \frac{P_{\text{ult}}}{A} + \frac{M_0}{Z} + \frac{P_{\text{ult}}(\delta_0+\Delta)}{\left(1-\frac{P_{\text{ult}}}{P_E}\right)Z}$$

- It can be expressed in terms of nondimensional parameters

$$(1-R-\mu)(1-\lambda^2 R) = \eta R$$

Homework #3 Derive left formula

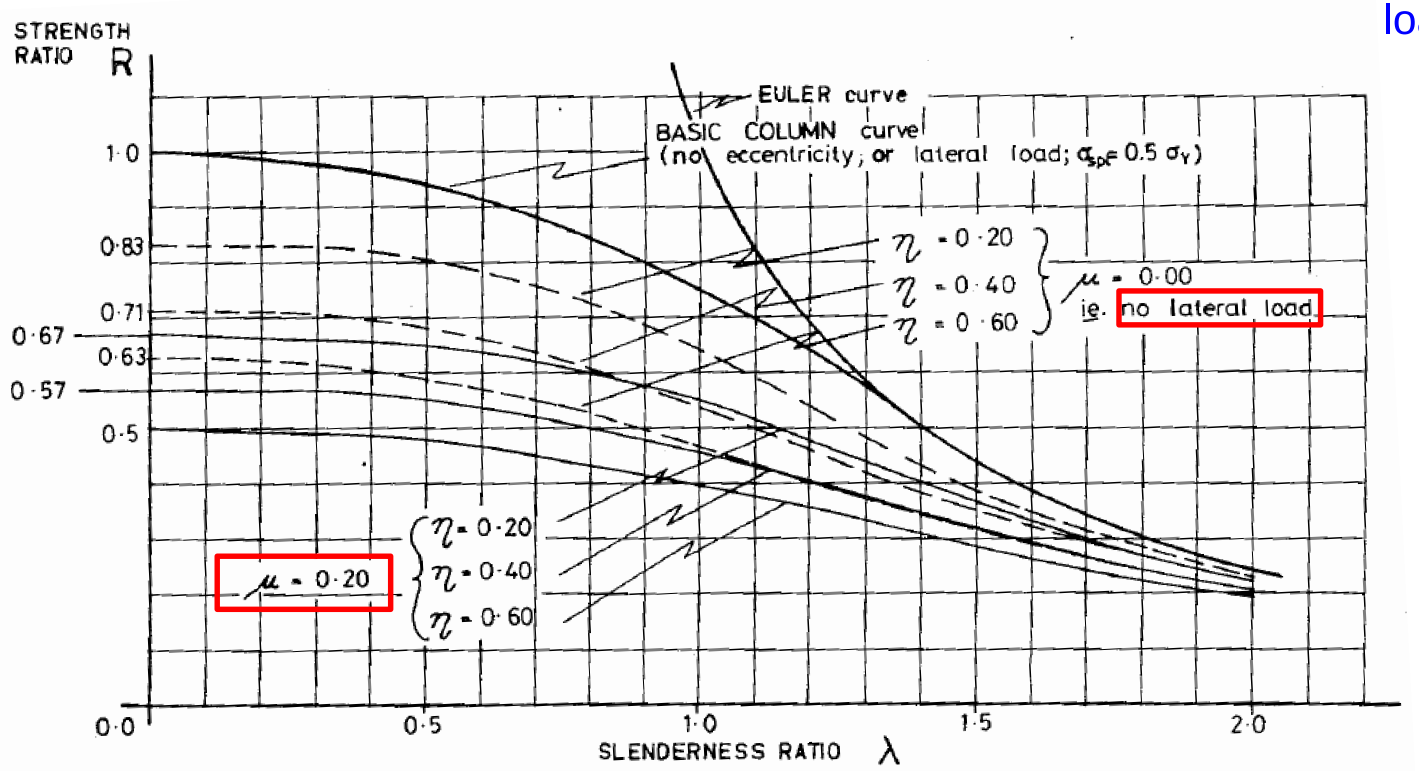
where

$$R = \frac{\sigma_{\text{ult}}}{\sigma_Y} \quad \lambda = \sqrt{\frac{\sigma_Y}{\sigma_E}} = \frac{L}{\pi\rho}\sqrt{\frac{\sigma_Y}{E}} \quad \eta = \frac{(\delta_0+\Delta)A}{Z} \quad \mu = \frac{\frac{M_0}{Z}}{\sigma_Y}$$

Use of Magnification Factor

- **The solution becomes**

$$R=\frac{1}{2}\left(1-\mu+\frac{1+\eta}{\lambda^2}\right)-\sqrt{\frac{1}{4}\left(1-\mu+\frac{1+\eta}{\lambda^2}\right)^2-\frac{1-\mu}{\lambda^2}}$$



Initial deflection and load eccentricity

$$\eta=\frac{(\delta_0+\Delta)A}{Z}$$

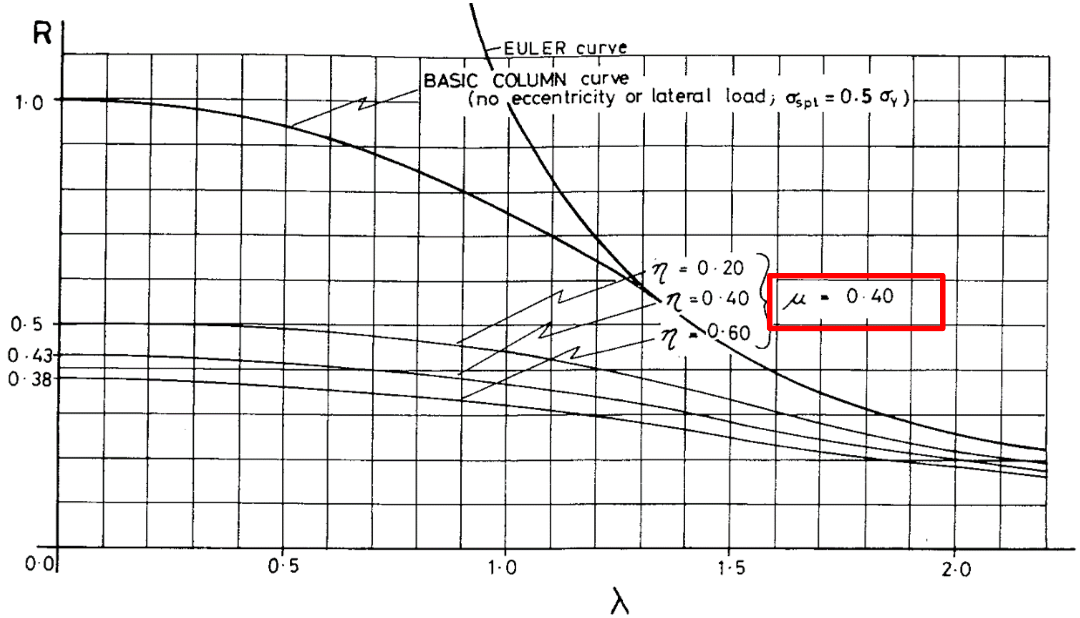
B.M due to lateral load

$$\mu=\frac{\dfrac{M_0}{Z}}{\sigma_Y}$$

A pinned beam column subjected to a specified lateral load.

11.3 Effect of Lateral Load: Beam Columns

Use of Magnification Factor



Initial deflection and load eccentricity

$$\eta = \frac{(\delta_0 + \Delta)A}{Z}$$

B.M due to lateral load

$$\mu = \frac{\frac{M_0}{Z}}{\sigma_Y}$$