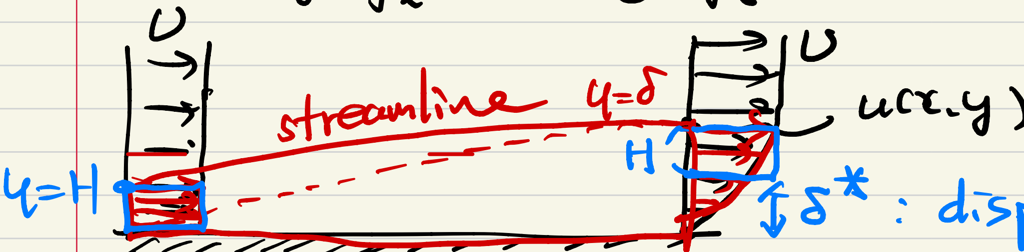
$$u(x,y) = U\left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)$$

$$\rightarrow \frac{\delta}{x} = \frac{5.5}{\sqrt{Re_x}}, \quad \frac{\theta}{x} = \frac{0.733}{\sqrt{Re_x}}, \quad \frac{\theta}{\delta} = \frac{2}{15}, \quad c_f = \frac{0.73}{\sqrt{Re_x}}$$

$$\delta \sim \sqrt{x} \qquad \theta \sim \sqrt{x} \qquad c_f \sim x^{-\frac{1}{2}}$$




$\delta^*$ : displacement thickness

배제 두께

$$\int_0^H U\,dy = \int_0^\delta u\,dy = \int_0^\delta (U + u - U)\,dy$$

$$= \underbrace{U\delta}_{} - \int_0^\delta (U - u)\,dy \qquad \theta = \frac{2}{15}\delta$$

$$U\cancel{H} = U(\cancel{H} + \delta^*)$$

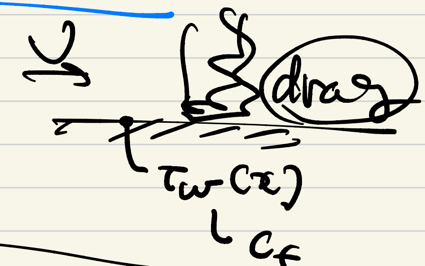
$$\rightarrow \boxed{\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy} = \frac{1}{3}\delta \qquad \delta > \delta^* > \theta$$

$$\frac{\delta^*}{x} = \frac{\delta^*}{\delta}\frac{\delta}{x} = \frac{1}{3}\cdot\frac{5.5}{\sqrt{Re_x}} = \frac{1.83}{\sqrt{Re_x}} \longrightarrow \text{6\% error}$$

$$\delta^* \sim \sqrt{x}$$

$$\frac{\delta^*}{x} = \frac{1.721}{\sqrt{Re_x}} \quad \text{Blasius}$$

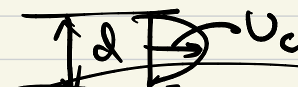
$$Re_x = \frac{Ux}{\nu}, \quad Re_\delta = \frac{U\delta}{\nu}, \quad Re_{\delta^*} = \frac{U\delta^*}{\nu}, \quad Re_\theta = \frac{U\theta}{\nu}$$




$$u^* = \sqrt{\frac{\tau_w}{\rho}} \quad : \text{wall-shear velocity}$$

$(u_\tau)$

$$Re_\tau = \frac{u^*\delta}{\nu}$$

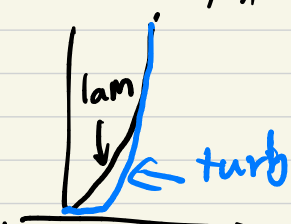



$$Re_d = \frac{Vd}{\nu}, \quad Re = \frac{U_c d}{\nu}, \quad \frac{VR}{\nu}, \; \ldots$$

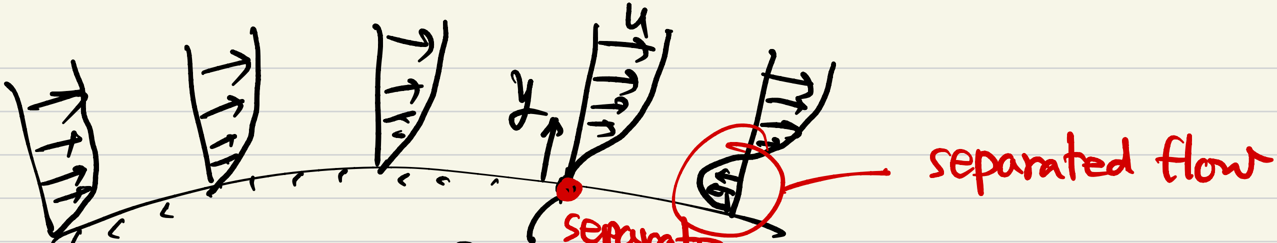
$$H \equiv \frac{\delta^*}{\theta} \quad : \text{shape factor}$$

$\simeq 2.5$ laminar     2.59 Blasius

1.3 turbulent




indicates whether or not bdry layer separation is about to occur.




separation point

박리점

$$\left.\frac{\partial u}{\partial y}\right|_w = 0$$

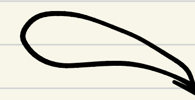
$H = 3.5$ laminar

2.4 turb

## 7.3 Boundary layer equations

Navier-Stokes eqs. ( steady 2D incomp. flow )

$$\begin{cases} \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0 & \text{— ①} \\ \rho\left(u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y}\right) = -\dfrac{\partial p}{\partial x} + \mu\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}\right) & \text{— ②} \\ \rho\left(u\dfrac{\partial v}{\partial x} + v\dfrac{\partial v}{\partial y}\right) = -\dfrac{\partial p}{\partial y} + \mu\left(\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2}\right) & \text{— ③} \end{cases}$$



No analytical sol. for external flow

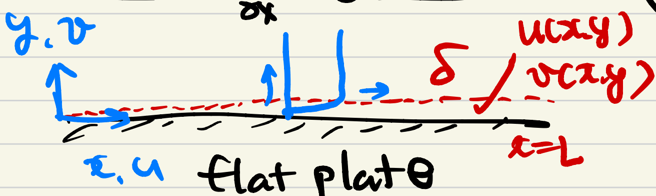
i) numerical sol. — CFD

ii) experiment — wind tunnel + velocimetries
water 〃

iii) $\boxed{\text{boundary layer theory}}$ — Ludwig Prandtl (1904)
경계층 이론

- Boundary layer approximation

$$\frac{\partial p}{\partial x} = 0$$




$\delta \ll L \leftarrow$ thin boundary layer

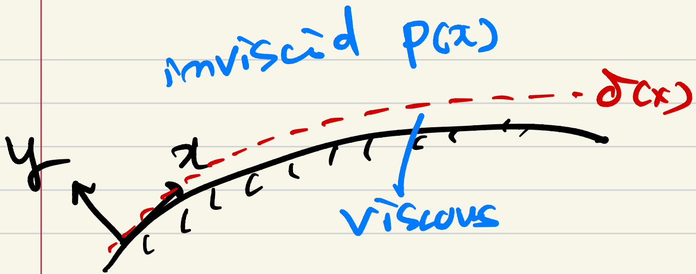
$$\boxed{\begin{array}{l} \dfrac{\partial}{\partial x}(\cdot) \ll \dfrac{\partial}{\partial y}(\cdot), \quad \dfrac{\partial^2}{\partial x^2}(\cdot) \ll \dfrac{\partial^2}{\partial y^2}(\cdot) \\ v \ll u \end{array}}$$

② + ③ $\rightarrow \dfrac{\partial p}{\partial y} \simeq 0 \Rightarrow \boxed{p = p(x) \text{ only}}$

$\dfrac{\partial p}{\partial x} = 0$ for flat plate

$\dfrac{\partial p}{\partial x} \neq 0$ otherwise

Pressure varies only along the bdry layer.

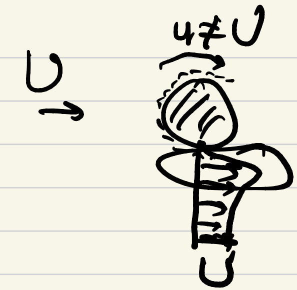



Bernoulli eq.

$$\frac{p}{\rho} + \frac{V^2}{2} = const$$

$$\rightarrow \frac{1}{\rho}\frac{dp}{dx} + U\frac{dU}{dx} = 0$$

$$\rightarrow \boxed{\frac{dp}{dx} = -U\frac{dU}{dx}} = 0 \text{ for flat plate}$$




$$\begin{aligned} ① &\rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \\ ② &\rightarrow u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\partial y^2} \end{aligned}$$

Boundary layer eq.
경계층 방정식

b.c : $u = v = 0$ @ $y = 0$
$u = U(x)$ @ $y = \delta$

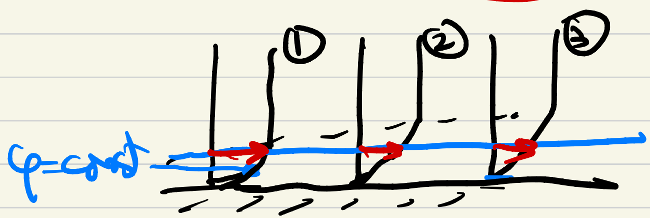
## 7.4 Flat-plate boundary layer → simple but the most important

$U = \text{const}$, $p = \text{const}$, $\frac{dp}{dx} = 0$, $\frac{dU}{dx} = 0$

### laminar flow

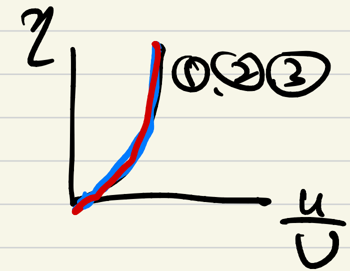
Blasius (1908) using coord. transformation showed that $\frac{u}{U}$ is a ft. only of a single dimensionless variable

$$\boxed{\eta = y\sqrt{\frac{U}{\nu x}}} \,. \Rightarrow \boxed{\frac{u}{U} = f'(\eta)}$$




① $y$, $x\uparrow \Rightarrow u\downarrow$

② $x$, $y\uparrow \Rightarrow u\uparrow$




→ $u = f'U$, $\eta = y\sqrt{\frac{U}{\nu x}}$

$$\boxed{f''' + \frac{1}{2} f f'' = 0}$$