

# Beginning

# Lecture 1

Equation from Newton's 2nd law

$$f = ma$$

force      mass      acceleration

But precisely

$$f \sim ma \quad (\text{It only gives proportionality})$$

$$\text{the constant of proportionality} = \frac{1}{g_c}$$

depend  
on unit.

Check unit!

$$[F] = \left[ \frac{M \cdot L}{T^2} \right]$$

example

$$1 \text{ Newton} = \frac{1 \text{ kg} \times 1 \text{ m}}{1 \text{ s}^2}$$

However,

$$1 \text{ kg}_f = \text{ gravity force on } 1 \text{ kg}_m \text{ mass}$$
$$= 9.8 \text{ N} \quad (\in 1 \text{ kg}_m \times 9.8 \text{ m/s}^2)$$

$$1 \text{ lbf} = \text{ gravity force on } 1 \text{ lb}_m \text{ mass}$$
$$= 32.2 \frac{\text{lb}_m \text{ ft}}{\text{s}^2} \quad (\in 1 \text{ lb}_m \times 32.2 \frac{\text{ft}}{\text{s}^2})$$

therefore, in general

$$f = \frac{1}{g_c} m a$$

$$g_c = 1 \quad \text{for N - m - s - kg}_m$$

$$= 9.8 \quad \text{for kg}_f - m - s - kg_m$$

$$= 32.2 \quad " \quad \text{lbf} - m - s - lb_m$$

In this lecture, I will use  
 $f = ma$ . but please keep in  
mind that  $f = \frac{1}{g_c} ma$  !

(You need to be careful  
on using units.)

# Fluid mechanics

## Lecture 1.

- Definition of fluid
  - (a) cannot support no shear stress in a state of static equilibrium
  - (b) deform continuously when subjected to any shear stress, no matter how small.
- Static equilibrium

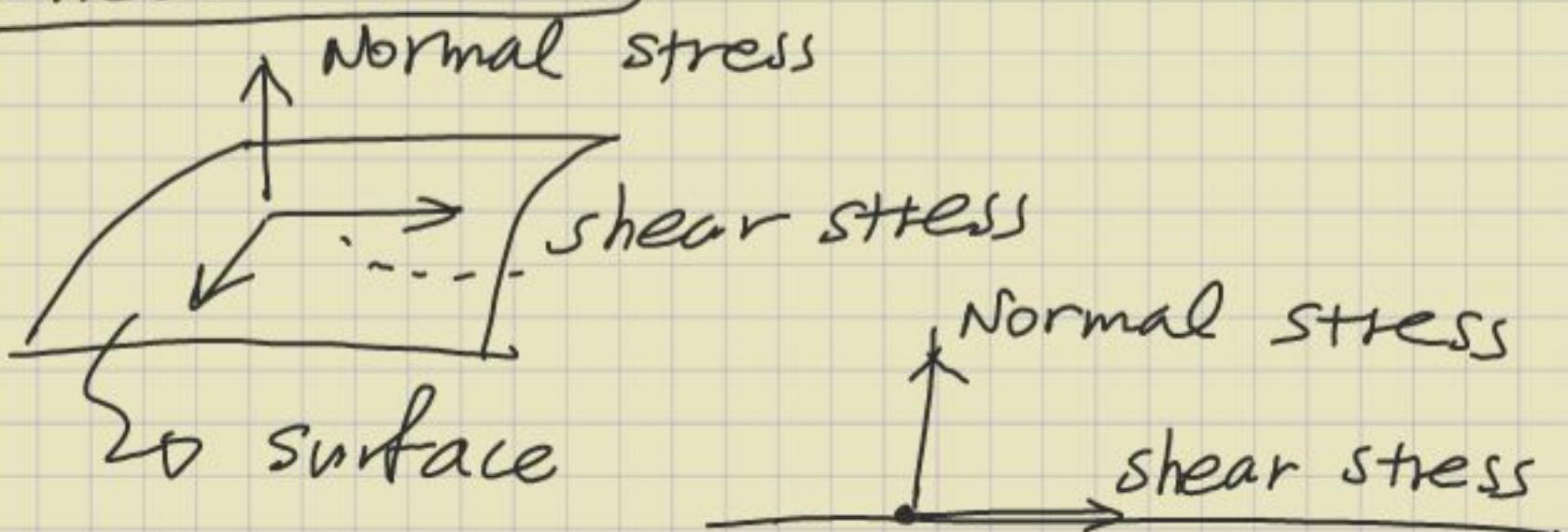
A state in which there is no movement of one part of a system related to any other part.  
However, the whole system can be in motion in solid body translation or rotation

- Normal & shear stress

Take a small, arbitrary oriented  
surface within a fluid

and consider the stress ( $\frac{\text{force}}{\text{Area}}$ )  
exerted by the fluid on  
one side of the surface on  
the fluid on the other side.

The component of the stress  
perpendicular to the surface  
is the normal stress, and  
the component parallel to the surface  
is shear stress.



cf)

$$\text{traction} = \left[ \frac{\text{force}}{\text{area}} \right] \dots \text{vector}$$

$$\text{stress} = \left[ \frac{\text{force}}{\text{area}} \right] \dots \text{tensor'}$$



→ Sometimes "state of stress" is used to emphasize that a given quantity is a tensor.

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show a slide for

Continuum theory

- Continuum --- a way to look at  
 filled a certain space      some mat'l  
Properties can vary at diff. position

For fluid mechanis

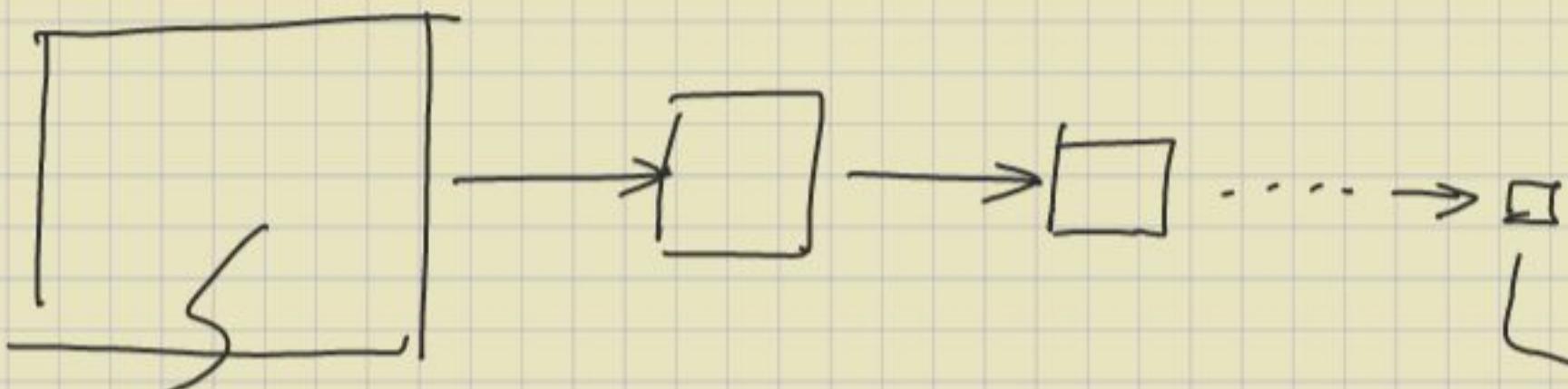
based on Continuum theory

- ① flow dimension  $\gg$  mean free path of molecules  
 ( $> \mu\text{m}$ )
- ② time scale  $\gg$  molecular vibration or collision  
 ( $> \text{ms}$ )

### • Concept of scale

ex) microscale flow vs macroscopic flow  
 (Sneezing)      (hurricane)

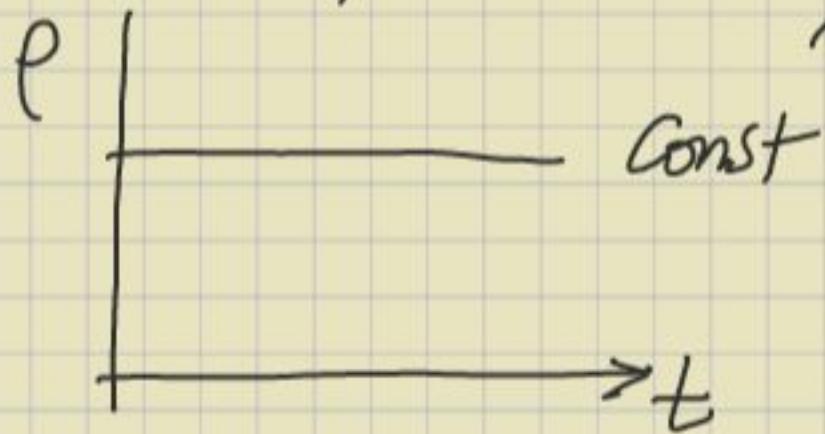
- Continuum vs. molecular theory
  - ① economic (easy to handle)
  - ② valid only for "large" length scale



Homogeneous

assumption (local equilibrium hypothesis)

introduced by Gibbs



at some point



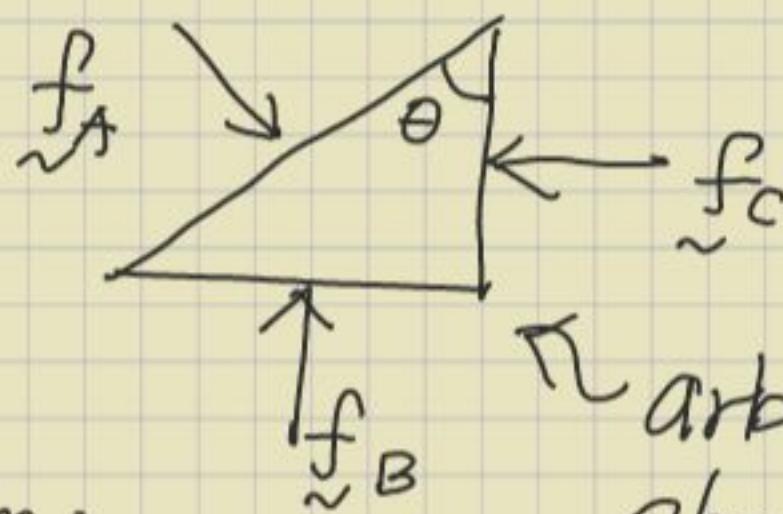
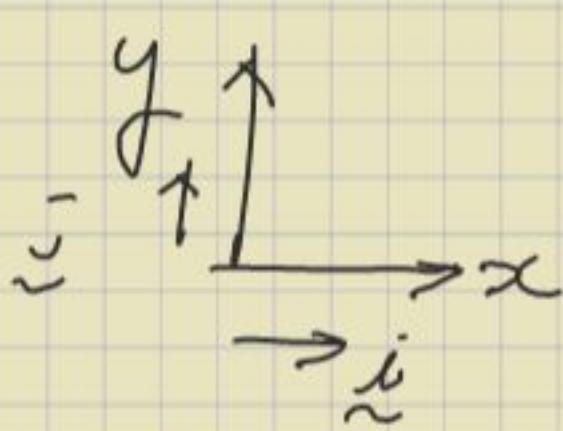
→ When it close to  
molecular level

## Pressure

It is a stress ( $\frac{\text{force}}{\text{area}}$ ) to surface  
; force that act perpendicular

In hydrostatics (at equilibrium)

Consider "Control volume"



At equilibrium:

arbitrary  
chosen  
Control volume

$$\sum \underline{f} = 0 \quad \underline{f} = \underline{i} \underline{f}_x + \underline{j} \underline{f}_y$$

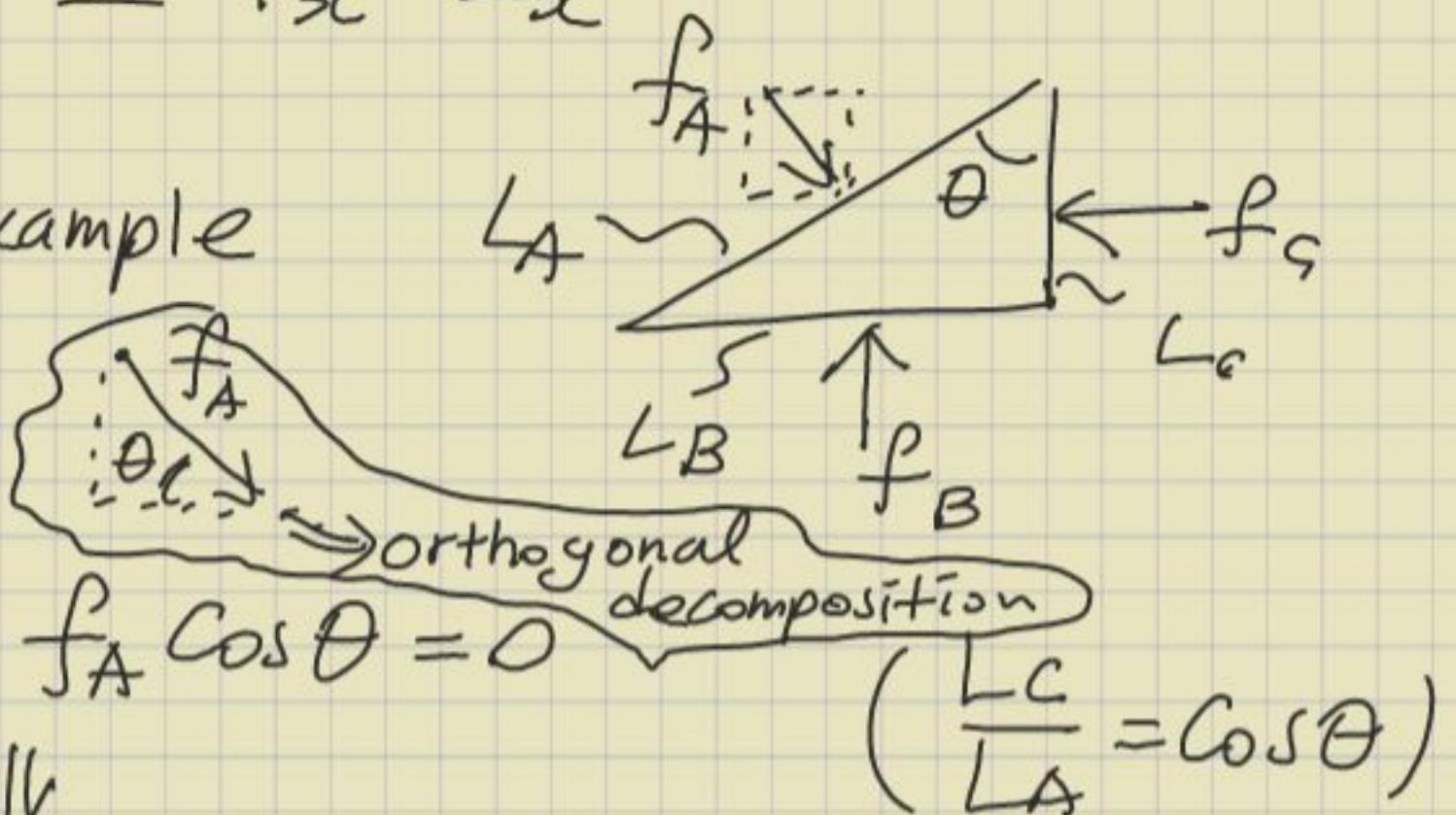
$$\underline{i} (\sum f_x) + \underline{j} (\sum f_y) = 0$$

$$\Rightarrow \sum f_x = 0 \quad \sum f_y = 0$$

$$\text{pressure} = \left[ \frac{\text{force}}{\text{area}} \right]$$

$$\sum f_x = \sum P_{xc} \cdot L_x$$

In this example



$$-P_C L_C + P_A L_A \cos \theta = 0$$

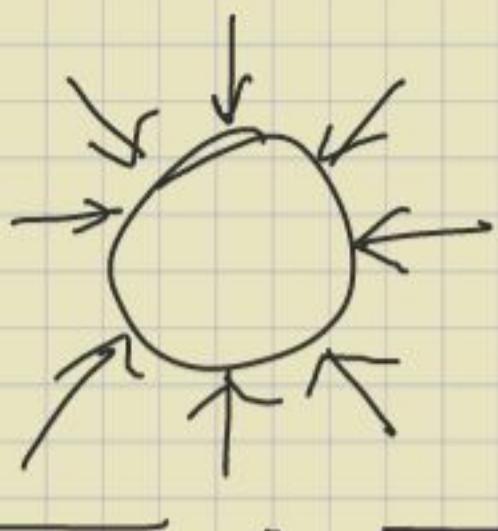
$$-P_C L_C + P_A L_C = 0$$

↓

$$\underline{P_A = P_C !}$$

It is a characteristic of pressure

: pressure around a "point"  
 (small element in continuum)  
 is the same in all direction.



$\Rightarrow \underline{\text{Isotropic!}}$

- density  $\left[ \frac{\text{mass}}{\text{volume}} \right] \frac{\text{kg}}{\text{m}^3}$

e.g.  $\rho_{\text{water}} \sim 1000 \text{ kg/m}^3$   
near atm

In general.

$\rho_{\text{liq}} \sim$  weak fn of pressure

$\rho_{\text{gas}} \sim$  strong fn of "  $\frac{\text{mass}}{\text{volume}}$ "

c.f.) Ideal gas law

$$PV = nRT$$

$$\rho = \frac{nM_w}{V}$$

$\therefore \frac{\rho}{M_w} \sim \frac{1}{V}$

$$\rightarrow P = \frac{n}{V} RT = \frac{\rho}{M_w} RT$$

Typically,  $\rho_{\text{gas}} \sim 1 \text{ kg/m}^3$

- Some important material properties related to density.

### ① Isothermal Compressibility

$$\beta = -\frac{1}{V} \left( \frac{\partial V_s}{\partial P} \right)_T$$

↑  
fix temp.

specific volume  
(Volume/mass)

$$\begin{cases} \text{liq: } \beta \uparrow \\ \text{gas: } \beta \downarrow \end{cases}$$

$$\beta = \frac{1}{\rho} \quad \begin{matrix} \text{for} \\ \text{ideal gas} \end{matrix}$$

### ② Coefficient of thermal expansion

$$\alpha = \frac{1}{V_s} \left( \frac{\partial V_s}{\partial T} \right)_P$$

$$[V_s \sim \frac{1}{\rho}]$$

↑  
fix pressure

In moderate  
temperature range

— — - reference density  
at  $T_0$

$$\rho = \rho_0 [1 - \alpha (T - T_0)]$$

- Specific gravity

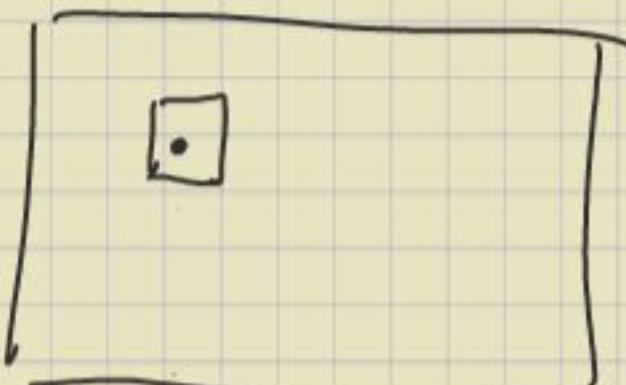
$$S = \frac{\rho}{\rho_{sc}}$$

usually reference  
Substance is used  
( $H_2O$ )

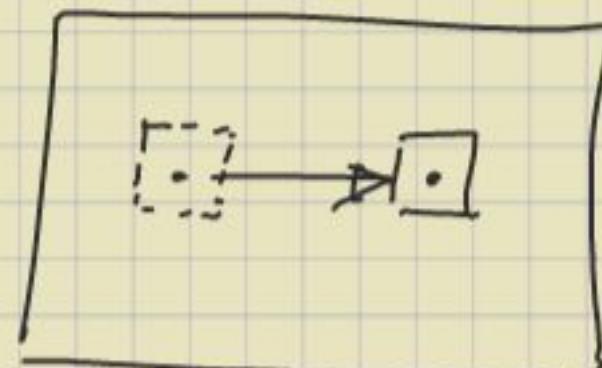
measured at  
standard Cond'n  
STP (273.15K  
1bar  
(100kPa)

## Velocity

= rate of change of position of  
a fluid particle.



at  $t = t_1$ ,



at  $t = t_2$

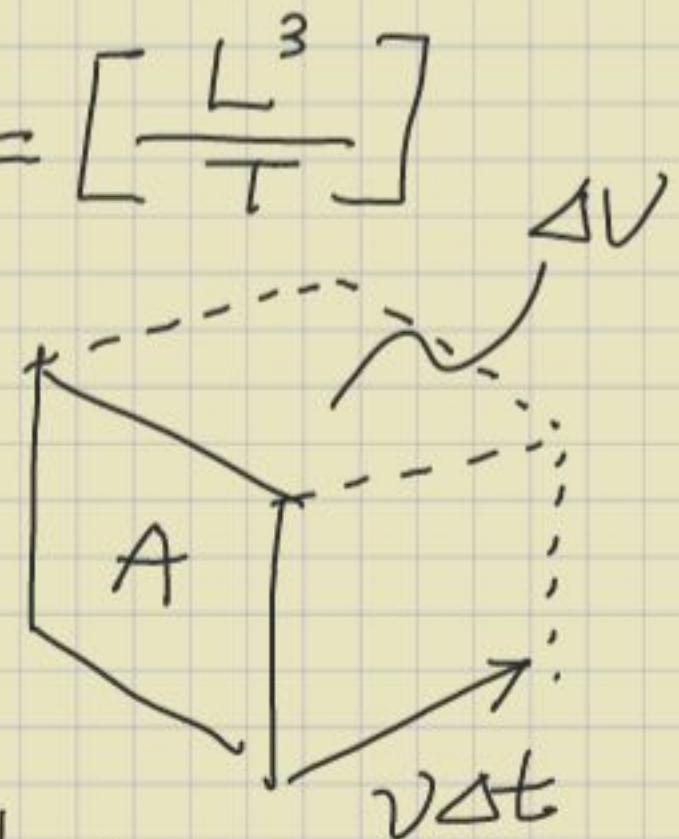
flow rate (rate of change  
caused by flow)  
 $\frac{\Delta \text{---(change)}}{\text{time}}$

$$\textcircled{1} \text{ volumetric flow rate} = \left[ \frac{L^3}{T} \right]$$

$$Q = v \times A$$

velocity  $\times$  area

$$= \left[ \frac{L}{T} \right] = [L^2]$$



volume change during  $\Delta t$

$$= \Delta V = (v\Delta t) \times A$$

$$\therefore \frac{\text{volume change}}{\Delta t} = Q = \frac{\Delta V}{\Delta t} = VA$$

velocity  $\sim$  volumetric flow rate  
area

② mass flow rate

$$\dot{m} = \rho Q = \left[ \frac{M}{L^3} \cdot \frac{L^3}{T} \right] = \left[ \frac{M}{T} \right]$$
$$= \rho v A$$

③ momentum flow rate

$$\dot{M} = \dot{m} v = \rho Q v = \rho v^2 A$$

cf) • momentum = mass  $\times$  velocity  
 $= m v$

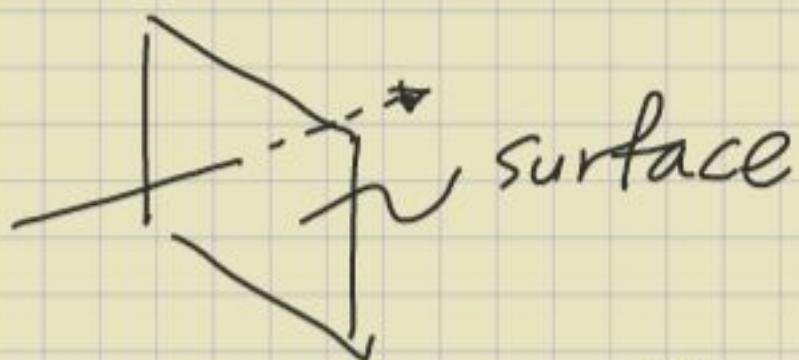
∴ momentum per unit mass

$$= \frac{m v}{m} = \textcircled{v}$$

↓  
another  
interpretation  
of velocity

∴  $\dot{M} = (\text{mass flow rate}) \times (\text{momentum per unit mass})$

flux: a rate of flow of a property per unit area



$$\text{flux} = \frac{\Delta}{(\text{area})(\text{time})}$$

① volumetric flux

$$\frac{Q}{A} = v \quad (= \text{velocity})$$

$$= \left[ \frac{L^3}{T} \cdot \frac{L}{L^2} \right] = \left[ \frac{L}{T} \right]$$

② mass flux

$$\frac{m}{A} = \rho v$$

③ momentum flux

$$\frac{\vec{m}}{A} = \rho \vec{v}^2$$

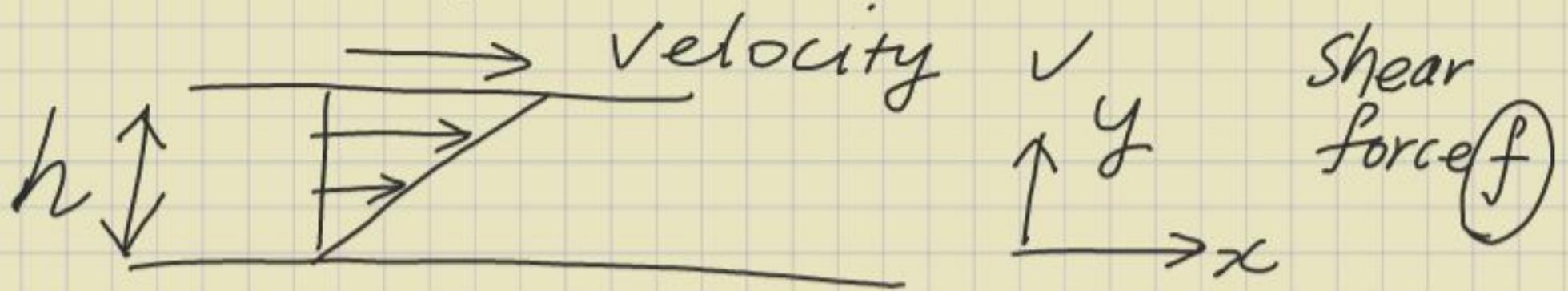
Note) force = rate of momentum change

$$= ma = m \frac{dv}{dt}$$

$$= \frac{d(mv)}{dt} = \frac{d \text{ momentum}}{dt}$$

when  $\rightarrow$  mass does not change

■ Viscosity ("resistance to flow")

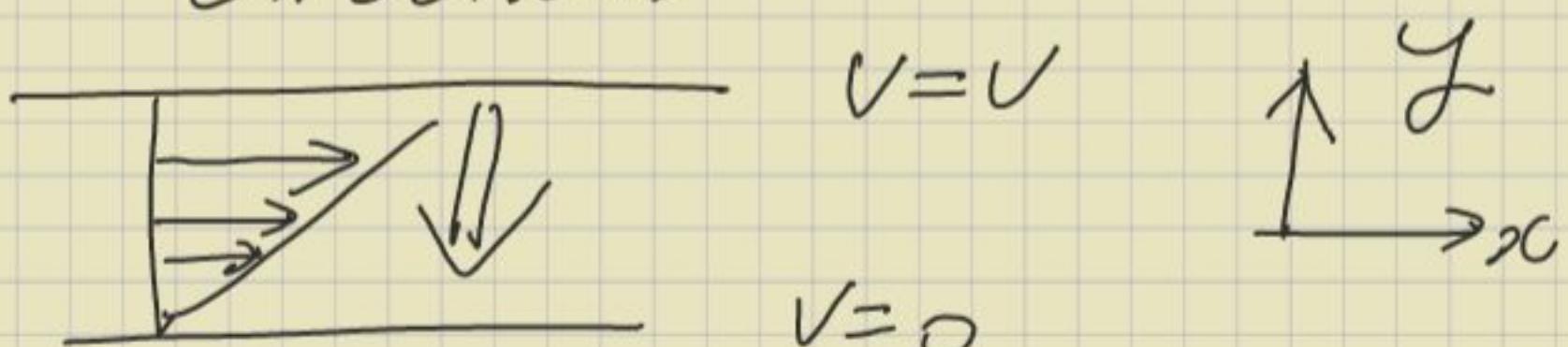


$$\tau = \frac{f}{A}$$

rate of velocity change

along direction

(momentum flux transfer direction)



$$\tau = \mu \frac{v}{h} = \mu \frac{\partial v}{\partial y}$$

If  $\mu$  is independent of  $\frac{v}{h}$

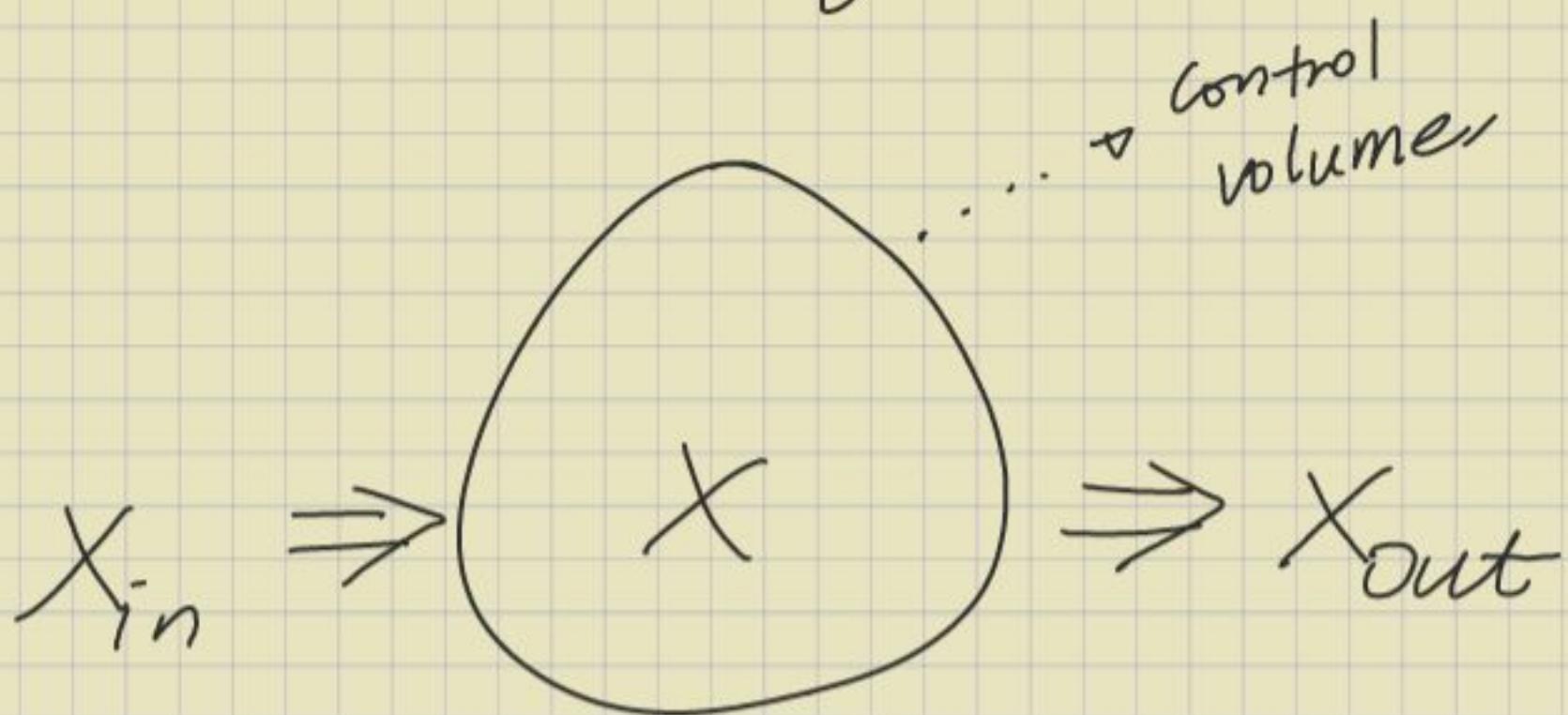
fluid is Newtonian

Otherwise it is non-Newtonian

\* liquid :  $\ln \mu = a + b \ln T$   
usually negative  
 $\mu = \mu_0 e^{a + b \ln T}$

\* Gas :  $\mu = \mu_0 \left( \frac{T}{T_0} \right)^n$   
 $n = 0.5 \sim 1$

# Balance equation



$$\frac{dX}{dt} = X_{in} - X_{out} + X_{gen} - X_{con}$$

X: conserved quantity.