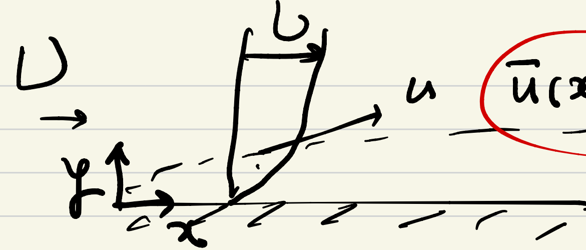
$$u = \bar{u} + u'$$

$$\overline{uv} = \bar{u}\bar{v} + \overline{u'v'}$$



$\bar{u}(x, y)$

$\frac{\partial}{\partial x} \ll \frac{\partial}{\partial y}$

N-S eq.

$$\Rightarrow \rho\frac{\partial}{\partial x}(\bar{u}\bar{u}) + \rho\frac{\partial}{\partial y}(\bar{u}\bar{v}) + \rho\frac{\partial}{\partial z}(\bar{u}\bar{w}) = -\frac{\partial \bar{p}}{\partial x}$$

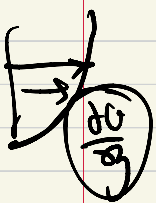
$$+ \frac{\partial}{\partial x}\left(\mu\frac{\partial \bar{u}}{\partial x} - \rho\overline{u'u'}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial \bar{u}}{\partial y} - \rho\overline{u'v'}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial \bar{u}}{\partial z} - \rho\overline{u'w'}\right)$$

viscous stress

turbulent stress

Reynolds stress

0



$$\tau = \mu\frac{\partial \bar{u}}{\partial y} - \rho\overline{u'v'}$$

$\leftarrow \tau_{turb}$ turbulent shear stress

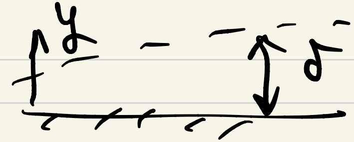
Reynolds " "

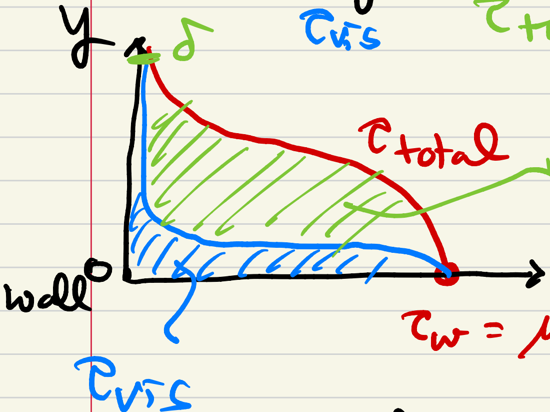
τ $\|$ τ_{total} total shear stress

$\mu\frac{\partial \bar{u}}{\partial y}$ $\|$ τ_{vis} viscous shear stress

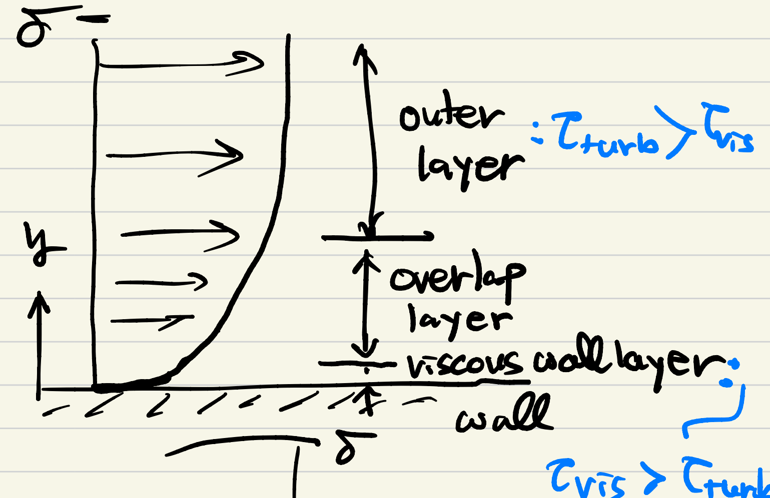
$$\tau = \mu \frac{\partial \bar{u}}{\partial y} - \rho \overline{u'v'}$$

(τ_{vis}, τ_{turb})





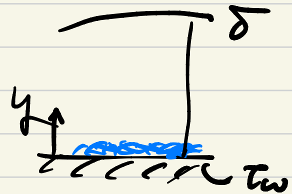
$$\tau_w = \mu \frac{\partial \bar{u}}{\partial y} - \rho \overline{u'v'} \quad (\rho\overline{u'v'} \to 0)$$



@ wall, $u = v = w = 0$

$u' = v' = w' = 0$

$u'v' = 0$

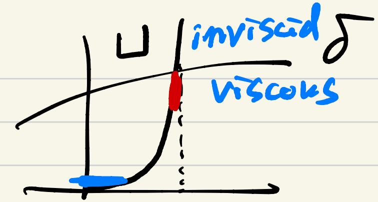


- wall layer : Prandtl (1930)

$$\bar{u} = f(\rho, \mu, y, \tau_w, \cancel{\delta}) \neq f(\delta)$$

dimensional analysis $\rightarrow \dfrac{\bar{u}}{u^*} = F\left(\dfrac{y u^*}{\nu}\right)$ $u^* = \sqrt{\dfrac{\tau_w}{\rho}}$: 벽전단속도 wall shear velocity

$u^+ \equiv \dfrac{\bar{u}}{u^*}$, $y^+ \equiv \dfrac{y u^*}{\nu}$ $\rightarrow$ $\boxed{u^+ = F(y^+)}$: law of the wall 벽법칙



- Outer layer : von Karman (1933)

$$U - \bar{u} = g(\rho, \cancel{\mu}, y, \tau_w, \delta) \neq g(\mu)$$

velocity defect → $$\boxed{\frac{U - \bar{u}}{u^*} = G\left(\frac{y}{\delta}\right)}$$: velocity defect law

- overlap layer (log layer) : Millikan (1937)

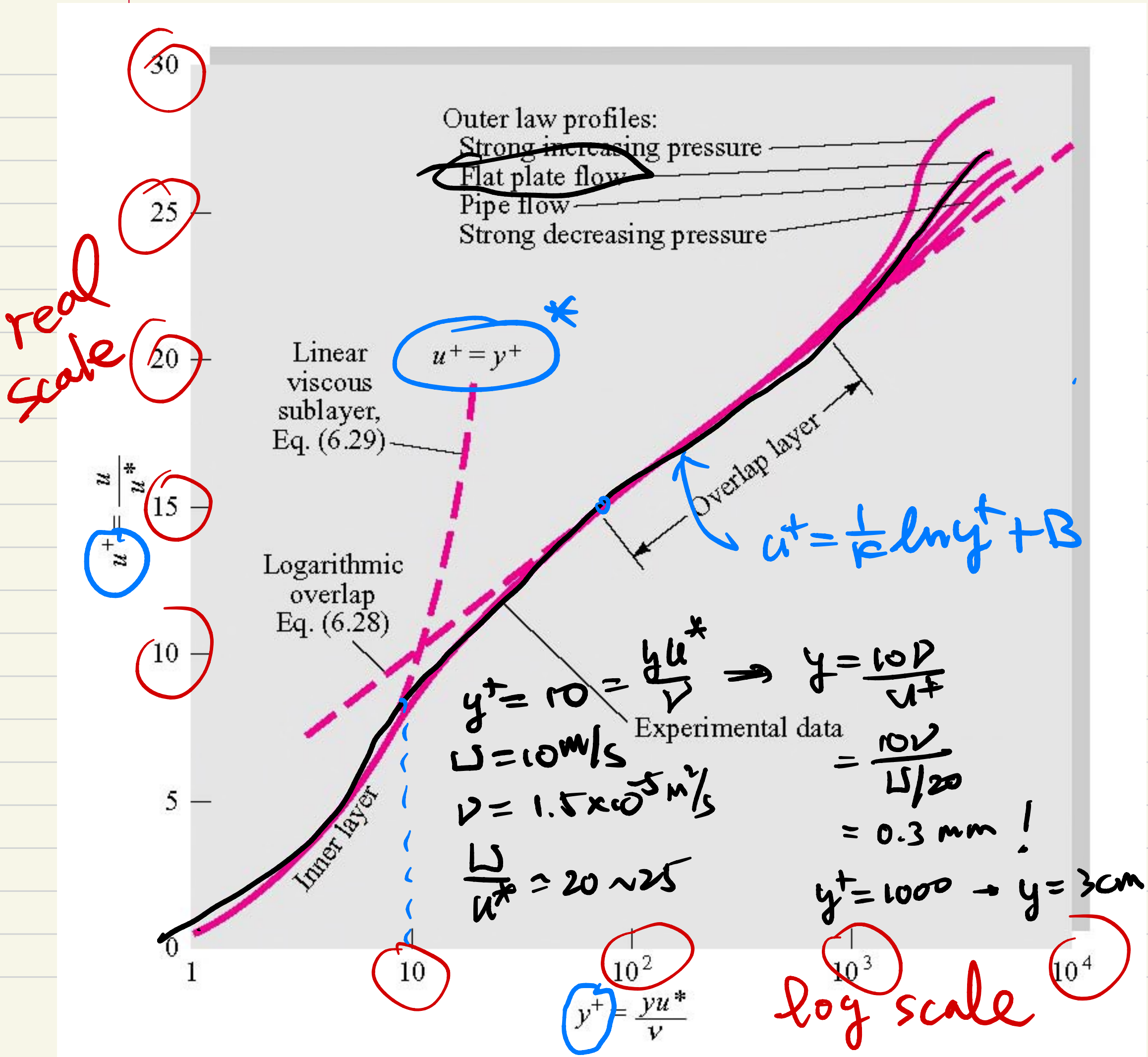
$$\frac{\bar{u}}{u^*} = \frac{1}{\kappa} \ln \frac{y u^*}{\nu} + B$$

→ $$\boxed{u^+ = \frac{1}{\kappa} \ln y^+ + B}$$: log law

$\kappa = 0.41$: von Karman constant

$B = 5.0$

($\kappa = 0.4$, $B = 5.5$)



$$u^+ = F(y^+)$$

$$\frac{U - \bar{u}}{u^*} = G\left(\frac{y}{\delta}\right)$$

$$u^+ = \frac{1}{\kappa} \ln y^+ + B$$

$$\bar{u}(y) = \bar{u}_w + y \left.\frac{\partial \bar{u}}{\partial y}\right|_w + \frac{1}{2} y^2 \left.\frac{\partial^2 \bar{u}}{\partial y^2}\right|_w + \cdots$$

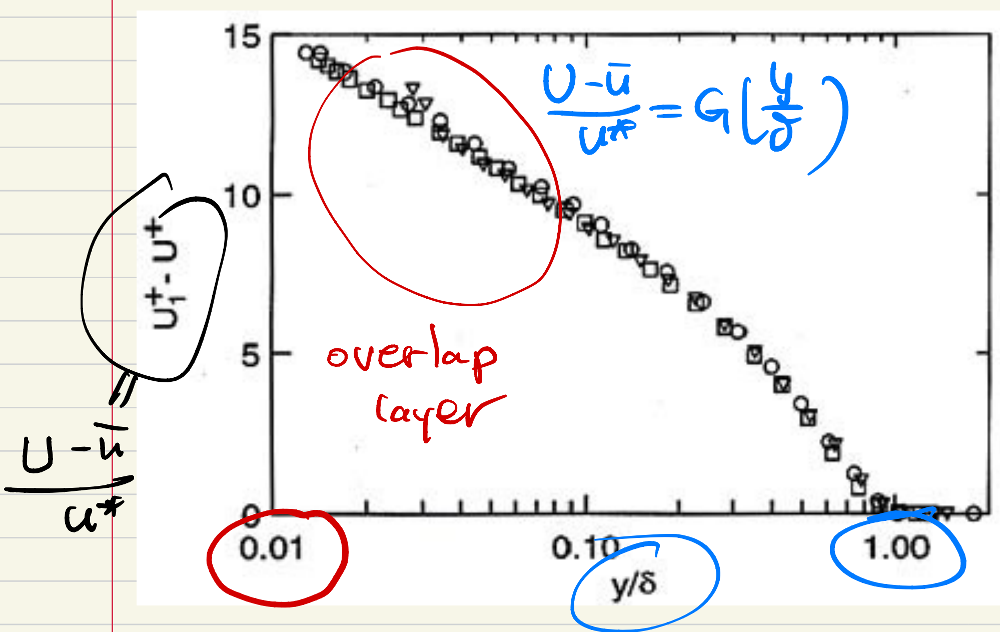
$$\tau_w = \mu \left.\frac{\partial \bar{u}}{\partial y}\right|_w$$

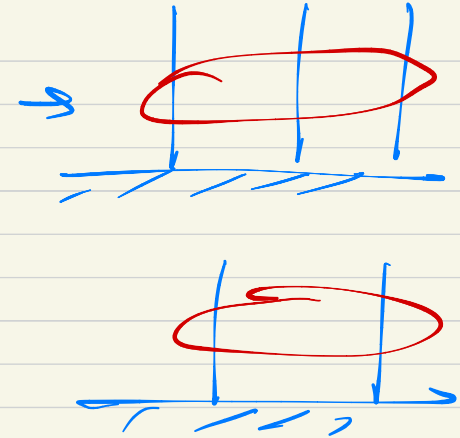
$$\rightarrow \bar{u} = y \cdot \frac{\tau_w}{\mu} \qquad u^* = \sqrt{\frac{\tau_w}{\rho}}$$

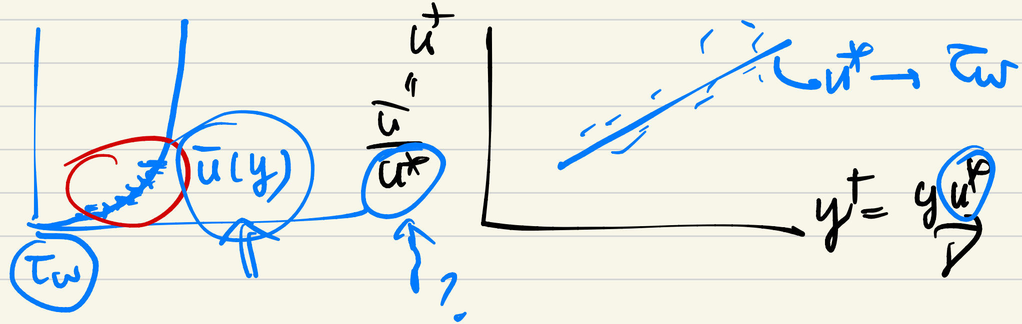
$$\frac{\bar{u}}{u^*} = y \frac{\tau_w}{\mu} \cdot \frac{1}{u^*} \qquad \tau_w = \rho u^{*2}$$

$$= y \cdot \frac{\rho u^*}{\mu} = \frac{y u^*}{\nu}$$

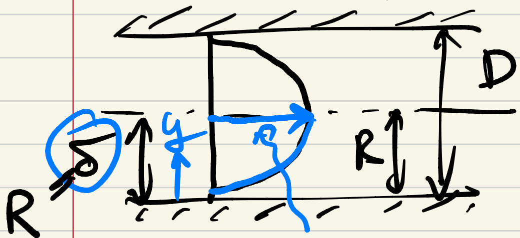
$$\rightarrow u^+ = y^+$$

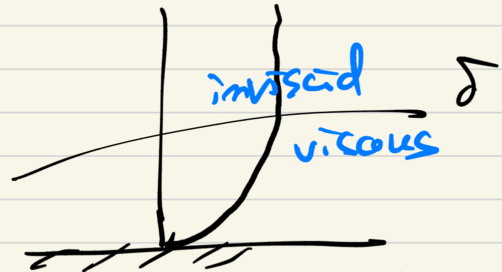

15
10
5
0
0.01
0.10
1.00
y/δ
$u_1^+ - u^+$
$\frac{U-\bar{u}}{u^*}$
overlap layer
$\frac{U-\bar{u}}{u^*} = G\left(\frac{y}{\delta}\right)$




$\bar{u}(y)$
τ_w
$\frac{\bar{u}}{u^*} = u^+$
?
$u^* \rightarrow \tau_w$
$y^+ = \frac{y u^*}{\nu}$

- turbulent pipe flow





viscous → log law is approx. satisfied @ $y = \delta$

$$\frac{\bar{u}}{u^*} = \frac{1}{\kappa} \ln \frac{y u^*}{\nu} + B$$

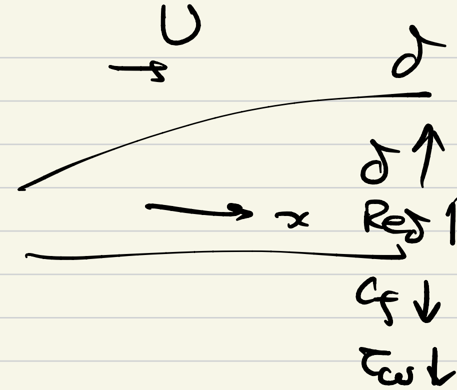
@ $y = \delta$ $\quad \frac{U}{u^*} = \frac{1}{\kappa} \ln \frac{\delta u^*}{\nu} + B$

$$u^* = \sqrt{\frac{\tau_w}{\rho}} \rightarrow u^{*2} = \frac{\tau_w}{\rho} = \frac{\frac{1}{2}\rho U^2 c_f}{\rho} = \frac{1}{2} U^2 c_f \rightarrow \frac{u^*}{U} = \sqrt{\frac{c_f}{2}}$$

$$\sqrt{\frac{2}{c_f}} = \frac{1}{\kappa} \ln \frac{\delta U \sqrt{\frac{c_f}{2}}}{\nu} + B = \frac{1}{\kappa} \ln \left(Re_\delta \sqrt{\frac{c_f}{2}} \right) + B$$

$$\rightarrow \boxed{\sqrt{\frac{2}{c_f}} = \frac{1}{\kappa} \ln \left(Re_\delta \sqrt{\frac{c_f}{2}} \right) + B} \quad : \text{skin-friction law}$$

Re_δ	10^4	10^5	10^6	10^7
c_f	0.00493	0.00315	0.00217	0.00188



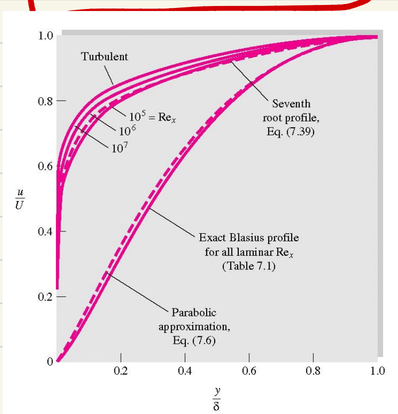
→ $$c_f \simeq 0.02\, Re_\delta^{-1/6}$$ Prandtl

$$\theta = \int_0^\delta \frac{\bar{u}}{U}\left(1 - \frac{\bar{u}}{U}\right) dy$$ ← log law? complicated!

$$\frac{\bar{u}}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$$ seventh law Prandtl

↕ very good at low Re # ($Re_x = 10^5 \sim 10^6$)

errors @ high " " ($Re_x > 10^6$)



$$\theta = \int_0^\delta \frac{\bar{u}}{U}\left(1 - \frac{\bar{u}}{U}\right) dy = \frac{7}{72}\delta$$

$$\delta^* = \int_0^\delta \left(1 - \frac{\bar{u}}{U}\right) dy = \frac{1}{8}\delta$$

$$H = \delta^* / \theta = 1.3$$

$$\frac{C_f}{2} = \frac{d\theta}{dx} = \frac{7}{72}\frac{d\delta}{dx} = \frac{7}{72}\frac{d\left(\frac{\delta U}{\nu}\right)}{d\left(\frac{xU}{\nu}\right)} = \frac{7}{72}\frac{dRe_\delta}{dRe_x}$$

$$\frac{1}{2}\cdot 0.02\, Re_\delta^{-1/6} \rightarrow Re_\delta = 0.16\, Re_x^{6/7} \rightarrow \boxed{\frac{\delta}{x} = \frac{0.16}{Re_x^{1/7}}}$$

$\delta \sim x^{6/7}$

lam. $\delta \sim x^{\frac{1}{2}}$

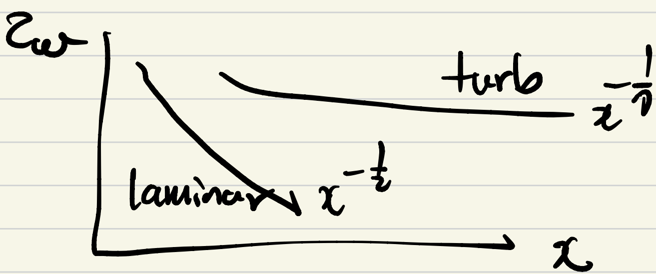
$$\frac{\theta}{x} = \frac{\theta}{\delta}\cdot\frac{\delta}{x} = \frac{7}{72}\cdot\frac{0.16}{Re_x^{1/7}} = \frac{0.0155}{Re_x^{1/7}}$$

$$C_f = 0.02\, Re_\delta^{-\frac{1}{6}} = \frac{0.027}{Re_x^{1/7}} = 0.027\, U^{-\frac{1}{7}} x^{-\frac{1}{7}} \nu^{\frac{1}{7}}$$

$$\tau_w = \frac{1}{2}\rho U^2 C_f = 0.0135\, \mu^{\frac{1}{7}} \rho^{\frac{6}{7}} U^{\frac{13}{7}} x^{-\frac{1}{7}}$$

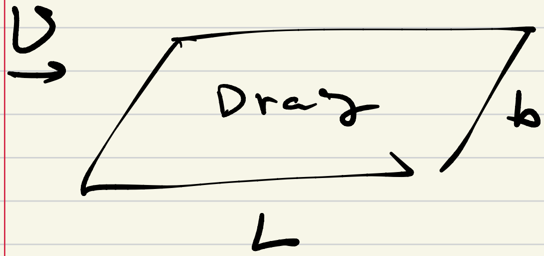
lam.

$$\tau_w = 0.332\, \rho^{\frac{1}{2}} \mu^{\frac{1}{2}} U^{\frac{3}{2}} x^{-\frac{1}{2}}$$



$$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_w \Leftarrow \frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$$

↳ $\tau_w \rightarrow \infty$ wrong!



Drag D

$$C_D \equiv \frac{D}{\frac{1}{2}\rho U^2 L b} = \frac{\rho b U^2 \theta(L)}{\frac{1}{2}\rho U^2 L b} = \frac{2\theta(x=L)}{L}$$

$$= \frac{0.031}{Re_L^{1/7}} = \frac{7}{6} c_f(L) \qquad Re_L = \frac{UL}{\nu}$$

lam. $C_D = 2 c_f(L)$