Boundary layer approximation $\left(\frac{\partial}{\partial x} \ll \frac{\partial}{\partial y}, \quad v \ll u\right)$

flat-plate

$$\begin{cases} \frac{\partial u}{\partial x} + \boxed{\frac{\partial v}{\partial y}} = 0 \\ u\frac{\partial u}{\partial x} + \boxed{v}\frac{\partial u}{\partial y} = \cancel{U\frac{dU}{dx}}^{\,0} + \nu\frac{\partial^2 u}{\partial y^2} \end{cases}$$

Boundary layer eq.

b.c. $u = v = 0$ @ $y = 0$
$u = U$ @ $y = \delta$

① Flat-plate bdry layer : $U = \text{const} \rightarrow \frac{dp}{dx} = 0, \frac{dU}{dx} = 0$

Blasius : $\frac{u}{U} = f'(\eta)$, $\eta = y\sqrt{\frac{U}{\nu x}}$

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial x} = Uf''\cdot y\sqrt{\frac{U}{\nu}}\left(-\frac{1}{2}\right)x^{-\frac{3}{2}}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial y} = Uf''\cdot\sqrt{\frac{U}{\nu x}} \Leftarrow$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(Uf''\sqrt{\frac{U}{\nu x}}\right) = \frac{\partial}{\partial \eta}\left(Uf''\sqrt{\frac{U}{\nu x}}\right)\frac{\partial \eta}{\partial y} = f'''U\sqrt{\frac{U}{\nu x}}\cdot\sqrt{\frac{U}{\nu x}}$$

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} \rightarrow v = -\frac{\partial}{\partial x}\int u\,dy = -\frac{\partial}{\partial x}\int f' U \sqrt{\frac{\nu x}{U}}\, d\eta$$

$$= \cdots = -\frac{1}{2}\sqrt{\frac{U\nu}{x}} f + \frac{1}{2} U x^{-1} f' y$$

$\eta = y\sqrt{\frac{U}{\nu x}}$

$\frac{u}{U} = f(\eta)$

$$\Rightarrow \boxed{f''' + \frac{1}{2} f f'' = 0}$$

Blasius eq. (exact eq.)

@ $y=0$, $u=v=0$ → @ $\eta = 0$, $f'(0) = 0$, $f(0) = 0$

@ $y \to \infty$ (δ), $u = U$ → @ $\eta \to \infty$, $f'(\infty) = 1$

Numerical method → obtain f, f', f'', ...

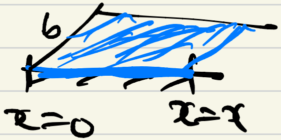
η = $y[U/(\nu x)]^{1/2}$, u/U = f'

$y[U/(\nu x)]^{1/2}$	u/U	$y[U/(\nu x)]^{1/2}$	u/U
0.0	0.0	2.8	0.81152
0.2	0.06641	3.0	0.84605
0.4	0.13277	3.2	0.87609
0.6	0.19894	3.4	0.90177
0.8	0.26471	3.6	0.92333
1.0	0.32979	3.8	0.94112
1.2	0.39378	4.0	0.95552
1.4	0.45627	4.2	0.96696
1.6	0.51676	4.4	0.97587
1.8	0.57477	4.6	0.98269
2.0	0.62977	4.8	0.98779
2.2	0.68132	5.0	0.99155
2.4	0.72899	∞	1.00000
2.6	0.77246		

$$\underset{5.0}{\eta} = \underset{\delta}{y}\sqrt{\frac{U}{\nu x}} \rightarrow \delta = \frac{5.0}{\sqrt{U/\nu x}}$$

$$\rightarrow \boxed{\frac{\delta}{x} = \frac{5.0}{\sqrt{\frac{Ux}{\nu}}} = \frac{5.0}{\sqrt{Re_x}}}$$

Drag $D(x) = \int_0^x \tau_w \, dx \cdot b = 0.664 \rho^{\frac{1}{2}} \mu^{\frac{1}{2}} U^{\frac{3}{2}} x^{\frac{1}{2}}$



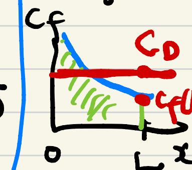
$$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_{y=0}$$

$$\frac{\partial u}{\partial y}\Big|_{y=0} = f''(0)\, U \sqrt{\frac{U}{\nu x}} \rightarrow \tau_w = \frac{0.332\sqrt{\rho \mu U^3}}{\sqrt{x}}$$

numerical sol.

Drag coefficient

$$C_D = \frac{D(L)}{\frac{1}{2}\rho U^2 b \cdot L} = \frac{1.328}{\sqrt{Re_L}} = 2c_f(L)$$



$$c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.664}{\sqrt{Re_x}} \qquad c_f \sim x^{-\frac{1}{2}}$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

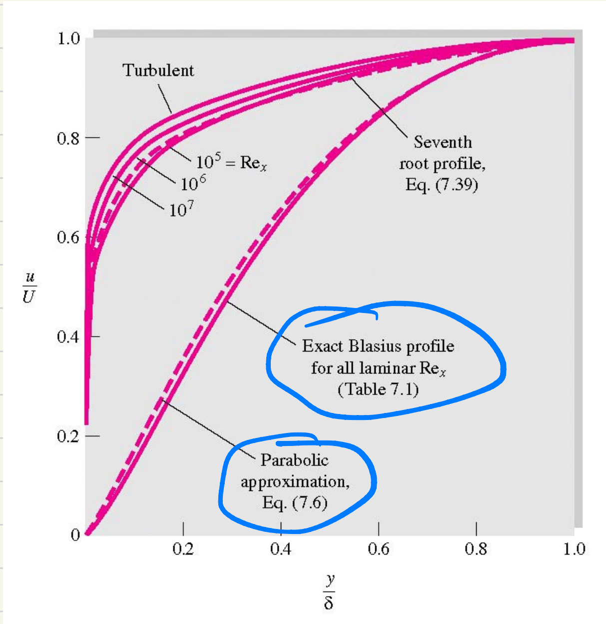
$$\int_0^\delta u \, dy = \int_0^{\eta(\delta)} f' U \sqrt{\frac{\nu x}{U}} \, d\eta = [\underbrace{f(\eta(\delta))}_{5.0} - \underbrace{f(0)}_{\to 0}]\, U \sqrt{\frac{\nu x}{U}}$$

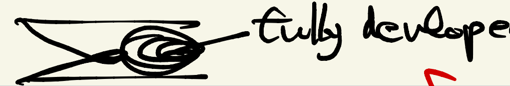
$$\frac{\delta^*}{x} = \frac{\delta}{x} - \frac{f(5.0)\sqrt{\frac{\nu x}{U}}}{x}$$

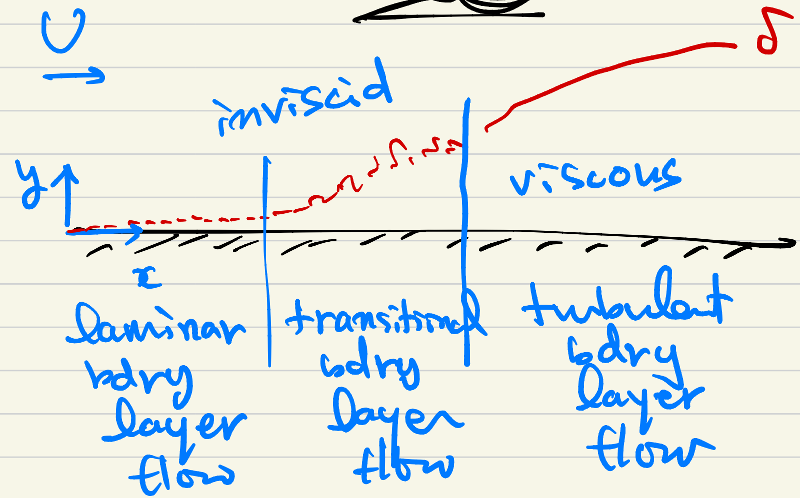
$$= \frac{5.0}{\sqrt{Re_x}} - \frac{f(5.0)}{\sqrt{Re_x}} = \frac{1.721}{\sqrt{Re_x}}$$

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \dashrightarrow \frac{\theta}{x} = \frac{0.664}{\sqrt{Re_x}}$$

$$H = \frac{\delta^*}{\theta} = \frac{1.721}{0.664} = 2.59$$







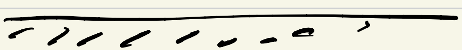
$$Re_x = \frac{Ux}{\nu}$$

$Re_{x,trans} \doteq 3\times10^6$ for smooth plate

5×10^5 for rough plate

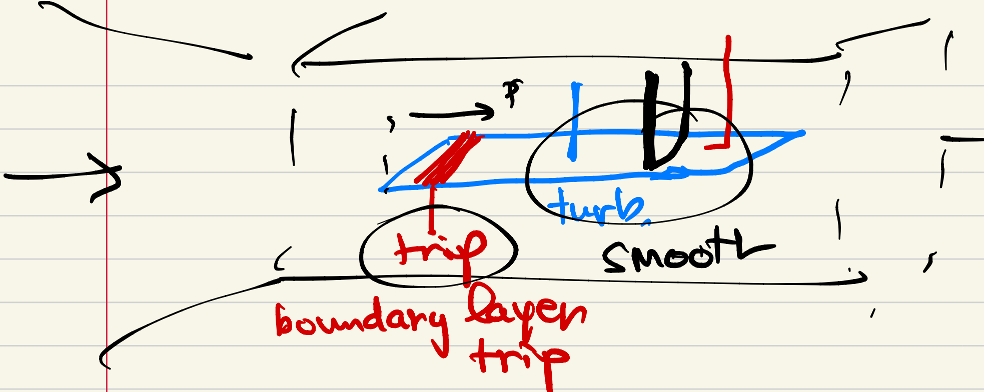
10 m/s →

air



$$\frac{U x_{tran}}{\nu} = 2\times10^6 \rightarrow x_{trans} = 4.5\,m$$

$$5\times10^5 \qquad = 0.75\,m$$

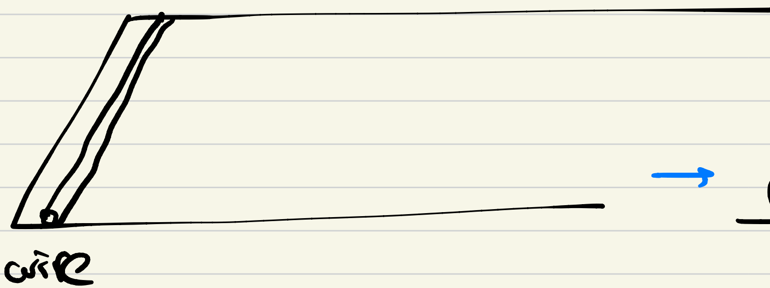


Re_x ? meaningless

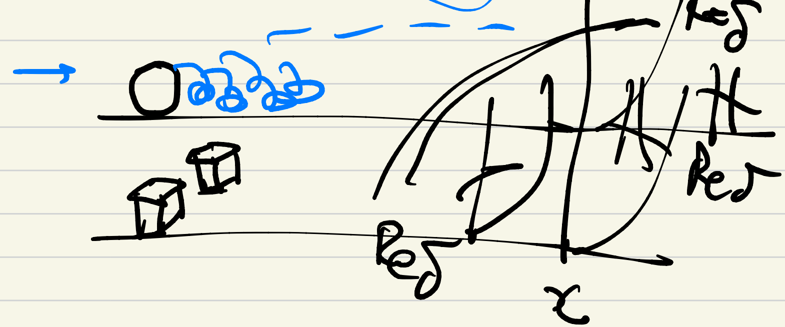
Re_δ

Re_{δ^*}

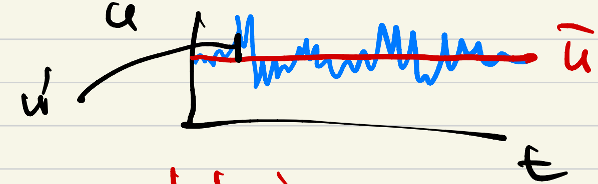
Re_θ



$u\frac{\partial u}{\partial x}$ $\quad$ $u\frac{\partial u}{\partial y}$

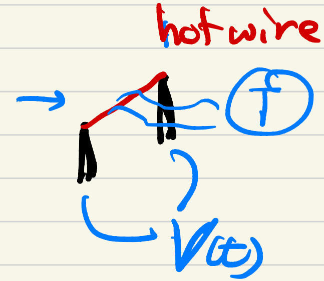


- Reynolds time averaging
  velocity decomposition



$$\boxed{u(t) = \bar{u} + u'(t)}$$

instantaneous velocity 순간속도 | mean velocity 평균속도 | fluctuating velocity 속도섭동



$$\overline{u} = \overline{\bar{u}} + \overline{u'} \quad \Rightarrow \quad \boxed{\overline{u'} = 0}$$

$$\overline{u} = \bar{u}, \qquad \overline{\bar{u}} = \bar{u}$$

Navier-Stokes eq.

$$\overline{\rho\frac{\partial u}{\partial t}} + \overline{\rho\frac{\partial}{\partial x}(uu)} + \overline{\rho\frac{\partial}{\partial y}(uv)} + \overline{\rho\frac{\partial}{\partial z}(uw)} = \overline{-\frac{\partial p}{\partial x}} + \overline{\mu\nabla^2 u}$$

$$\overline{-\frac{\partial p}{\partial x}} = -\frac{\partial \bar{p}}{\partial x}, \qquad \overline{\mu\nabla^2 u} = \mu\nabla^2\bar{u}$$

$$\overline{\rho\frac{\partial u}{\partial t}} = 0$$

$$\overline{\frac{\partial}{\partial x}(uu)} = \frac{\partial}{\partial x}\left(\overline{uu}\right) = \frac{\partial}{\partial x}\left(\overline{(\bar{u}+u')(\bar{u}+u')}\right) = \frac{\partial}{\partial x}\left(\overline{\bar{u}\bar{u}} + \cancel{\overline{2\bar{u}u'}} + \overline{u'u'}\right)$$

$$= \frac{\partial}{\partial x}\left(\bar{u}\bar{u} + \overline{u'u'}\right)$$

$$\overline{\frac{\partial}{\partial y}(uv)} = \frac{\partial}{\partial y}(\bar{u}\bar{v} + \overline{u'v'})$$

Reynolds-averaged Navier-Stokes eq.

$$\overline{\frac{\partial}{\partial z}(uw)} = \frac{\partial}{\partial z}(\bar{u}\bar{w} + \overline{u'w'})$$

$$\Rightarrow \boxed{\rho\frac{\partial}{\partial x}(\bar{u}\bar{u}) + \rho\frac{\partial}{\partial y}(\bar{u}\bar{v}) + \rho\frac{\partial}{\partial z}(\bar{u}\bar{w}) = -\frac{\partial \bar{p}}{\partial x} + \frac{\partial}{\partial x}\left(\mu\underline{\frac{\partial \bar{u}}{\partial x}} - \rho\overline{u'u'}\right) + \frac{\partial}{\partial y}\left(\mu\underline{\frac{\partial \bar{u}}{\partial y}} - \rho\overline{u'v'}\right) + \frac{\partial}{\partial z}\left(\mu\underline{\frac{\partial \bar{u}}{\partial z}} - \rho\overline{u'w'}\right)}$$

viscous stress

turbulent stress or Reynolds stress

$$\boxed{f(\bar{u}, \bar{v}, \bar{w})}$$

$$\overline{\frac{\partial u}{\partial x}} + \overline{\frac{\partial v}{\partial y}} + \overline{\frac{\partial w}{\partial y}} \to 0 \quad \to \quad \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0$$

$\boxed{\text{turbulence modeling}}$ closure problem

$$\underset{\text{g.e}}{\overline{u'v'}} \longrightarrow \frac{\partial}{\partial t}(\overline{u'v'}) + \cdots = \frac{\partial}{\partial x}(\overline{u'u'u'}) + \cdots$$