재료의 기계적 거동
(Mechanical Behavior of Materials)

Lecture 5 – Overview of Mechanical Behavior
(Concept of strain, yielding and fracture)

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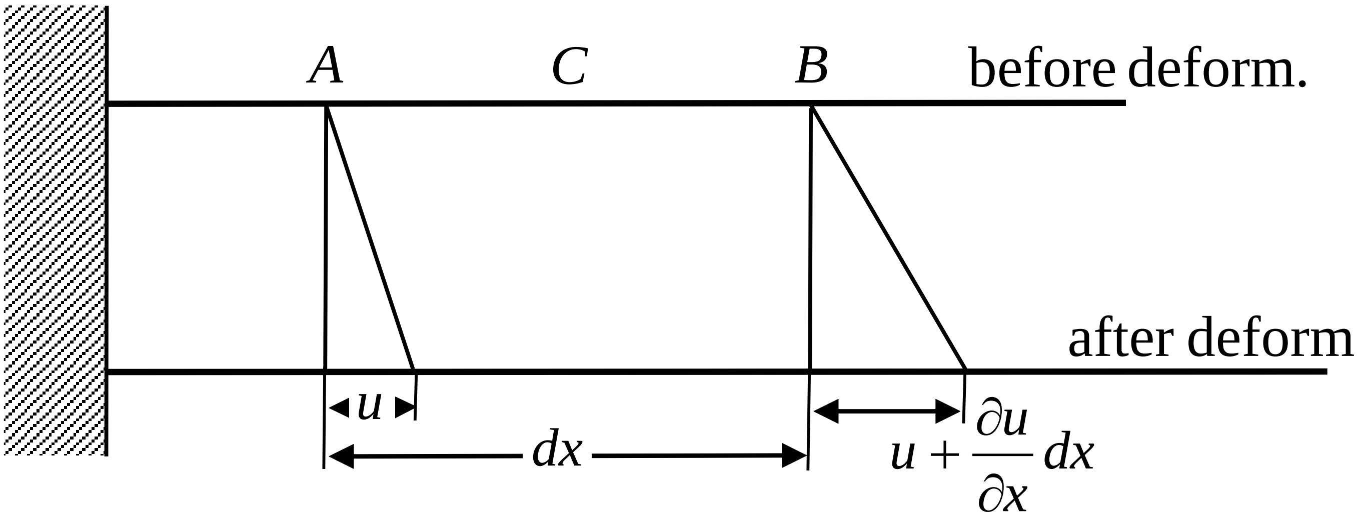
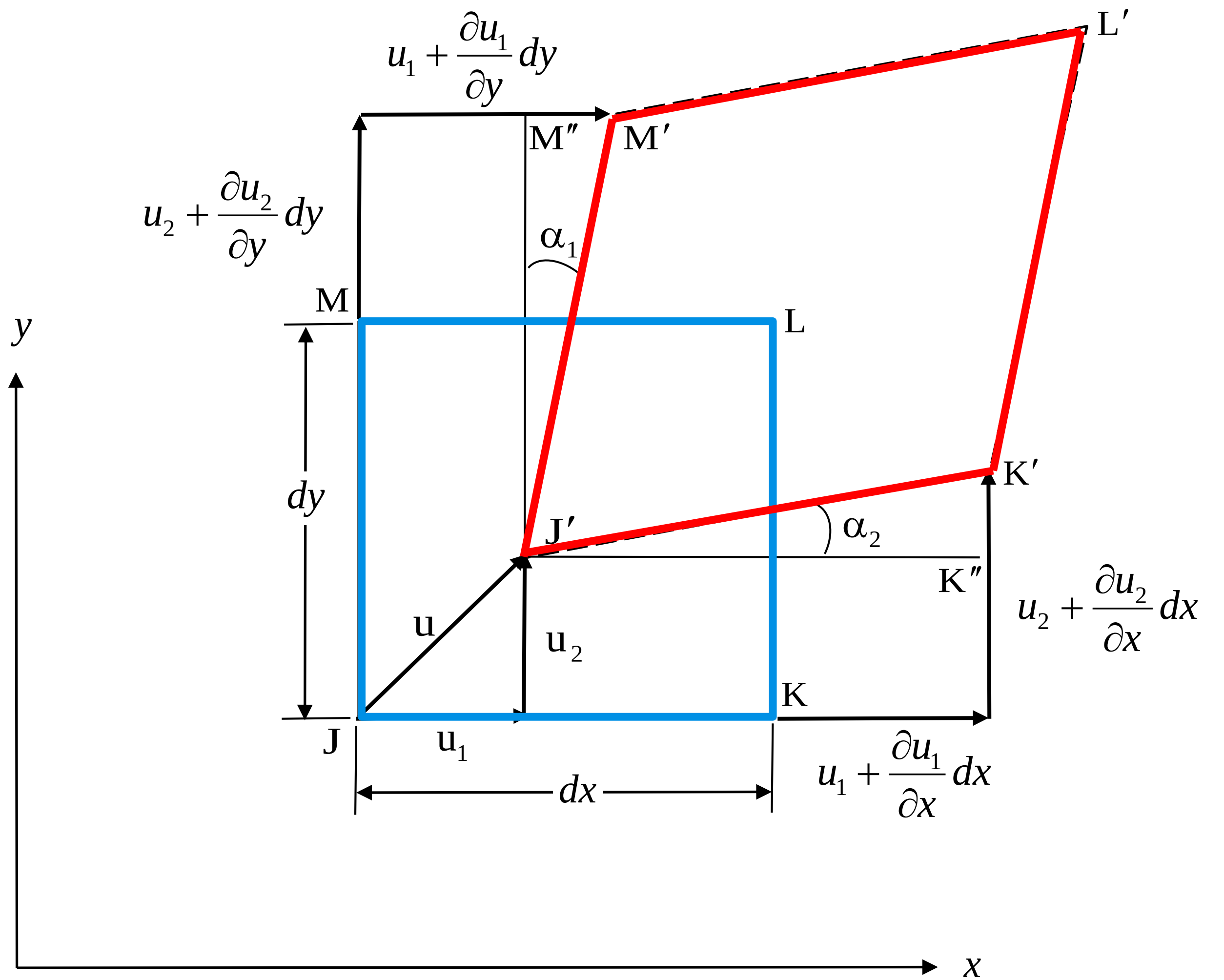
Concept of strain



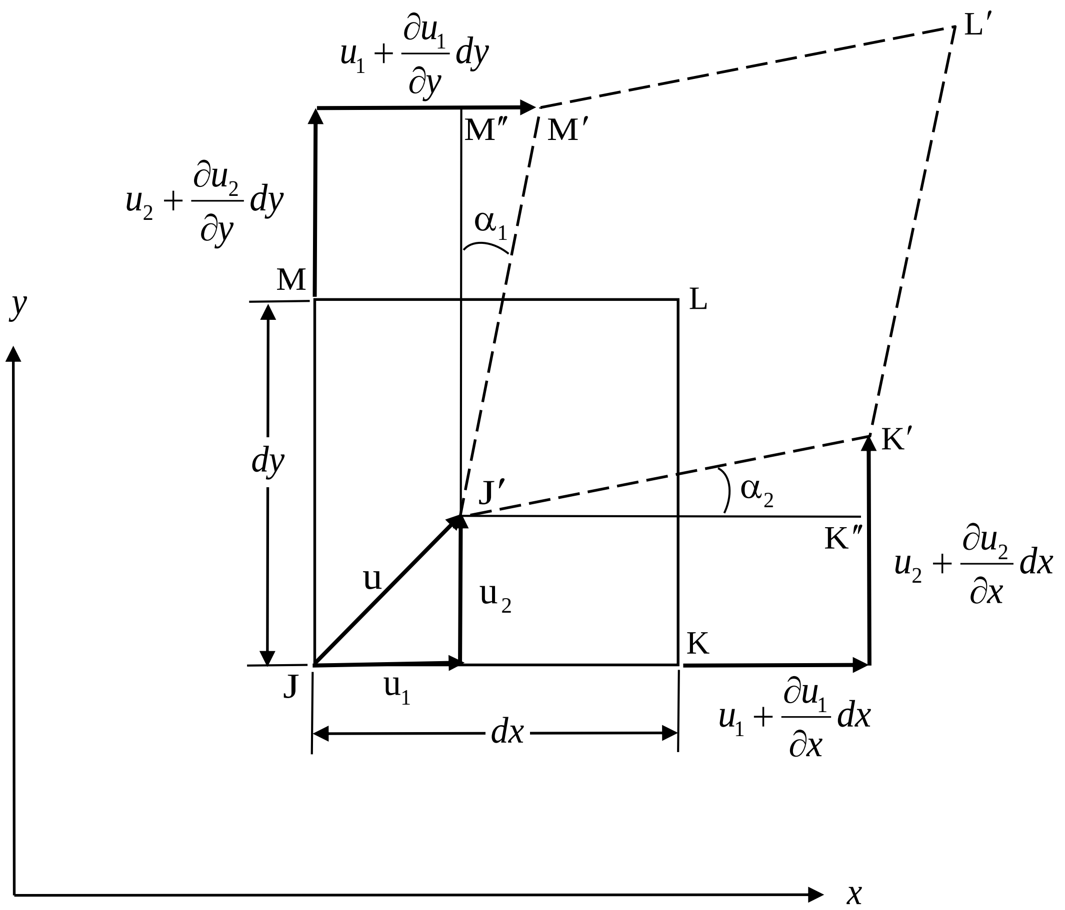
$$\frac{\partial u}{\partial x}$$: 1-dimensional strain

Concept of strain



2-dimensional strain

Concept of strain



x-directional normal

$$e_{11} = \frac{dx + \frac{\partial u_1}{\partial x} dx - dx}{dx} = \frac{\partial u_1}{\partial x}$$

y-directional normal

$$e_{22} = \frac{dy + \frac{\partial u_2}{\partial y} dy - dy}{dy} = \frac{\partial u_2}{\partial y}$$

Shear

$$\gamma_{12} = \alpha_1 + \alpha_2 \cong \frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x}$$

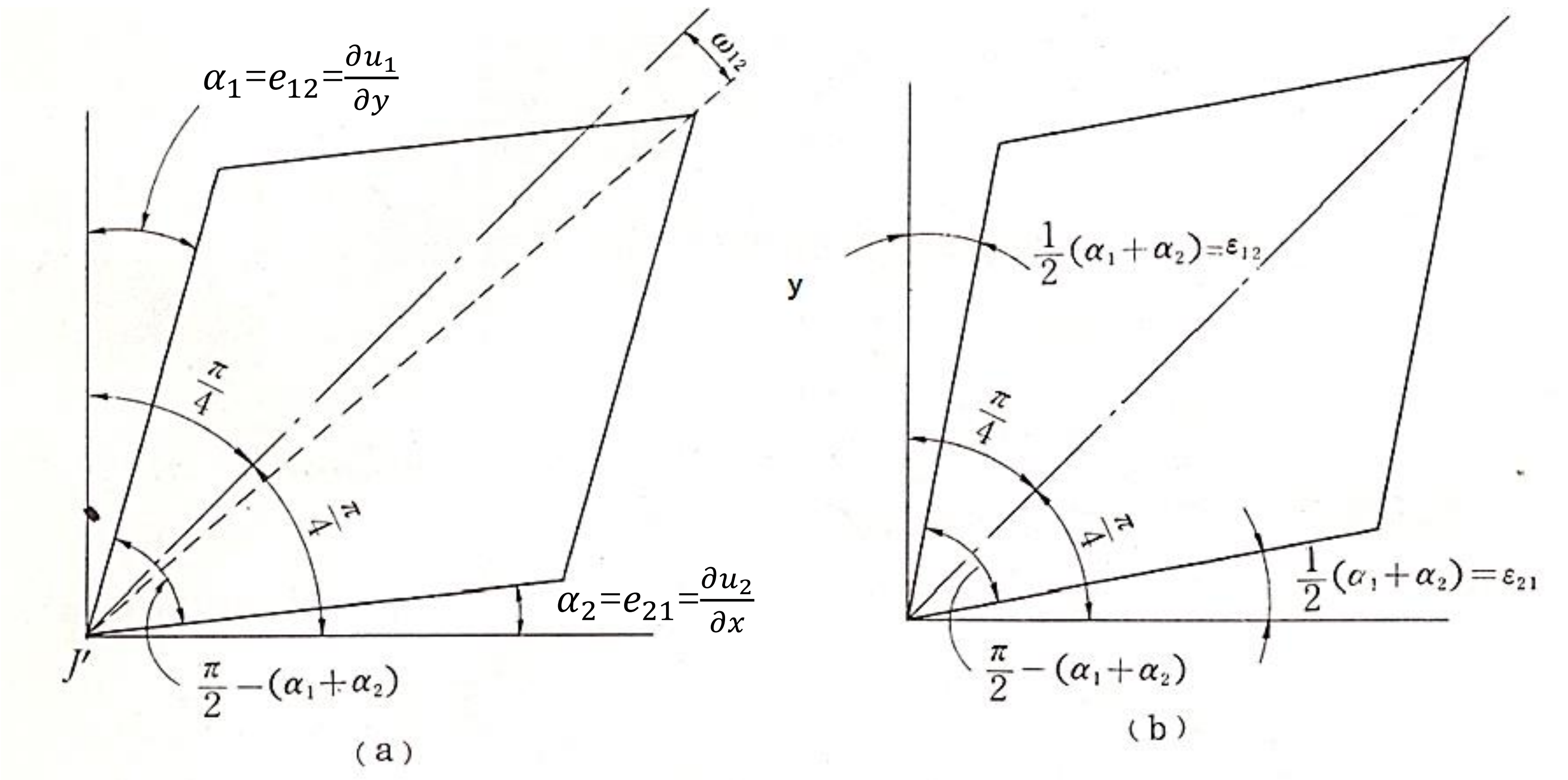
2-dimensional strain

Concept of strain

3-dimensional strain

$$e_{ij}=\begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix}=\begin{bmatrix} \dfrac{\partial u_1}{\partial x} & \dfrac{\partial u_1}{\partial y} & \dfrac{\partial u_1}{\partial z} \\ \dfrac{\partial u_2}{\partial x} & \dfrac{\partial u_2}{\partial y} & \dfrac{\partial u_2}{\partial z} \\ \dfrac{\partial u_3}{\partial x} & \dfrac{\partial u_3}{\partial y} & \dfrac{\partial u_3}{\partial z} \end{bmatrix}$$

Concept of strain



(a) → (b)

Rigid body rotation by ω_{12}

Concept of strain

Strain tensor

$$\varepsilon_{ij} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\partial u_1}{\partial x} & \dfrac{1}{2}\left(\dfrac{\partial u_1}{\partial y} + \dfrac{\partial u_2}{\partial x}\right) & \dfrac{1}{2}\left(\dfrac{\partial u_1}{\partial z} + \dfrac{\partial u_3}{\partial x}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial u_1}{\partial y} + \dfrac{\partial u_2}{\partial x}\right) & \dfrac{\partial u_2}{\partial y} & \dfrac{1}{2}\left(\dfrac{\partial u_2}{\partial z} + \dfrac{\partial u_3}{\partial y}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial u_1}{\partial z} + \dfrac{\partial u_3}{\partial x}\right) & \dfrac{1}{2}\left(\dfrac{\partial u_2}{\partial z} + \dfrac{\partial u_3}{\partial y}\right) & \dfrac{\partial u_3}{\partial z} \end{bmatrix}$$

Spin tensor

$$\varpi_{ij} = \begin{bmatrix} \varpi_{11} & \varpi_{12} & \varpi_{13} \\ \varpi_{21} & \varpi_{22} & \varpi_{23} \\ \varpi_{31} & \varpi_{32} & \varpi_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \dfrac{1}{2}\left(\dfrac{\partial u_1}{\partial y} - \dfrac{\partial u_2}{\partial x}\right) & \dfrac{1}{2}\left(\dfrac{\partial u_1}{\partial z} - \dfrac{\partial u_3}{\partial x}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial u_2}{\partial x} - \dfrac{\partial u_1}{\partial y}\right) & 0 & \dfrac{1}{2}\left(\dfrac{\partial u_2}{\partial z} - \dfrac{\partial u_3}{\partial y}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial u_3}{\partial x} - \dfrac{\partial u_1}{\partial z}\right) & \dfrac{1}{2}\left(\dfrac{\partial u_3}{\partial y} - \dfrac{\partial u_2}{\partial z}\right) & 0 \end{bmatrix}$$

Concept of strain

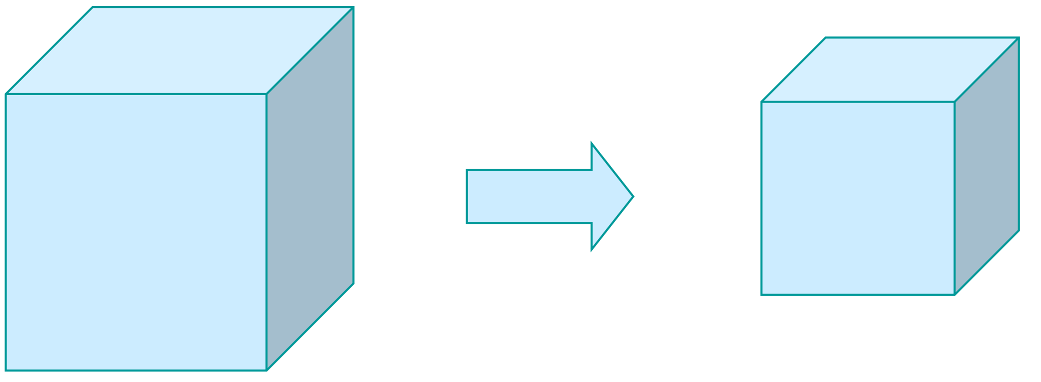
$$e'_{ij} = a_{ki} a_{lj} e_{kl}$$

$$\varepsilon'_{ij} = a_{ki} a_{lj} \varepsilon_{kl}$$

$$\omega'_{ij} = a_{ki} a_{lj} \omega_{kl}$$

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{12} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{13} & \varepsilon_{23} & \varepsilon_{33} \end{bmatrix} \Rightarrow \boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{\mathrm{I}} & 0 & 0 \\ 0 & \varepsilon_{\mathrm{II}} & 0 \\ 0 & 0 & \varepsilon_{\mathrm{III}} \end{bmatrix}$$

Volumetric Strain



Initial length of side :
dx, dy, dz

Finial length of side :
$(1+\varepsilon_{11})dx$, $(1+\varepsilon_{22})dy$, $(1+\varepsilon_{33})dz$

$$\varepsilon_V = \frac{\Delta V}{V_0} = \frac{V - V_0}{V_0} = \frac{(1+\varepsilon_{11})(1+\varepsilon_{22})(1+\varepsilon_{33})dxdydz - dxdydz}{dxdydz}$$

$$\varepsilon_V \cong \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = 3\varepsilon_m$$

Deviatoric strain

Mean strain :

$$\varepsilon_m = \frac{\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}}{3} = \frac{\varepsilon_I + \varepsilon_{II} + \varepsilon_{III}}{3} = \frac{\varepsilon_{ii}}{3} = \frac{\varepsilon_V}{3}$$

Deviatoric strain :

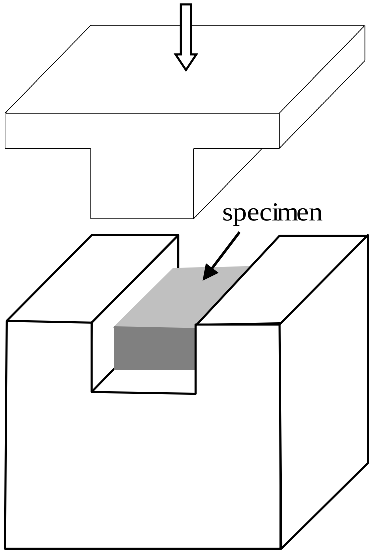
$$\varepsilon'_{ij} = \begin{bmatrix} \varepsilon'_{11} & \varepsilon'_{12} & \varepsilon'_{13} \\ \varepsilon'_{12} & \varepsilon'_{22} & \varepsilon'_{23} \\ \varepsilon'_{13} & \varepsilon'_{23} & \varepsilon'_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{12} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{13} & \varepsilon_{23} & \varepsilon_{33} \end{bmatrix} - \begin{bmatrix} \varepsilon_m & 0 & 0 \\ 0 & \varepsilon_m & 0 \\ 0 & 0 & \varepsilon_m \end{bmatrix}$$

$$\varepsilon'_{ij} = \varepsilon_{ij} - \varepsilon_m \delta_{ij}$$

Special Types of Strain

Plane strain



Pure shear

$$\begin{bmatrix} 0 & 0 & \varepsilon_1 \\ 0 & 0 & 0 \\ \varepsilon_1 & 0 & 0 \end{bmatrix}$$

Strain compatibility

General condition

$$\frac{\partial^2 \varepsilon_{11}}{\partial y \partial z} = \frac{\partial}{\partial x}\left(-\frac{\partial \varepsilon_{23}}{\partial x} + \frac{\partial \varepsilon_{31}}{\partial y} + \frac{\partial \varepsilon_{12}}{\partial z} \right)$$

$$\frac{\partial^2 \varepsilon_{22}}{\partial z \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial \varepsilon_{23}}{\partial x} - \frac{\partial \varepsilon_{31}}{\partial y} + \frac{\partial \varepsilon_{12}}{\partial z} \right)$$

$$\frac{\partial^2 \varepsilon_{33}}{\partial x \partial y} = \frac{\partial}{\partial z}\left(\frac{\partial \varepsilon_{23}}{\partial x} + \frac{\partial \varepsilon_{31}}{\partial y} - \frac{\partial \varepsilon_{12}}{\partial z} \right)$$

$$2\frac{\partial^2 \varepsilon_{12}}{\partial x \partial y} = \frac{\partial^2 \varepsilon_{11}}{\partial y^2} + \frac{\partial^2 \varepsilon_{22}}{\partial x^2}$$

$$2\frac{\partial^2 \varepsilon_{23}}{\partial y \partial z} = \frac{\partial^2 \varepsilon_{22}}{\partial z^2} + \frac{\partial^2 \varepsilon_{33}}{\partial y^2}$$

$$2\frac{\partial^2 \varepsilon_{31}}{\partial z \partial x} = \frac{\partial^2 \varepsilon_{33}}{\partial x^2} + \frac{\partial^2 \varepsilon_{11}}{\partial z^2}$$

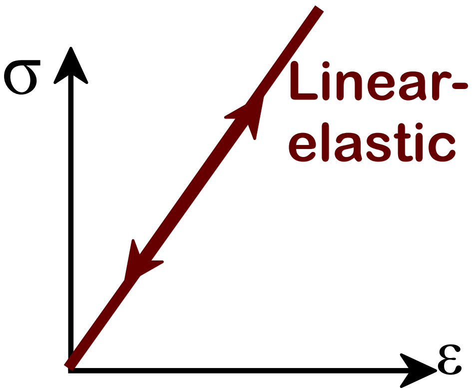
Plane strain condition

$$\varepsilon_{33} = \varepsilon_{13} = \varepsilon_{23} = 0$$

$$2\frac{\partial^2 \varepsilon_{12}}{\partial x \partial y} = \frac{\partial^2 \varepsilon_{11}}{\partial y^2} + \frac{\partial^2 \varepsilon_{22}}{\partial x^2}$$

Mathematical Theory of Elasticity
I.S. Solkolnikoff, P. 25

Stress-Strain Relationship in Elastic Region (Linear Elastic)



Robert Hooke [1635~1702] suggested a law.

$$\sigma = E\varepsilon$$ **Hooke's law**

Elastic modulus or Young's modulus

$$\sigma_{11} = E\varepsilon_{11}$$

$$\boxed{\nu = -\frac{\varepsilon_{22}}{\varepsilon_{11}} = -\frac{\varepsilon_{33}}{\varepsilon_{11}}}$$ Isotropic conditon

Poisson's ratio

Robert Hooke

FRS (Fellow of Royal Society)

Stress-Strain Relationship in Elastic Region (Linear Elastic)

Elastic constitutive equation under isotropic condition

$$\varepsilon_{11} = \frac{1}{E}[\sigma_{11} - \nu(\sigma_{22} + \sigma_{33})]$$

$$\varepsilon_{22} = \frac{1}{E}[\sigma_{22} - \nu(\sigma_{33} + \sigma_{11})]$$

$$\varepsilon_{33} = \frac{1}{E}[\sigma_{33} - \nu(\sigma_{11} + \sigma_{22})]$$

$$\gamma_{12} = \frac{1}{G}\sigma_{12}$$

$$\gamma_{23} = \frac{1}{G}\sigma_{23}$$

$$\gamma_{31} = \frac{1}{G}\sigma_{31}$$

or

$$\sigma_{12} = G\gamma_{12}$$

$$\sigma_{23} = G\gamma_{23}$$

$$\sigma_{31} = G\gamma_{31}$$

Shear modulus

$$G = \frac{E}{2(1+\nu)}$$

Stress-Strain Relationship in Elastic Region (Linear Elastic)

Elastic constitutive equation under isotropic condition

$$\sigma_{ij} = \frac{E}{1+\nu}\varepsilon_{ij} + \frac{\nu E}{(1+\nu)(1-2\nu)}\varepsilon_{kk}\delta_{ij}$$

$$\sigma_{ij} = 2G\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij}$$

$$\sigma_{ij}' = 2G\varepsilon_{ij}'$$

λ, Lame's constant

$$P = -B\varepsilon_v$$

$$\sigma_{11} + \sigma_{22} + \sigma_{33} = 3B(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33})$$

Bulk modulus

$$B = \frac{E}{3(1-2\nu)}$$

Stress-Strain Relationship in Elastic Region (Linear Elastic)

Elastic Constant \ Constant to be used	**E, ν**	**E, G**	**B, ν**	**B, G**	**λ, G**
E	E	E	$3(1-2\nu)B$	$\dfrac{9B}{1+3B/G}$	$\dfrac{G(3\lambda+2G)}{\lambda+G}$
ν	ν	$-1+\dfrac{E}{2G}$	ν	$\dfrac{1-2G/3B}{2+2G/3B}$	$\dfrac{1}{2(1+G/\lambda)}$
G	$\dfrac{E}{2(1+\nu)}$	G	$\dfrac{3(1-2\nu)B}{2(1+\nu)}$	G	G
B	$\dfrac{E}{3(1-2\nu)}$	$\dfrac{E}{9-3E/G}$	B	B	$\lambda+\dfrac{2G}{3}$
λ	$\dfrac{\nu E}{(1+\nu)(1-2\nu)}$	$\dfrac{E(1-2G/E)}{3-E/G}$	$\dfrac{3B\nu}{1+\nu}$	$B-\dfrac{2G}{3}$	λ

The Elastic Constants of Isotropic Elements at Room Temperature.

Element	Bulk modulus B	Young's modulus E	Shear modulus G	Poisson's Ratio v
Li	13.6	11.5	4.2	0.36
Be	125.5	309.0	146.8	0.05
B(fibres)		379		
C(graphite fibres)		475		0.16
Na	8.16	8.92	3.38	0.32
Mg	33.25	44.3	17.35	0.29
Al	73.1	70.5	26.7	0.34
Si	316	113	39.7	
K	3.98	3.53	1.27	0.35
α -Ti	123.5	106	39.8	0.34
V	162	127	46.7	0.36
Cr	162	286		
α -Fe	166.0	208.2	80.65	0.29
Co	183	200	74.8	
Ni	192	213	81.3	0.31
Cu	137	122.5	45.5	0.34
Zn	60.5	92.2	37.2	0.29
Ge	69.7	99.0	39	0.28
Y	46.8	65.0	25.7	0.27
α -Zr	89.7	95.6	36.1	0.33
Nb	173	104	36.6	0.38
Mo	275	340	120	0.30
Pd	187	125.5	45.2	0.39
Ag	100	79	28.8	0.38
Cd	47.5	64.7	24.1	0.30
Hf	109.5	138	53	
Ta	207	184.5	68.7	0.35
W	313	388	148.5	0.29
Re		452		
Ir	371	527	210	0.26
Pt	275	171	61	0.39
Au	171	78.37	27.7	0.42
Tl	36.5	7.95	2.75	
Pb	41.4	16.2	5.6	0.44

Ref : Texture and Related Phenomena, D. N. Lee, 2006

Unit : GPa

Yielding under multiaxial loading

- **Plastic deformation (yielding)**
  - Slip process
  - (Maximum) Shear stress

- **Yield Criteria**
  - Tresca (Maximum-Shear-Stress) Yield Criteria

$$\tau_{\max} = \frac{\sigma_{\mathrm{I}} - \sigma_{\mathrm{III}}}{2} = \mathbf{k}$$

  - Von-Mises Yield Criteria

$$J_2 = \boldsymbol{k}^2$$

Yield Criteria

Tresca (Maximum-Shear-Stress) Yield Criteria

$$\tau_{\max} = \frac{\sigma_{\mathrm{I}} - \sigma_{\mathrm{III}}}{2} = \mathbf{k}$$

We can determine **k** from a simple tensile test. In uniaxial tension, yielding occurs when $\sigma_{\mathrm{I}}=\sigma_0$ (yield stress), $\sigma_{\mathrm{II}} = \sigma_{\mathrm{III}} = 0$.

$$\mathbf{k} = \frac{\sigma_0}{2}$$

$$\boxed{\sigma_{\mathrm{I}} - \sigma_{\mathrm{III}} = \sigma_0}$$

Yield Criteria

Recall:

$$\sigma^3 - (\sigma_{11} + \sigma_{22} + \sigma_{33})\sigma^2 + (\sigma_{11}\sigma_{22} + \sigma_{22}\sigma_{33} + \sigma_{33}\sigma_{11} - \sigma_{12}{}^2 - \sigma_{23}{}^2 - \sigma_{13}{}^2)\sigma$$
$$-(\sigma_{11}\sigma_{22}\sigma_{33} + 2\sigma_{12}\sigma_{23}\sigma_{13} - \sigma_{11}\sigma_{23}{}^2 - \sigma_{22}\sigma_{13}{}^2 - \sigma_{33}\sigma_{12}{}^2) = 0$$

or

$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0$$

For a given volume element of a material, we can resolve the stresses acting on the element into a mean(average) stress:

$$\sigma_m = P = \frac{\sigma_{11} + \sigma_{22} + \sigma_{33}}{3} = \frac{\sigma_I + \sigma_{II} + \sigma_{III}}{3} = \frac{I_1}{3}$$

Yield Criteria

This stress is invariant meaning that its value is independent of the choice of axes. We can consider σ_m to be "hydrostatic". The average stress represents PART OF the total stress; thus we can define the total stress as follows:

$$\underbrace{\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix}}_{\text{Total Stress}} = \underbrace{\begin{bmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{bmatrix}}_{\text{Hydrostatic Stress}} + \underbrace{\begin{bmatrix} \sigma_{11}-\sigma_m & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22}-\sigma_m & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33}-\sigma_m \end{bmatrix}}_{\text{Stress Deviator}}$$

The stress deviator is what causes distortion of the material element.

Deviatoric stress is a symmetric second rank tensor.

Yield Criteria

If we take the determinant of the stress deviator in the same way that we took the determinant of the total stress tensor earlier, we generate a new cubic equation that has three new invariants:

$$\sigma'^3 - J_1\sigma'^2 + J_2\sigma' - J_3 = 0$$

Two of the new invariants, the invariants of the stress deviator, are of importance:

$$J_1 = (\sigma_{11} - \sigma_m) + (\sigma_{22} - \sigma_m) + (\sigma_{33} - \sigma_m)$$

$$J_2 = \frac{1}{6}\left[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{12}^2 - \sigma_{23}^2 - \sigma_{31}^2)\right]$$

$$= \frac{1}{6}\left[(\sigma_{\mathrm{I}} - \sigma_{\mathrm{II}})^2 + (\sigma_{\mathrm{II}} - \sigma_{\mathrm{III}})^2 + (\sigma_{\mathrm{III}} - \sigma_{\mathrm{I}})^2\right]$$

Yield Criteria

We can determine **k²** from a simple tensile test. In uniaxial tension, yielding occurs when $\sigma_I=\sigma_0$ (yield stress), $\sigma_{II} = \sigma_{III} = 0$. Thus J2 becomes:

$$J_2 = \frac{1}{6}\left[(\sigma_{\mathrm{I}} - \sigma_{\mathrm{II}})^2 + (\sigma_{\mathrm{II}} - \sigma_{\mathrm{III}})^2 + (\sigma_{\mathrm{III}} - \sigma_{\mathrm{I}})^2\right] = \frac{1}{6}\left[(\sigma_o)^2 + (\text{-}\sigma_o)^2\right] = \frac{\sigma_o^2}{3}$$

It represents the condition required to cause yielding. Therefore:

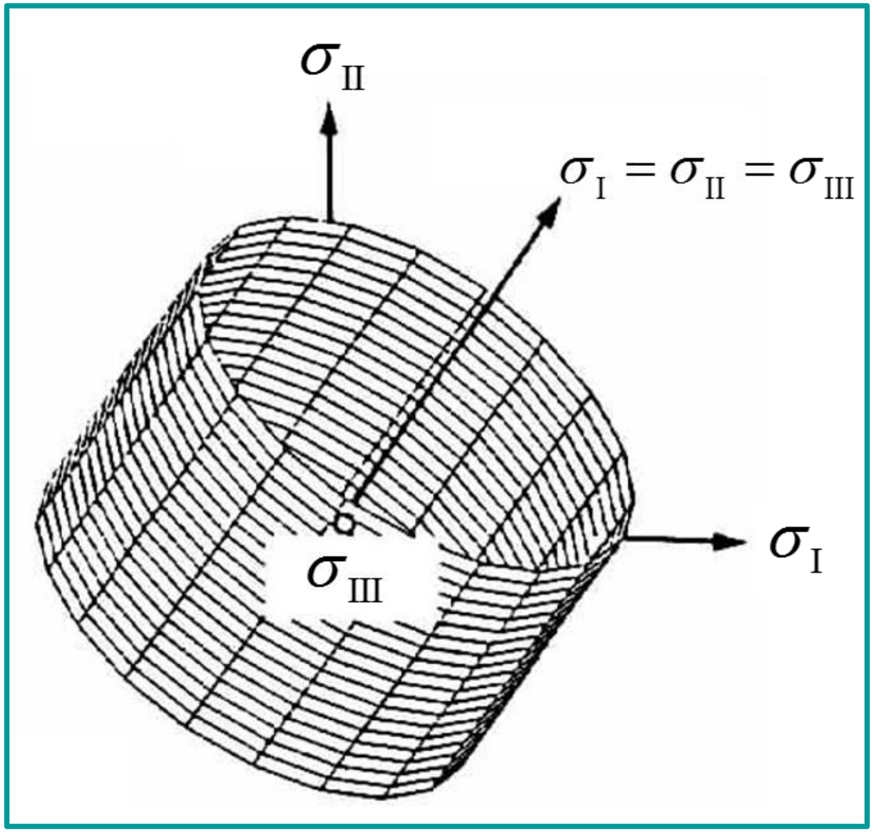
$$k^2 = \frac{1}{6}[\sigma_o^2 + \sigma_o^2] = \frac{\sigma_o^2}{3} = \frac{YS^2}{3}$$

The von Mises criterion then becomes:

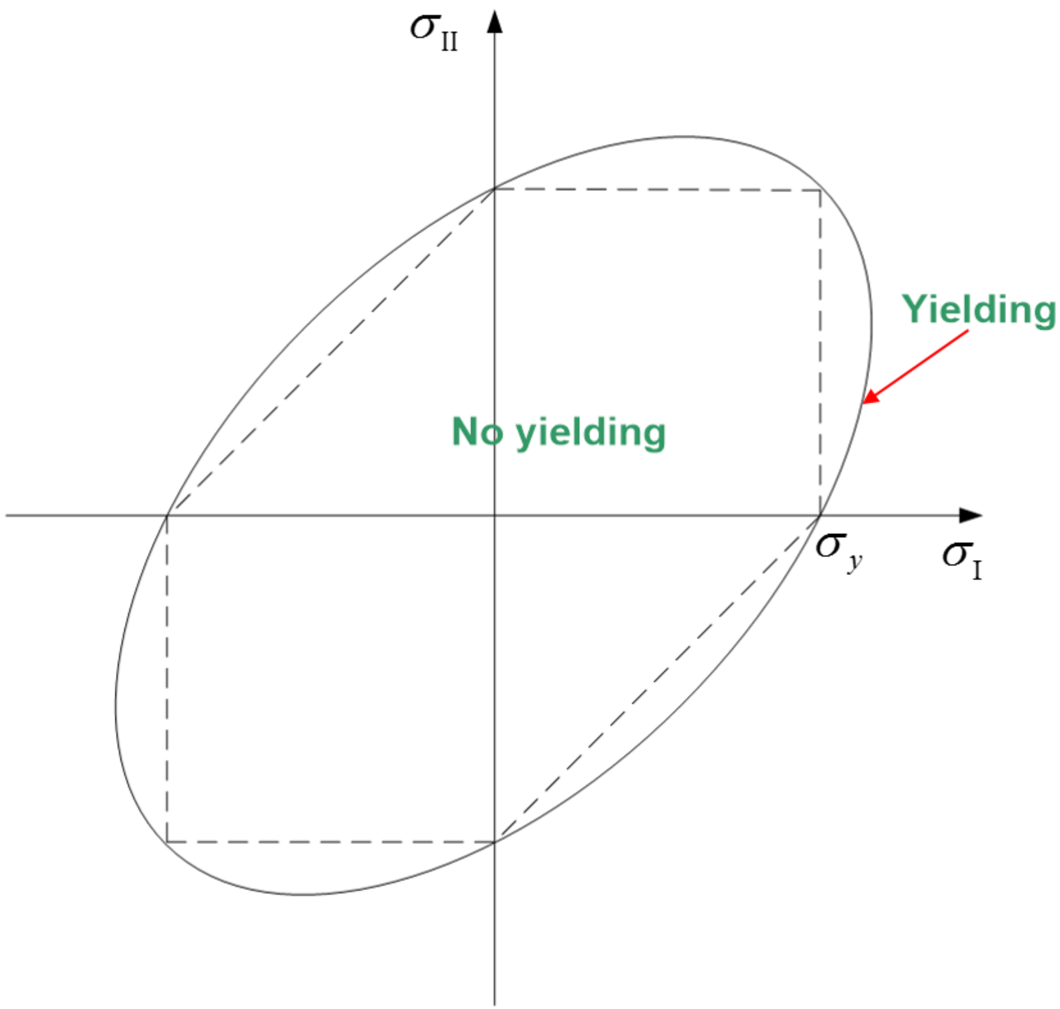
$$\frac{1}{\sqrt{2}}\left[(\sigma_{\mathrm{I}} - \sigma_{\mathrm{II}})^2 + (\sigma_{\mathrm{II}} - \sigma_{\mathrm{III}})^2 + (\sigma_{\mathrm{III}} - \sigma_{\mathrm{I}})^2\right]^{\frac{1}{2}} = \sigma_{ys} = \sigma_o$$

Yield Surface

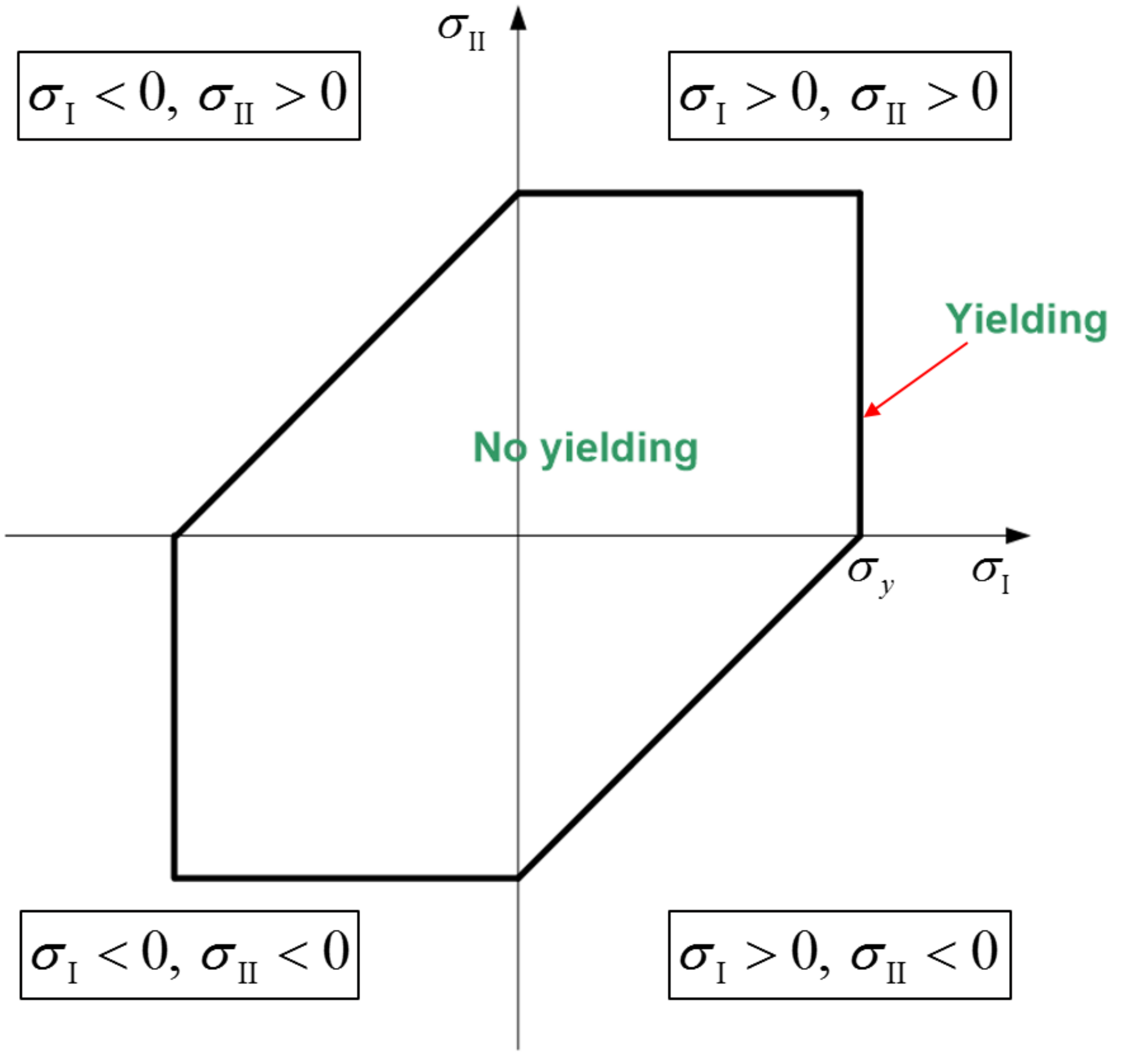
$$\frac{1}{\sqrt{2}}\left[(\sigma_{\mathrm{I}}-\sigma_{\mathrm{II}})^2+(\sigma_{\mathrm{II}}-\sigma_{\mathrm{III}})^2+(\sigma_{\mathrm{III}}-\sigma_{\mathrm{I}})^2\right]^{\frac{1}{2}}=\sigma_{ys}=\sigma_o$$



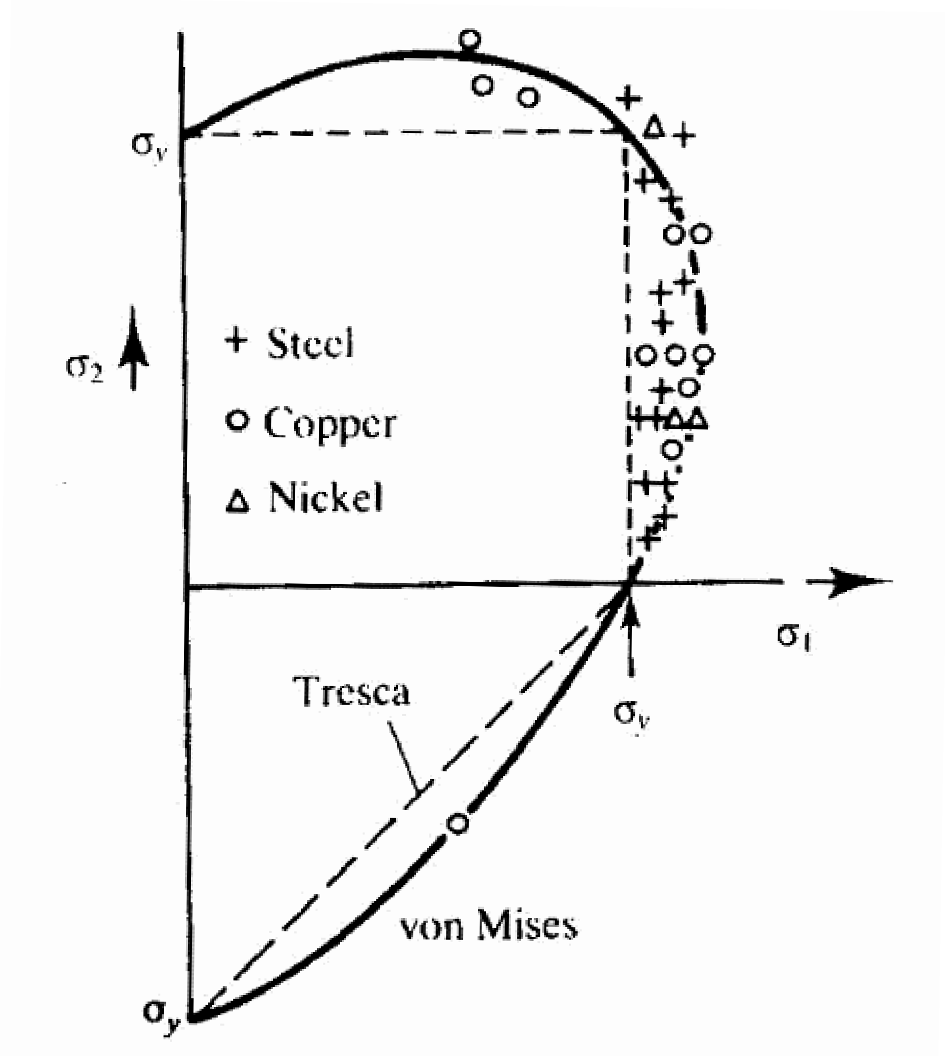
von-Mises Yield Locus



Tresca Yield Locus



Experimental comparison of Yield Locus



Effective stress

Under von-Mises Yield Criterion

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\left[(\sigma_{\mathrm{I}} - \sigma_{\mathrm{II}})^2 + (\sigma_{\mathrm{II}} - \sigma_{\mathrm{III}})^2 + (\sigma_{\mathrm{III}} - \sigma_{\mathrm{I}})^2\right]^{\frac{1}{2}}$$

$$\bar{\sigma} = \frac{1}{\sqrt{2}}[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2)]^{\frac{1}{2}}$$

$$\bar{\sigma} = \sqrt{\frac{3}{2}\,\sigma'_{ij}\,\sigma'_{ij}}$$

Plastic constitutive equation (flow rule)

Levy and von Mises suggested this relationship under von-Mises yield criterion. dλ : a positive constant.

$$\frac{d\varepsilon_{11}}{\sigma'_{11}} = \frac{d\varepsilon_{22}}{\sigma'_{22}} = \frac{d\varepsilon_{33}}{\sigma'_{33}} = \frac{d\varepsilon_{12}}{\sigma_{12}} = \frac{d\varepsilon_{23}}{\sigma_{23}} = \frac{d\varepsilon_{31}}{\sigma_{31}} = d\lambda$$

$$d\varepsilon_{11} = \frac{2}{3} d\lambda [\sigma_{11} - \frac{1}{2}(\sigma_{22} + \sigma_{33})]$$

$$d\varepsilon_{22} = \frac{2}{3} d\lambda [\sigma_{22} - \frac{1}{2}(\sigma_{33} + \sigma_{11})]$$

$$d\varepsilon_{33} = \frac{2}{3} d\lambda [\sigma_{33} - \frac{1}{2}(\sigma_{11} + \sigma_{22})]$$

$$d\varepsilon_{12} = d\lambda \sigma_{12}$$

$$d\varepsilon_{23} = d\lambda \sigma_{23}$$

$$d\varepsilon_{31} = d\lambda \sigma_{31}$$

$$\Rightarrow \quad d\lambda = \frac{3}{2} \frac{d\bar{\varepsilon}}{\bar{\sigma}}$$

Effective strain

Under von-Mises Yield Criterion

$$d\bar{\varepsilon} = A[(d\varepsilon_1 - d\varepsilon_2)^2 + (d\varepsilon_2 - d\varepsilon_3)^2 + (d\varepsilon_3 - d\varepsilon_1)^2]^{1/2}$$

$$d\bar{\varepsilon} = A[(d\varepsilon_{11} - d\varepsilon_{22})^2 + (d\varepsilon_{22} - d\varepsilon_{33})^2 + (d\varepsilon_{33} - d\varepsilon_{11})^2 + 6(d\varepsilon_{12} + d\varepsilon_{23} + d\varepsilon_{31})]^{1/2}$$

$$A = \frac{\sqrt{2}}{3} \longleftarrow d\varepsilon_{11} + d\varepsilon_{22} + d\varepsilon_{33} = 0$$

$$d\bar{\varepsilon} = \sqrt{\frac{2}{3}\, d\varepsilon'_{ij}\, d\varepsilon'_{ij}}$$

Example

A region on the surface of an alloy component has the following stresses:

$$\sigma_{11}=70\ \mathit{MPa},\ \sigma_{22}=120\ \mathit{MPa},\ \sigma_{12}=60\ \mathit{MPa}$$

Determine the yielding for both Tresca and von Mises criteria, given that $\sigma_y=150\ \mathit{MPa}$ (yield stress).