Topics in Ship Structural Design
(Hull Buckling and Ultimate Strength)

Lecture 9 Ultimate Strength of Ship Hulls

Reference : CSR Rule
Ultimate Limit State Design of Steel-Plated Structures Ch. 8
Ultimate Limit State Design of Ship Structures Ph.D Thesis of B.J. Kim

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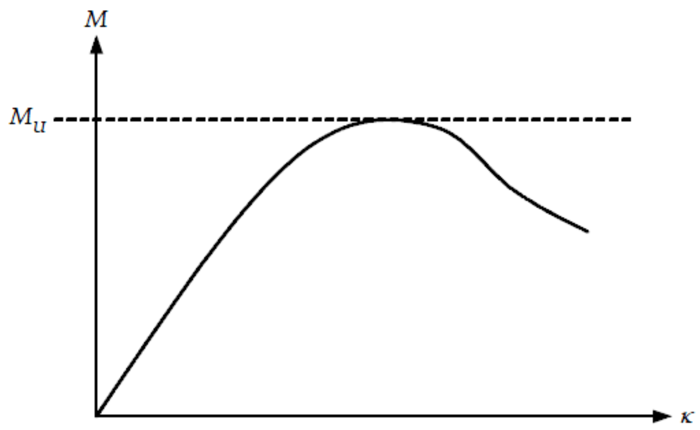
1. General

❖ Hull girder bending moment capacity

- The hull girder ultimate bending moment capacity, M_U, : the maximum sagging bending capacity of the hull girder beyond which the hull will collapse.
- Hull girder failure is controlled by buckling, ultimate strength and yielding of longitudinal structural elements.
- The maximum value on the static non-linear bending moment-curvature relationship M-κ. The curve represents the progressive collapse behavior of hull girder under vertical bending.



Bending Moment - Curvature Curve M-κ

1. General

- The curvature of the critical inter-frame section, κ, is defined as

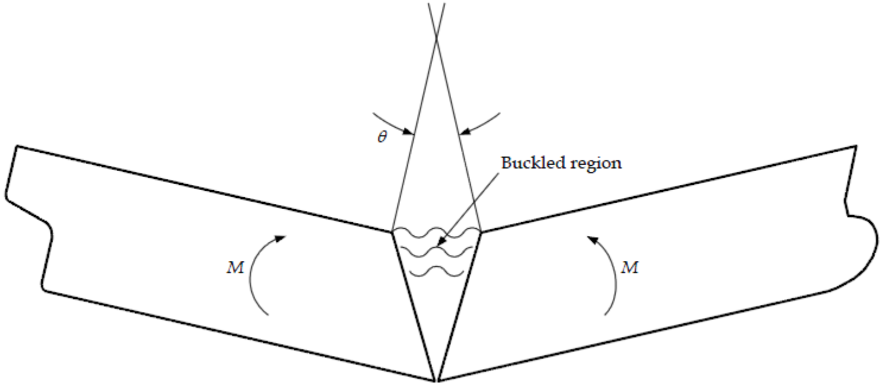
$$\kappa = \frac{\theta}{l}$$

- Where:

  θ : the relative angle rotation of the two neighboring cross-sections at transverse frame positions

  l : the transverse frame spacing, i.e. span of longitudinal

- The critical failure mode : Inter-frame buckling failure in sagging
- Only vertical bending is considered. The effects of shear force, torsional loading, horizontal bending moment and lateral pressure are neglected

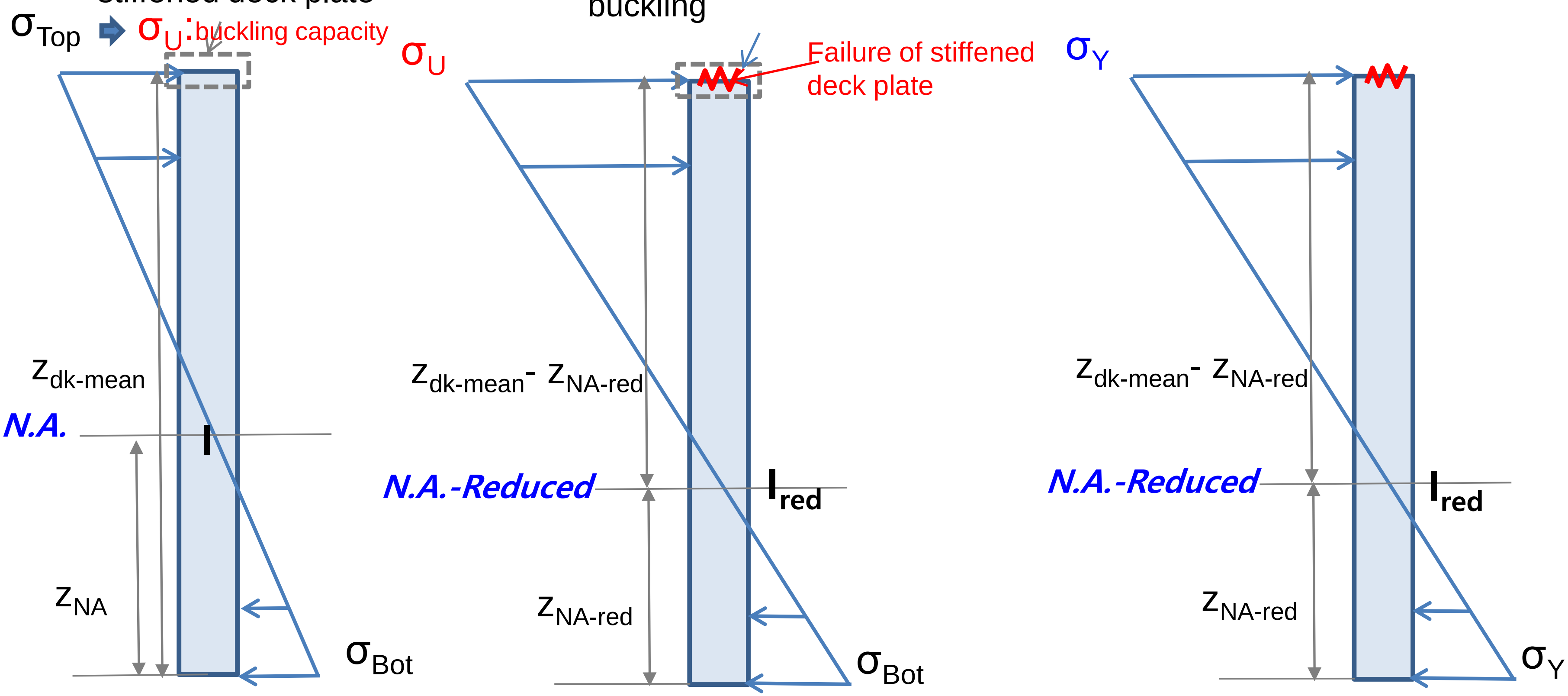


Ship in Extreme Sagging Inter-Frame Buckling Failure

2. Single Step Ultimate Capacity Method

A_{net50}: net area of the stiffened deck plate

A_{eff}: effective net area after buckling



$$Z = \frac{I}{z_{dk\text{-}mean} - z_{NA}}$$ at upper Deck

$$Z_{red}\downarrow = \frac{I_{red}\downarrow}{z_{dk\text{-}mean} - z_{NA\text{-}red}\uparrow}$$

Ultimate hull girder capacity

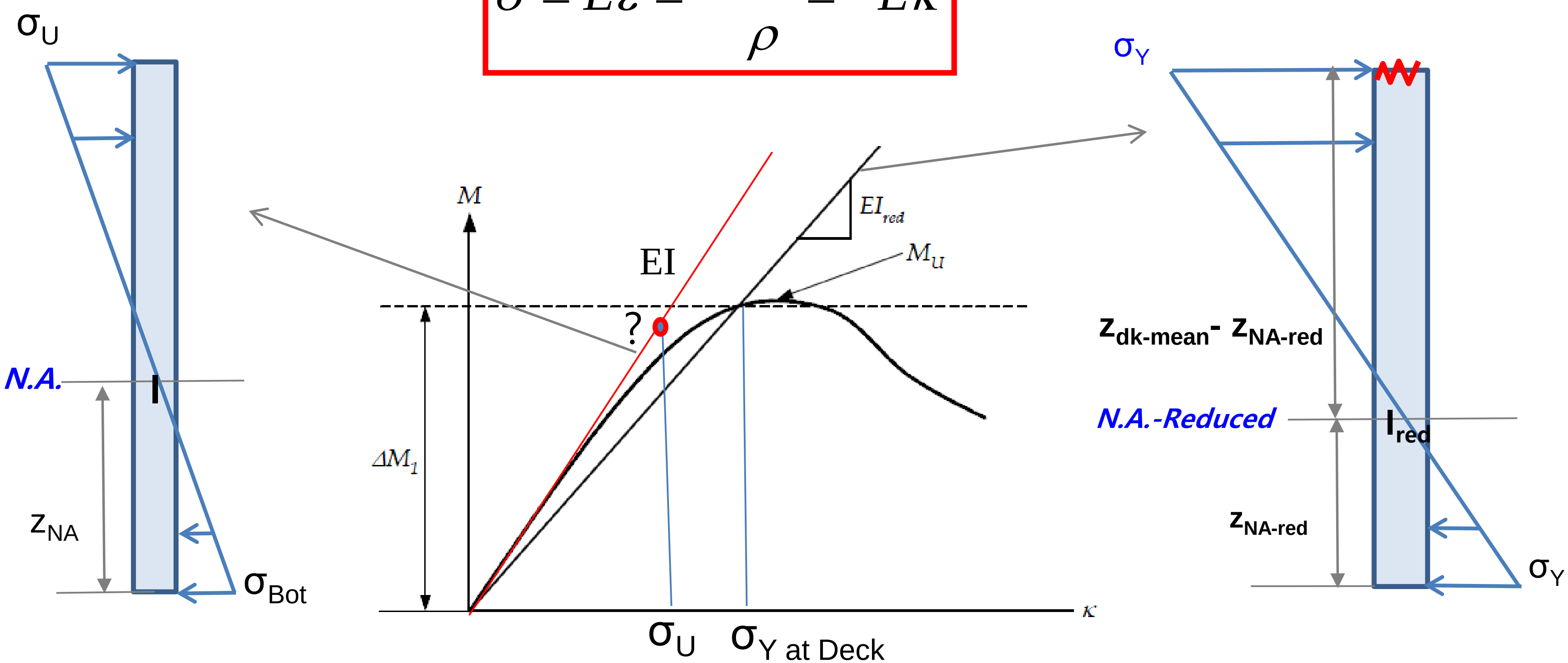
$M_u = Z_{red}\sigma_{yd}$

: B.M. when yield stress occurs on reduced upper deck

$$M_u < \sigma_{yd}\frac{I_{red}}{z_{NA\text{-}red}}$$

2. Single Step Ultimate Capacity Method

$$\boxed{\sigma = E\varepsilon = -\frac{Ey}{\rho} = -E\kappa}$$



Moment – Curvature of Hull Girder Single Step Procedure

2. Single Step Ultimate Capacity Method

❖ Single Step Ultimate Capacity Method

- The assumption : that the ultimate sagging capacity of tankers is the point at which the ultimate capacity of the stiffened deck panels is reached

- The single step procedure for calculation of the sagging hull girder ultimate bending capacity is a simplified method based on a reduced hull girder bending stiffness accounting for buckling of the deck

$$M_U = Z_{red}\sigma_{yd} \cdot 10^3 \text{ kNm}$$

- Z_{red} : reduced section modulus of deck

$$Z_{red} = \frac{I_{red}}{z_{dk-mean} - z_{NA-red}} m^3$$

- I_{red} : reduced hull girder moment of inertia using
  - a hull girder net thickness of t_{net50} for all longitudinally
  - effective members the effective net area after buckling of each stiffened panel of the deck, A_{eff}.

2. Single Step Ultimate Capacity Method

- A_{eff} : effective net area after buckling of the stiffened deck panel. The effective area is the proportion of stiffened deck panel that is effectively able to be stressed to yield:

$$A_{eff} = \frac{\sigma_U}{\sigma_{yd}} A_{net50} \ \mathrm{m}^2$$

- σ_U : buckling capacity of stiffened deck panel. To be calculated for each stiffened panel using:
  - the advanced buckling analysis method
  - the net thickness t_{net50}

- σ_{yd} : specified minimum yield stress of the material

- $z_{dk\text{-}mean}$: vertical distance to the mean deck height, taken as the mean of the deck at side and the deck at centreline, measured from the baseline.

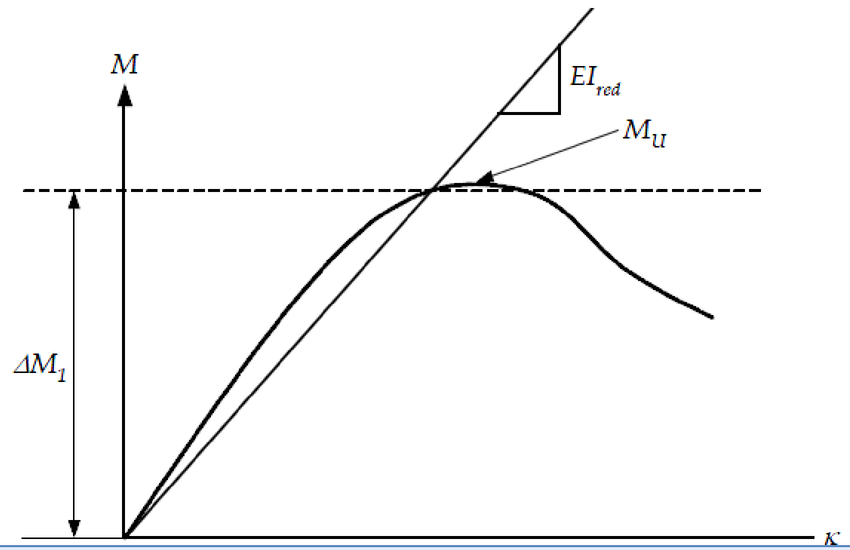
- $z_{NA\text{-}mean}$: vertical distance to the neutral axis of the reduced section measured from the baseline.

2. Single Step Ultimate Capacity Method

- M_U, does not give stresses exceeding the specified minimum yield stress of the material, σ_{yd}, in the bottom shell plating. M_U, is not to be greater than

$$M_U = \sigma_{yd} \frac{I_{red}}{Z_{NA-red}} 10^3 \text{ kNm}$$

$$\boxed{\sigma = E\varepsilon = -\frac{Ey}{\rho}}$$



Moment – Curvature of Hull Girder Single Step Procedure

2. Single Step Ultimate Capacity Method

❖ Vertical hull girder ultimate bending capacity

$$\gamma_S M_{sw} + \gamma_W M_{wv-sag} \leq \frac{M_U}{\gamma_R}$$

- M_{sw} sagging still water bending moment
- $M_{wv\text{-}sag}$ sagging vertical wave bending moment
- M_U sagging vertical hull girder ultimate bending capacity
- γ_S partial safety factor for the sagging still water bending moment
- γ_W partial safety factor for the sagging vertical wave bending moment
- γ_R partial safety factor for the sagging vertical hull girder bending capacity covering material, geometric and strength prediction uncertainties

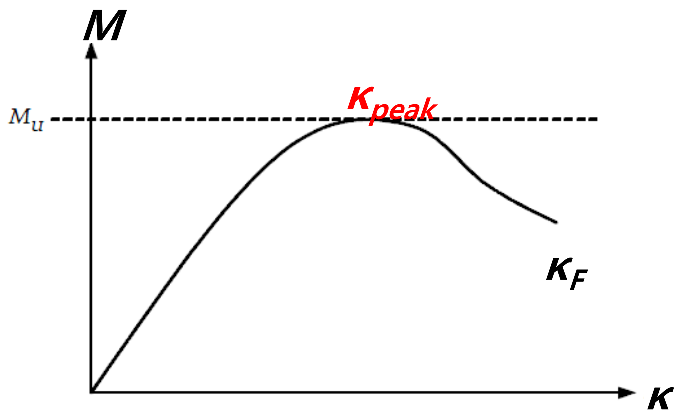
Design load combination	Definition of Still Water Bending Moment, Msw	γ_S	γ_W	γ_R
a)	Permissible sagging still water bending moment, $M_{sw\text{-}perm\text{-}sea}$,	1.0	1.2	1.1
b)	Maximum sagging still water bending moment for operational seagoing homogeneous full load condition, $M_{sw\text{-}full}$ ($<M_{sw\text{-}perm\text{-}sea}$)	1.0	1.3	1.1

3. Simplified Method Based on an Incremental-iterative Approach

- Ultimate hull girder bending moment capacity M_U is defined as the peak value of the curve with vertical bending moment *M* versus the curvature κ
- The curve *M-κ* is obtained by means of an incremental-iterative approach;

The expected maximum required curvature, κ_F,

$$\kappa_F = 3\frac{M_{yd}}{EI_{v-net50}}10^{-3}\ \mathrm{m}^{-1}$$



- M_{yd} : vertical bending moment given by a linear elastic bending stress of yield in the deck or keel. To be taken as the greater of :

$Z_{v\text{-}net50\text{-}dk}\sigma_{yd}$
$Z_{v\text{-}net50\text{-}kl}\ \sigma_{yd}$

$Z_{v\text{-}net50\text{-}dk}, Z_{v\text{-}net50\text{-}kl}$: section modulus at deck or bottom,

Assumptions and modeling of the hull girder cross-section

❖ Assumptions

(a) The ultimate strength is calculated at a hull girder transverse section between two adjacent transverse webs.

(b) The hull girder transverse section remains plane during each curvature increment.

(c) The material properties of steel are assumed to be elastic, perfectly plastic.

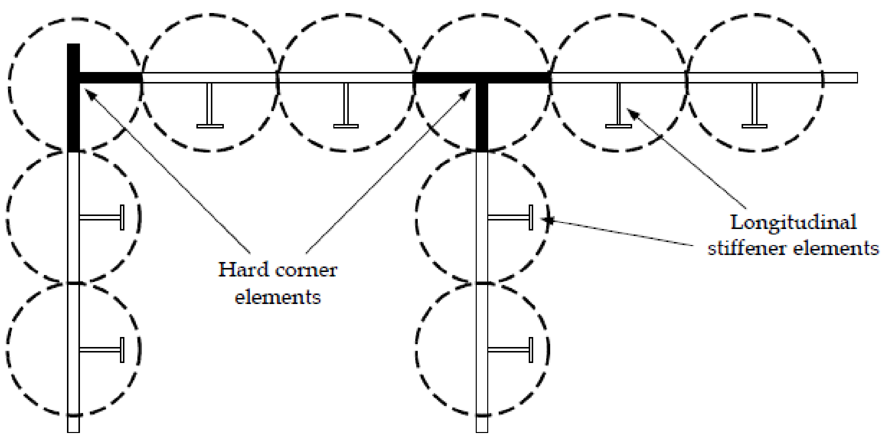
(d) The hull girder transverse section can be divided into a set of elements which act independently of each other.

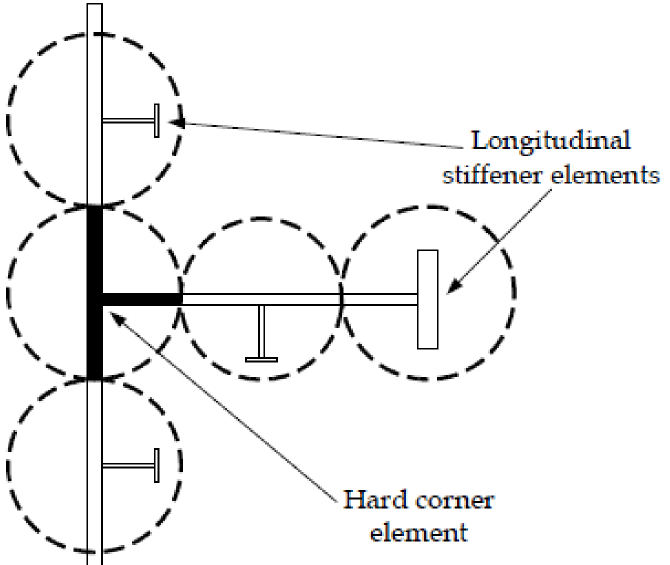
❖ The elements making up the hull girder transverse section are:

(a) longitudinal stiffeners with attached plating

(b) transversely stiffened plate panels

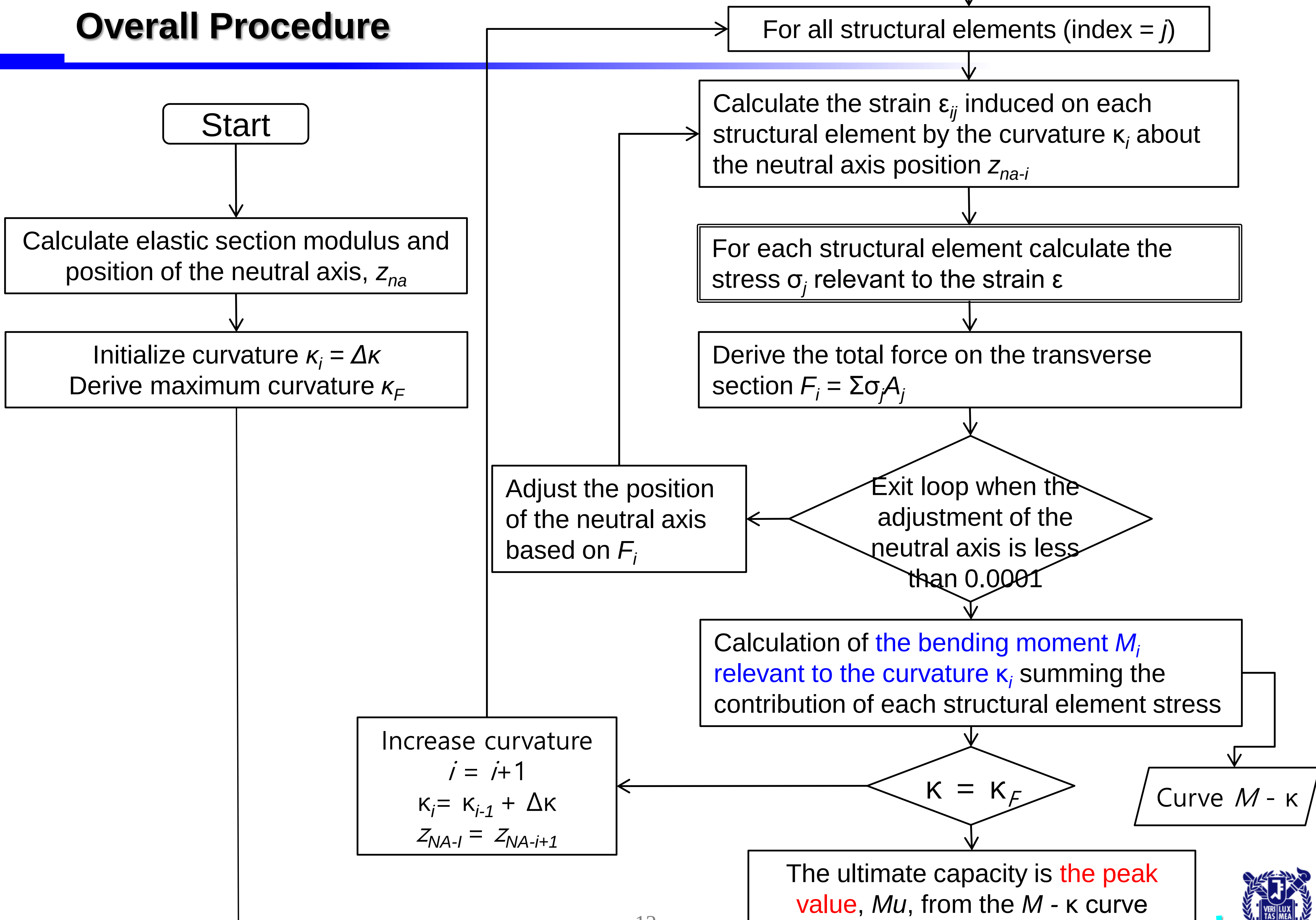
(c) hard corners





CSR – Calculation of hull girder ultimate capacity

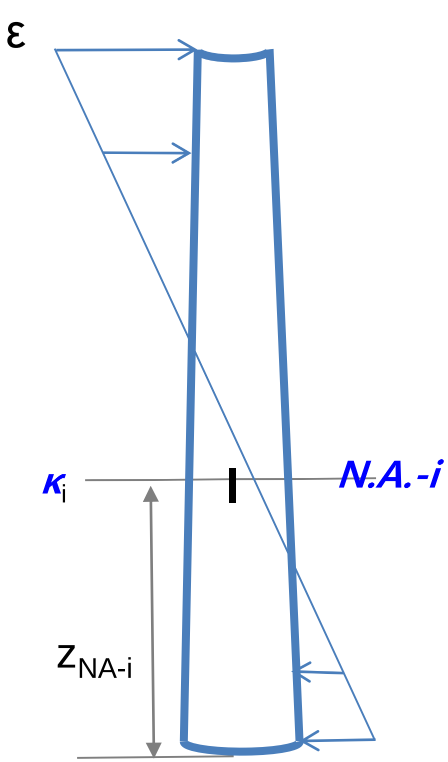
Overall Procedure

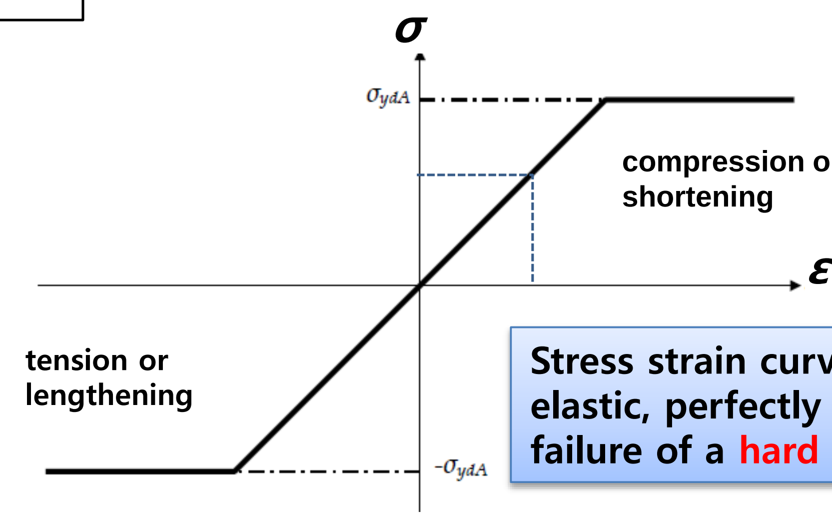


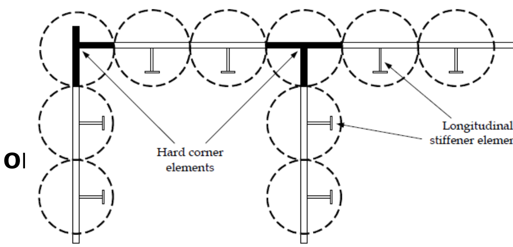
Simplified Method Based on an Incremental-iterative Approach

Calculate the strain ε_{ij} induced on each structural element by the curvature κ_i about the neutral axis position $z_{na\text{-}i}$

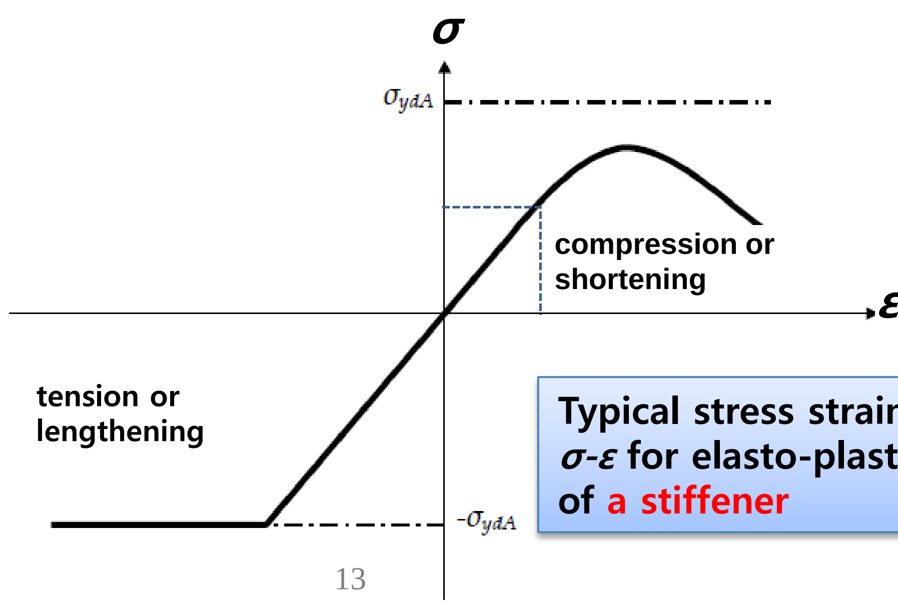
For each structural element calculate the stress σ_j relevant to the strain ε







Stress strain curve *σ-ε* for elastic, perfectly plastic failure of a hard corner

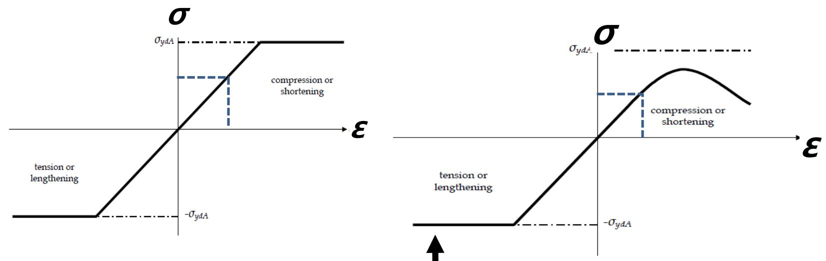


Typical stress strain curve *σ-ε* for elasto-plastic failure of a stiffener

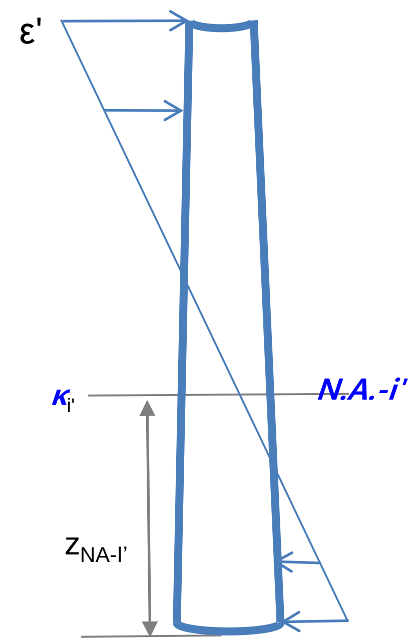
CSR – Calculation of hull girder ultimate capacity

Simplified Method Based on an Incremental-iterative Approach

For each structural element calculate the stress σ_j relevant to the strain ε



Calculate the strain ε_{ij} induced on each structural element by the curvature κ_i about the neutral axis position $z_{na\text{-}i}$

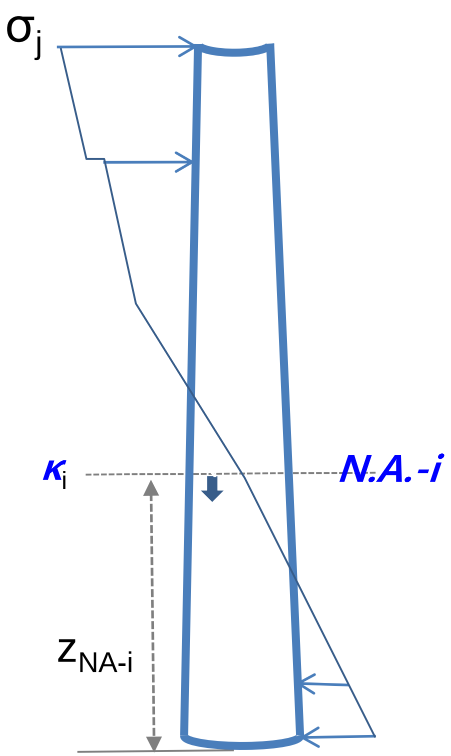


Exit loop when the adjustment of the neutral axis is less than 0.0001

No

Yes

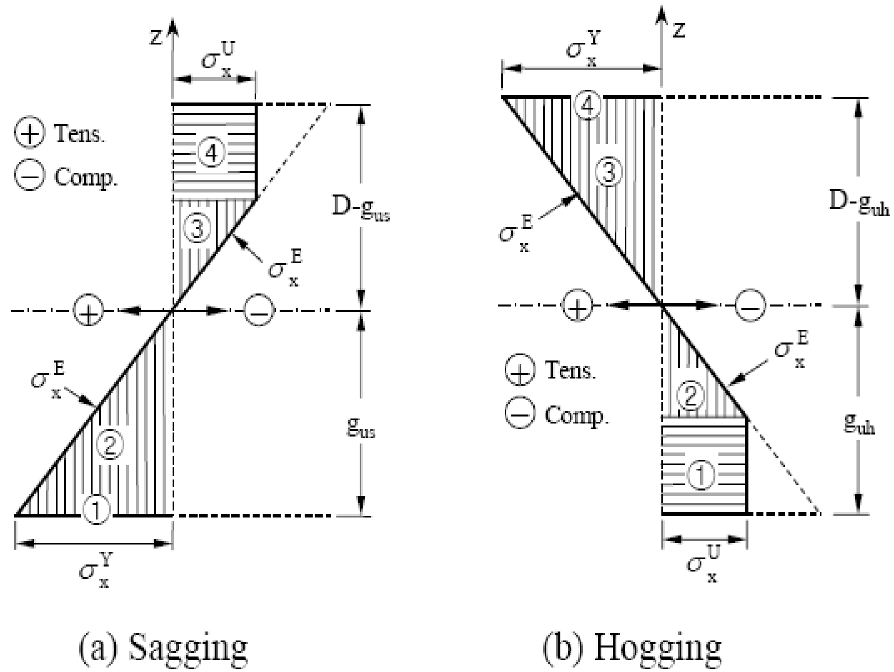
Derive the total force on the transverse section $F_i = \Sigma \sigma_j A_j$



Simplified Method Based on an Incremental-iterative Approach

- Neutral axis above base line

$$g_{us} = \frac{\sum_1 \sigma_{xi}^{Y} A_i z_i + \sum_2 \sigma_{xj}^{E} A_j z_j + \sum_3 \sigma_{xk}^{E} A_{ek} z_k + \sum_4 \sigma_{xl}^{U} A_{el} z_l}{\sum_1 \sigma_{xi}^{Y} A_i + \sum_2 \sigma_{xj}^{E} A_j + \sum_3 \sigma_{xk}^{E} A_{ek} + \sum_4 \sigma_{xl}^{U} A_{el}}$$



- Ultimate hull girder moment

$$M_{us} = -\sum_1 \sigma_{xi}^{Y} A_i (g_{us} - z_i) - \sum_2 \sigma_{xj}^{E} A_j (g_{us} - z_j) + \sum_3 \sigma_{xk}^{E} A_{ek} (z_k - g_{us}) + \sum_4 \sigma_{xl}^{U} A_{el} (z_l - g_{us})$$

① : yield region
② : elastic tension region
③ : elastic compression region
④ : collapsed compression region

Simplified Method Based on an Incremental-iterative Approach

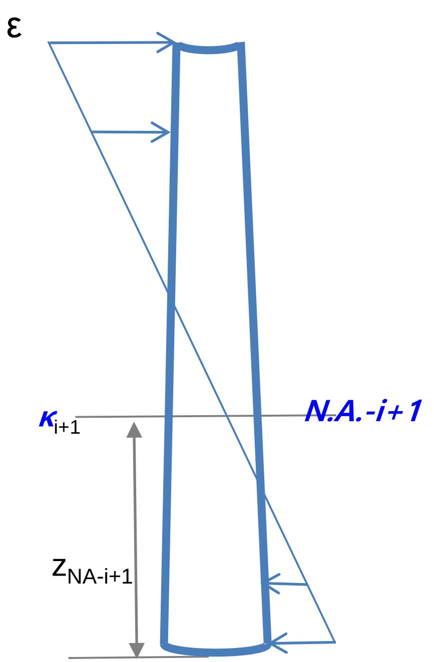
Exit loop when the adjustment of the neutral axis is less than 0.0001

Yes

Increase curvature
$i = i+1$
$\kappa_i = \kappa_{i-1} + \Delta\kappa$
$z_{NA\text{-}I} = z_{NA\text{-}i+1}$

Calculation of the bending moment M_i relevant to the curvature κ_i summing the contribution of each structural element stress

Curve M - κ

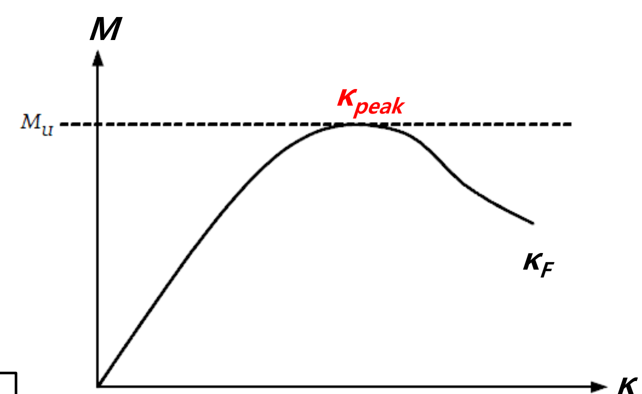


$\kappa = \kappa_F$

No

Yes

The ultimate capacity is the peak value, Mu, from the M - κ curve

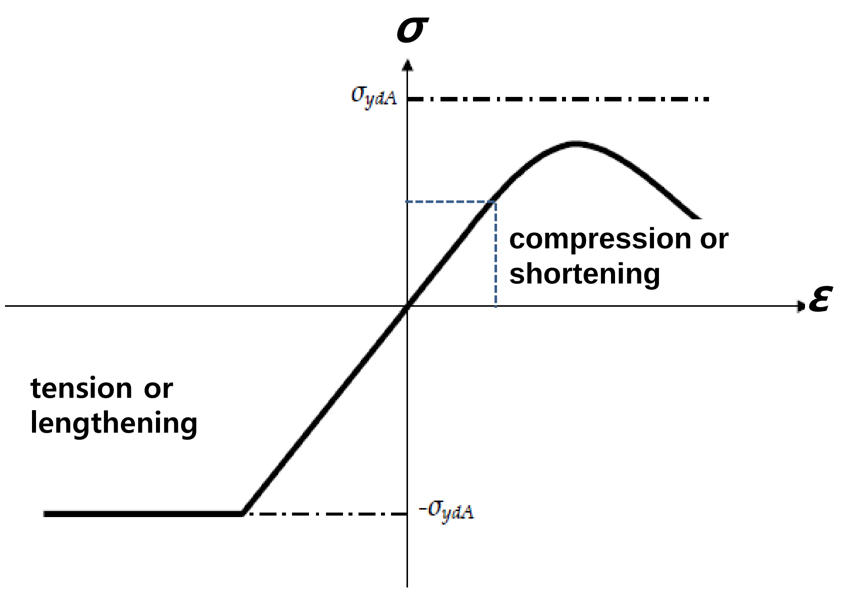


Stress-strain Curves *σ-ε* (or Load-end Shortening Curves)

❖ Plate panels and stiffeners

- Plate panels and stiffeners are assumed to fail according to one of the modes of failure
- The relevant stress-strain curve *σ-ε* is to be obtained for lengthening and shortening strains

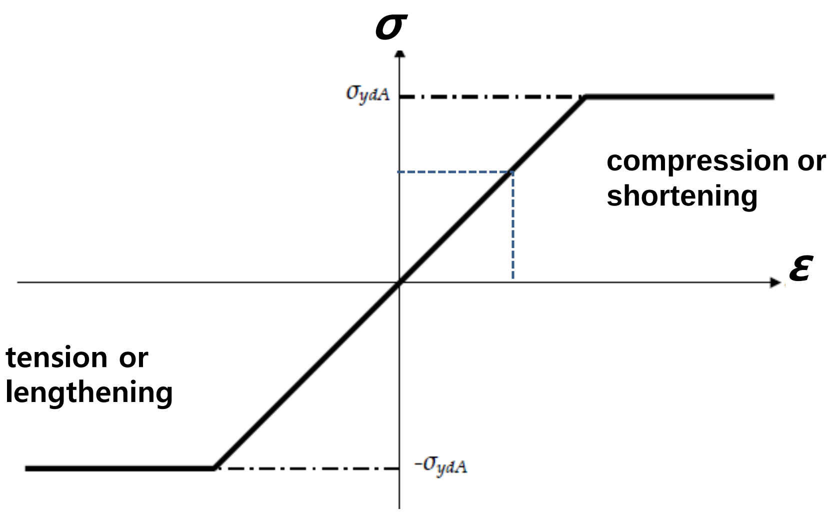
Element	Mode of failure
Lengthened transversely framed plate panels or stiffeners	Elastic, perfectly plastic failure
Shortened stiffeners	Beam column buckling Torsional buckling Web local buckling of flanged profiles Web local buckling of flat bars
Shortened transversely framed plate panels	Plate buckling



Stress-strain Curves *σ-ε* (or Load-end Shortening Curves)

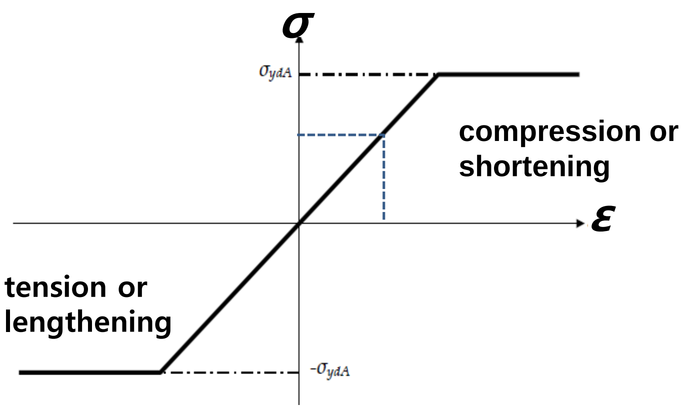
❖ Hard Corners

- Hard corners are sturdier elements which are assumed to buckle and fail in an elastic, perfectly plastic manner.



Stress-strain Curves σ-ε (or Load-end Shortening Curves)

❖ Elasto-plastic failure of structural elements



❖ Beam column buckling

- Euler buckling with plasticity correction

σ_{C1} critical stress, in N/mm²:

$$\sigma_{C1} = \frac{\sigma_{E1}}{\varepsilon} \qquad for \quad \sigma_{E1} \le \frac{\sigma_{ydB}}{2}\varepsilon$$

$$\sigma_{C1} = \sigma_{ydB}\left(1 - \frac{\sigma_{ydB}\varepsilon}{4\sigma_{E1}}\right) \qquad for \quad \sigma_{E1} > \frac{\sigma_{ydB}}{2}\varepsilon$$

ε relative strain defined in *2.3.3.1*

σ_{E1} Euler column buckling stress, in N/mm²:

$$\sigma_{E1} = \pi^2 E \frac{I_{E-net50}}{A_{E-net50} l_{stf}^2} 10^{-4}$$

Stress-strain Curves σ-ε (or Load-end Shortening Curves)

❖ Torsional buckling of stiffeners

- Euler torsional buckling with plasticity correction

σ_{C2} critical stress, in N/mm²:

$$\sigma_{C2} = \frac{\sigma_{E2}}{\varepsilon} \quad for \quad \sigma_{E2} \le \frac{\sigma_{yds}}{2}\varepsilon$$

$$\sigma_{C2} = \sigma_{yds}\left(1 - \frac{\sigma_{yds}\varepsilon}{4\sigma_{E2}}\right) \quad for \quad \sigma_{E2} > \frac{\sigma_{yds}}{2}\varepsilon$$

σ_{E2} Euler torsional buckling stress, in N/mm²

$\sigma_{E2} = \sigma_{ET}$

❖ Web local buckling of stiffeners with flanged profiles

- Effective breadth concept

$$\sigma_{CR3} = \Phi \frac{b_{eff-p} t_{net50} \sigma_{ydp} + \left(d_{w-eff} t_{w-net50} + b_f t_{f-net50}\right)\sigma_{yds}}{s t_{net50} + d_w t_{w-net50} + b_f t_{f-net50}}$$

$d_{w\text{-}eff}$ effective depth of the web, in mm:

$$d_{w-eff} = \left(\frac{2.25}{\beta_w} - \frac{1.25}{\beta_w^2}\right) d_w \quad \text{for} \quad \beta_w > 1.25$$

$$d_{w-eff} = d_w \quad \text{for} \quad \beta_w \le 1.25$$

β_w $$\frac{d_w}{t_{w-net50}}\sqrt{\frac{\varepsilon\sigma_{yds}}{E}}$$

Stress-strain Curves σ-ε (or Load-end Shortening Curves)

❖ Web local buckling of flat bar stiffeners

- Euler torsional buckling with plasticity correction

σ_{C4} critical stress, in N/mm²:

$$\sigma_{C4} = \frac{\sigma_{E4}}{\varepsilon} \qquad for \quad \sigma_{E4} \le \frac{\sigma_{yds}}{2}\varepsilon$$

$$\sigma_{C4} = \sigma_{yds}\left(1 - \frac{\sigma_{yds}\varepsilon}{4\sigma_{E4}}\right) \qquad for \quad \sigma_{E4} > \frac{\sigma_{yds}}{2}\varepsilon$$

σ_{E4} Euler buckling stress, in N/mm²:

$$\sigma_{E4} = 160000\left(\frac{t_{w-net\,50}}{d_w}\right)^2$$

❖ Web local buckling of flat bar stiffeners

$$\sigma_{CR5} = \min\begin{cases} \Phi\sigma_{ydp}\left[\dfrac{s}{1000 l_{stf}}\left(\dfrac{2.25}{\beta_p} - \dfrac{1.25}{\beta_p^2}\right) + 0.1\left(1 - \dfrac{s}{1000 l_{stf}}\right)\left(1 + \dfrac{1}{\beta_p^2}\right)^2\right] \\ \sigma_{ydp}\Phi \end{cases} \quad \text{N/mm}^2$$

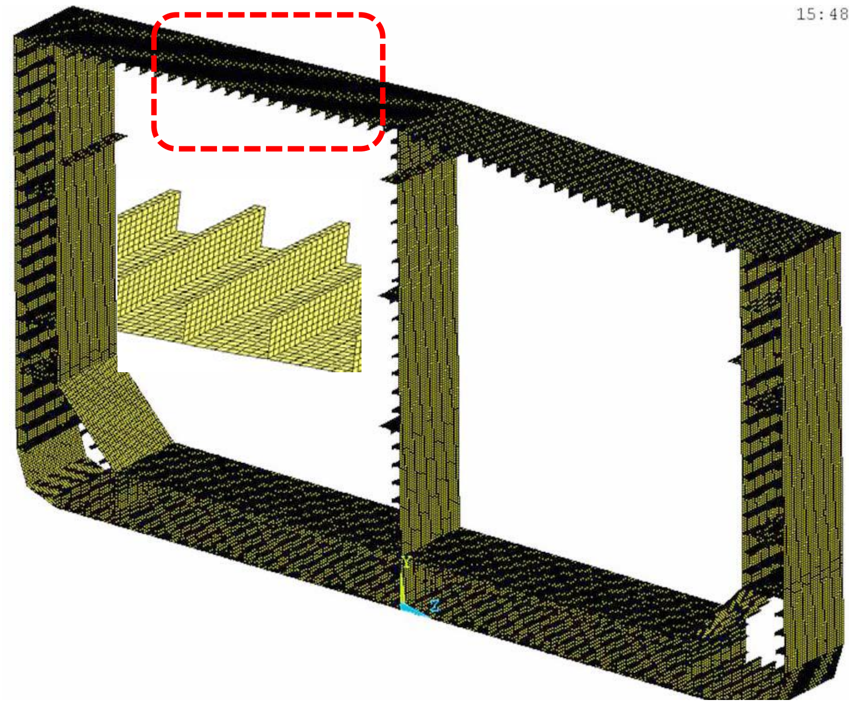
4. Alternative Methods

- **Considerations for alternative models**
- The bending moment-curvature relationship, M-κ, may be established by alternative methods. Such models are to consider all the relevant effects important to the non-linear response with due considerations of:
  a. non-linear geometrical behaviour
  b. inelastic material behaviour
  c. geometrical imperfections and residual stresses (geometrical out-of flatness of plate and stiffeners)
  d. simultaneously acting loads:
  e. bi-axial compression
  f. bi-axial tension
  g. shear and lateral pressure
  h. boundary conditions
  i. interactions between buckling modes
  j. interactions between structural elements such as plates, stiffeners, girders etc.
  k. post-buckling capacity.

4. Alternative Methods

❖ Non-linear finite element analysis

- FE models are to consider the relevant effects important to the non-linear responses with due consideration of the items listed in.
- Particular attention is to be given to modeling the shape and size of geometrical imperfections. It is to be ensured that the shape and size of geometrical imperfections trigger the most critical failure modes.

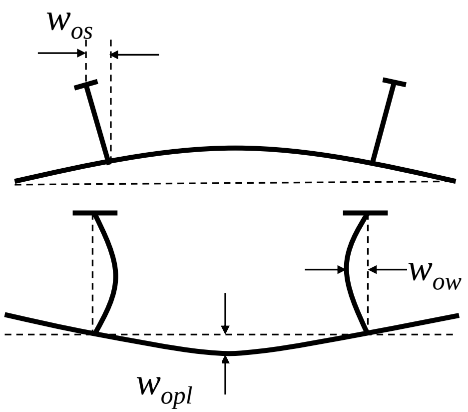


5. Nonlinear FE analysis

❖ Non-linear finite element analysis

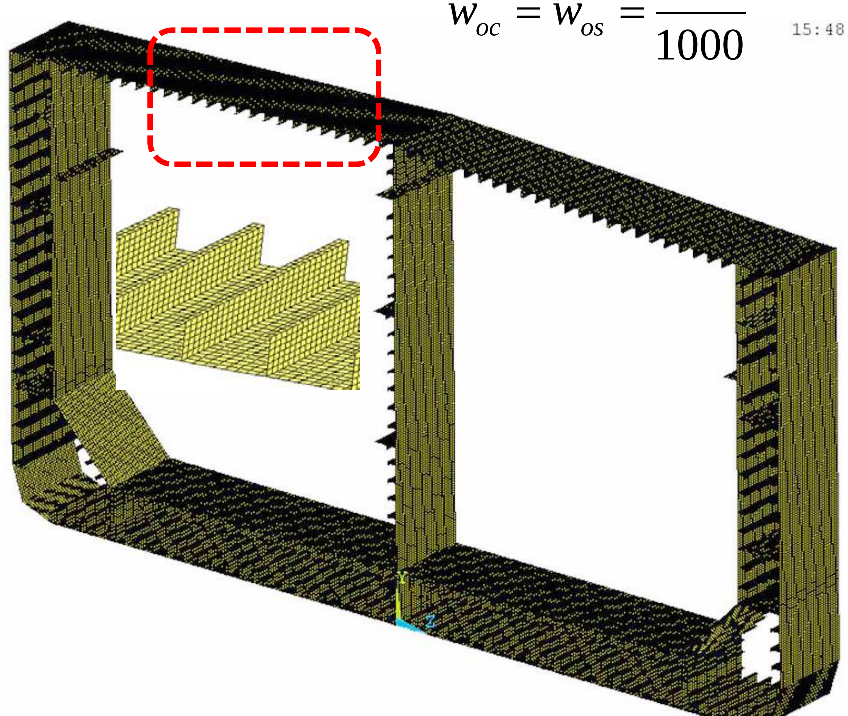
- 50% corrosion margin
- Initial deflection of plating of plating(w_{opl}) and stiffener web(w_{ow}), elastic buckling mode

$$w_{opl} = \frac{b}{200}, w_{ow} = \frac{h_w}{200}$$



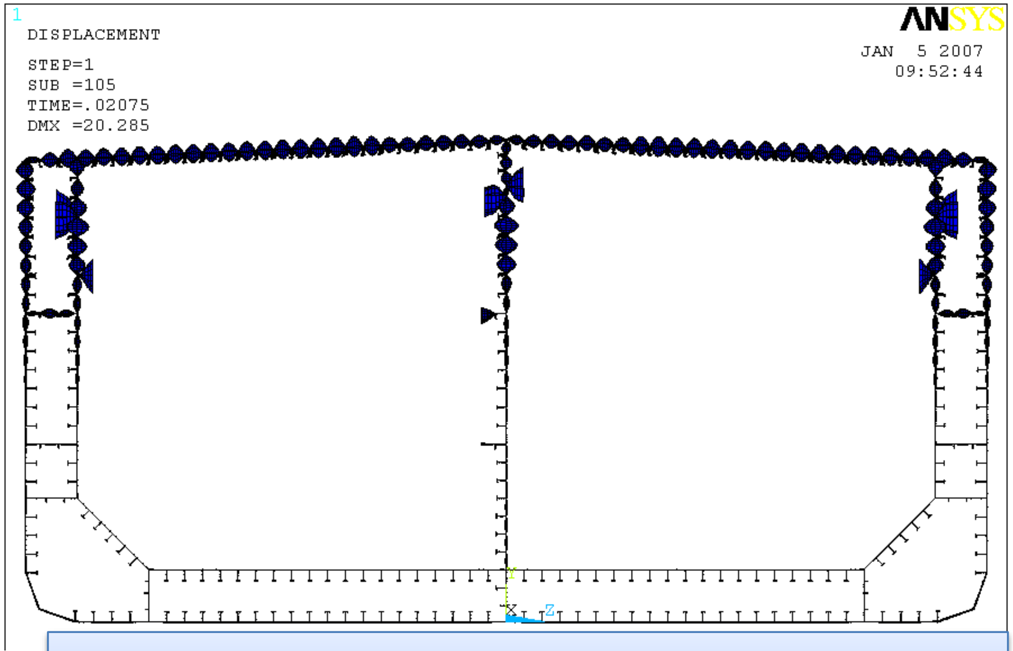
- the fabrication related initial distortions of stiffeners

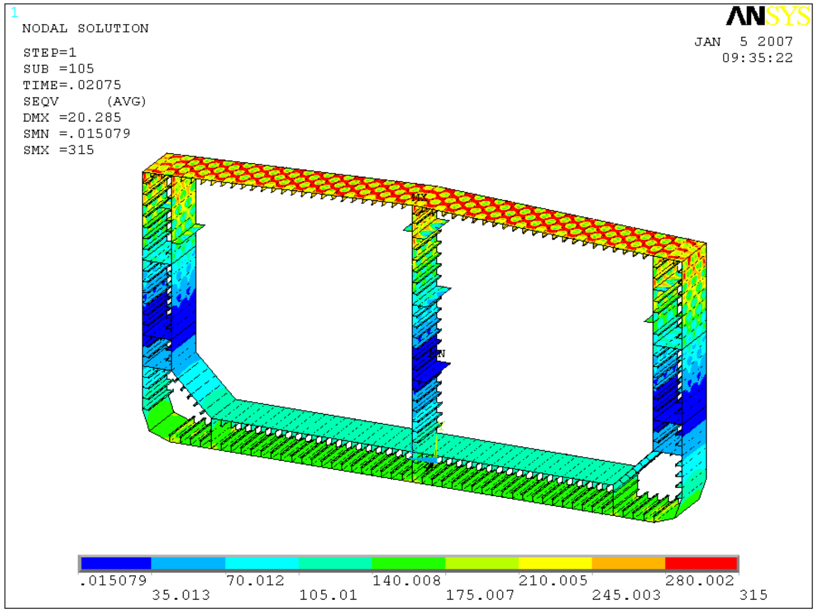
$$w_{oc} = w_{os} = \frac{a}{1000}$$



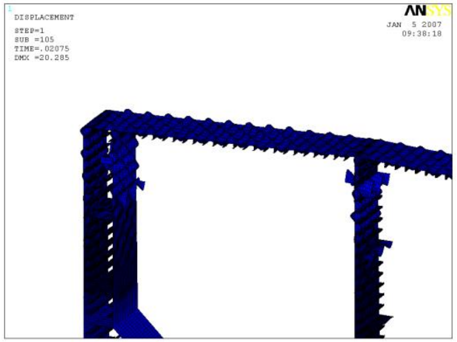
- buckling collapse may take place in vertical members of the hull structure until the hull girder reaches the ultimate limit state → fine mesh modeling for vertical member
- Changing neutral axis of the hull cross-section due to the progressive collapse of individual structural components is to be considered.

5. Nonlinear FE analysis



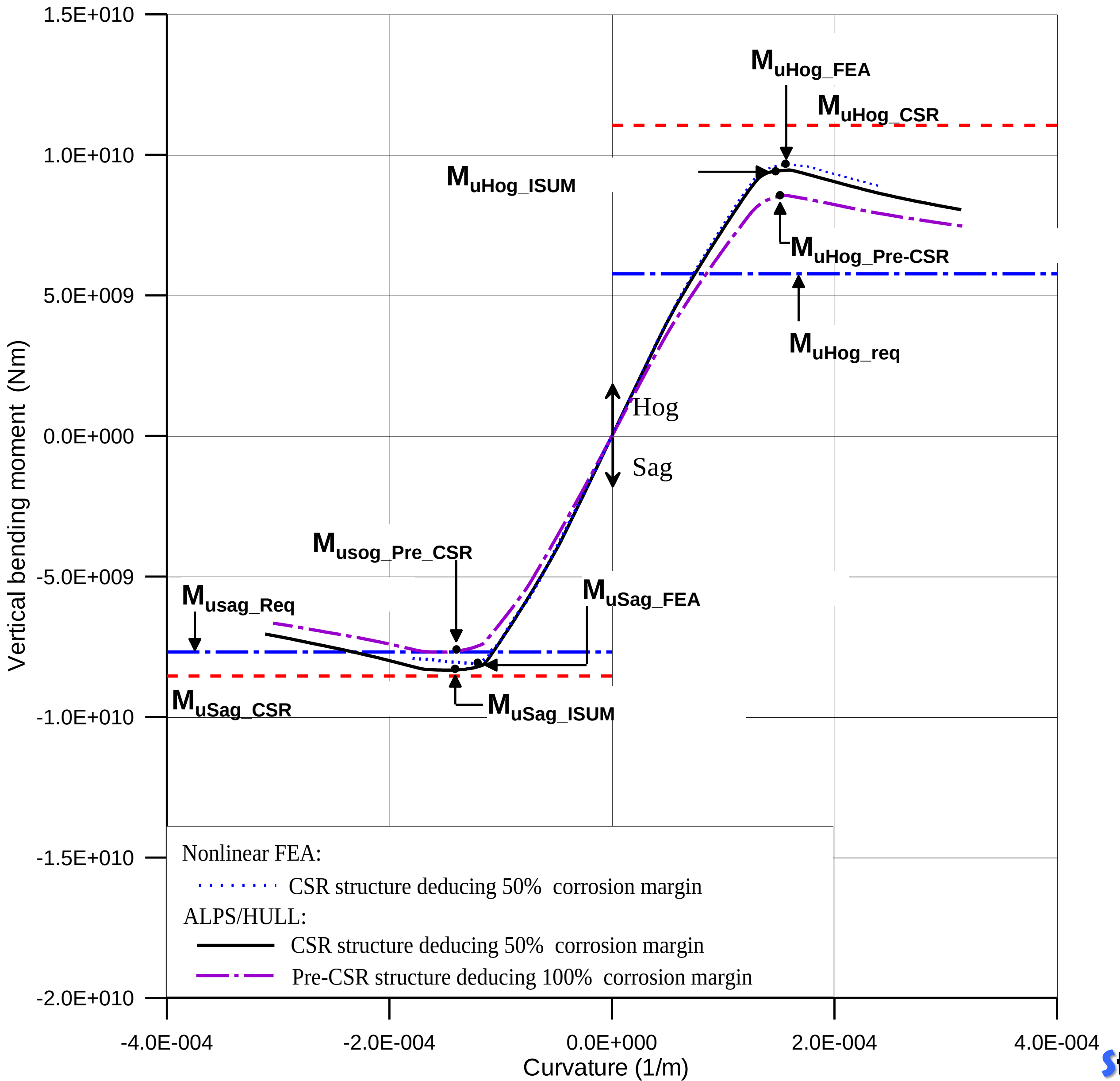


Deformed shape of the hull at the ultimate limit state under sagging moment



CSR – Calculation of hull girder ultimate capacity

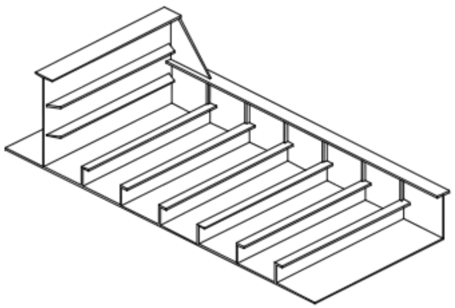
FE analysis results



Structural Lab

ISUM Modeling strategy

- Ueda & Rashed (1974, 1984) suggested
- Idealized Structural Unit Method (ISUM)
- Several different types of ISUM units
  - the beam-column unit
  - the rectangular plate unit
  - the stiffened panel unit



A typical steel plated structure

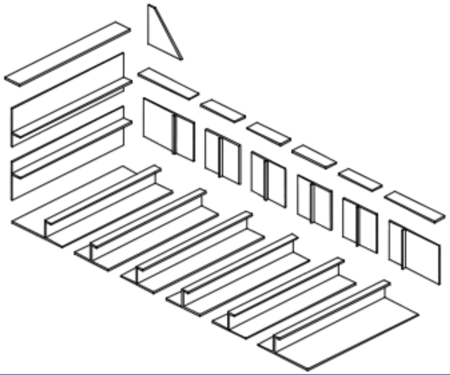


Plate-stiffener combination units

When stiffeners are relatively weak and strong,

- Plate → the rectangular plate unit
- Stiffeners without attached plating →beam-column unit

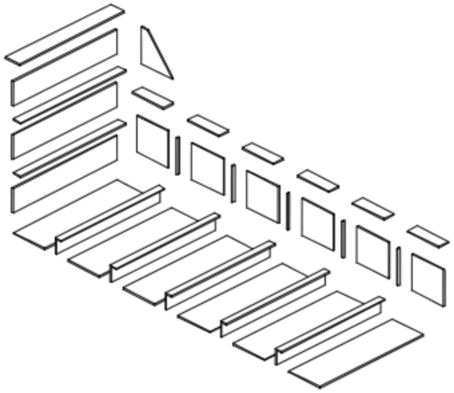
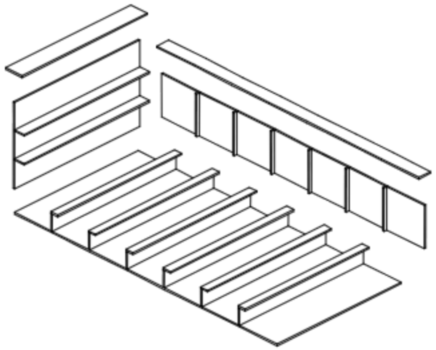


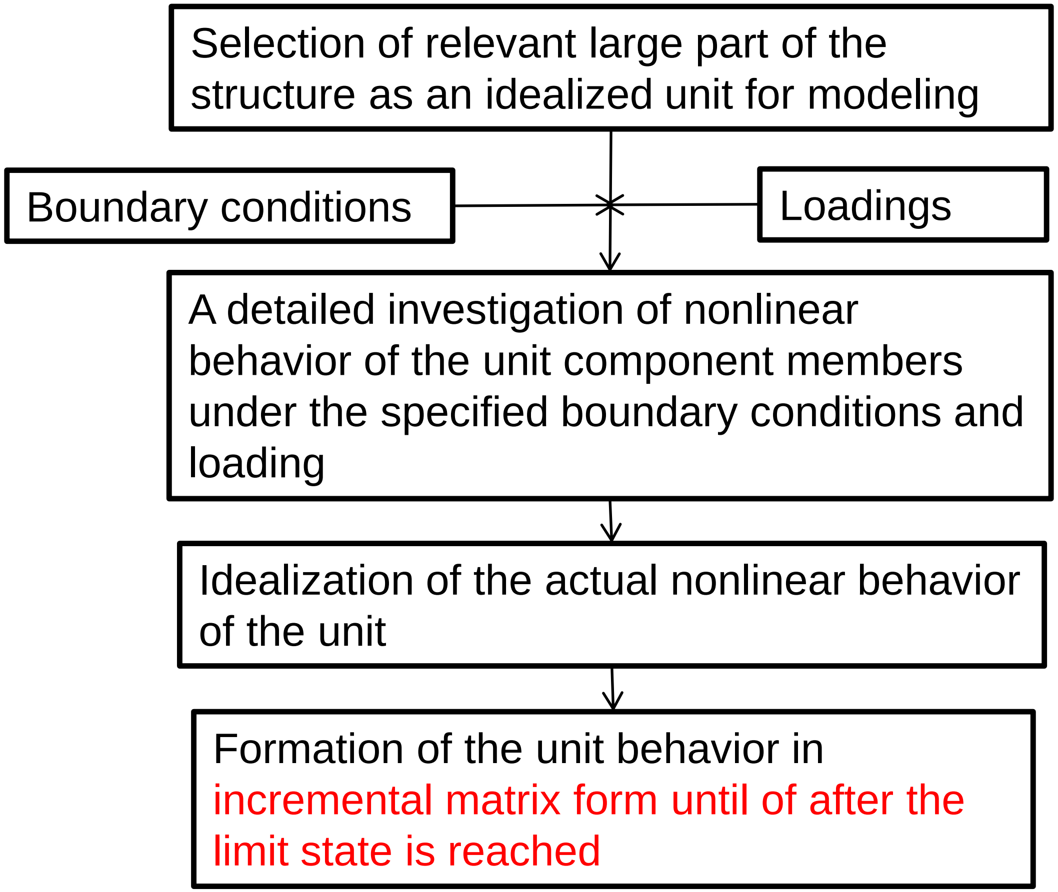
Plate-stiffener Separation units



Assembly of stiffened panels

ISUM Units Development Procedure

- ISUM units(elements) are used within the framework of the nonlinear matrix displacement procedure, by applying the incremental method, much like in the case of conventional FEM

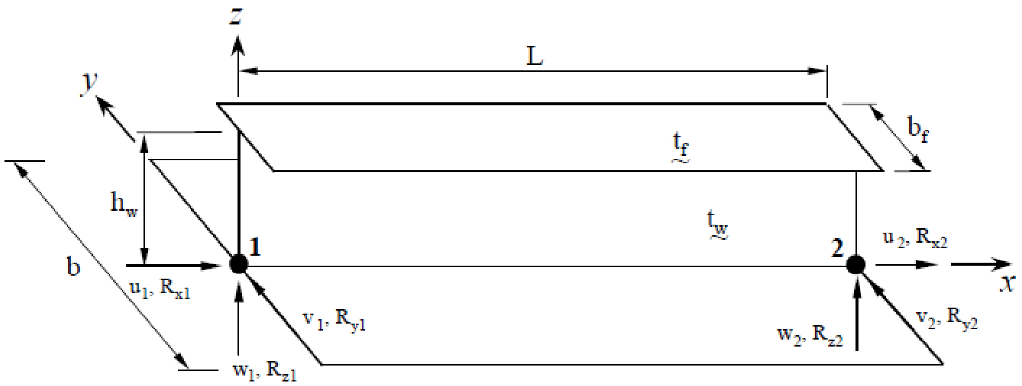


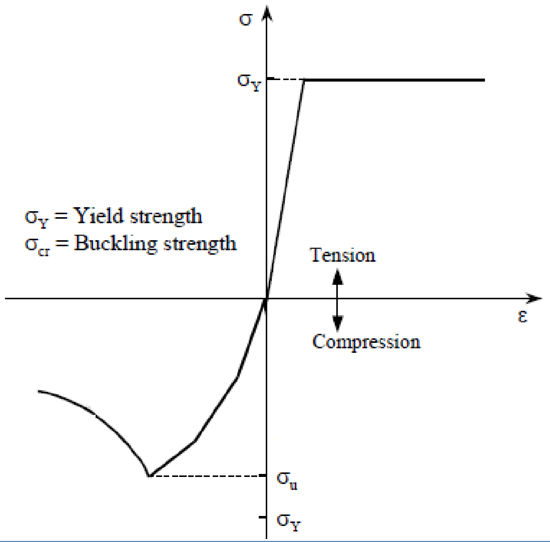
Procedure for the development of an idealized structural unit

"The behavior will differ depending on User's insight and knowledge"

ISUM Beam-Column Unit

- Four nodes, three translational DOF





Idealized structural behavior of the ISUM beam – column unit

$$\{\Delta R\} = \{\Delta R_{x1}\ \ \Delta R_{y1}\ \ \Delta R_{z1}\ \ \Delta R_{x2}\ \ \Delta R_{y2}\ \ \Delta R_{z2}\}^T$$

$$\{\Delta U\} = \{\Delta u_1\ \ \Delta v_2\ \ \Delta w_1\ \ \Delta u_2\ \ \Delta v_2\ \ \Delta w_2\}^T$$

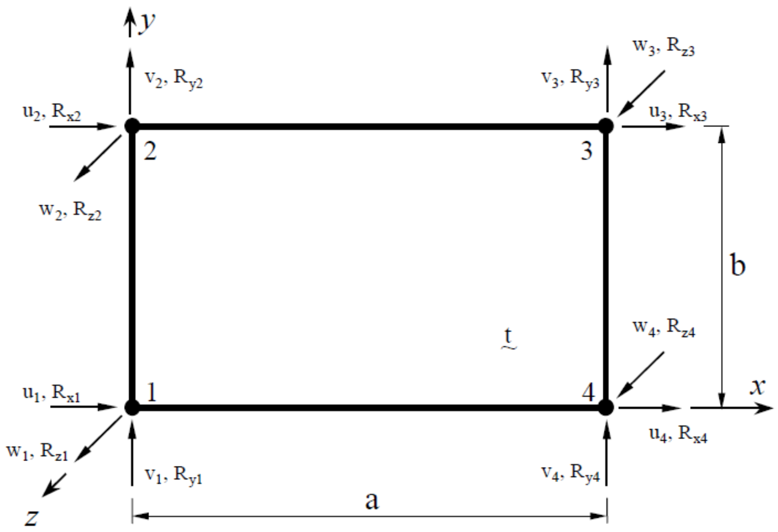
$$\{\varDelta R\} = [K]\{\varDelta U\}$$

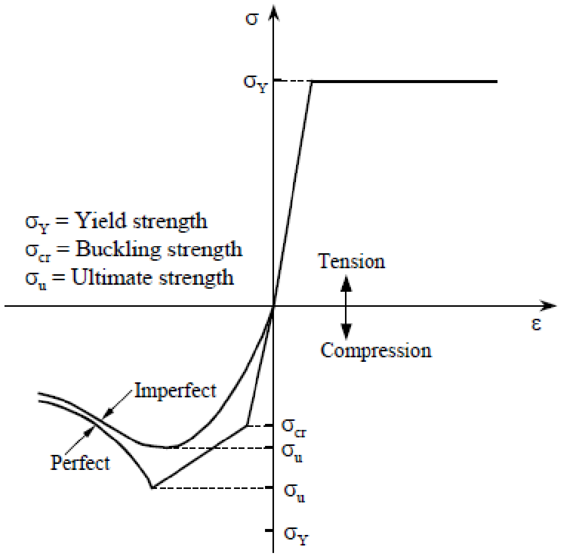
$\{\varDelta R\}$= nodal force increment vector

$\{\varDelta U\}$= nodal displacement increment vector

$[K]$: tangent stiffness matrix

ISUM Rectangular Plate Unit





$$\{\Delta R\} = \{\Delta R_{x1}\ \Delta R_{y1}\ \Delta R_{z1}\ \ldots \Delta R_{x4}\ \Delta R_{y4}\ \Delta R_{z4}\}^T$$

$$\{\Delta U\} = \{\Delta u_1\ \Delta v_2\ \Delta w_1\ \ldots \Delta u_4\ \Delta v_4\ \Delta w_4\}^T$$

Idealized structural behavior of the ISUM rectangular unit

$$\{\varDelta R\} = [K]\{\varDelta U\}$$

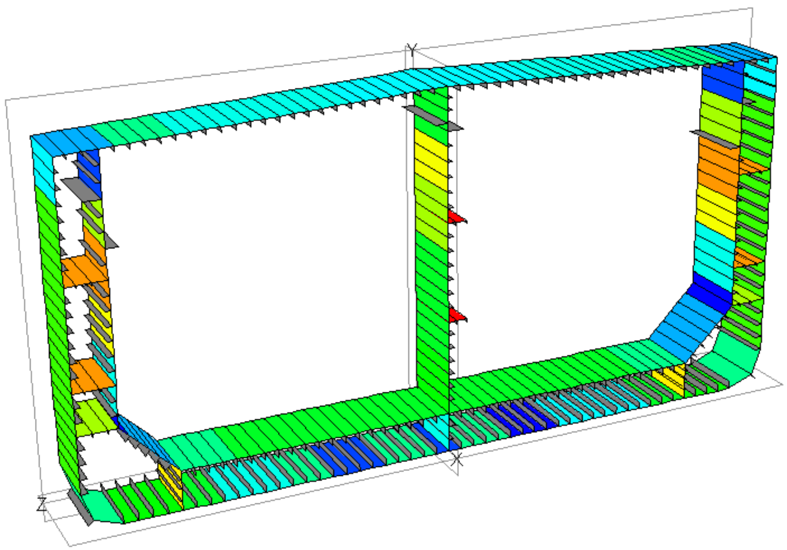
$\{\varDelta R\}$= nodal force increment vector

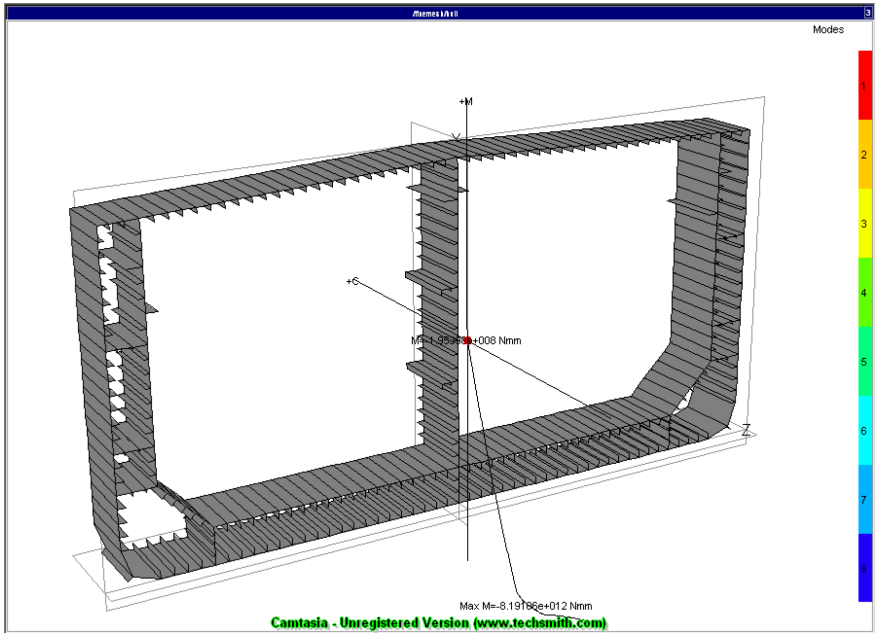
$\{\varDelta U\}$= nodal displacement increment vector

$[K]$: tangent stiffness matrix

ISUM Method

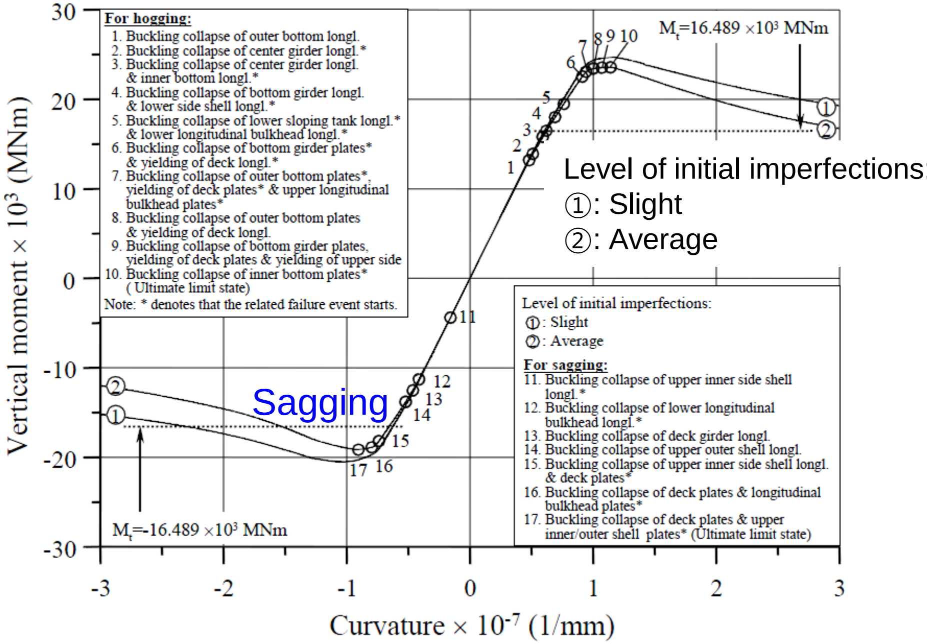
- Modeling using Beam-Column Unit and Rectangular Plate Unit

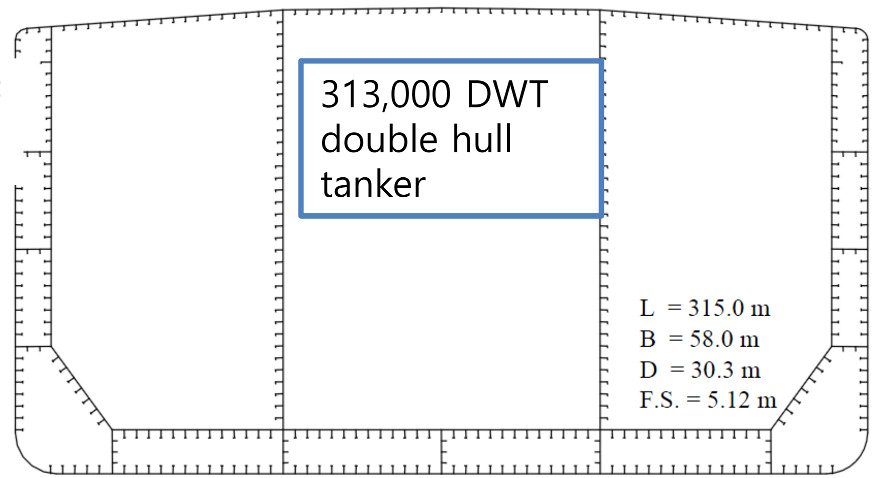




Double Hull Tanker Example

- The collapse of the compression flange of the tanker hulls takes place prior to the yielding of the tension flange as in design of usual ship structures



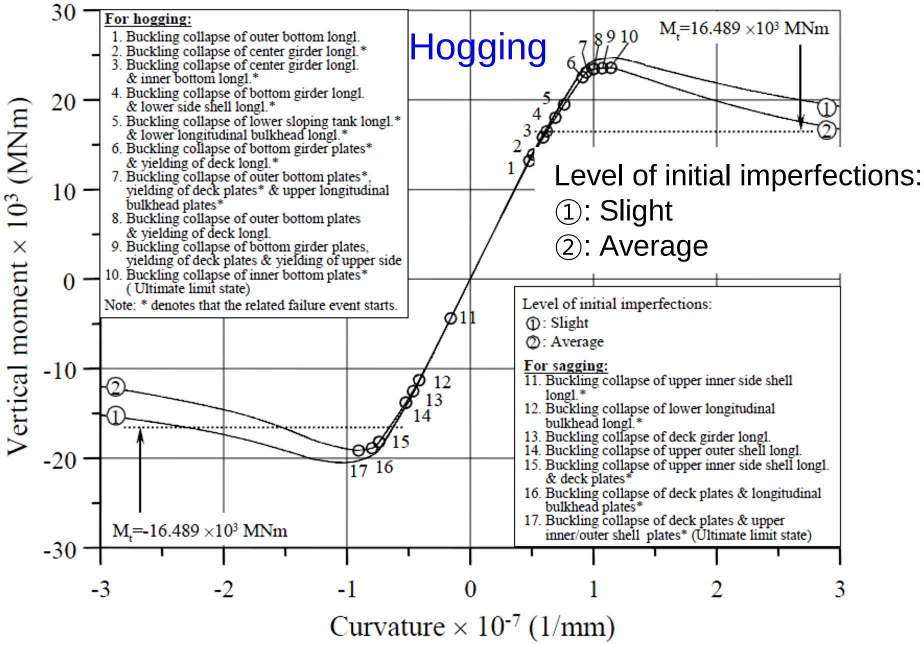


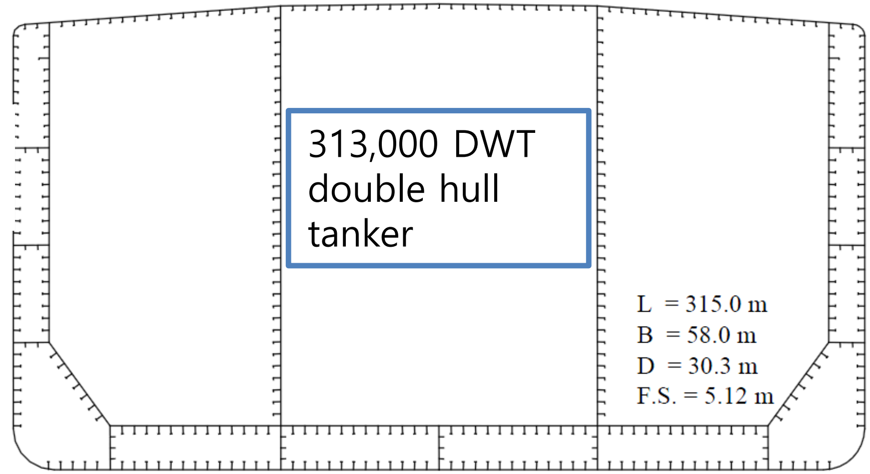
For sagging:

11. Buckling collapse of upper inner side shell longl.*
12. Buckling collapse of lower longitudinal bulkhead longl.*
13. Buckling collapse of deck girder longl.
14. Buckling collapse of upper outer shell longl.
15. Buckling collapse of upper inner side shell longl. & deck plates*
16. Buckling collapse of deck plates & longitudinal bulkhead plates*
17. Buckling collapse of deck plates & upper inner/outer shell plates* (Ultimate limit state)

Note: * denotes that the related failure event starts.

Double Hull Tanker Example





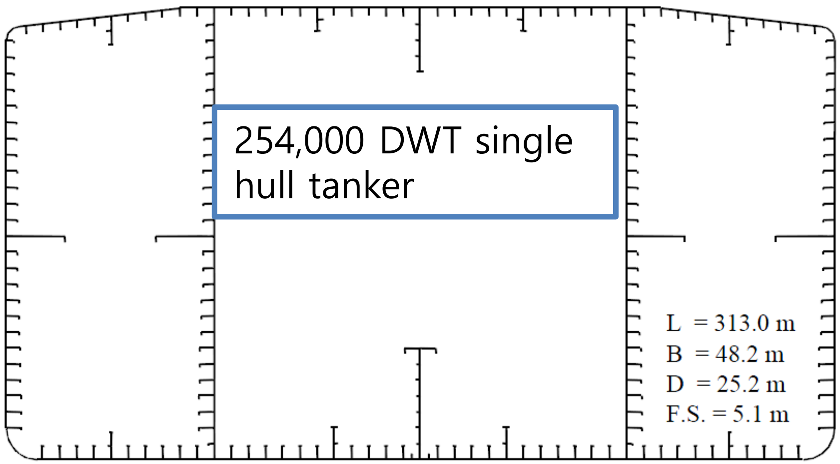
For Hogging:

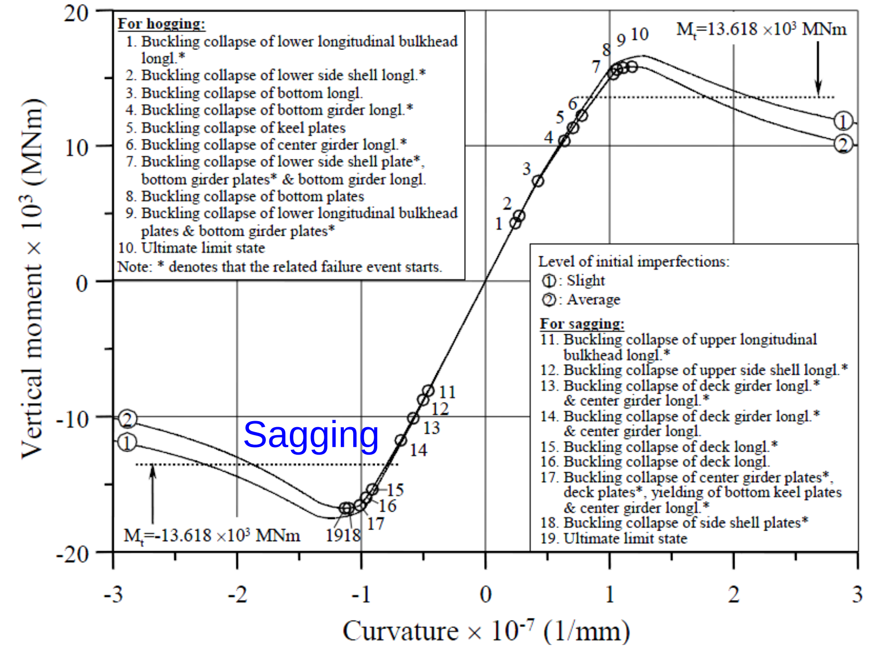
1. Buckling collapse of outer bottom longl.
2. Buckling collapse of center girder longl.*
3. Buckling collapse of center girder longl. & inner bottom longl.*
4. Buckling collapse of bottom girder longl. & lower side shell longl.*
5. Buckling collapse of lower sloping tank longl.* & lower longitudinal bulkhead longl.*
6. Buckling collapse of bottom girder plates* & yielding of deck longl.*
7. Buckling collapse of outer bottom plates*, yielding of deck plates* & upper longitudinal bulkhead plates*
8. Buckling collapse of outer bottom plates & yielding of deck longl.
9. Buckling collapse of bottom girder plates, yielding of deck plates & yielding of upper side
10. Buckling collapse of inner

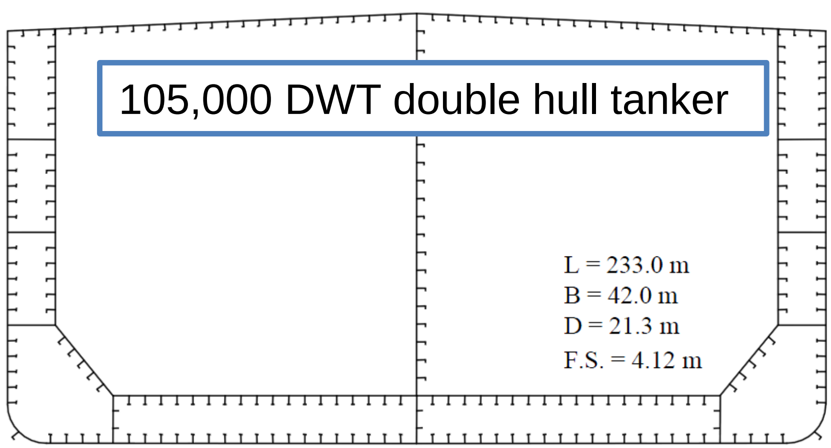
Note: * denotes that the related failure event starts.

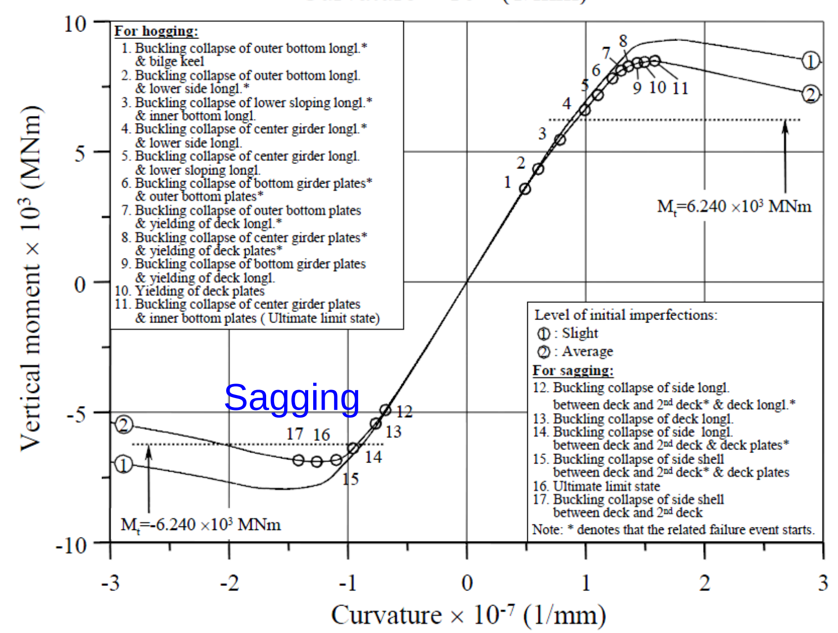
ISUM Method

Other Examples









Other Examples