Topics in Ship Structural Design (Hull Buckling and Ultimate Strength)

Lecture 5 Buckling and Ultimate Strength of Plates

Reference : Ship Structural Design Ch.12

NAOE

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Facts about *MSC Napoli*

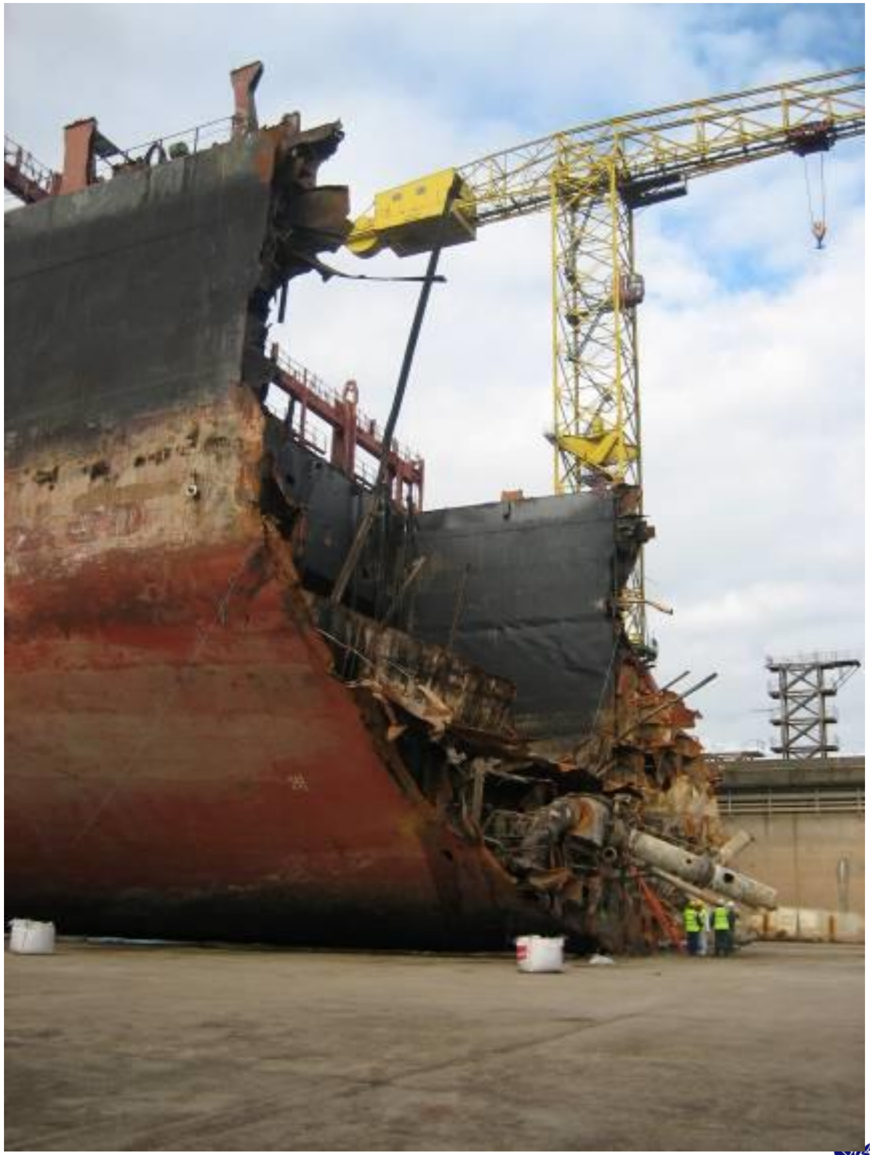
- One of the world's largest container ships when built (1991)
- Built to BV Class and changed to DNV 2002
- Last renewal survey carried out in 2004 in Singapore
- Built 1991
- Length over all 275.66 m
- Breadth 37.13 m
- Draught 13.50 m
- Gross tonnage 53,409 GRT
- Capacity 4419 TEU

Accident January 2007 – MSC Napoli

- Ship left Antwerp 17 January 2007 heading for Sines in Portugal
- 18 January - water ingress in engine room reported
- All 26 crew members safely rescued
- Ship beached in Lyme Bay near Branscombe, UK on 19 January 2007

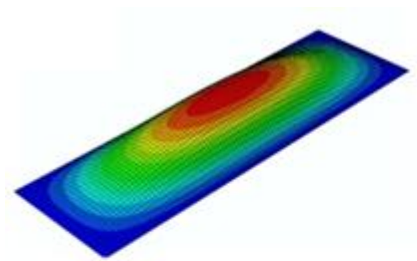
Accident January 2007 – MSC Napoli

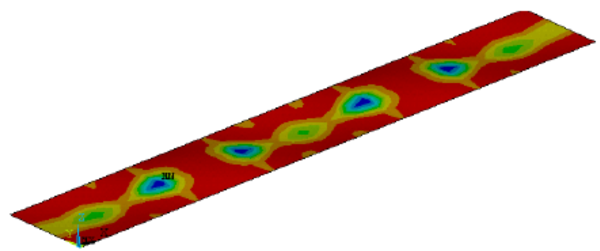
- The vessels was split into two in July 2007
- Forward part was towed to Belfast for recycling



B200 Plate panel in uni-axial compression

- Unstiffened Plate (Plating between stiffeners)
  - Elastic and Inelastic Buckling
  - Post-Buckling and Ultimate strength





Classification Rule

Ideal elastic buckling stress

$$\sigma_{el} = 0.9kE\left(\frac{t}{1000\,s}\right)^2 \ (\mathrm{N/mm^2})$$

For plating with longitudinal stiffeners (in direction of compression stress): k=4

Johnson-Ostenfeld plasticity correction formula

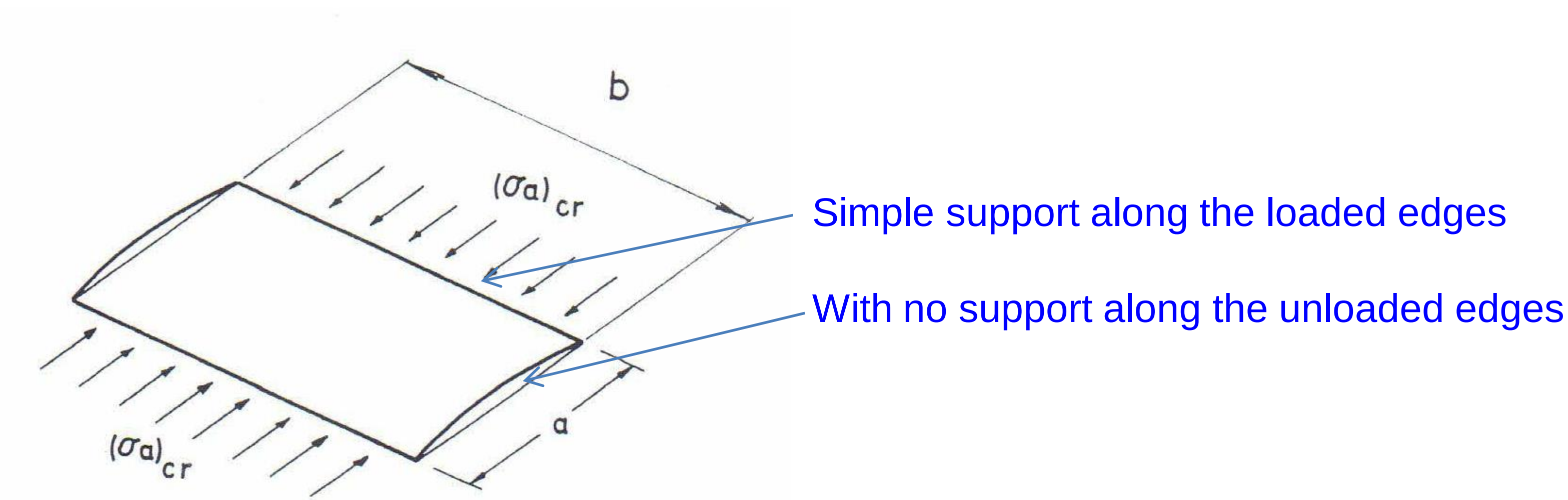
$$\begin{aligned}\sigma_c &= \sigma_{el} \quad \text{when} \quad \sigma_{el} < \frac{\sigma_f}{2} \\ &= \sigma_f\left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \quad \text{when} \quad \sigma_{el} > \frac{\sigma_f}{2}\end{aligned}$$

Buckling of a Wide Column

- The plate is acting more as a wide column than as a plate. The product EI is replaced by the plate flexural rigidity D.

$$P_{cr} = \frac{\pi^2 Db}{a^2} \qquad \sigma_{cr} = \frac{\pi^2 D}{a^2 t} = \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{a}\right)^2$$

- The thickness / length ratio plays the same role as the slenderness ratio for columns.
- The width b plays no part, no support along the unloaded edge → It is inefficient to use



Buckling of wide column

Large-Deflection Plate Theory by von Karman

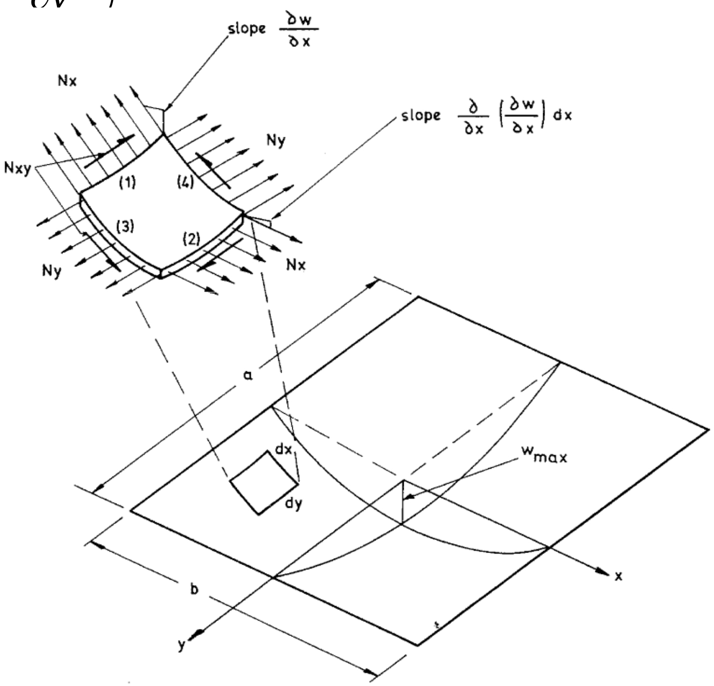
- **Small-Deflection Plate Theory**

$$\boxed{\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{p}{D}}$$

- **Large-Deflection Plate Theory**

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{1}{D}\left(p+N_x\frac{\partial^2 w}{\partial x^2}+2N_{xy}\frac{\partial^2 w}{\partial x \partial y}+N_y\frac{\partial^2 w}{\partial v^2}\right)$$

$$\boxed{\nabla^4 w=\frac{1}{D}\left(p+N_x\frac{\partial^2 w}{\partial x^2}+2N_{xy}\frac{\partial^2 w}{\partial x \partial y}+N_y\frac{\partial^2 w}{\partial y^2}\right)}$$



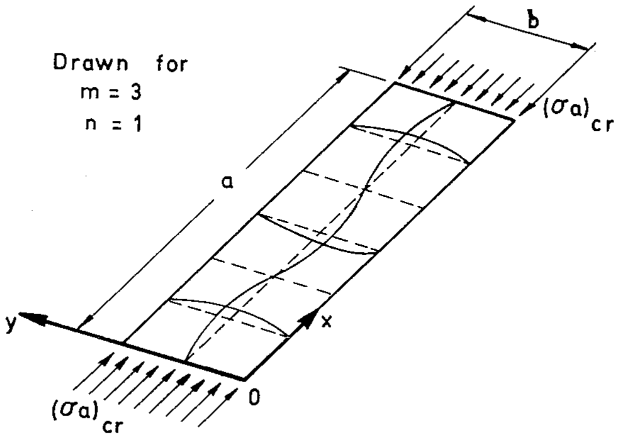
Buckling of a Simply Supported Plate

- From large-deflection plate theory

$$N_x = -\sigma_a t,\ p = N_y = N_{xy} = 0 \qquad \nabla^4 w = -\frac{\sigma_a t}{D}\frac{\partial^2 w}{\partial x^2}$$

- Since the edges are simply supported, the deflected shape can be expressed in the form:

$$w = \sum_m \sum_n w_{mn} = \sum_m \sum_n C_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$



Buckled shape of long plate

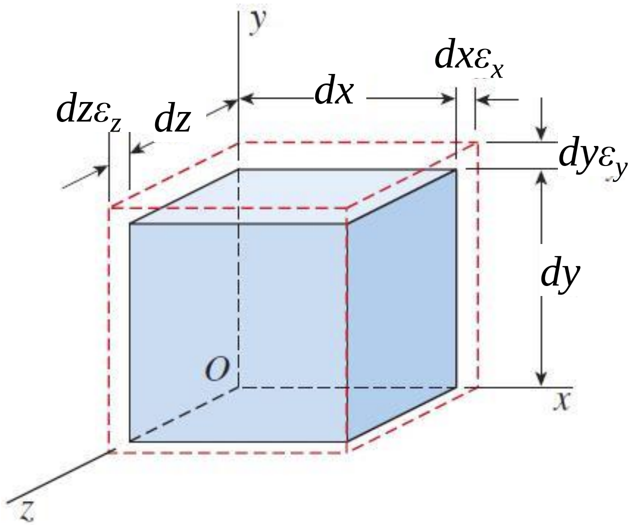
which satisfies both the boundary conditions and the general biharmonic equation.

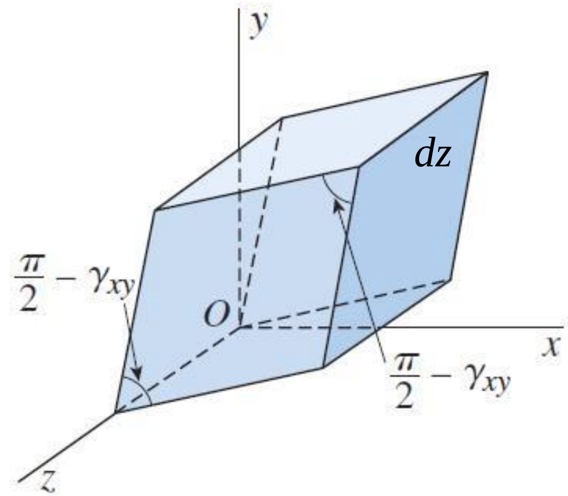
- Plate is assumed to be free to move inward under the action of the in-plane compression. → The strain energy of deformation is due to bending only

$$U = \frac{D}{2}\int_0^a\int_0^b \left\{ \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2(1-\nu)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] \right\} dxdy$$

$$U = \frac{\pi^4 ab}{8} D \sum_m \sum_n C_{mn}^2 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2$$

Strain Energy Density for plane stress (σ_z=0)





Load applied on dydz area

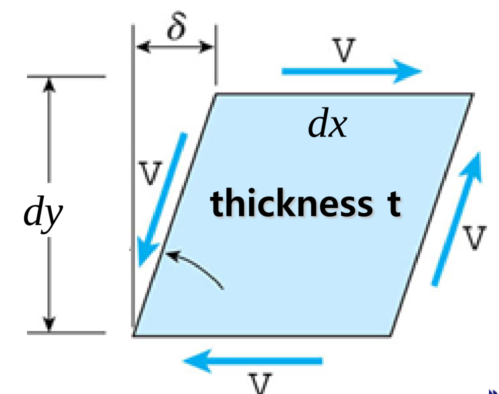
Elongation in x-direction

$$du_1 = \frac{1}{2}(\sigma_x dydz)(dx\varepsilon_x) + \frac{1}{2}(\sigma_y dxdz)(dy\varepsilon_y)$$

$$= \frac{1}{2}(\sigma_x \varepsilon_x + \sigma_y \varepsilon_y)dxdydz$$

$$du_2 = \frac{V\delta}{2} = \frac{1}{2}(\tau_x dxdz)(\gamma_{xy} dy)$$

$$= \frac{1}{2}\tau_x \gamma_{xy} dxdydz$$



$$\therefore du = \frac{1}{2}(\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \tau_x \gamma_{xy})dxdydz$$

Strain Energy for plane stress (σ_z=0)

- In Chapter 9 (Lecture 03), Plate bending (Derivation of Plate Bending Equation), the followings are derived

$$\sigma_x = \frac{E}{1-v^2}(-z)\left(\frac{\partial^2 w}{\partial x^2} + v\frac{\partial^2 w}{\partial y^2}\right) \qquad \varepsilon_x = (-z)\left(\frac{\partial^2 w}{\partial x^2}\right)$$

$$\sigma_y = \frac{E}{1-v^2}(-z)\left(\frac{\partial^2 w}{\partial y^2} + v\frac{\partial^2 w}{\partial x^2}\right) \qquad \varepsilon_y = (-z)\left(\frac{\partial^2 w}{\partial y^2}\right)$$

$$\tau = -2Gz\frac{\partial^2 w}{\partial x \partial y}$$

$$\gamma = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

$$u = -z\frac{\partial w}{\partial x}$$

$$v = -z\frac{\partial w}{\partial y}$$

$$\Rightarrow \gamma = (-2z)\left(\frac{\partial^2 w}{\partial x \partial y}\right)$$

$$U = \int du = \int_0^a \int_0^b \int_{-t/2}^{t/2} \frac{1}{2}(\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \tau_x \gamma_{xy})dzdxdy$$

Strain Energy Density for plane stress (σ_z=0)

$$U=\int du=\frac{1}{2}\int_0^a\int_0^b\int_{-t/2}^{-t/2}\frac{E}{(1-v^2)}\left(\frac{\partial^2 w}{\partial x^2}+v\frac{\partial^2 w}{\partial y^2}\right)\left(\frac{\partial^2 w}{\partial x^2}\right)(z^2)$$

$$+\frac{E}{(1-v^2)}\left(\frac{\partial^2 w}{\partial y^2}+v\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right)(z^2)+4Gz^2\left(\frac{\partial^2 w}{\partial x\partial y}\right)dzdydx$$

$$\boxed{G=\frac{E}{2(1+v)}}$$

$$=\int_0^a\int_0^b\frac{Et^3}{12(1-v^2)}\left\{\left(\frac{\partial^2 w}{\partial x^2}\right)^2+\left(\frac{\partial^2 w}{\partial y^2}\right)^2+2v\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right\}+\frac{4}{12}\frac{E(1-v)}{2(1+v)(1-v)}t^3\left(\frac{\partial^2 w}{\partial x\partial y}\right)dydx$$

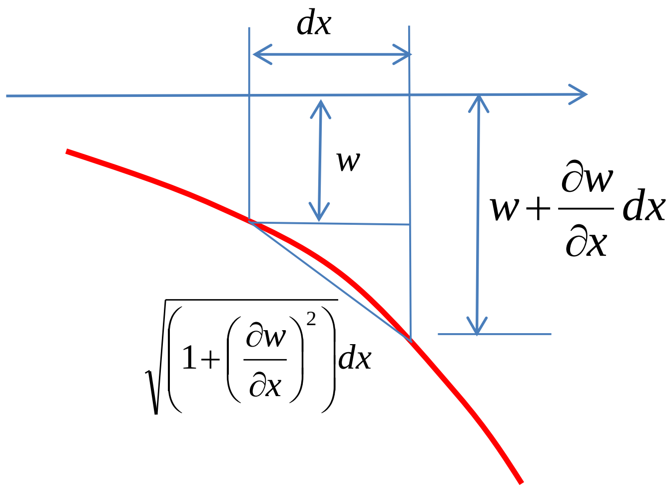
$$=\frac{D}{2}\int_0^a\int_0^b\left(\frac{\partial^2 w}{\partial x^2}\right)^2+\left(\frac{\partial^2 w}{\partial y^2}\right)^2+2v\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}+2(1-v)\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2dydx$$

$$\boxed{U=\frac{D}{2}\int_0^a\int_0^b\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)^2-2(1-v)\left(\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\frac{\partial^2 w}{\partial x\partial y}\right)^2dydx}$$

Work done for plane stress

For unit-width strip in Section 9.2

$$\delta_x = \int_0^b \left(\sqrt{1+\left(\frac{\partial w}{\partial x}\right)^2} - 1 \right) dx \cong \int_0^b \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 dx$$



$$W = \int_0^a \int_0^b N_x \delta_x dxdy = \int_0^a \int_0^b \sigma_a t \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 dxdy$$

$\sqrt{(1+a)} \cong 1+\frac{a}{2}$ for small a

Buckling of a Simply Supported Plate

- Likewise, the work done by the in-plane compressive stress is

$$W = \frac{\sigma_a t}{2}\int_0^a\int_0^b\left(\frac{\partial w}{\partial x}\right)^2 dxdy \qquad W = \frac{\pi^4 b\sigma_a t}{8a}\sum_m\sum_n C_{mn}^2 m^2$$

- Because of W=U, and hence,

$$\sigma_a = \frac{\pi^2 a^2 D\sum_m\sum_n C_{mn}^2\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)}{t\sum_m\sum_n m^2 C_{mn}^2}$$

$$\min(c_1/d_1, c_2/d_2, \ldots, c_n/d_n) \le \frac{c_1+c_2+\ldots+c_n}{d_1+d_2+\ldots+d_n} \le \max(c_1/d_1, c_2/d_2, \ldots, c_n/d_n)$$

- The minimum value of σ_a is given by taking only one term, say C_{mn},

$$(\sigma_a)_{cr} = \frac{\pi^2 a^2 D}{tm^2}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2$$

where m and n indicate the number of half-waves in each direction in the buckled shape.

- When $n=1$, σ_a gives the smallest value. Hence the plate will buckle into only one half-wave transversely.

$$(\sigma_a)_{cr} = \frac{\pi^2 D}{a^2 t}\left[m+\frac{1}{m}\left(\frac{a}{b}\right)^2\right]^2$$

Buckling of a Simply Supported Plate

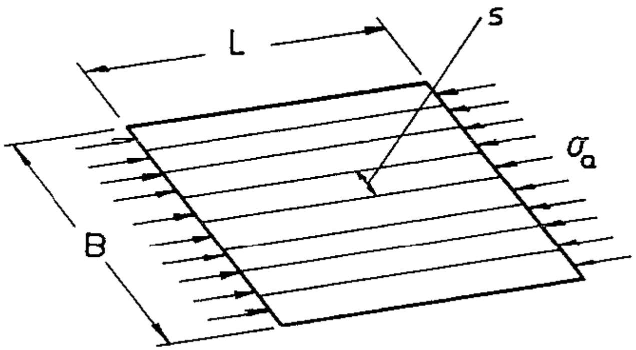
- A buckling coefficient k is generally used. It depends on the type of boundary support.

$$(\sigma_a)_{cr} = k\frac{\pi^2 D}{b^2 t} \qquad k = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2$$

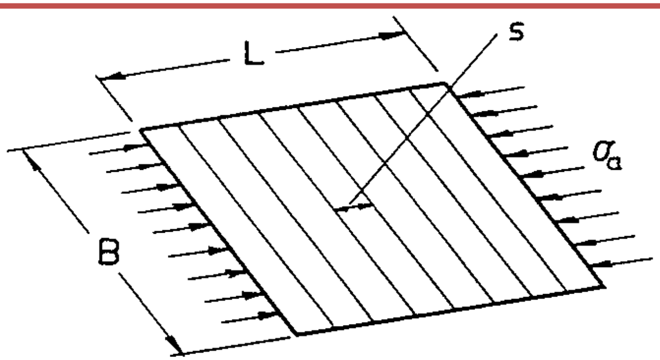
- For design applications, in which the plate thickness is to be determined, it is usually written like this:

$$(\sigma_a)_{cr} = KE\left(\frac{t}{b}\right)^2 \qquad K = \frac{\pi^2 k}{12(1-\nu^2)} \Leftarrow \boxed{D = \frac{Et^3}{12(1-\nu^2)}}$$

Q: Which critical stress will be higher?, which stiffener arrangement is better against in-plane compression?



(a)



(b)

Buckling of a Simply Supported Plate

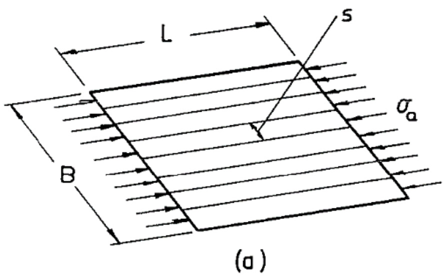
- For long simply supported plates it is usually assumed that *k=4.*

$$(\sigma_a)_{cr} = k\frac{\pi^2 D}{b^2 t} \qquad k = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2$$

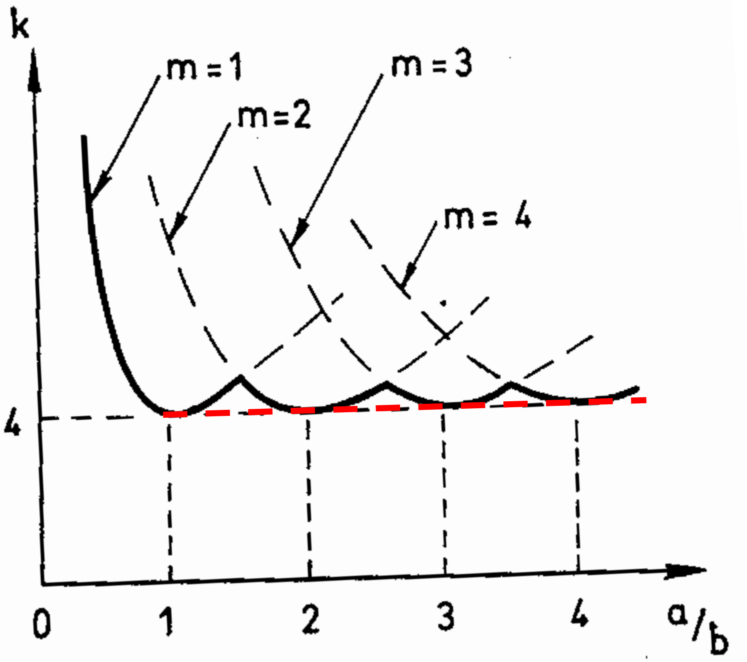
- Assuming *v=0.3*

$$(\sigma_a)_{cr} = 4\frac{\pi^2 D}{b^2 t}$$

$$(\sigma_a)_{cr} = 3.62E\left(\frac{t}{b}\right)^2$$



Homework #1 Plot this curve



Buckled shape of long plate

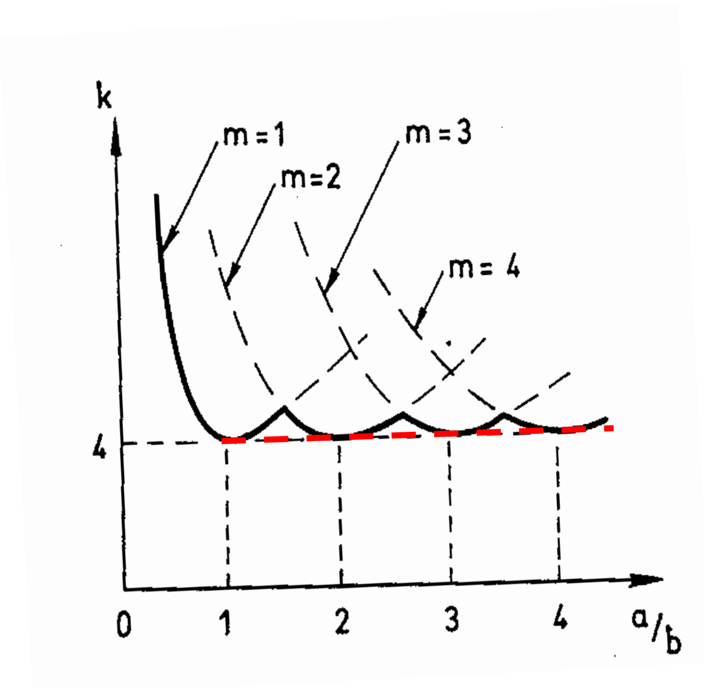
Classification Rule

$$\sigma_{el} = 0.9kE\left(\frac{t}{1000s}\right)^2 \quad (\mathrm{N/mm^2})$$

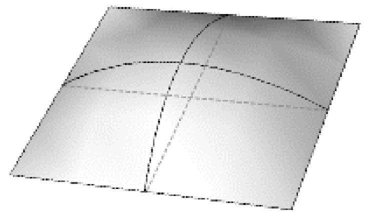
k=4, s=b (m)

12.1 Elastic Plates Subjected to Uniaxial Compression

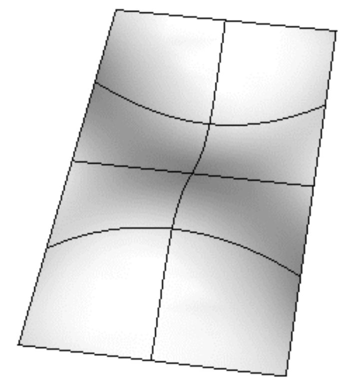
Buckling of a Simply Supported Plate



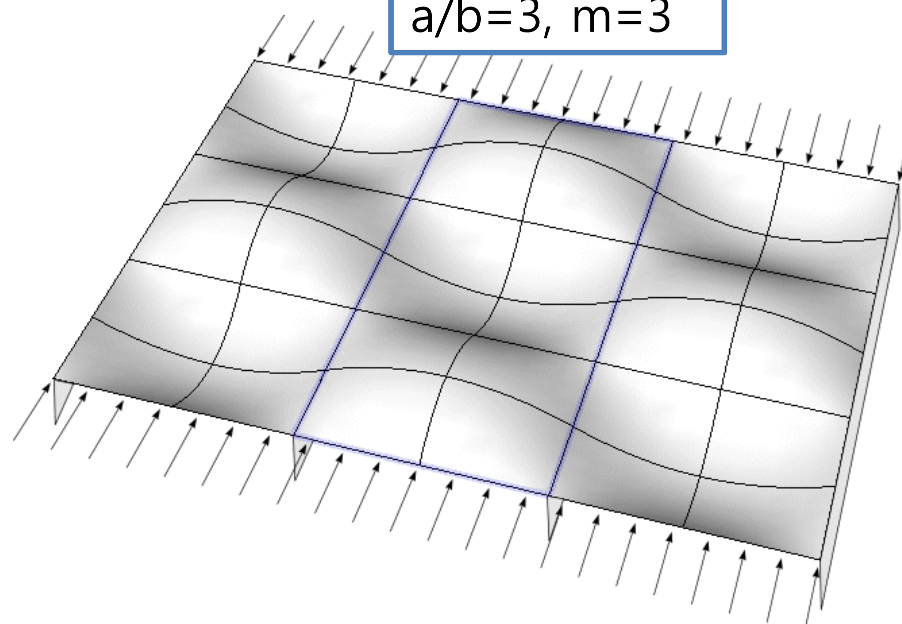
a/b=1, m=1



a/b=2, m=2



a/b=3, m=3

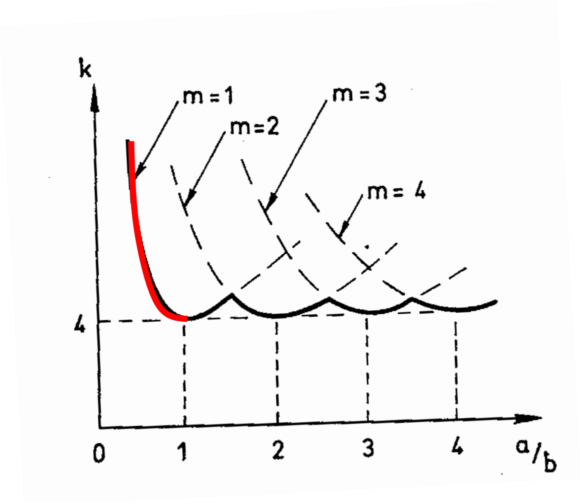


Buckling of a Simply Supported Plate

- For a wide plate, in which the aspect ratio(*a/b*) is less than 1.0, *m=1*

$$(\sigma_a)_{cr} = \frac{\pi^2 D}{a^2 t}\left[1+\left(\frac{a}{b}\right)^2\right]^2$$

- For a general "wide plate“, in terms of a because a<b



$$(\sigma_a)_{cr} = \bar{k}\frac{\pi^2 D}{a^2 t} \qquad \bar{k} = \left(\frac{a}{b}\right)^2 k \qquad k = \left(\frac{b}{a}+\frac{a}{b}\right)^2$$

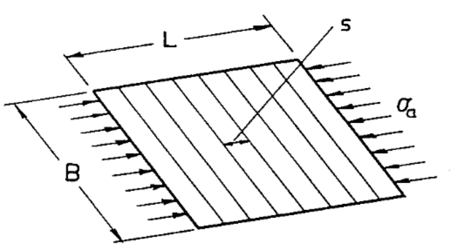
- For design purposes it may be written as:

$$(\sigma_a)_{cr} = \overline{K}E\left(\frac{t}{a}\right)^2 \qquad \overline{K} = \frac{\pi^2}{12(1-v^2)}\left[1+\left(\frac{a}{b}\right)^2\right]^2$$

$$\boxed{D = \frac{\pi^2 E t^3}{12(1-v^2)}}$$

For $v=0.30$

$$\overline{K} = 0.905\left[1+\left(\frac{a}{b}\right)^2\right]^2$$

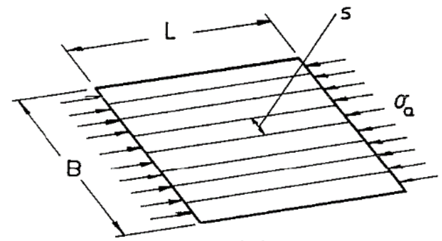


(b)

Buckling of a Simply Supported Plate

- Longitudinal stiffeners: a>>b(=s),

$$(\sigma_a)_{cr} = \frac{4\pi^2 D}{s^2 t}$$



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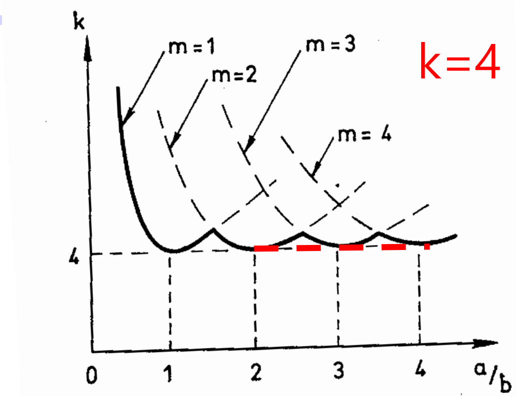
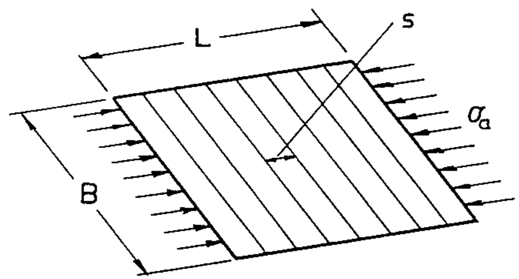


- Transverse stiffeners: a<<b, a=s, b=B,

$$(\sigma_a)_{cr} = \frac{\pi^2 D}{s^2 t}\left[1+\underbrace{\left(\frac{s}{B}\right)^2}_{<<1}\right]^2$$



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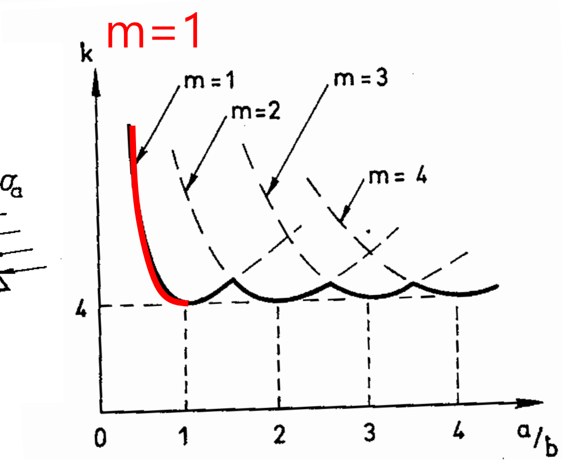
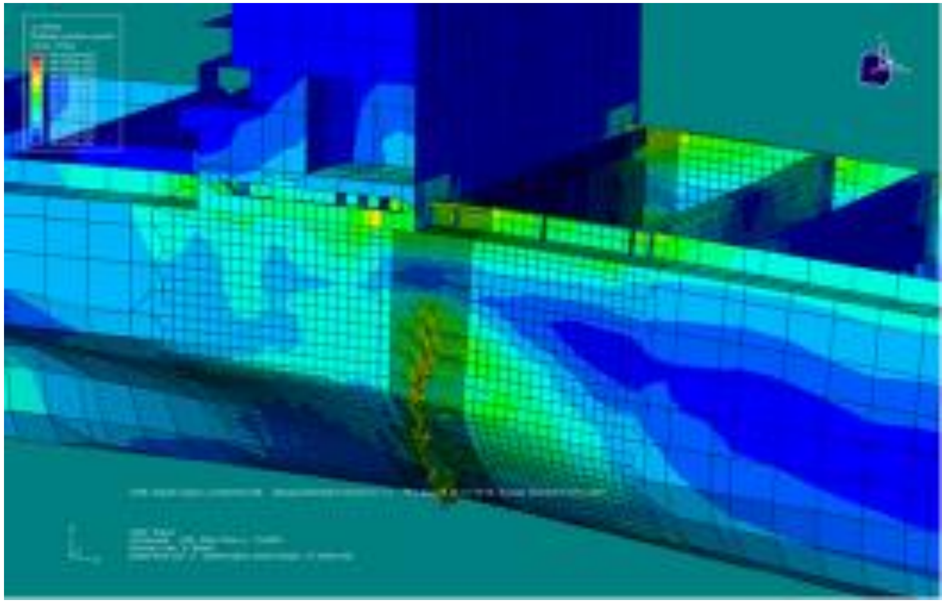


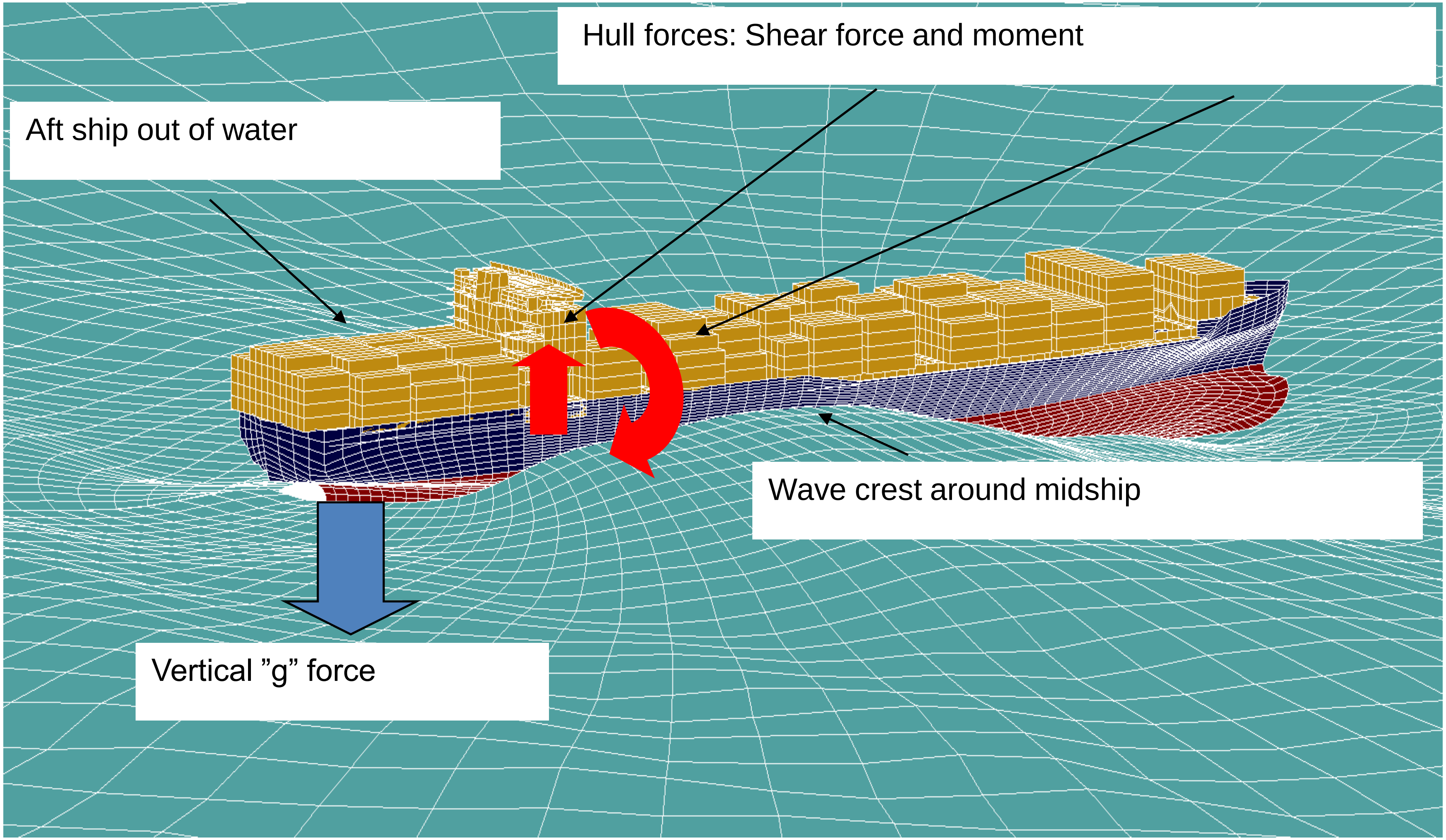
- Longitudinally stiffened plating have the great advantage over transversely stiffened plating in ship structures, and the former is used wherever possible.

Reproducing the event in a computer model

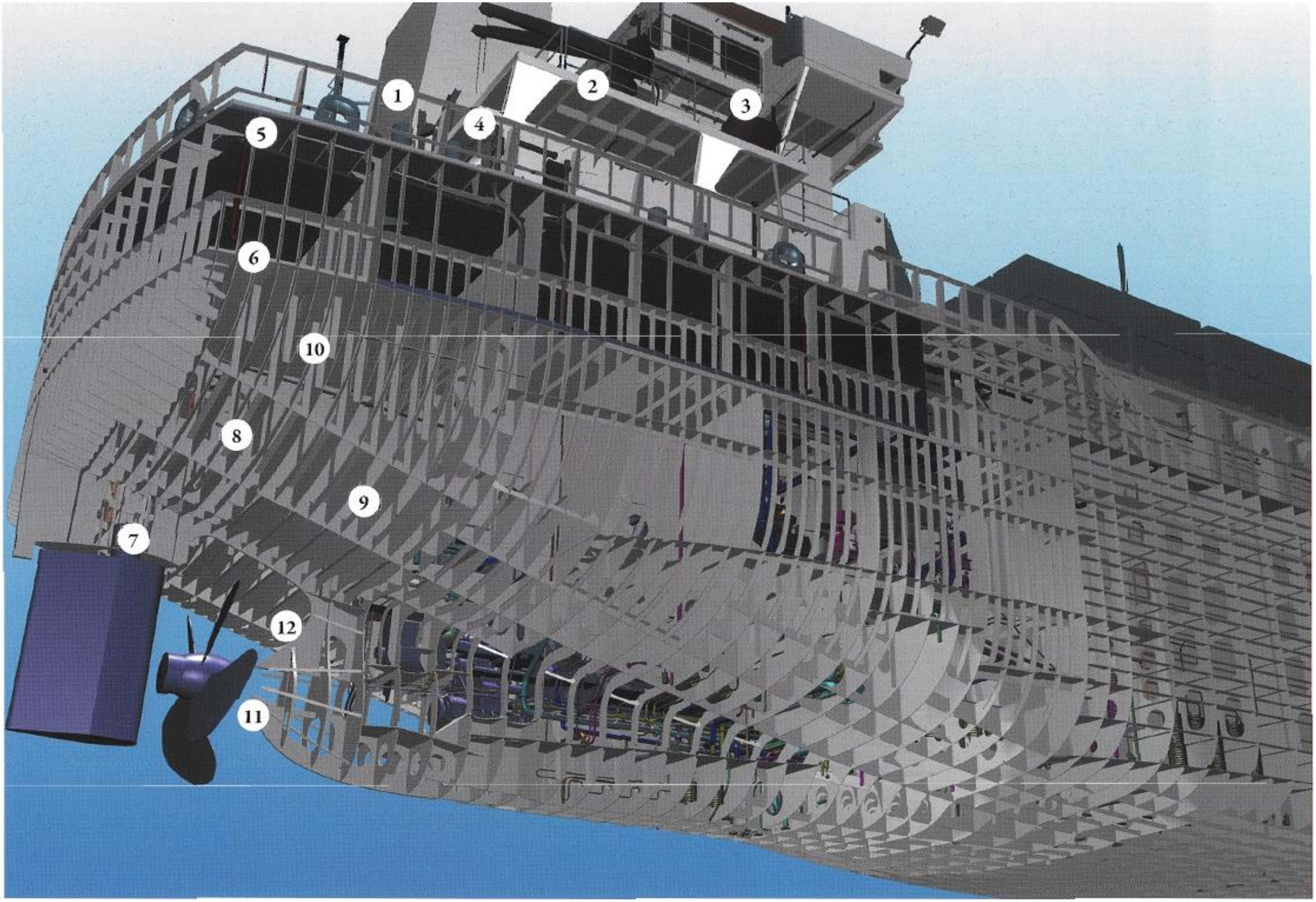
- Direct wave load calculations
- Linear strength analysis
- Non-linear strength analysis
- Load and strength comparisons
- Simulation of crack propagation



Most severe wave for engine room area

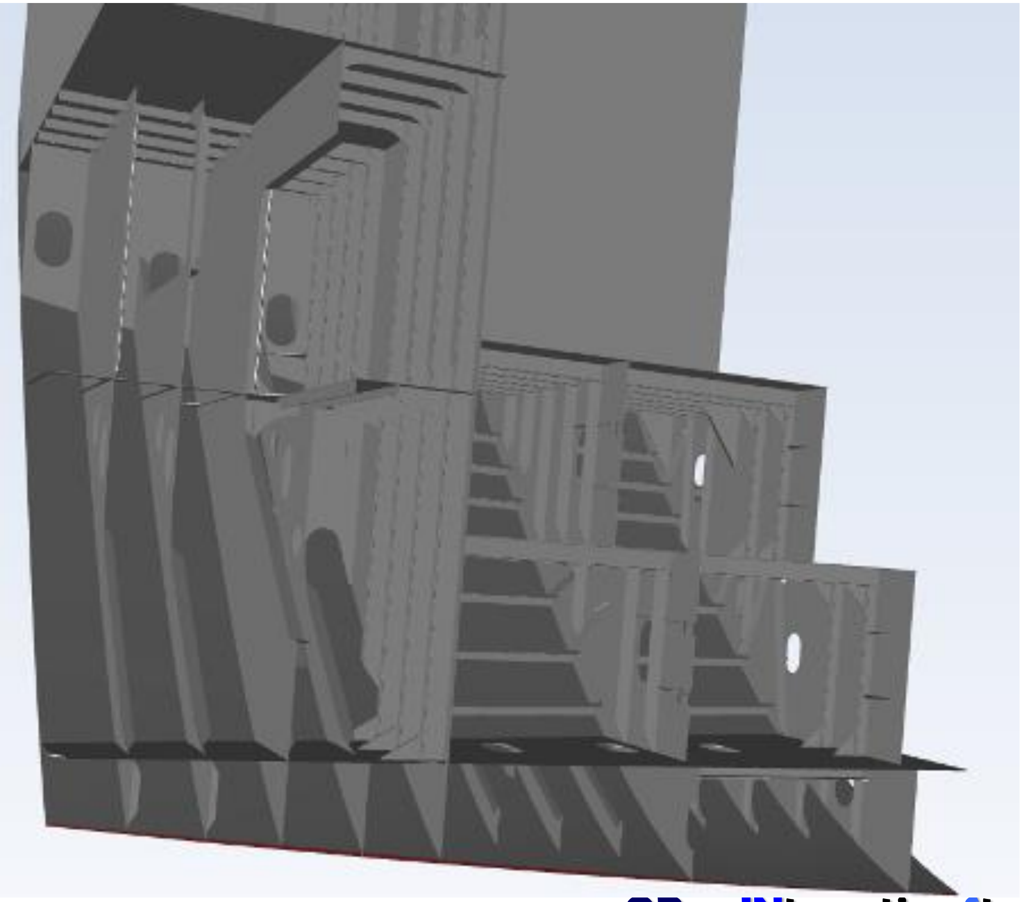


Structural arrangement in Engine room zone

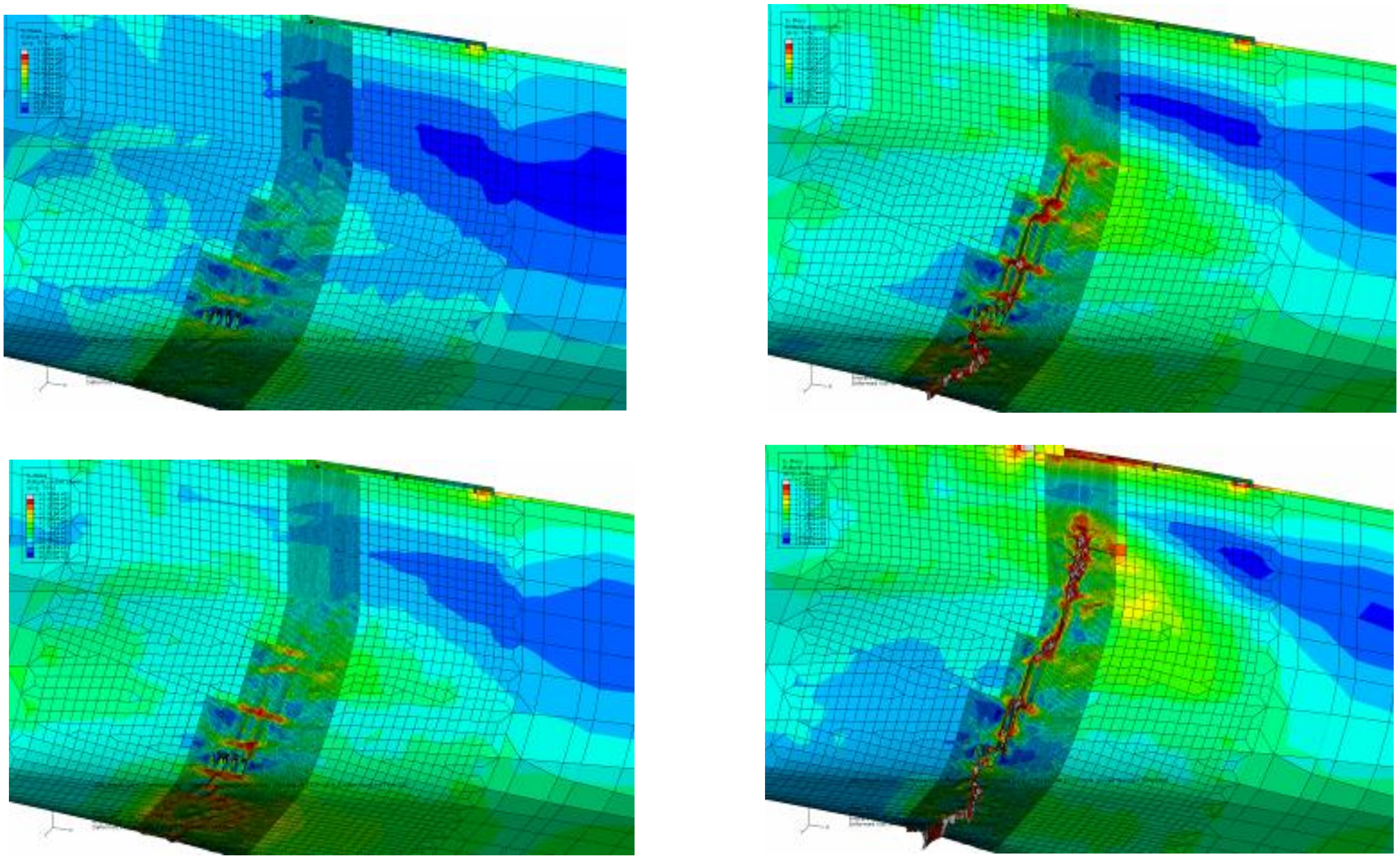


Not sufficient buckling capacity

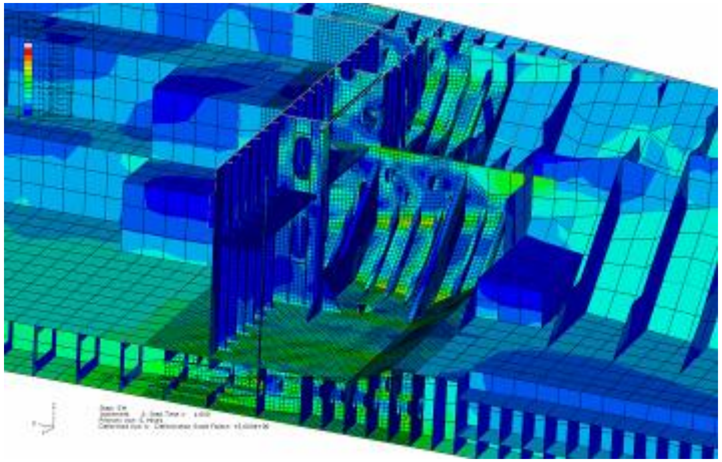
- The buckling capacity might not have been checked sufficiently when the ship was built
- Potentially insufficient buckling strength in the engine room bulkhead

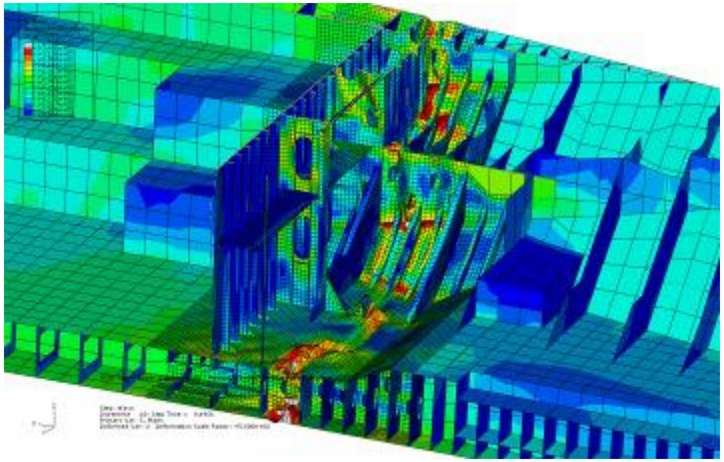


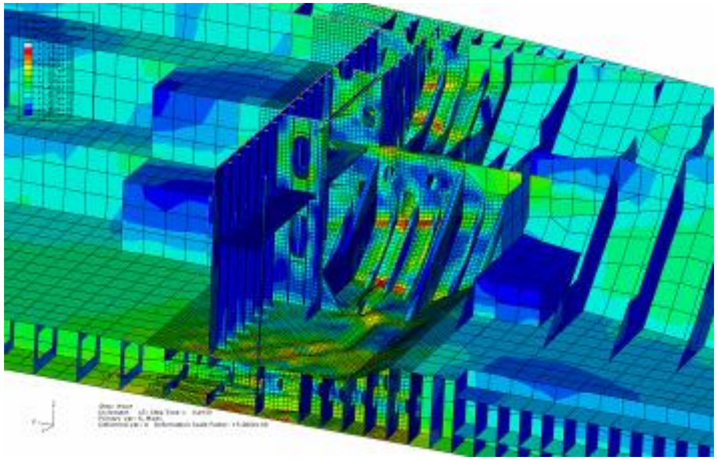
Four stages of progressive collapse Outer shell

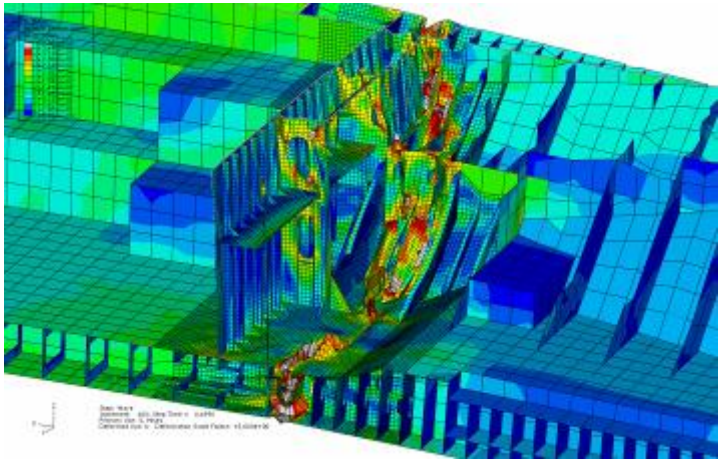


Four stages of progressive collapse Inner structure

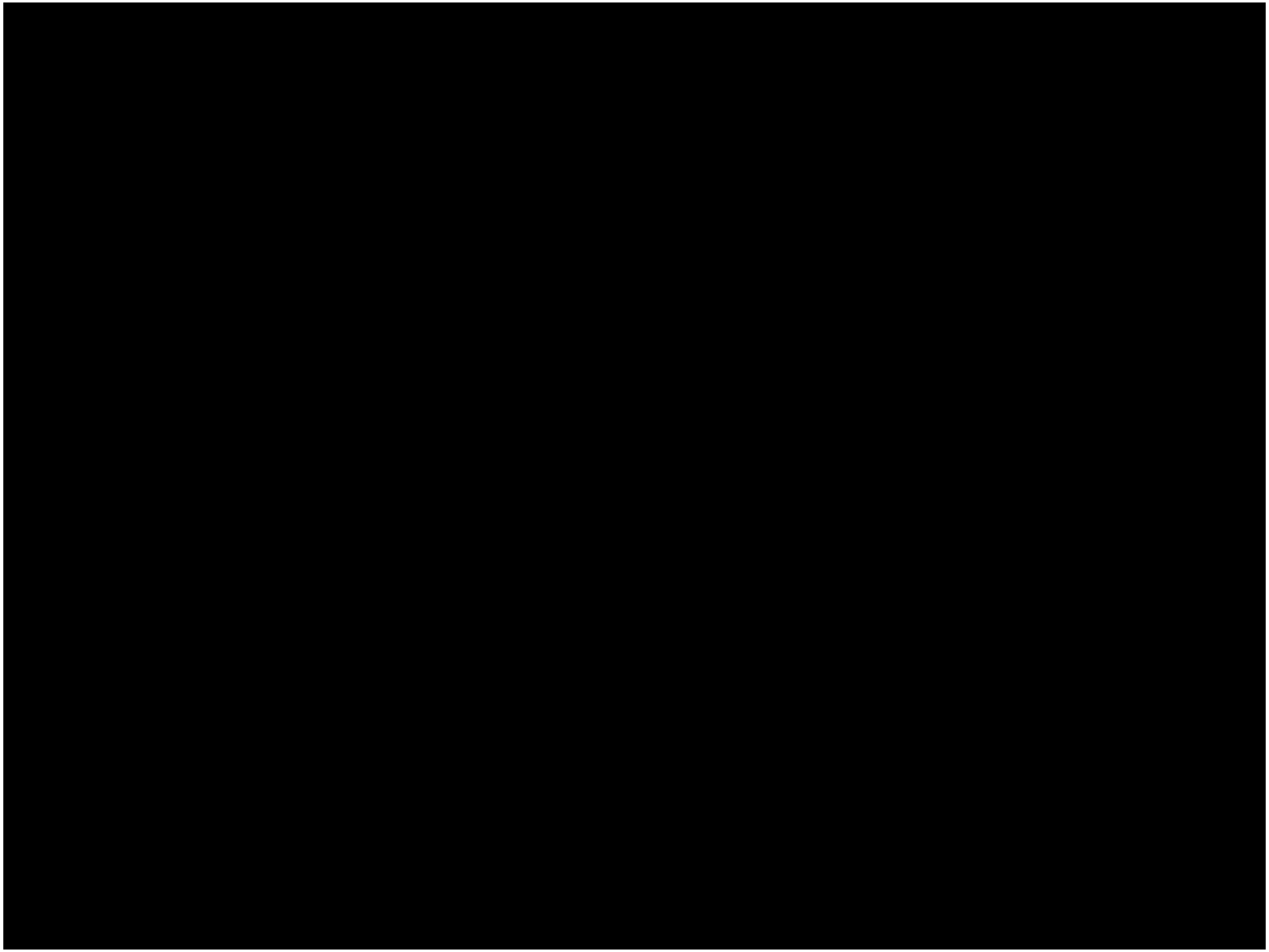








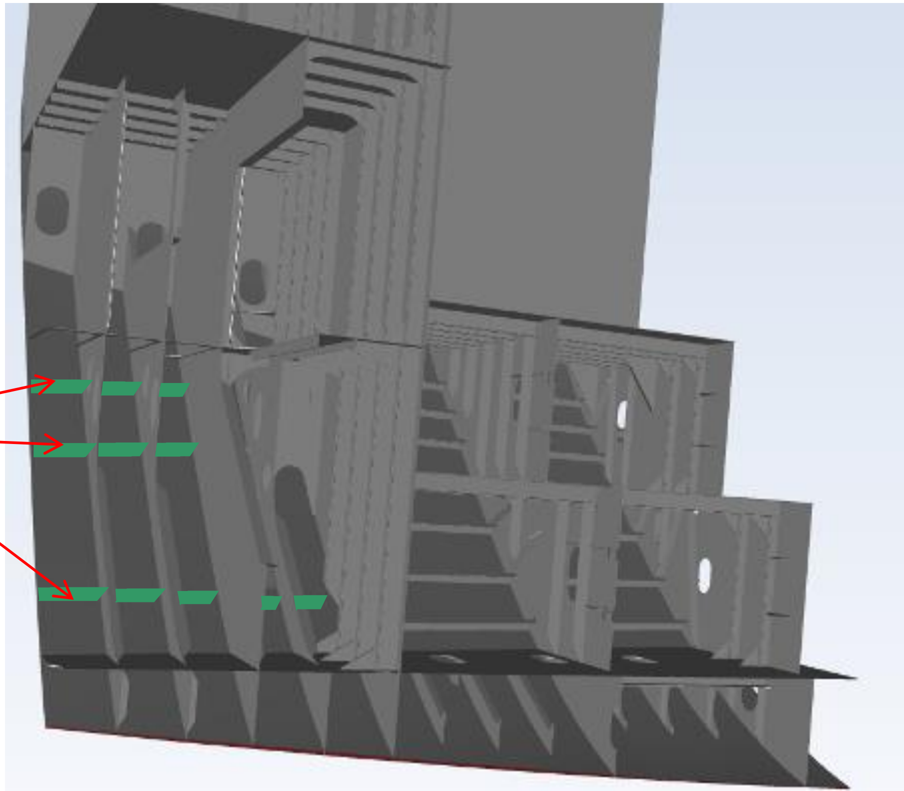
Four stages of progressive collapse Inner structure



Alternative correcting actions

- The likelihood of reoccurrence is very low:
  - ✓ Damage statistics are very good
  - ✓ Little likelihood of such a harsh sea state
  - ✓ The ship's strength was below the strength of similar ships
  - ✓ Maybe not all ships checked in this area
  - ✓ However – the consequences are major

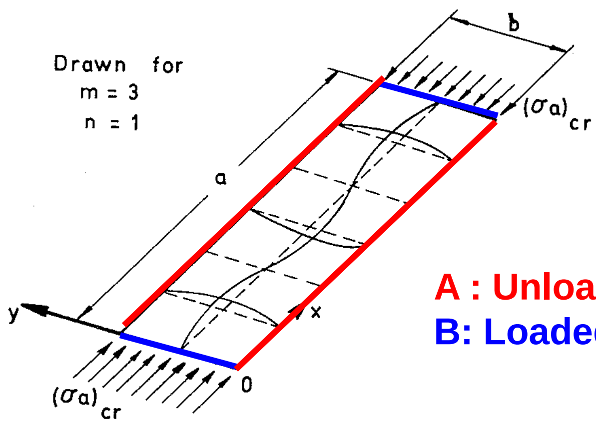
- Increase buckling strength
  - ✓ Minor modifications – small amount of steel to be added
  - ✓ Aft of the engine room bulkhead
  - ✓ Can be done while in service



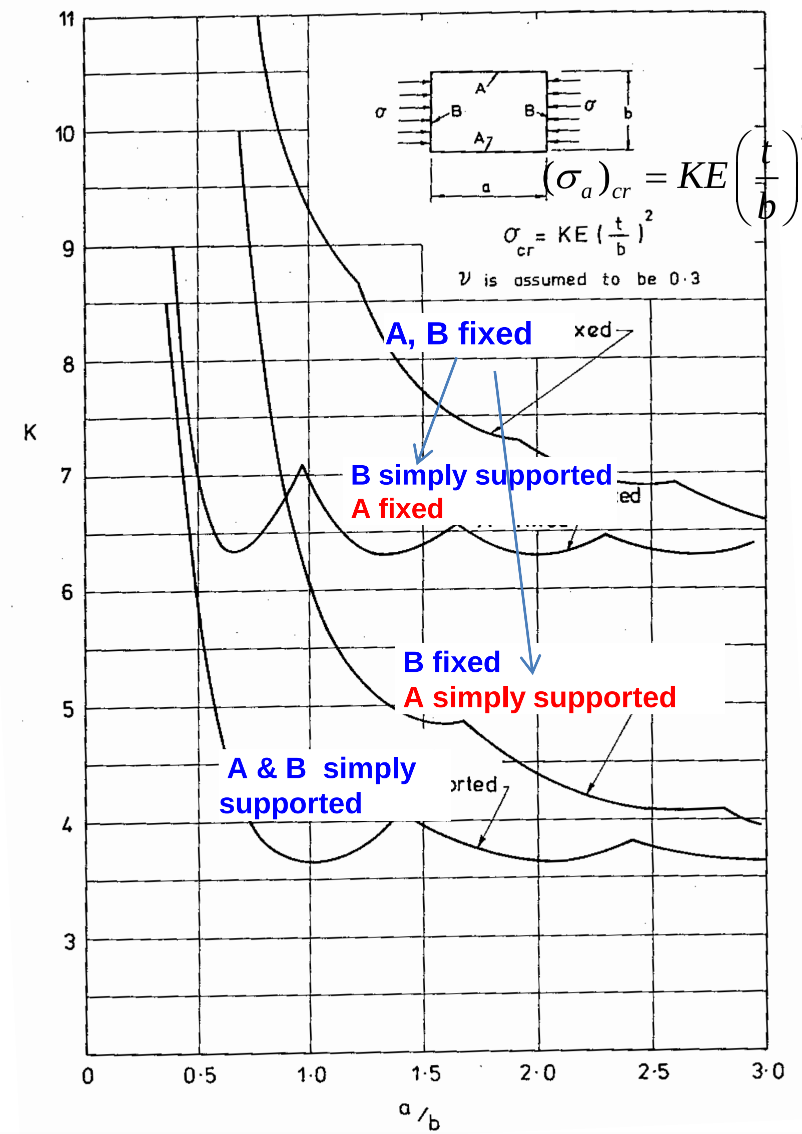
12.2 Other Boundary Conditions

Solutions for Some Principal Cases

Q: Which edge is more effective to in-plane buckling? Loaded edge or unloaded edge?



- When **unloaded edge (A)** is replaced by simply supported, the critical buckling stress drops more than when **loaded edge (B)** is by simply supported.

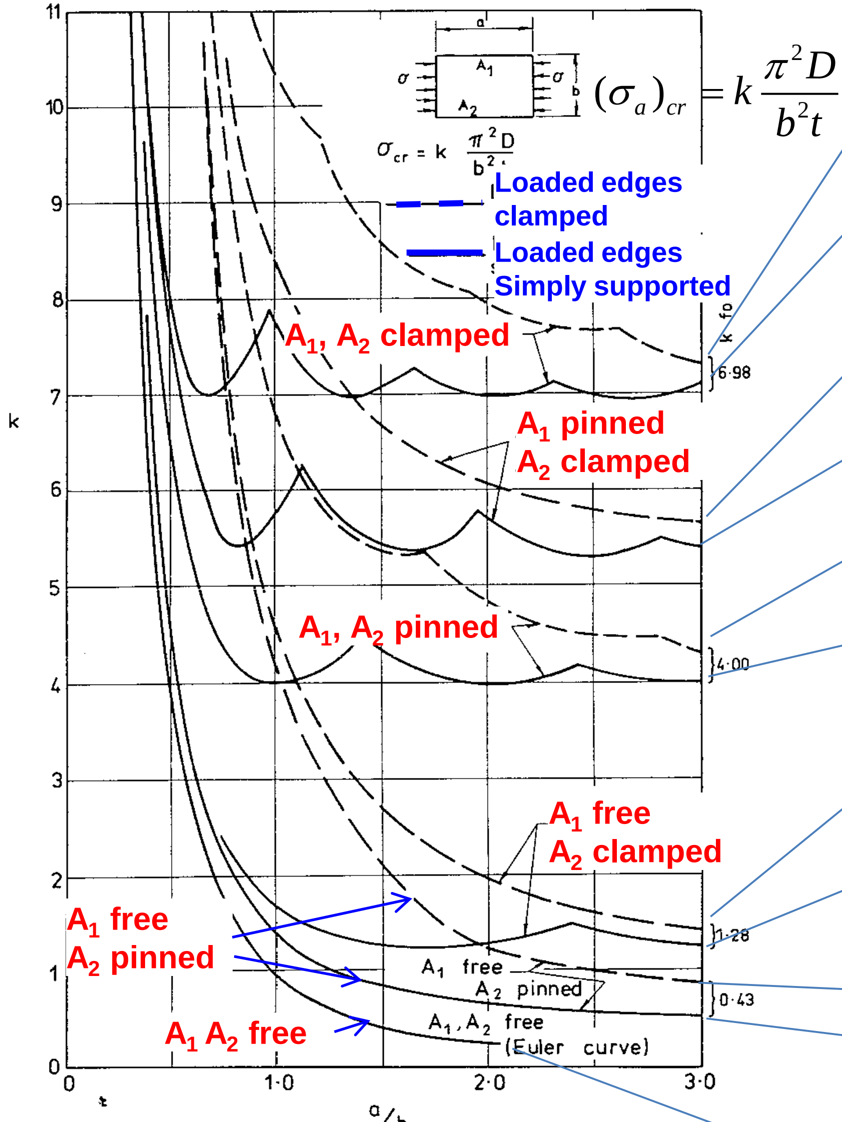


Buckling coefficient k in the design formula for flat plates in uniaxial compress

12.2 Other Boundary Conditions

Solutions for Some Principal Cases

In general, b≈ 800mm,
a≈3300mm, a/b≈3~4



Buckling stress coefficient k for flat plates in uniaxial compression

Unloaded edge : clamped
Loaded edge : clamped

Unloaded edge : clamped
Loaded edge : simply supported

Unloaded edge : pinned and clamped
Loaded edge : clamped

Unloaded edge : pinned and clamped
Loaded edge : simply supported

Unloaded edge : simply supported
Loaded edge : clamped

Unloaded edge : simply supported
Loaded edge : simply supported

Unloaded edge : free & clamped
Loaded edge : clamped

Unloaded edge : free & clamped
Loaded edge : simply supported

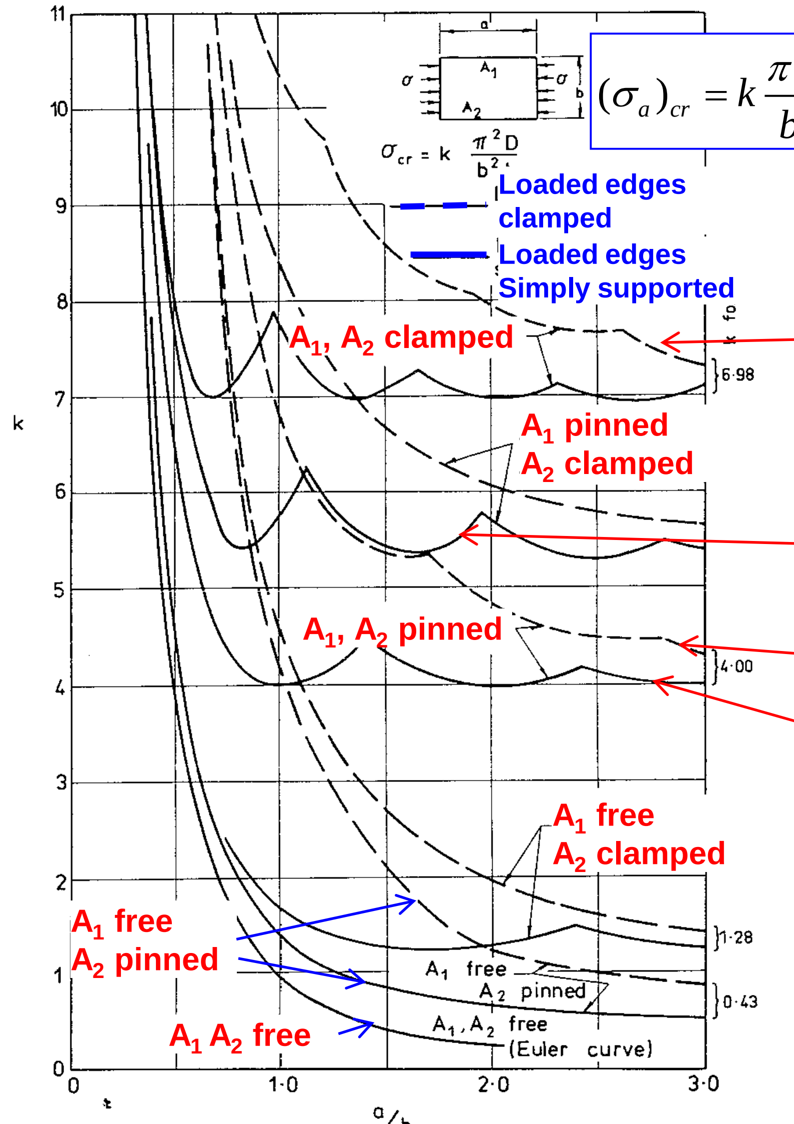
Unloaded edge : free & pinned
Loaded edge : clamped

Unloaded edge : free & pinned
Loaded edge : simply supported

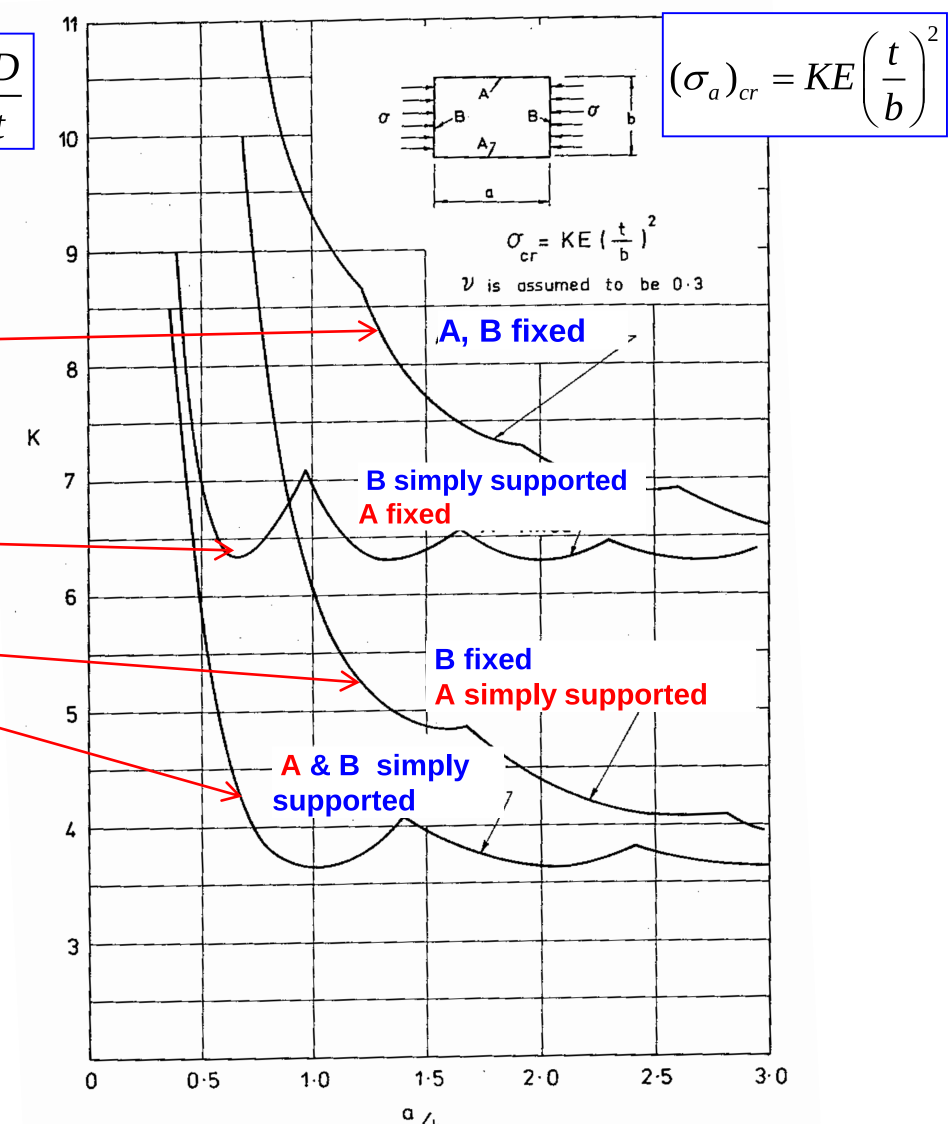
Unloaded edge : free
Loaded edge : simply supported

12.2 Other Boundary Conditions

Solutions for Some Principal Cases



Buckling stress coefficient k for flat plates in uniaxial compression



Buckling coefficient k in the design formula for flat plates in uniaxial compress

12.2 Other Boundary Conditions

Clamped Edges

- For in-plane loads, as in the case of lateral loads, it is not possible to obtain finite expressions for the solution of clamped plates.
- Numerical solutions by Faxen, Maulbetsch, and Levy.

Buckling coefficient k for clamped plates under uniaxial compression

Aspect Ratio a/b	Number of Buckles	Faxen	Maulbetsch	Levy
0.50	1	18.30	—	—
0.75	1	11.39	12.77	11.66
1.00	1	10.07	10.48	10.07
1.25	2	9.2	9.38	9.25
1.50	2	8.30	8.45	8.33
1.75	2	8.18	8.17	8.11
2.00	3	7.87	8.06	7.88
2.25	3	—	7.96	7.63
2.50	3	—	7.99	7.57
2.75	4	—	7.76	7.44
3.00	4	—	7.59	7.37
3.25	4	—	7.86	7.35
3.50	5	—	7.37	7.27
3.75	5	—	7.40	7.24
4.00	5	—	7.45	7.23
∞	∞	—	—	—

12.2 Other Boundary Conditions

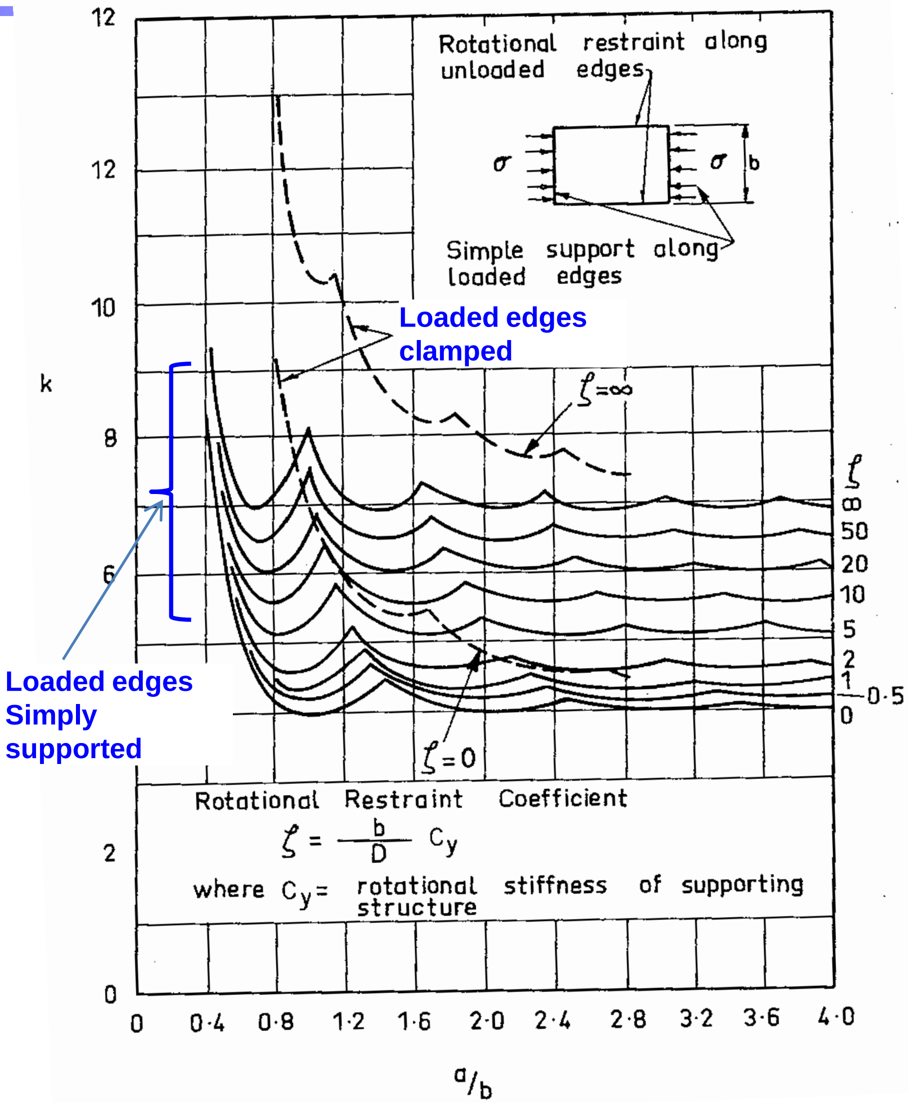
Unloaded Edges Rotationally Restrained

- Lundquist and Stowell have investigated the case in which the support along the unloaded edges is intermediate between simply supported and clamped.

- The degree of rotational restraint is specified in terms of a coefficient of restraint, defined as

$$\zeta = \frac{b}{D} C_y$$

- C_y : rotational stiffness of the supporting structure along the unloaded edge



Buckling coefficient k for plates with loaded edges simply supported and longitudinal edges rotationally restrained

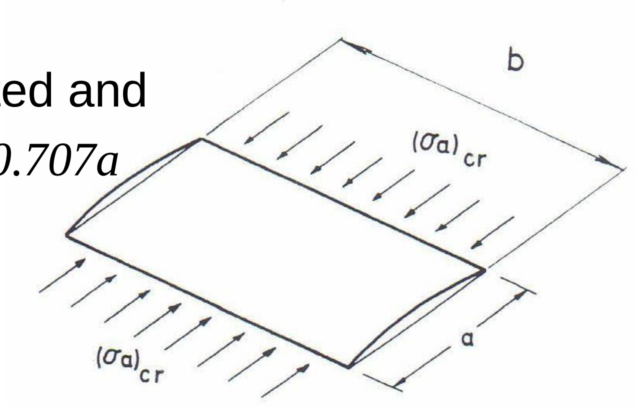
Loaded Edges Rotationally Restrained

- The important boundary conditions are those along the longer edges of the plate. Thus, for short wide plates the edge restraint along the loaded edges becomes significant.
- Similar to end conditions in a column, by using an effective length a_e:

$$(\sigma_a)_{cr} = \frac{\pi^2 D}{a_e^2 t}\left[1+\left(\frac{a_e}{b}\right)^2\right]^2$$

for clamped ends $a_e = 1/2a$

for one end simply supported and the other clamped $a_e = 0.707a$



- Using a coefficient of restraint ζ :

$$\zeta = \frac{a}{D} C_x$$

C_x : rotational stiffness of the supporting structure along the unloaded edge

- The solution to this case is obtained from

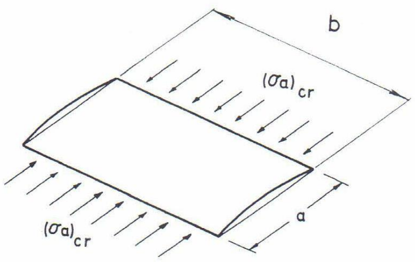
$$K_1 \tan\frac{\alpha k_1}{2} - K_2 \tan\frac{\alpha k_2}{2} + \frac{\alpha}{\zeta}(K_1^2 - K_2^2) = 0$$

in which K_1 and K_2 are related to the buckling coefficient k.

$$K_{1,2} = \frac{\pi}{2}(\sqrt{k} \pm \sqrt{k-4})$$

Loaded Edges Rotationally Restrained

Buckling coefficient $\bar{k}$ for wide plates in compression elastically restrained on the loaded edges



Coefficient ζ	Aspect Ratio a/b a<b							
	0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0
clamped	4.02	4.08	4.19	4.34	4.55	4.82	5.59	6.74
40	3.35	3.40	3.51	3.63	3.88	4.15	4.94	6.18
20	2.88	2.93	3.04	3.18	3.42	3.70	4.51	5.78
13.2	2.56	2.62	2.72	2.86	3.10	3.38	4.21	5.48
10	2.32	2.38	2.49	2.62	2.86	3.15	3.98	5.26
8	2.15	2.21	2.32	2.45	2.68	2.97	3.80	5.09
6.67	2.01	2.07	2.18	2.32	2.55	2.84	3.67	4.98
5.92	1.90	1.96	2.08	2.22	2.44	2.73	3.57	4.80
5	1.81	1.88	1.99	2.13	2.35	2.65	3.48	4.80
4.46	1.74	1.81	1.91	2.06	2.28	2.58	3.41	4.73
4	1.69	1.75	1.85	2.00	2.23	2.52	3.35	4.68
2	1.39	1.46	1.55	1.71	1.92	2.22	3.06	4.37
1	1.22	1.27	1.38	1.55	1.76	2.04	2.89	4.19
pinned	1.02	1.08	1.19	1.35	1.56	1.85	2.69	4.00

$$\bar{k} = (a/b)^2 k$$

Loaded Edges Rotationally Restrained

- The corresponding coefficient   in the "design" version of the wide plate formula

$$(\sigma_a)_{cr} = \overline{K}E\left(\frac{t}{a}\right)^2$$

- In ship structures the rotational restraint is usually provided by flange-and-web type of transverse stiffeners.

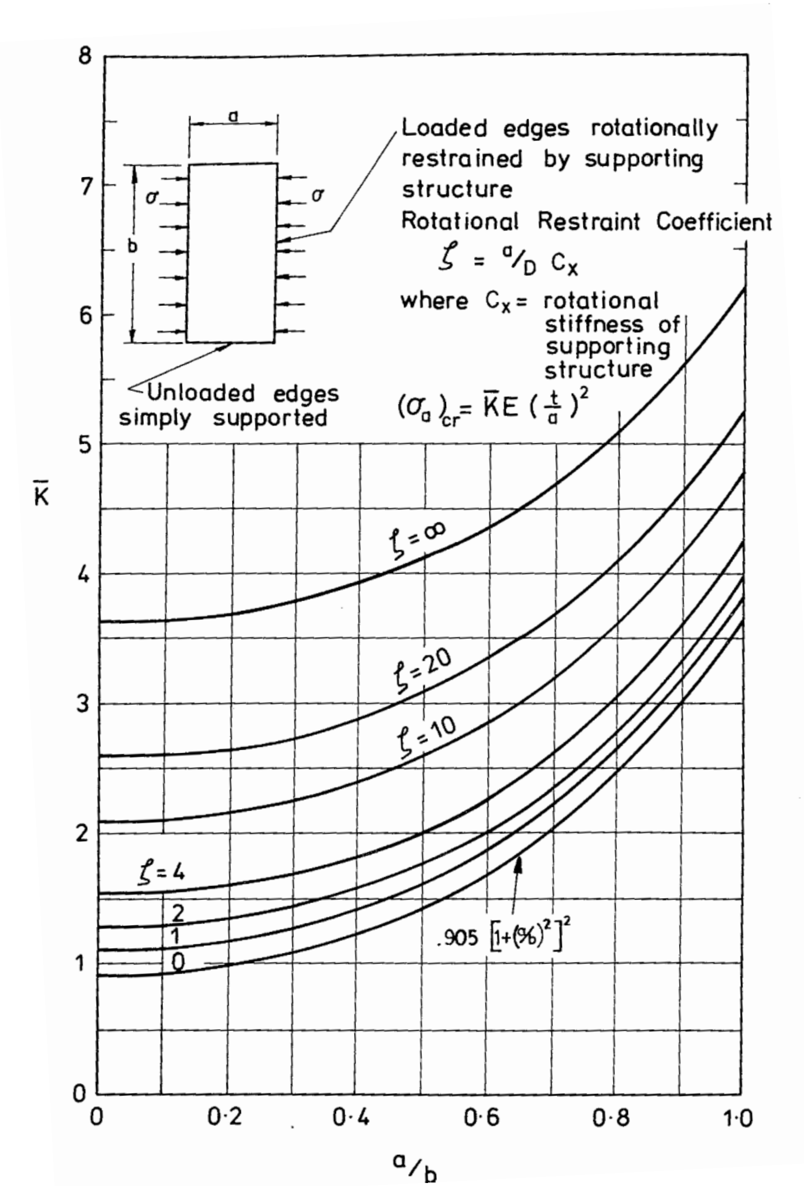
- In this case ζ is given approximately by

$$\zeta = \frac{27a}{t^3b^2}\left(\frac{\pi^2 I d^2}{b^2} + \frac{J}{2.6}\right)$$

d : depth of the web

I : second moment of area of the stiffener about the midthickness of the web

J : Saint-Venant's torsion constant for the stiffener



Buckling coefficient k for plates with loaded edges simply supported and longitudinal edges rotationally restrained

All Edges Simply Supported

- a is parallel to σ_{ax} and b to σ_{ay}. Aspect ratio $\alpha = a/b$.
- Applying the energy method yields the following expression for the critical combination:

$$W = \frac{\pi^4 t}{8} \sum_m \sum_m C_{mn}^2 \left(\frac{b\sigma_{ax}}{a} m^2 + \frac{a\sigma_{ay}}{b} n^2 \right) = \frac{\pi^4 at}{8b} \sum_m \sum_m C_{mn}^2 \left(\frac{m^2}{\alpha^2} \sigma_{ax} + n^2 \sigma_{ay} \right)$$

$$U = \frac{\pi^4 ab}{8} D \sum_m \sum_m C_{mn}^2 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2 = \frac{\pi^4 ab}{8b^4} D \sum_m \sum_m C_{mn}^2 \left(\frac{m^2}{\alpha^2} + n^2 \right)^2$$

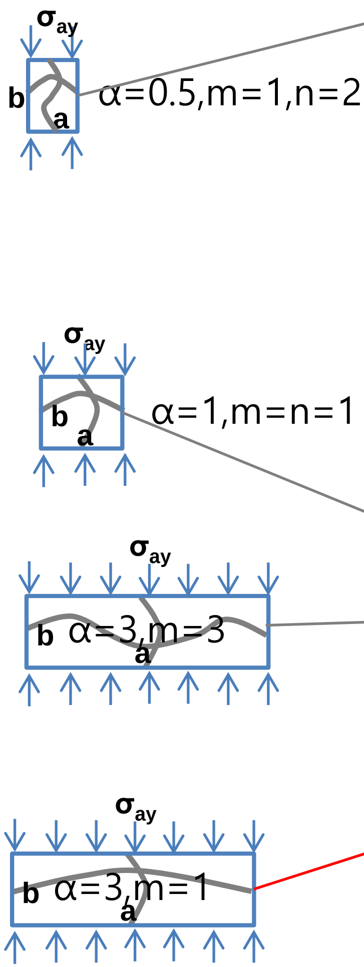
- $U = W$ ➡ $$\left[\left(\frac{m}{\alpha} \right)^2 \sigma_{ax} + n^2 \sigma_{ay} \right]_{cr} = \frac{\pi^2 D}{b^2 t} \left[\left(\frac{m}{\alpha} \right)^2 + n^2 \right]^2$$

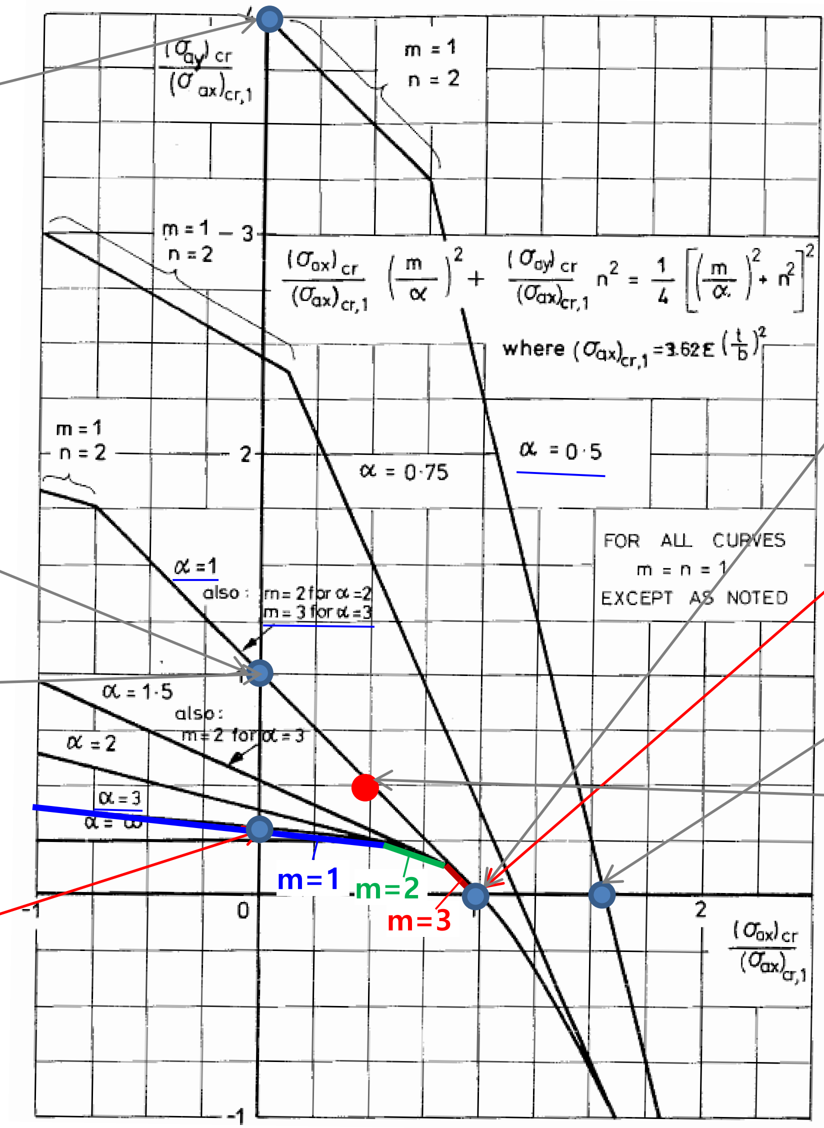
- If we denote the square plate critical stress and nondimensional form

$$(\sigma_{ax})_{cr,1} = 3.62E \left(\frac{t}{b} \right)^2 \quad ➡ \quad \left[\left(\frac{m}{\alpha} \right)^2 \frac{\sigma_{ax}}{(\sigma_{ax})_{cr,1}} + n^2 \frac{\sigma_{ay}}{(\sigma_{ax})_{cr,1}} \right]_{cr} = \frac{1}{4} \left[\left(\frac{m}{\alpha} \right)^2 + n^2 \right]^2$$

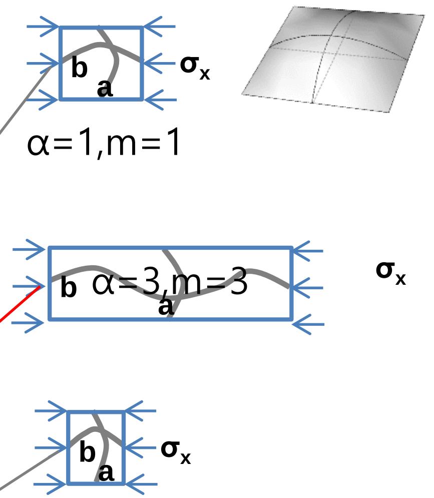
12.3 Biaxial Compression

All Edges Simply Supported





$$(\sigma_{ax})_{cr,1} = 3.62E\left(\frac{t}{b}\right)^2$$



When $\sigma_{ax} \approx \sigma_{ay}$, α=1

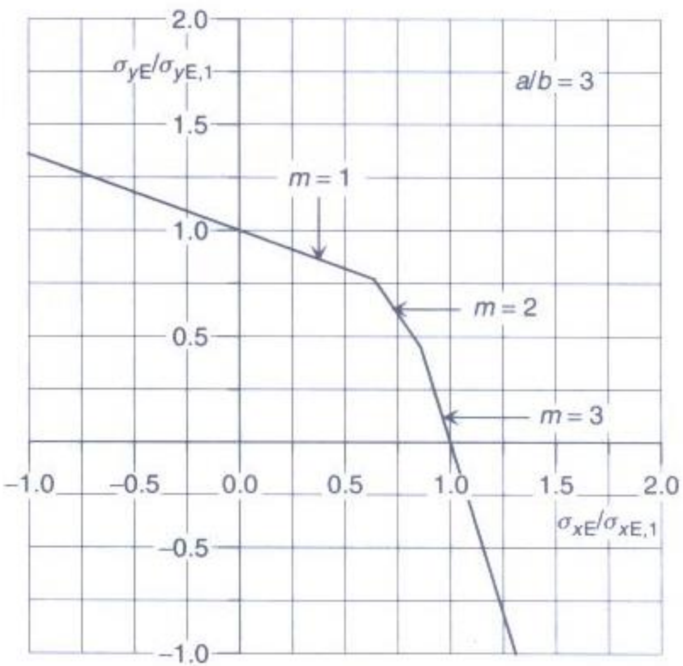
$$(\sigma_{ax} + \sigma_{ay})_{cr} = (0.5 + 0.5) \times \boxed{3.62}E\left(\frac{t}{b}\right)^2$$

$$= 3.62E\left(\frac{t}{b}\right)^2$$

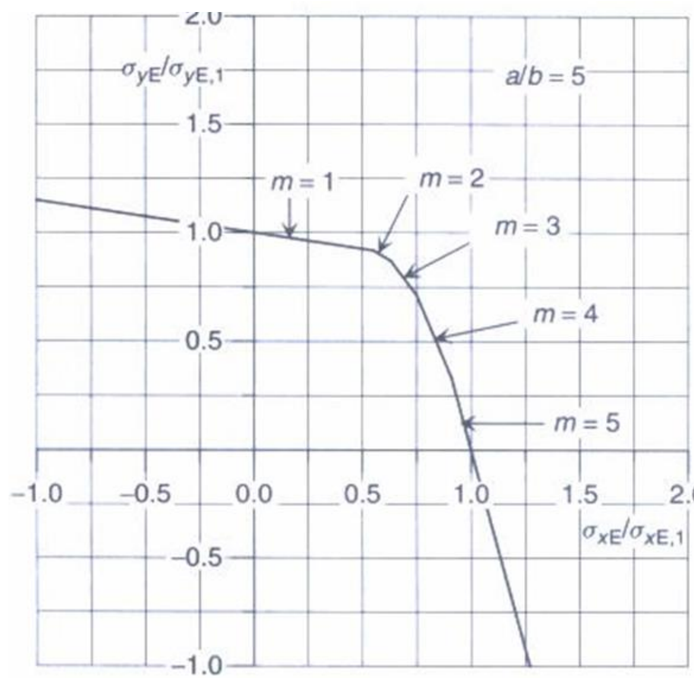
Buckling stresses of biaxially loaded simply supported plates

All Edges Simply Supported

Plate under biaxial load



a/b = 3



a/b = 5

B400 Plate panel in bi-axial compression

- For plate panels subject to bi-axial compression the interaction between the longitudinal and transverse buckling strength ratios is given by

$$\frac{\sigma_{ax}}{\eta_x \sigma_{cx}} - K \frac{\sigma_{ax}\sigma_{ay}}{\eta_x \eta_y \sigma_{cx}\sigma_{cy}} + \left(\frac{\sigma_{ay}}{\eta_y \sigma_{cy}}\right)^n \leq 1$$

σ_{ax} = compressive stress in longitudinal direction (perpendicular to stiffener spacing s)

σ_{ay} = compressive stress in transverse direction (perpendicular to the longer side l of the plate panel)

σ_{cx} = critical buckling stress in longitudinal direction as calculated in 200

σ_{cy} = critical buckling stress in transverse direction as calculated in 200

η_x, η_y = 1.0 for plate panels where the longitudinal stress σ_{al} (as given in 205) is incorporated in σ_{ax} or σ_{ay}
= 0.85 in other cases

K = c β^a

c and a are factors given in Table B1.

$$\beta = 1000 \frac{s}{t - t_k} \sqrt{\frac{\sigma_f}{E}}$$

n = factor given in Table B1.

Table B1 Values for c, a, n

	c	a	n
1.0 < l/s < 1.5	0.78	minus 0.12	1.0
1.5 ≤ l/s < 8	0.80	0.04	1.2

Homework #2 Plot DNV bi-axial interaction curve and compare with the previous interaction curve (Fig. 12.8)

All Edges Clamped

- For plates subjected to approximately equal compressive stresses ($\sigma_{ax} \approx \sigma_{ay}$) the interaction formula is

$$(\sigma_{ax} + \alpha^2 \sigma_{ay})_{cr} = 1.20E\left(\frac{t}{b}\right)^2 \left(\frac{3}{\alpha^2} + 3\alpha^2 + 2\right)$$

- When α=1, $(\sigma_{ax} + \sigma_{ay})_{cr} = \boxed{9.6}E\left(\frac{t}{b}\right)^2$

- For square plates(α=1), critical combinations are given for particular values of $\sigma_{ax} / \sigma_{ay}$, including cases in which σ_{ay} is tensile.

σ_{ay}/σ_{ax}	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	0	$-\frac{1}{4}$	$-\frac{1}{2}$	-1	$-\frac{3}{2}$	-2
σ_{ax}/σ_e	5.61	6.41	7.48	8.40	8.80	10.65	13.12	15.20	17.4	18.75	20.0
σ_{ay}/σ_e	5.61	4.81	3.74	2.80	2.20	0	−3.28	−7.60	−17.4	−28.1	−40.0

$\sigma_e = (\pi^2 D)/(b^2 t) = 0.905E(t/b)^2$.

- When $\sigma_{ax} = \sigma_{ay}$

$$(\sigma_{ax} + \sigma_{ay})_{cr} = (5.61 + 5.61) \times 0.905E\left(\frac{t}{b}\right)^2 = \boxed{10.15}E\left(\frac{t}{b}\right)^2$$

12.4 Other Types of In-plane Loads

Pure Shear

- In ship structures the plating is commonly subjected to large shear loads. The shearing load can cause buckling since it gives rise to in-plane compressive stress.
  - For the case of pure shear, in-plane compressive stress is equal to the shear stress and acts at 45° to the shear axis.

$$N_x = p = N_y = 0, N_{xy} = \tau t \quad \Rightarrow \quad \nabla^4 w = \frac{2\tau t}{D}\frac{\partial^2 w}{\partial x \partial y}$$

  - In shear buckling, the coefficients are denoted as ks and Ks.

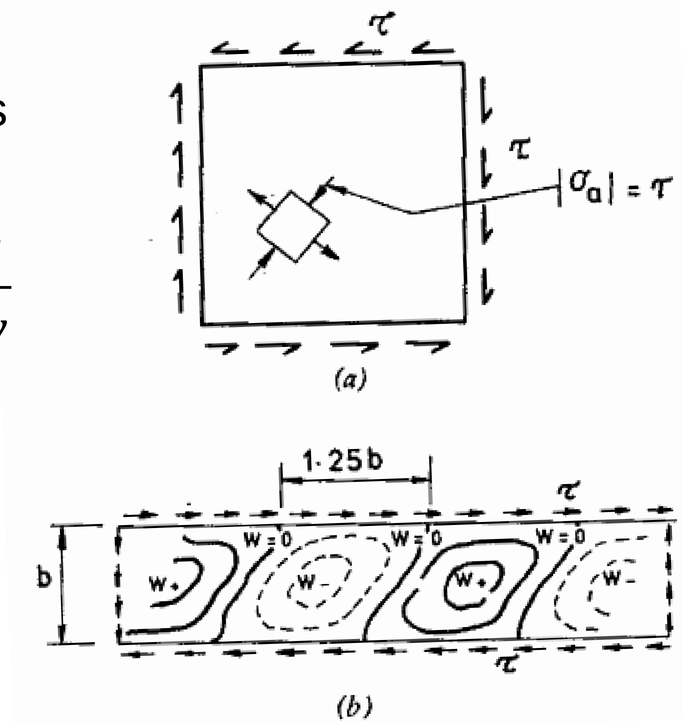
$$\tau_{cr} = k_s \frac{\pi^2 D}{b^2 t} \qquad \tau_{cr} = K_s E\left(\frac{t}{b}\right)^2$$

  - For simply supported plates

$$k_s = 5.35 + 4(b/a)^2$$

  - For clamped plates

$$k_s = 8.98 + 5.6(b/a)^2$$



Buckling of an infinitely long, simply supported plate

Pure Shear

- For simply supported plates

$$\tau_{cr} = k_s \frac{\pi^2 D}{b^2 t} \qquad k_s = 5.35 + 4(b/a)^2$$

$$\Rightarrow \quad \tau_{cr} = \frac{\pi^2}{12(1-v^2)} k_s E \left(\frac{t}{b}\right)^2 = 0.90 k_s E \left(\frac{t}{b}\right)^2$$

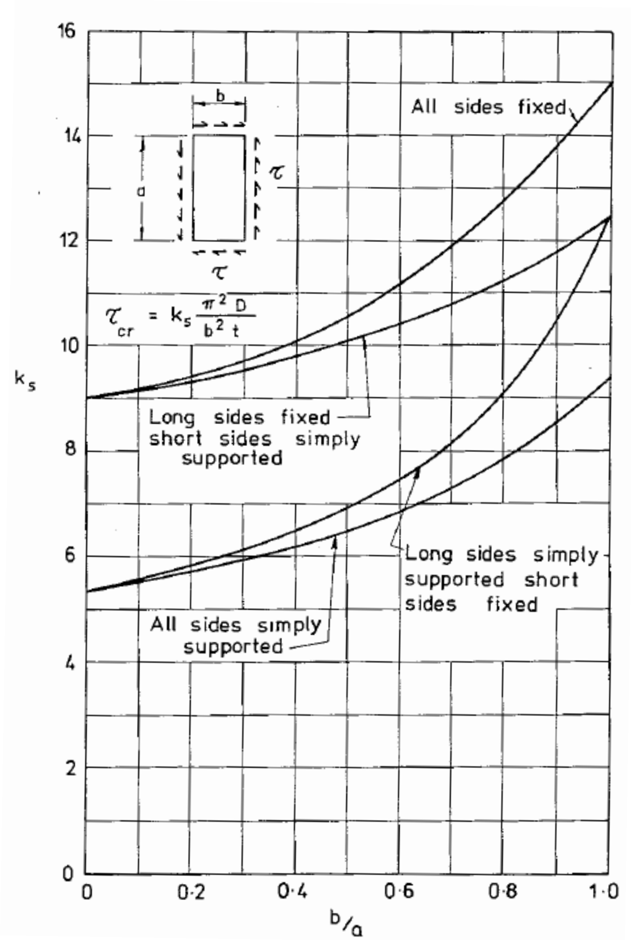
B300 Plate panel in shear

(DNV Rule for Classification of Ships Part 3 Chapter 1, Section 13)

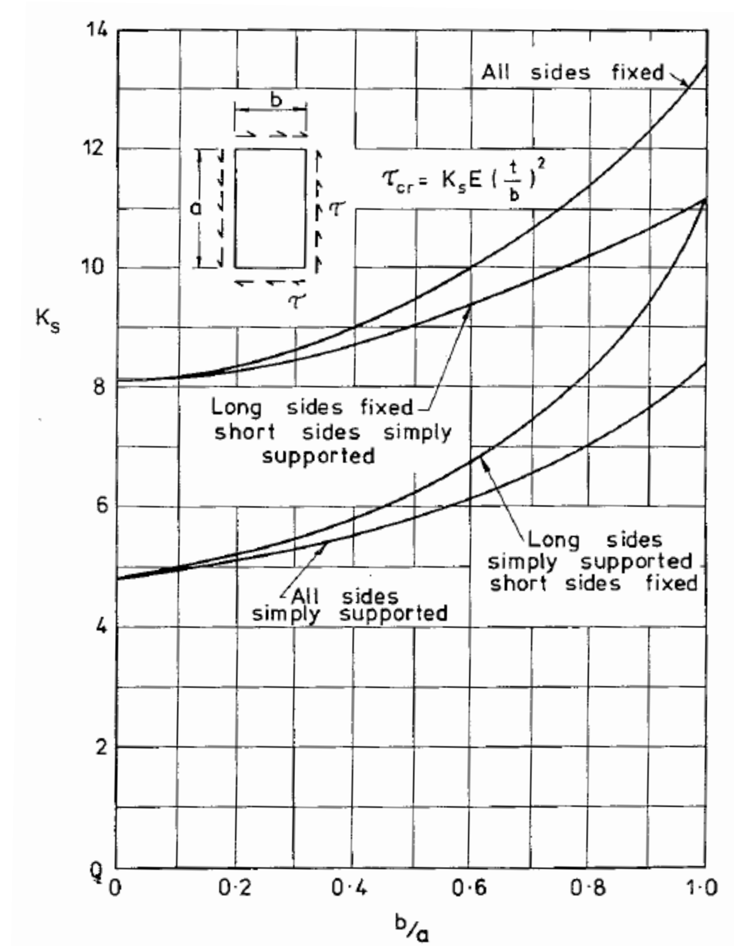
$$\tau_{el} = 0.9 k_t E \left(\frac{t}{1000\,s}\right)^2 \ (\mathrm{N/mm^2}), \ \ k_t = 5.34 + 4\left(\frac{s}{l}\right)^2$$

Pure Shear

- k_s and K_s are given for various types of boundary conditions. Because of the symmetry of the pure shear loading , the choice of a and b is independent of the load.



Buckling coefficient of flat plates in shear



Buckling coefficient of flat plates in shear (Design formula)

Biaxial Compression and Shear

- For long plates k_s is given approximately by:
  - All edges simply supported:

$$k_s = \left[2\left(1-\frac{\sigma_{ay}}{\sigma_e}\right)^{1/2} + 2 - \frac{\sigma_{ax}}{\sigma_e} \right]^{1/2} \times \left[2\left(1-\frac{\sigma_{ay}}{\sigma_e}\right)^{1/2} + 6 - \frac{\sigma_{ax}}{\sigma_e} \right]^{1/2}$$

  - All edges clamped:

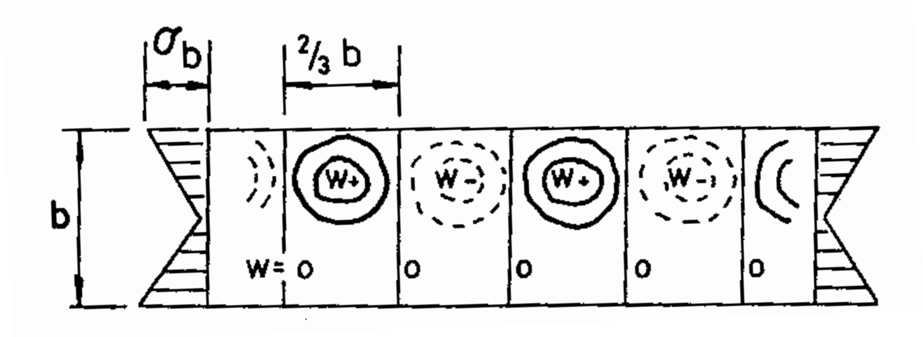
$$k_s = \left[\frac{4}{\sqrt{3}}\left(4-\frac{\sigma_{ay}}{\sigma_e}\right)^{1/2} + \frac{8}{3} - \frac{\sigma_{ax}}{\sigma_e} \right]^{1/2} \times \left[\frac{4}{\sqrt{3}}\left(4-\frac{\sigma_{ay}}{\sigma_e}\right)^{1/2} + 8 - \frac{\sigma_{ax}}{\sigma_e} \right]^{1/2}$$

where $\sigma_e = \dfrac{\pi^2 D}{b^2 t}$

In-plane Bending

- σ_b denotes the largest or edge value of the applied stress.

$$(\sigma_b)_{cr} = k_b \frac{\pi^2 D}{b^2 t}$$



- Some approximate formulas to calculate the values of k_b
  - simply supported edges:

    for $a/b \le 2/3$ $\quad k_b = 15.87 + 1.87(b/a)^2 + 8.6(a/b)^2$

    for $a/b > 2/3$ $\quad k_b = 23.9$
  - clamped edges:

    for $a/b \ge 1$ $\quad k_b = 41.8$
  - one unloaded edge clamped; the others simply supported

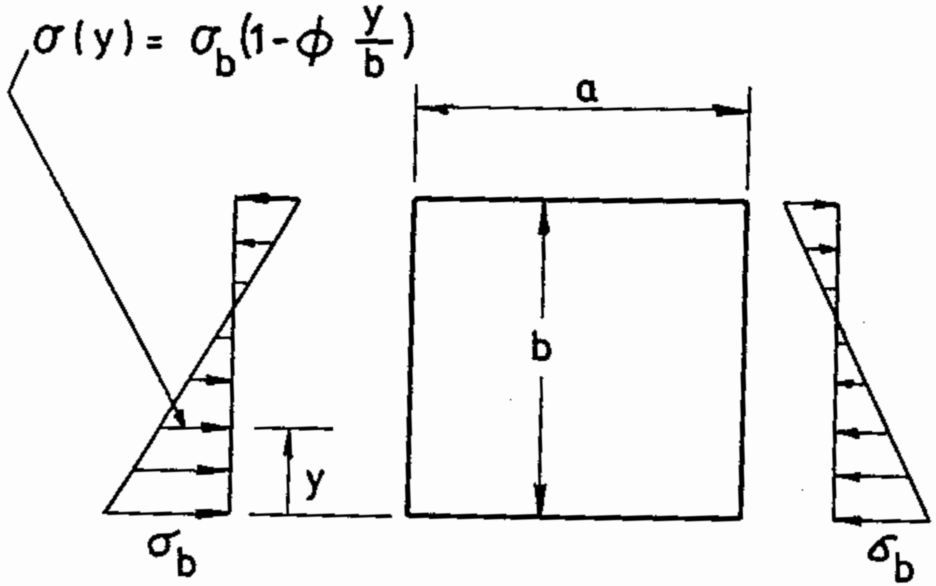
    for $a/b \ge 1/2$ $\quad k_b = 25$
  - unloaded edges clamped; loaded edges simply supported

    for $a/b \ge 0.4$ $\quad k_b = 40$

In-plane Bending

- The figure illustrates the case in which the bending is unsymmetric. For simply supported edges the value of k_b is given approximately by

$$k_b = 5\phi^2 + 4 \qquad \left(\frac{a}{b} > \frac{2}{3} \text{ simply supported edges only} \right)$$



φ = 0 for pure compression

φ = 2 for pure bending

Combined In-plane Loads: Interaction Formulas

❖ Uniaxial compression and in-plane bending

- $(\sigma_a)_{cr}$: critical values of axial loading
- $(\sigma_b)_{cr}$: critical values of and in-plane bend

$$\left(\frac{\sigma_a}{(\sigma_a)_{cr}}\right)+\left(\frac{\sigma_b}{(\sigma_b)_{cr}}\right)^{1.75}=1$$

❖ Uniaxial load(compressive or tensile) and shear

- For convenience we adopt the symbol R to denote a critical load ratio. In the present case the strength ratios are

$$R_c=\frac{\sigma_a}{(\sigma_a)_{cr}} \qquad R_s=\frac{\tau}{\tau_{cr}}$$

- The interaction formula is

$$R_c+R_s^2=1 \qquad \alpha\geq 1$$

$$\left(\frac{1+0.6\alpha}{1.6}\right)R_c+R_s^2=1 \qquad \alpha<1$$

❖ In-plane bending and shear

$$R_b=\frac{\sigma_b}{(\sigma_b)_{cr}} \qquad R_b^2+R_s^2=1 \;\; (\alpha>1/2)$$

Combined In-plane Loads: Interaction Formulas

❖ Biaxial compression, in-plane bending, and shear

- The two compression strength ratios are

$$R_x = \frac{\sigma_{ax}}{(\sigma_{ax})_{cr}} \qquad R_y = \frac{\sigma_{ay}}{(\sigma_{ay})_{cr}}$$

- By performing a series of four-variable curve-fitting solutions,

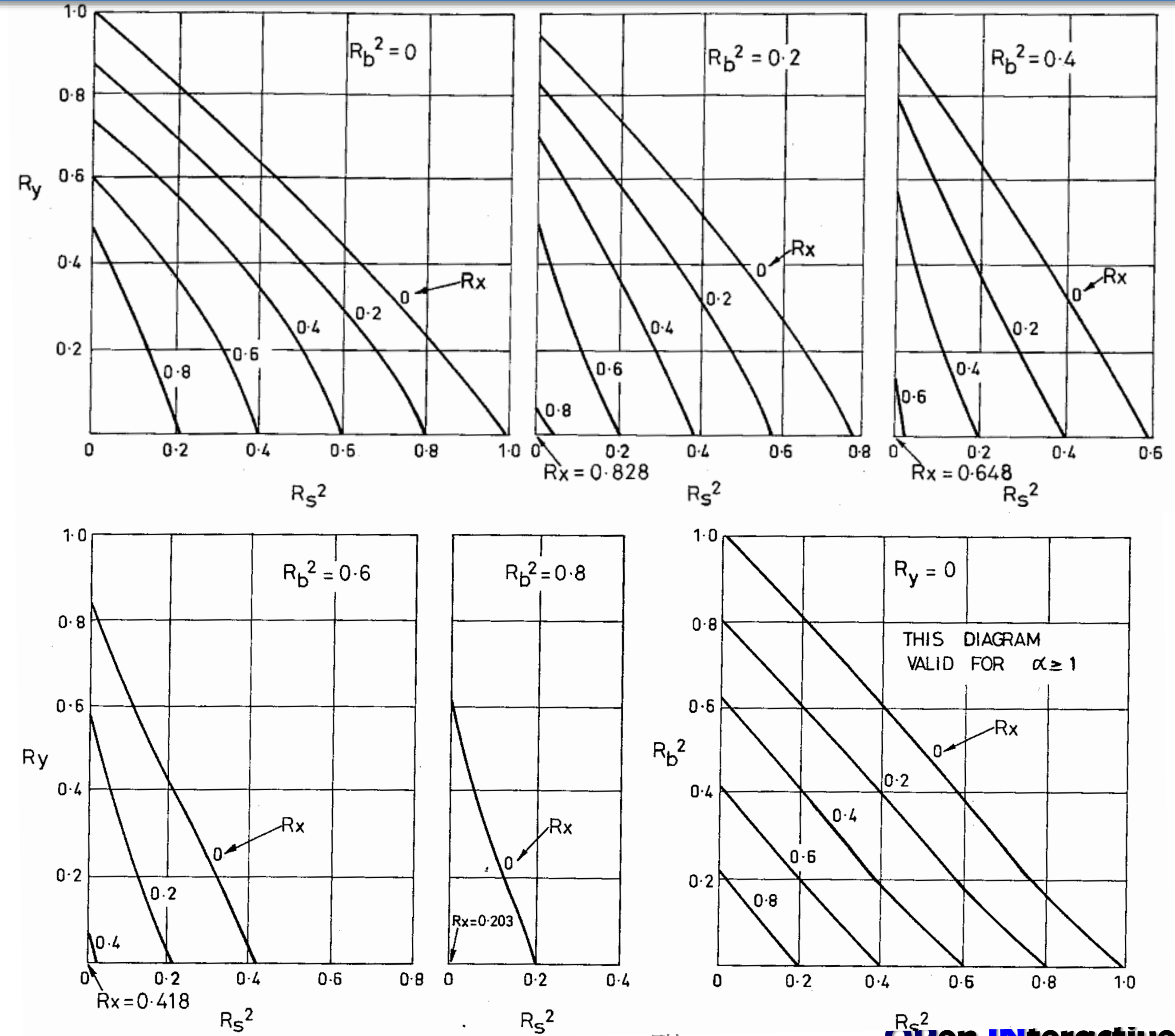
$$\frac{0.625(1+0.6/\alpha)R_y}{(1-0.625R_x)\left[1-\frac{R_b^4}{(1-R_x)^2}\right]} + \frac{R_s^2}{1-R_x} = 1 \qquad \alpha \geq 1$$

B500 Plate panel in bi-axial compression and shear
(DNV Rule for Classification of Ships Part 3 Chapter 1, Section 13)

$$\frac{\sigma_{ax}}{\eta_x \sigma_{cx} q} - K\frac{\sigma_{ax}\sigma_{ay}}{\eta_x \eta_y \sigma_{cx}\sigma_{cy} q} + \left(\frac{\sigma_{ay}}{\eta_y \sigma_{cy} q}\right)^n \leq 1 \qquad q = 1 - \left(\frac{\tau_a}{\tau_a}\right)^2$$

Combined In-plane Loads: Interaction Formulas

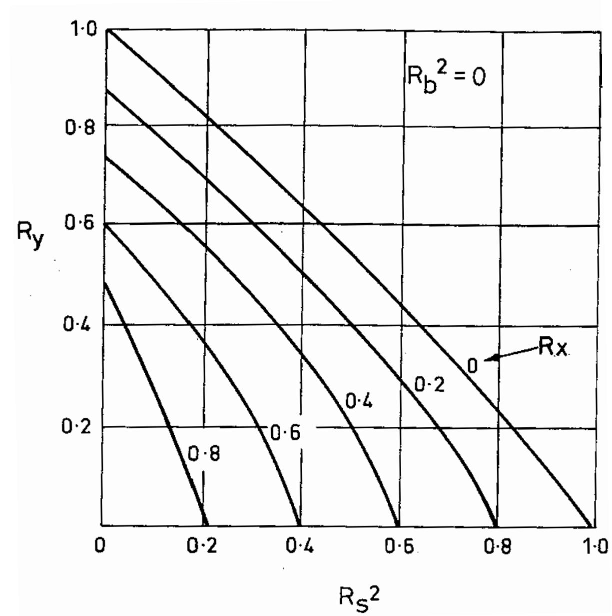
Interaction curves for biaxial compression, in-plane bending, and shear drawn for α=2



Combined In-plane Loads: Interaction Formulas

Homework #3 Plot DNV bi-axial interaction curve like the right figure and compare with the following curve for R_b=0

$$\frac{0.625(1+0.6/\alpha)R_y}{(1-0.625R_x)\left[1-\frac{R_b^4}{(1-R_x)^2}\right]}+\frac{R_s^2}{1-R_x}=1 \qquad \alpha \geq 1$$



B500 Plate panel in bi-axial compression and shear
(DNV Rule for Classification of Ships Part 3 Chapter 1, Section 13)

$$\frac{\sigma_{ax}}{\eta_x \sigma_{cx} q} - K\frac{\sigma_{ax}\sigma_{ay}}{\eta_x\eta_y\sigma_{cx}\sigma_{cy}q} + \left(\frac{\sigma_{ay}}{\eta_y\sigma_{cy}q}\right)^n \leq 1 \qquad q = 1-\left(\frac{\tau_a}{\tau_a}\right)^2$$

Plates Without Residual Stress

- Uniaxially loaded, simply supported square plate, with sides free to pull in. some typical initial distortion in the form of a half wave in each direction.
- Plate slenderness $\beta = \frac{b}{t}\sqrt{\frac{\sigma_Y}{E}}$
- The relationship between the applied load (σ_a) and the axial shortening

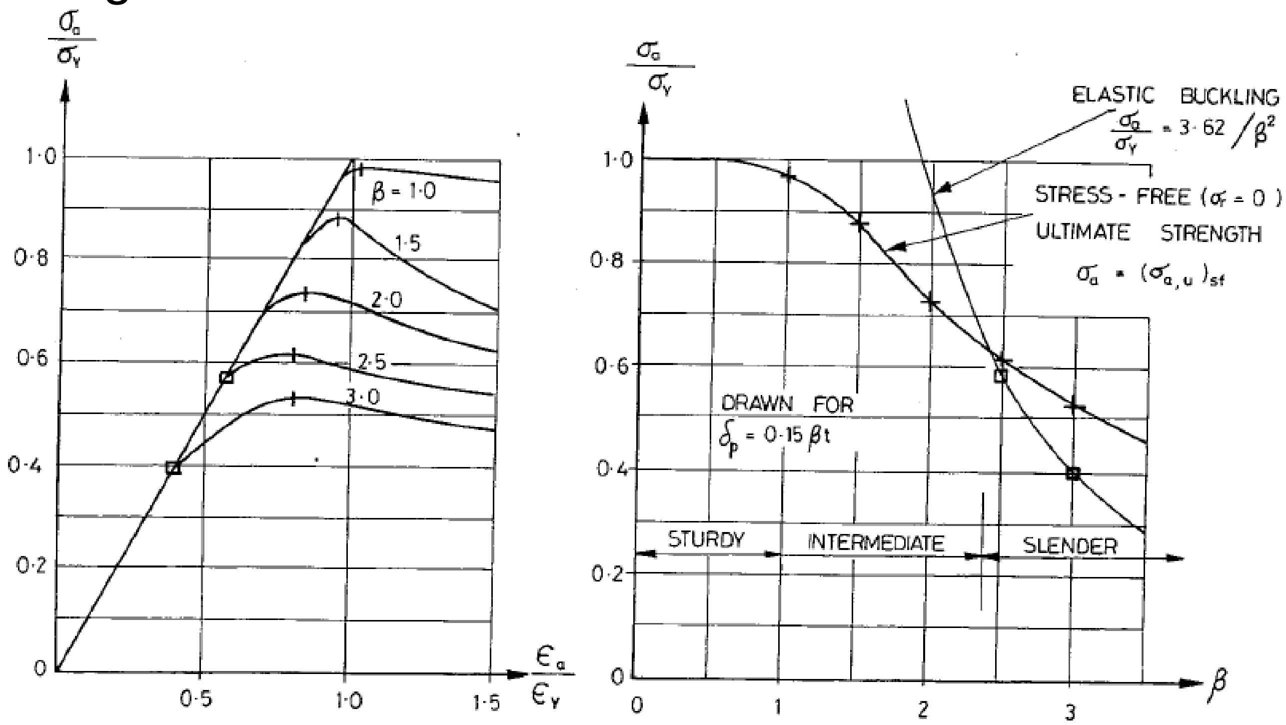


Plate strength without welding (σ_r=0)

Plates Without Residual Stress

❖ Slender plate (β>2.4)

- Buckling stress is well below yield stress and below the curve of collapse stress.
- After buckling (σ_a) a greater proportion of the load is taken by the region of plating near the sides → Non-uniform compressive stress distribution
- Deflected shape of the buckled portion → overall stiffness of the plate ($d\sigma_a/d\varepsilon_a$) is reduced.
- The center region becomes more pronounced and the maximum stress at the sides increases. When the maximum stress = yield stress → collapse.

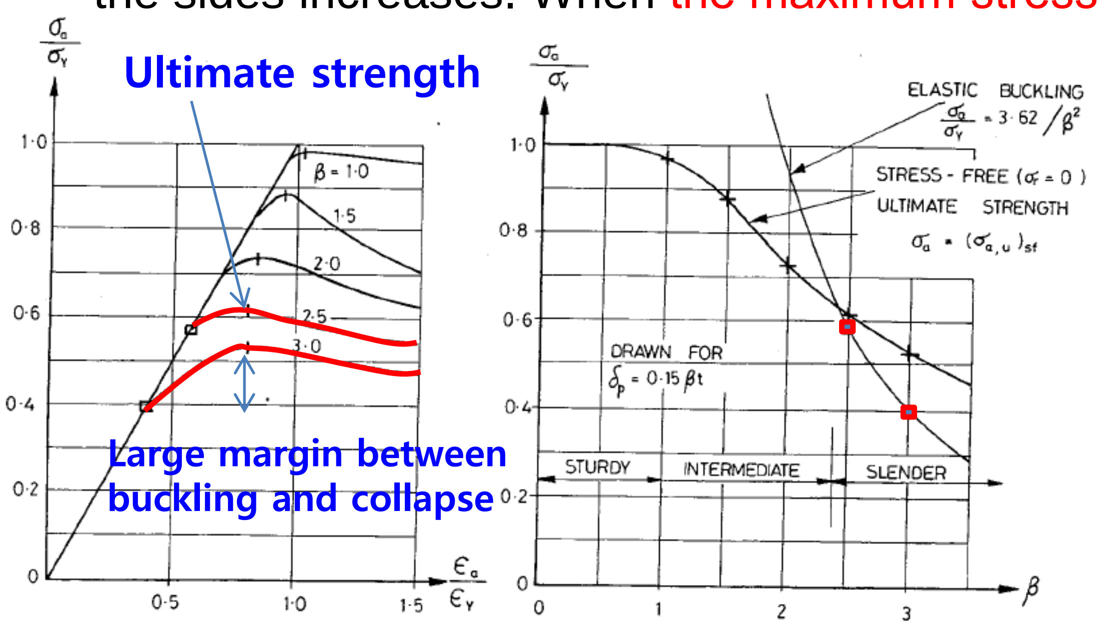
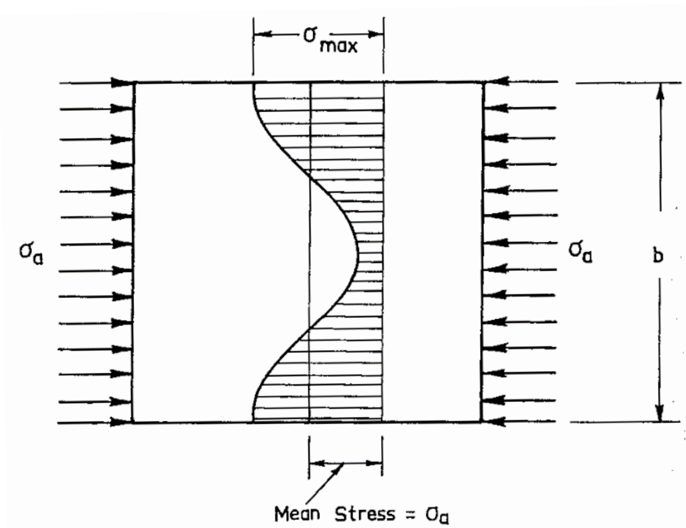


Plate strength without welding (σ_r=0)

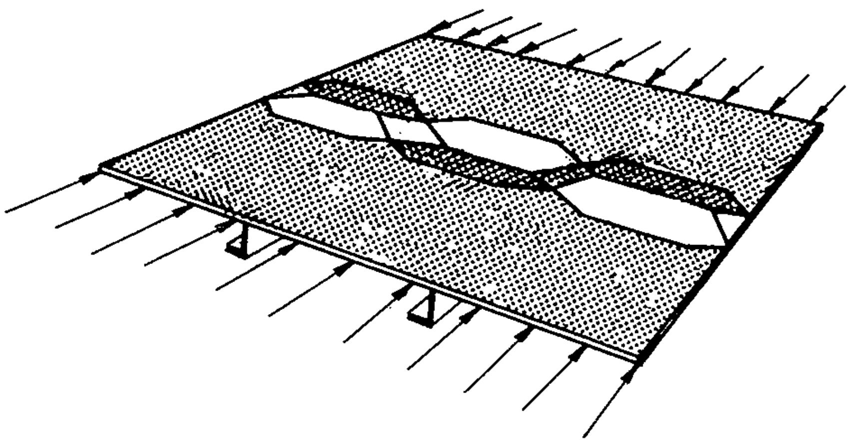


Post-buckling stress distribution

Plates Without Residual Stress

❖ **Plates of intermediate slenderness (1<β<2.4)**

- Buckling stress ≈ yield stress
- For a rigorous analysis, elasto-plastic large deflection theory to be used.
- As applied stress increases → magnification of the initial distortion → loss of stiffness → some local yield → stress redistribution → yielding of the side → sudden collapse.
- Pitched roof : allows large axial shortening with minimum strain energy.



Typical post buckling behavior

Plates Without Residual Stress

❖ Sturdy plates (1>β)

- The initial distortion is smaller and the magnification is less because the elastic buckling stress is very large.
- Plates can carry a load equal to the full "squash load" $\sigma_{a,u} = \sigma_Y$.
- After the peak load, the load carrying capacity remains approximately constant up to very large strains.

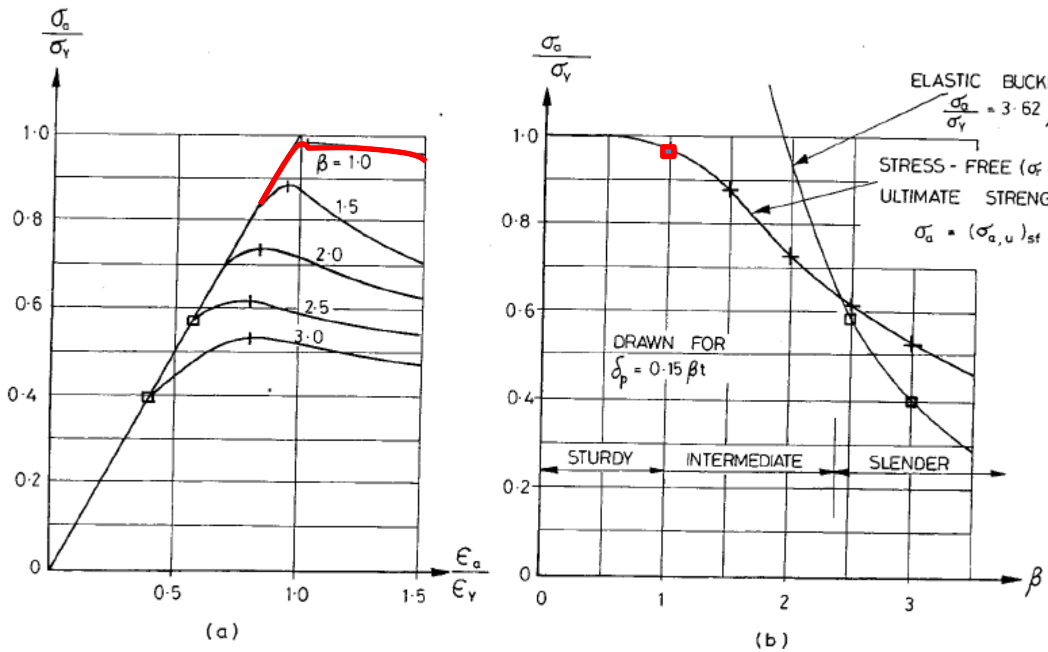
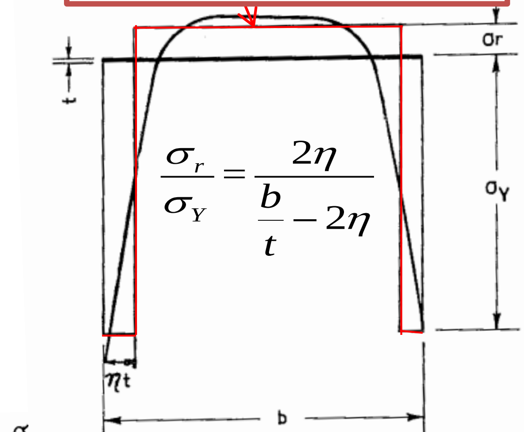


Plate strength without welding (σ_r=0)

Plates With Residual Stress

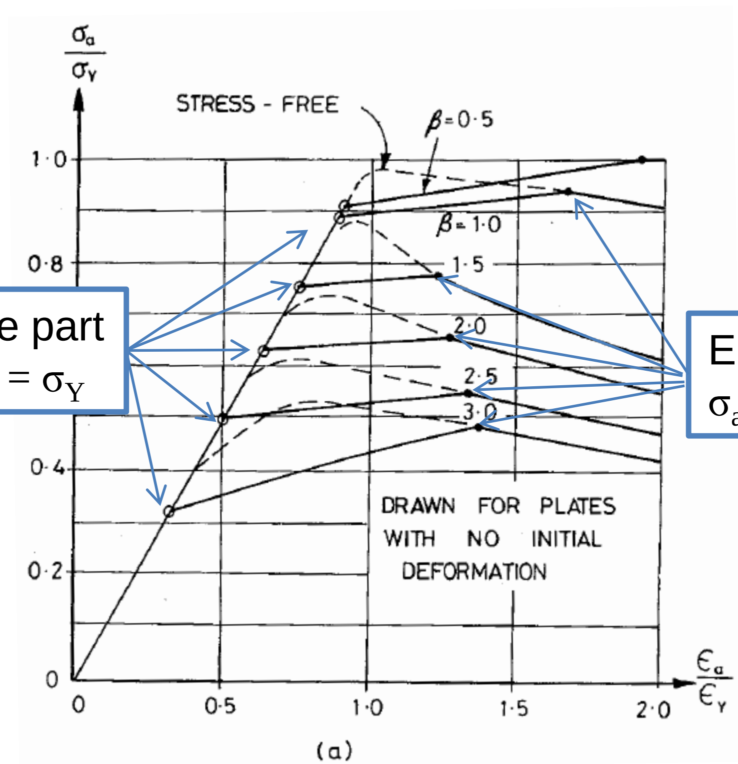
- Departure from linearity occur at the stress which is less σ_a less than for a stress-free plate.
- Sturdy plate ($1>\beta$) : no load shedding, but large reduction in stiffness → regarded collapse.
- Intermediately slender and slender plate ($1<\beta$) : the loss of ultimate strength ≈ σ_r

Idealized residual stress distribution



Middle part

$\sigma_a + \sigma_r = \sigma_Y$



Edge part

$\sigma_a - \sigma_Y = \sigma_Y$

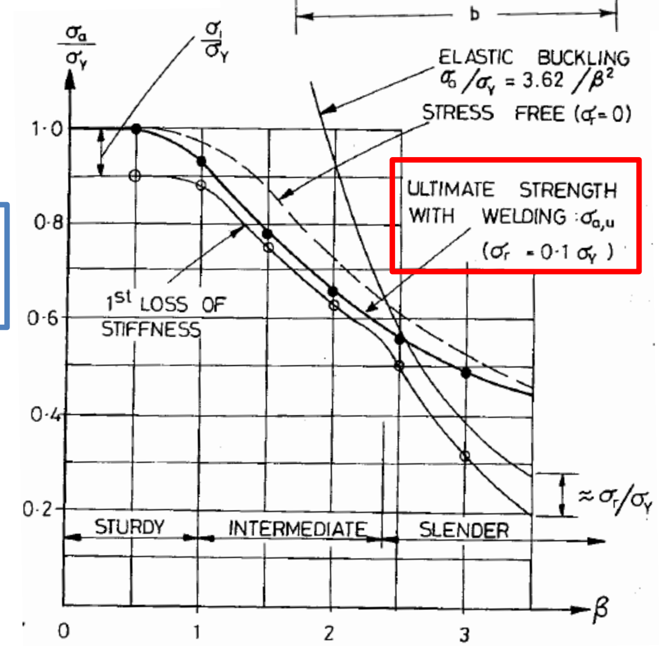


Plate strength with welding (σ_r=0.1σ_Y)

Effects of Other Parameters

❖ Restraint at Sides

- Clamping the sides of a plate increase the elastic buckling stress by 75%, however, the increase in buckling stress even in slender plate ≈ 10% at most.
- Stiffeners surrounding the panel is not clamped edge → this restraint can be ignored.

❖ Initial Deformation

- The effect of initial deformation removes sharp knuckle in curve of σ_a and ε_a. The increasing lateral deflection causes a progressive reduction in the in-plane stiffness of the plate.
- However, the ultimate strength is slightly decreased.

❖ Shear stress

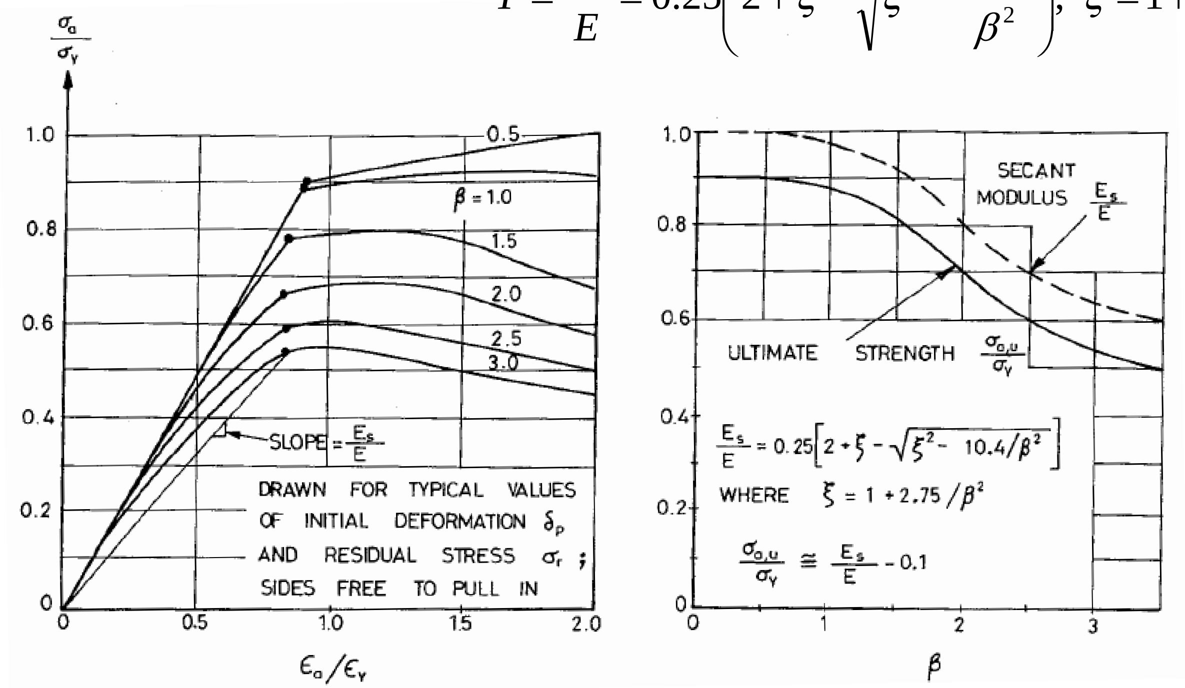
- In –plane shear stress tends to lower the resistance to longitudinal compression.
- Reduced yield stress $r_\tau \sigma_Y$

$$r_\tau = \sqrt{1 - 3\left(\frac{\tau}{\sigma_Y}\right)^2} \Leftarrow \boxed{\sigma_{eq} = \sqrt{\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2} = \sigma_Y}$$

Ultimate Strength of Uniaxial Loaded Plates

- Plating of uniaxially loaded, longitudinally stiffened,
  initial deformation ($\delta_p < 0.2\beta t$), residual stress ($\sigma_r \approx 0.1\sigma_Y$)
  side constrained to remain straight but free to pull in
- For sturdy plate, first loss of stiffness is taken as collapse.
- For plates of greater slenderness : loss of stiffness is gradual.
- Secant modulus ratio

$$T = \frac{E_s}{E} = 0.25\left(2 + \xi - \sqrt{\xi^2 - \frac{10.4}{\beta^2}}\right), \quad \xi = 1 + \frac{2.75}{\beta^2}$$



Design curves of ultimate strength and secant modulus

Ultimate Strength of Uniaxial Loaded Plates

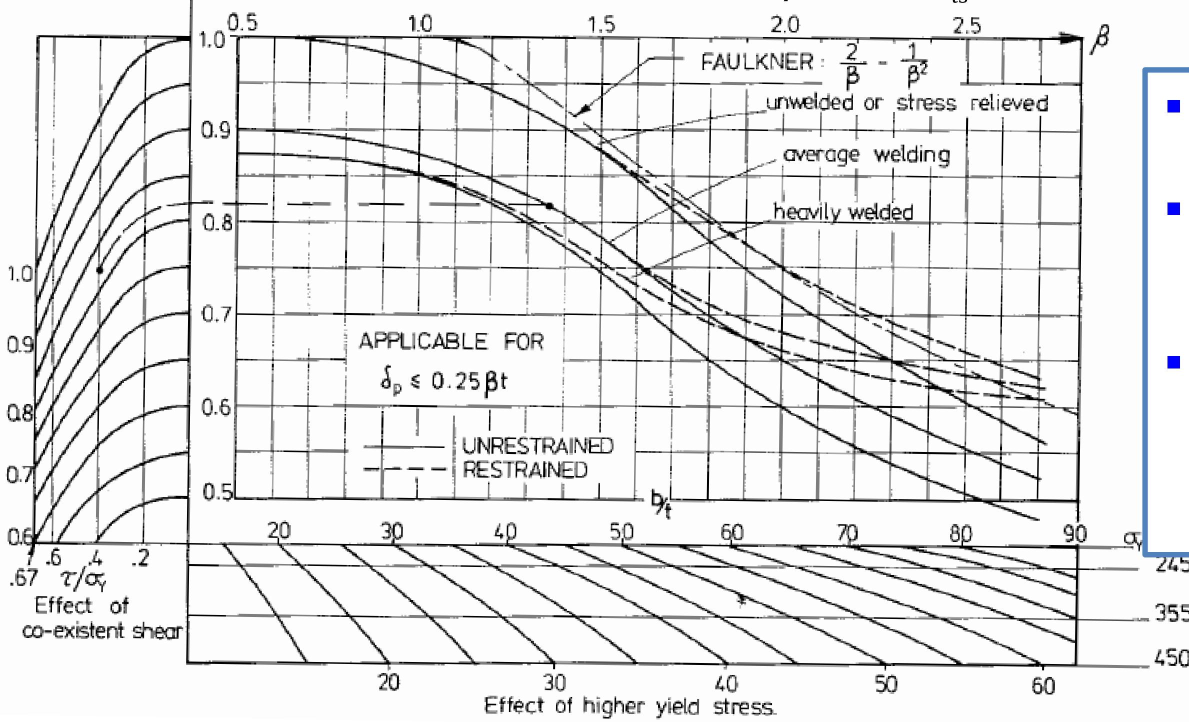
- Faulkner's formula for the ultimate strength of unwedded plates : good agreement with extensive experimental data.

$$\frac{\sigma_{a,u}}{\sigma_Y} = \frac{2}{\beta} - \frac{1}{\beta^2}$$

- The effect of residual stress → strength reduction factor R_r

$$R_r = 1 - \frac{\sigma_r E_{ts}}{\sigma_Y E} \qquad \frac{E_{ts}}{E} = \frac{2\beta - 1}{\beta} \ (1 < \beta < 2.5)$$

$$\text{for } \beta > 2.5 \ E_{ts} \approx E \ \text{ and for } \beta < 1.0 \ E_{ts} \approx 0$$



- Restrained : the sides remain straight and do not pull in.
- Unrestrained : both types of transverse deformations can occur.
- Stress relived (σ_r=0), average welding ($\sigma_r \le 0.1\sigma_r$), heavily welded ($\sigma_r \le 0.33\sigma_r$)

Curves for ultimate strength of plates