445.204

Introduction to Mechanics of Materials

(재료역학개론)

Chapter 2: Stress

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- Basic terminology
- Definition of stress
- Stress tensor
- Symmetry of stress tensor
- Equilibrium equation

Statically determinate or indeterminate

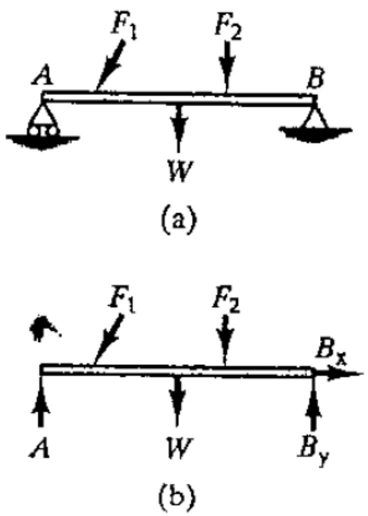


Figure 2.1. Statically determinate beam.

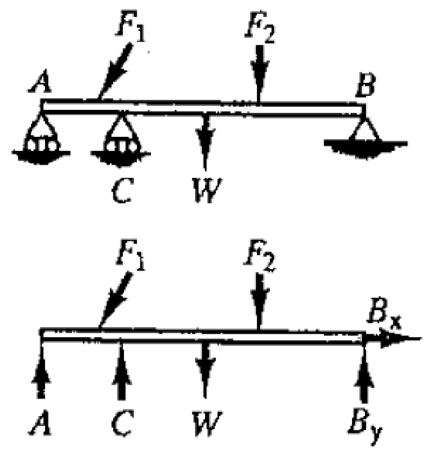


Figure 2.2. Statically indeterminate beam.

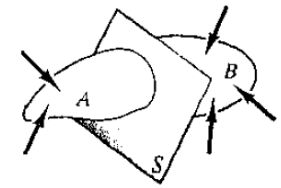


Figure 2.3. Body with a mathematical cut.

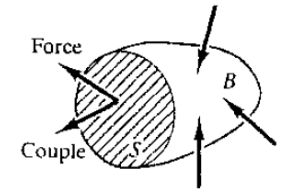


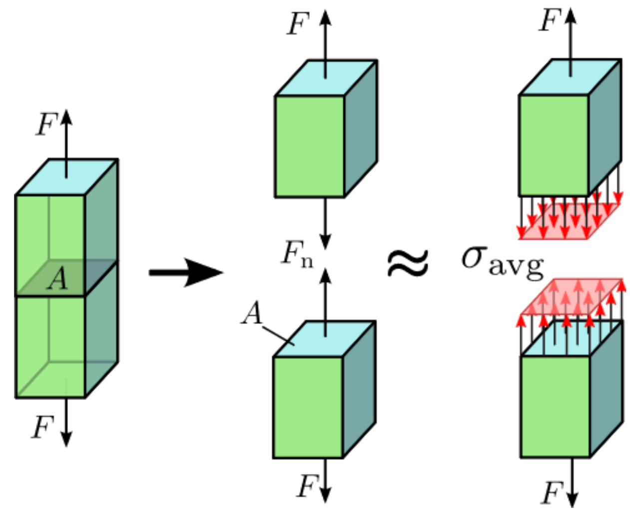
Figure 2.4. Free body showing cut surface.

- Free body diagram
- Statistically determinate: can be solved (in terms of force components) by only Newton’s law
- Statistically indeterminate: cannot be readily solved by only Newton’s law + deformation

Stress

Stress is the force per unit area on a body that tends to cause it to change shape.
Stress is a measure of the internal forces in a body between its particles. These internal forces are a reaction to the external forces applied on the body that cause it to separate, compress or slide. External forces are either surface forces or body forces. Stress is the average force per unit area that a particle of a body exerts on an adjacent particle, across an imaginary surface that separates them. (Wikipedia)

- Force acting on unit area
- Unit: $N/m^2 = Pa$



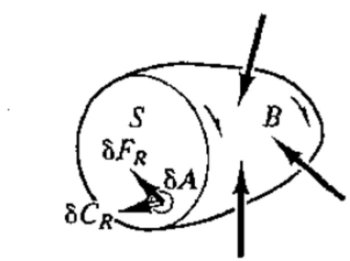


Figure 2.5. Force system on δA.

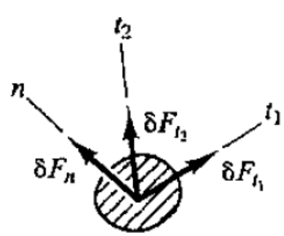


Figure 2.6. Rectangular components of force on δA.

$$\tau_n = \lim_{\delta A \to 0} \frac{\delta F_n}{\delta A} = \frac{dF_n}{dA}$$

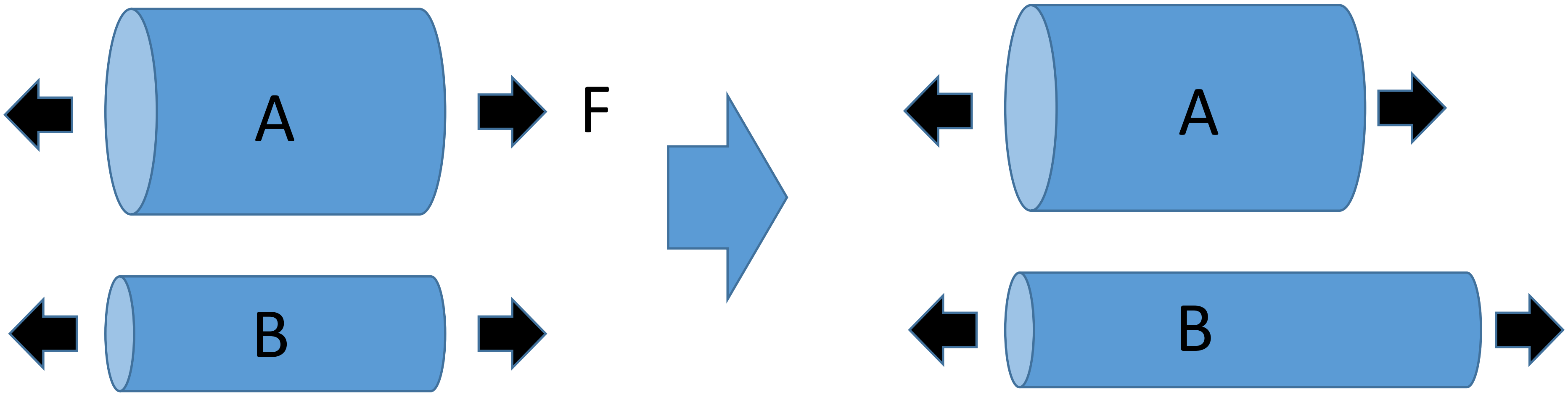
Normal stress
(Stretch or contraction)

$$\tau_{t_1} = \lim_{\delta A \to 0} \frac{\delta F_{t_1}}{\delta A} = \frac{dF_{t_1}}{dA}$$

$$\tau_{t_2} = \lim_{\delta A \to 0} \frac{\delta F_{t_2}}{\delta A} = \frac{dF_{t_2}}{dA}$$

Shear stress
(Twist)

Stress



- For the same force, F, A is less elongated.
- Therefore, A is stronger or A is material with higher strength??

Exercise – thin walled cylinder

- Torsion of thin sectioned cylinder
- Question: what is the stress acting on the cross-section of the thin walled cylinder?

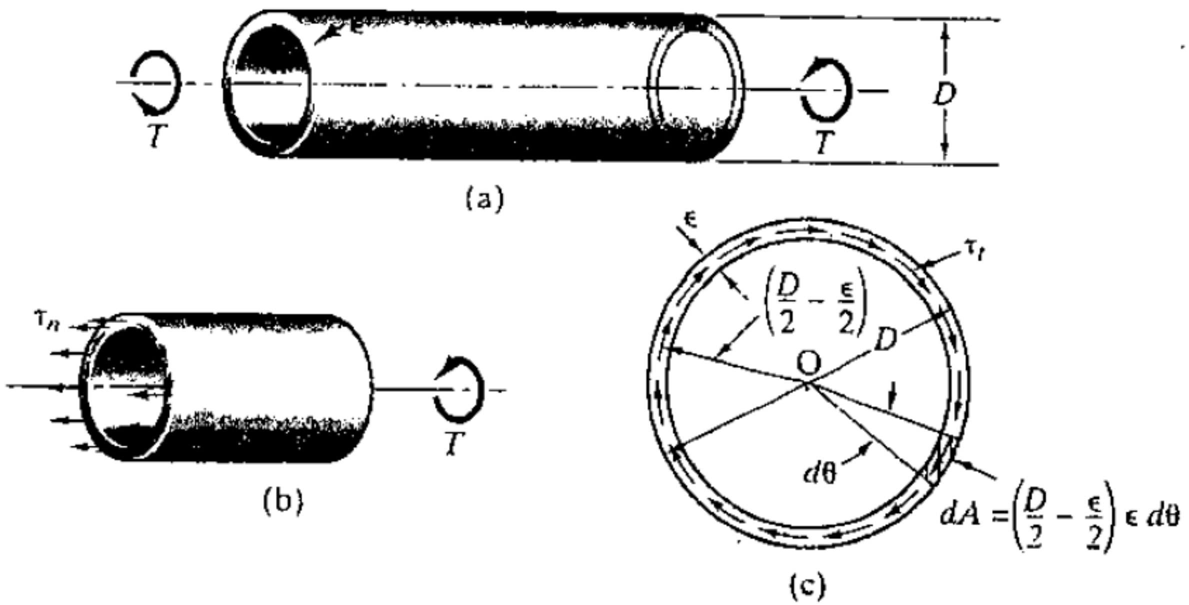


Figure 2.7. Thin-walled cylinder under torsion.

Exercise – pressurized vessel

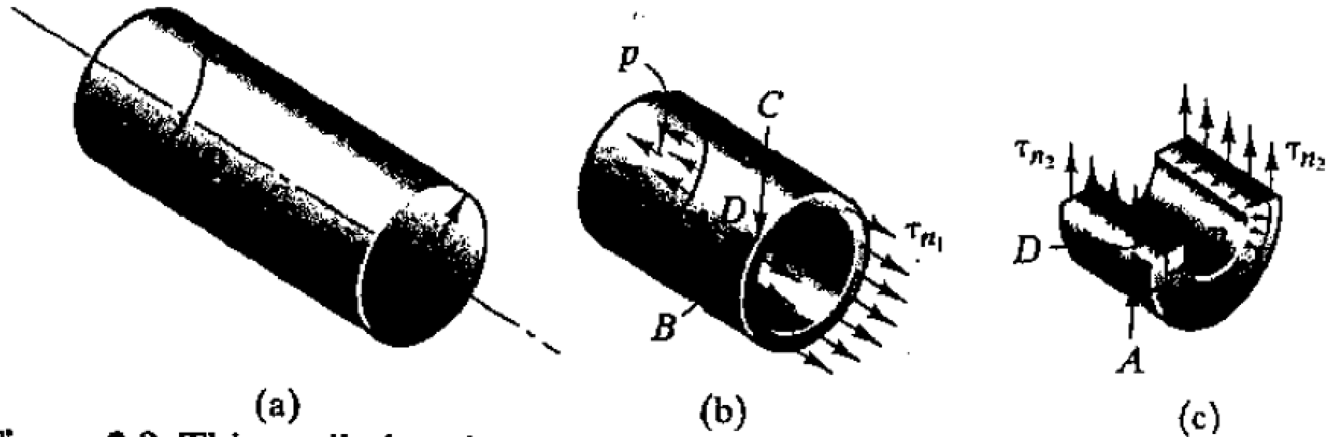


Figure 2.8. Thin-walled tank with inside gauge pressure p.

- Question: What are the stress components in the thin-walled tank under internal pressure p?

- Stress by the force acting on two side walls
- And, the stress by pressure acting on the circumferential (hoop) direction.

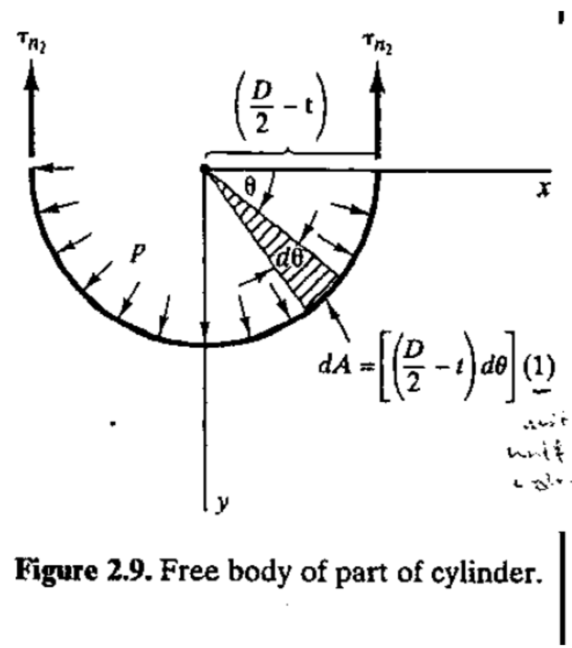
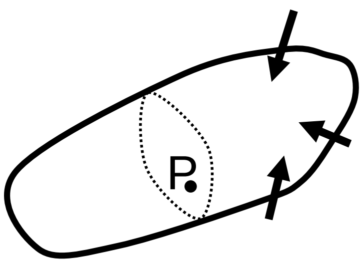


Figure 2.9. Free body of part of cylinder.

Stress

Stress on different cuts in the material

Continuum body



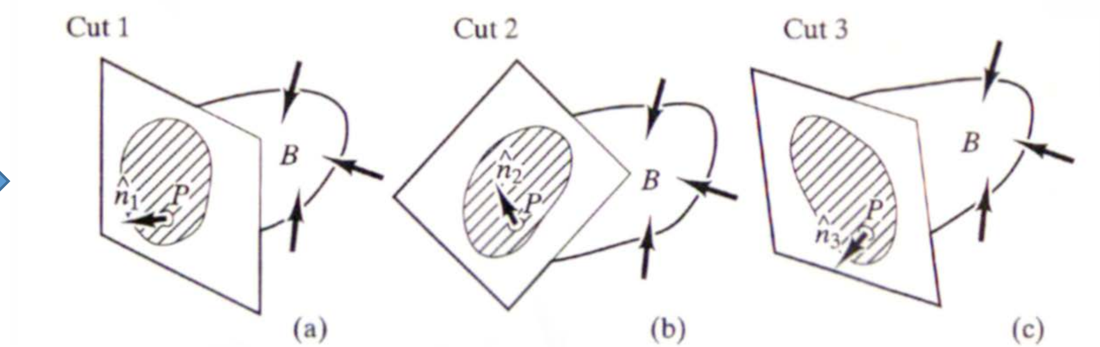


Figure 2.21. Different cuts through the point *P*.

- There are numerous acting planes including point P
- All different acting planes have different acting force
- Because of different acting forces, normal and shear stresses are also different
- To describe stress at point P, both acting plane normal direction and stress direction are needed

Exercise – stress component on various planes

$$\tau_n = \frac{P}{A}$$

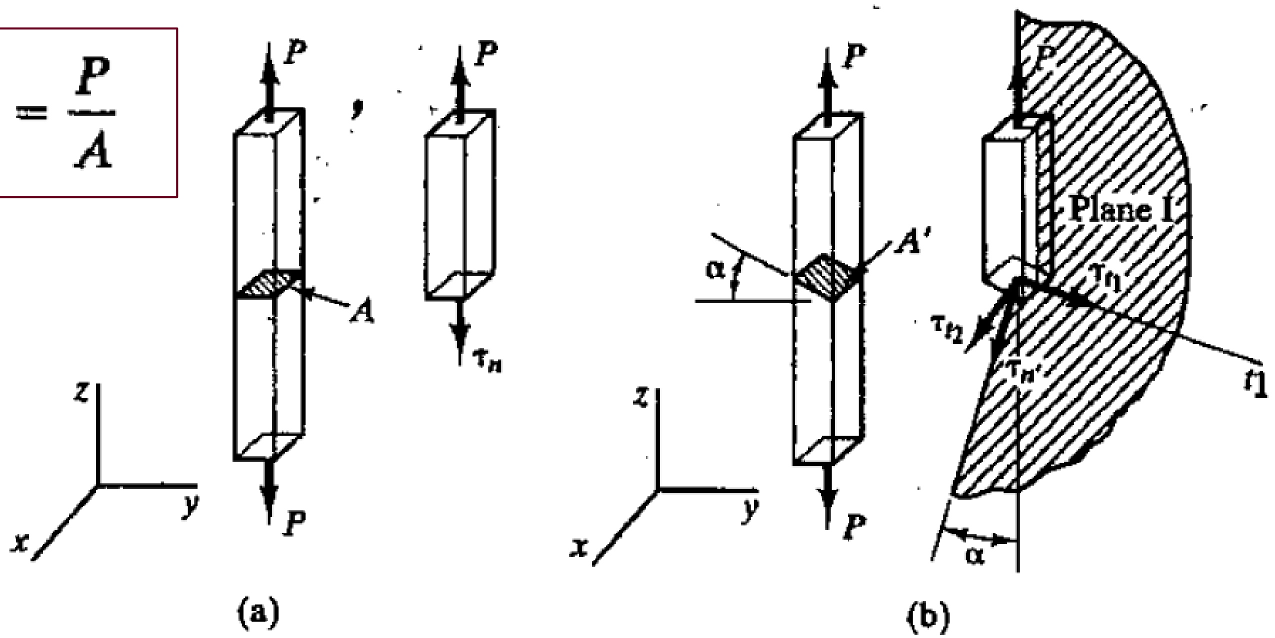


Figure 2.22. One-dimensional member with different cuts.

$\sum F_z = 0$:

$$P - \tau_{n'} A' \cos\alpha - \tau_{t_1} A' \sin\alpha = 0$$

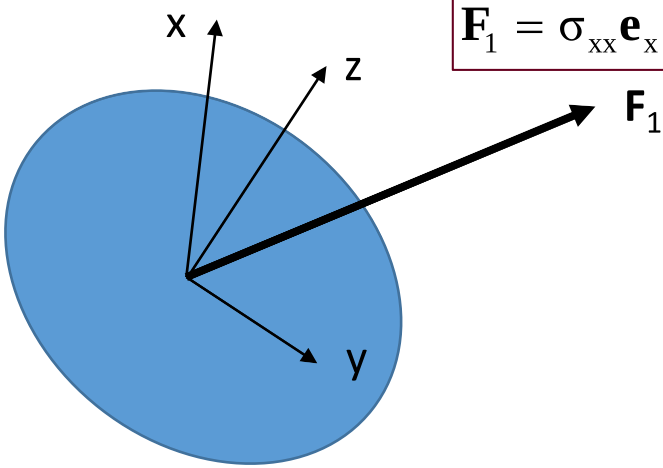
$\sum F_y = 0$:

$$-\tau_{n'} A' \sin\alpha + \tau_{t_1} A' \cos\alpha = 0$$

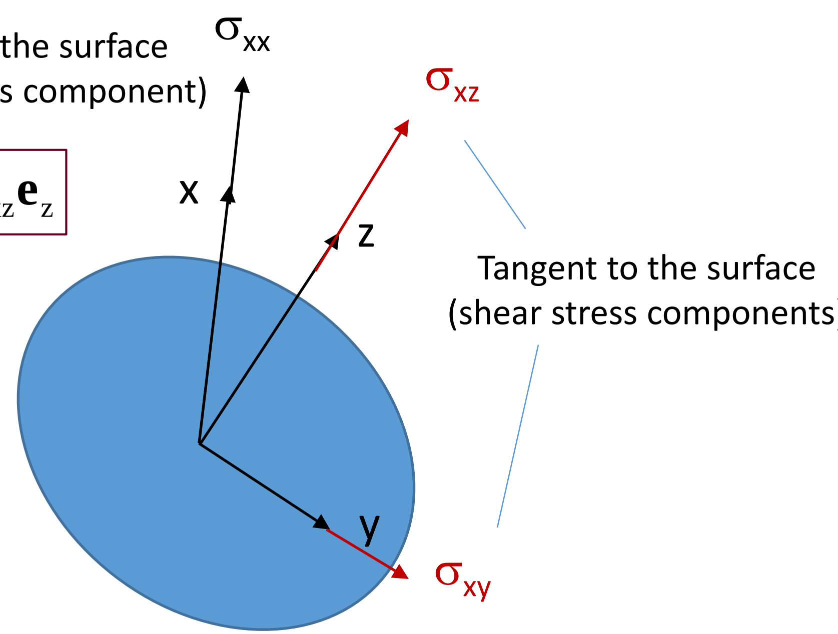
Stress tensor

- Suppose a force $\mathbf{F}_1$ acts on a surface of area A
- This force acts uniformly over the surface
- The force can be resolved in three directions; one acting in the normal and two in the directions parallel to the surface

$$\mathbf{F}_1 = \sigma_{xx}\mathbf{e}_x + \sigma_{xy}\mathbf{e}_y + \sigma_{xz}\mathbf{e}_z$$



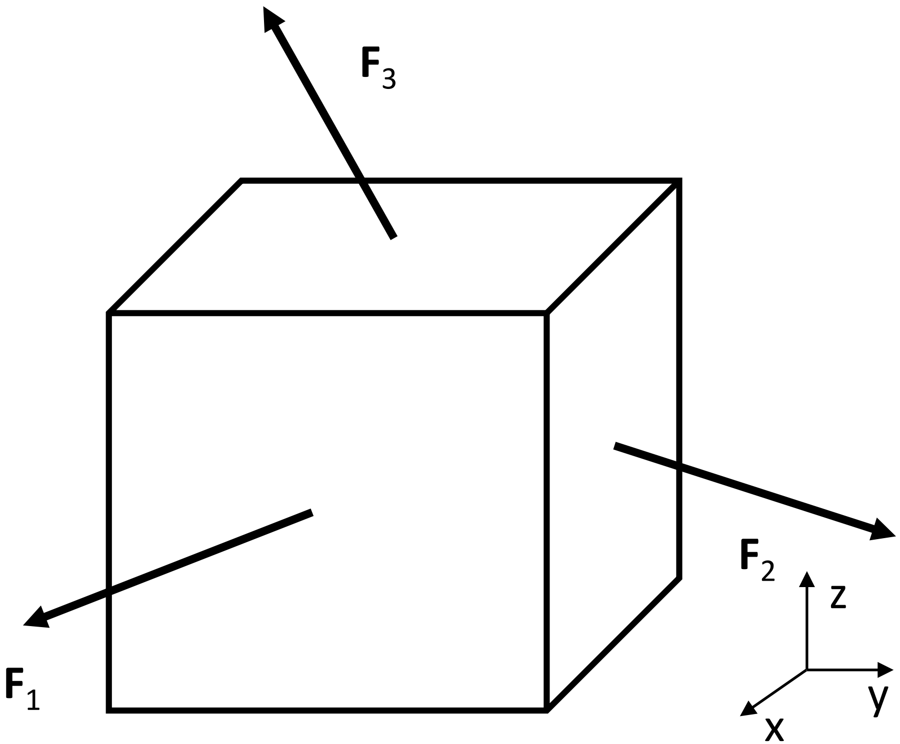
Force on the surface normal to x coordinate

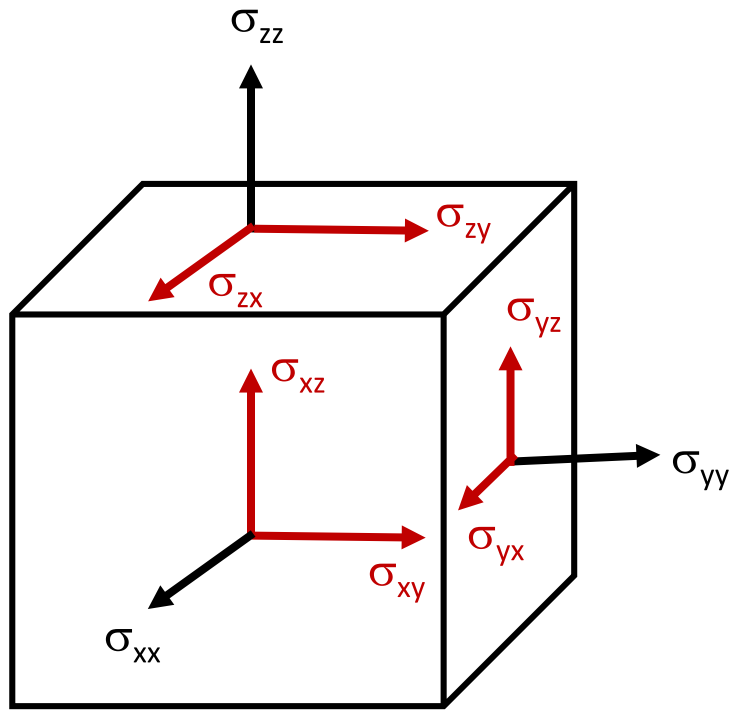


Three components of **force per unit area (= stress)**

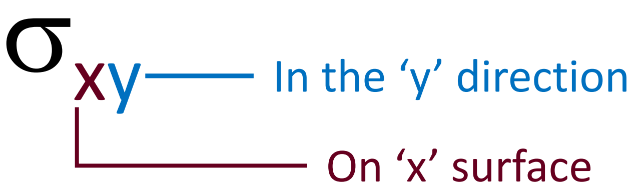
Stress tensor

Cartesian coordinate system

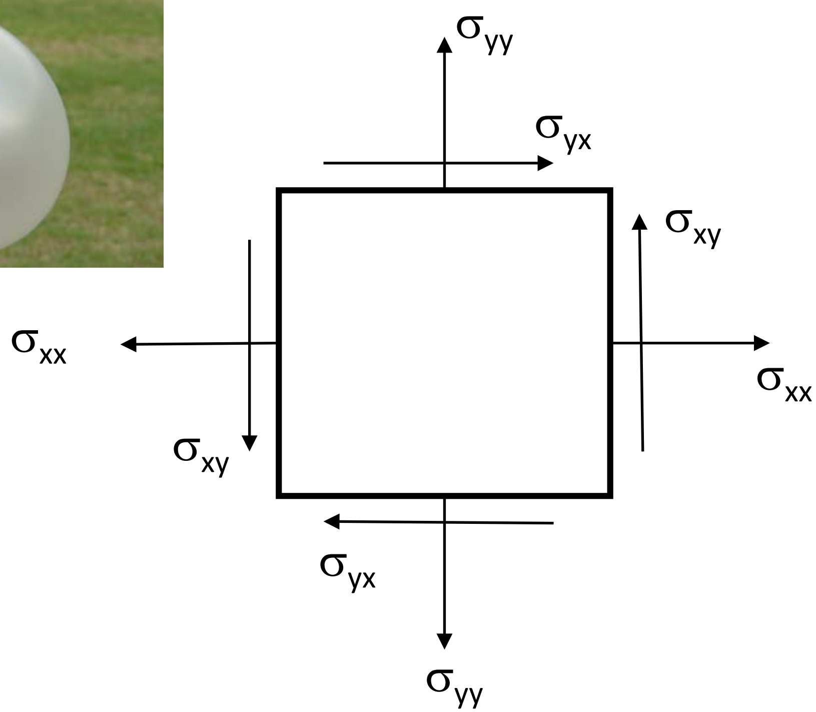


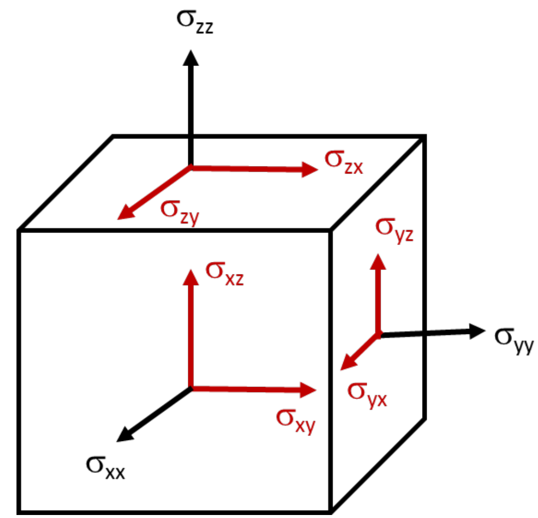


Nomenclature of stress component



Stress tensor: notation





c.f. Textbook

$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

Stress tensor

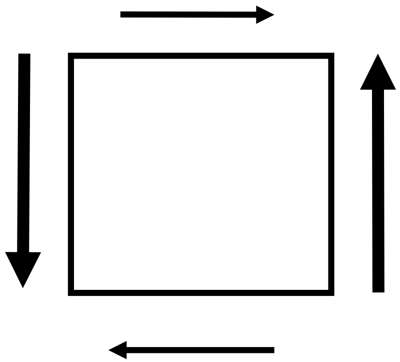
$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix}$$

2D simplification

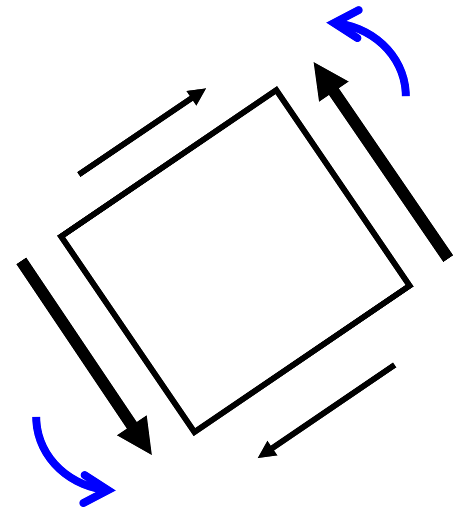
$$\tau_{ij} = \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix}$$

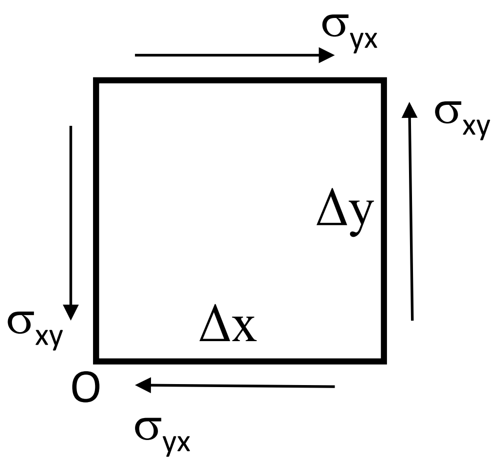
Stress tensor

Symmetry of Tensor using moment equilibrium



What happens?





thickness Δz

Moment (or couple) at the reference O should be zero

$$\sigma_{xy}(\Delta y \Delta z)\Delta x - \sigma_{yx}(\Delta x \Delta z)\Delta y = 0$$

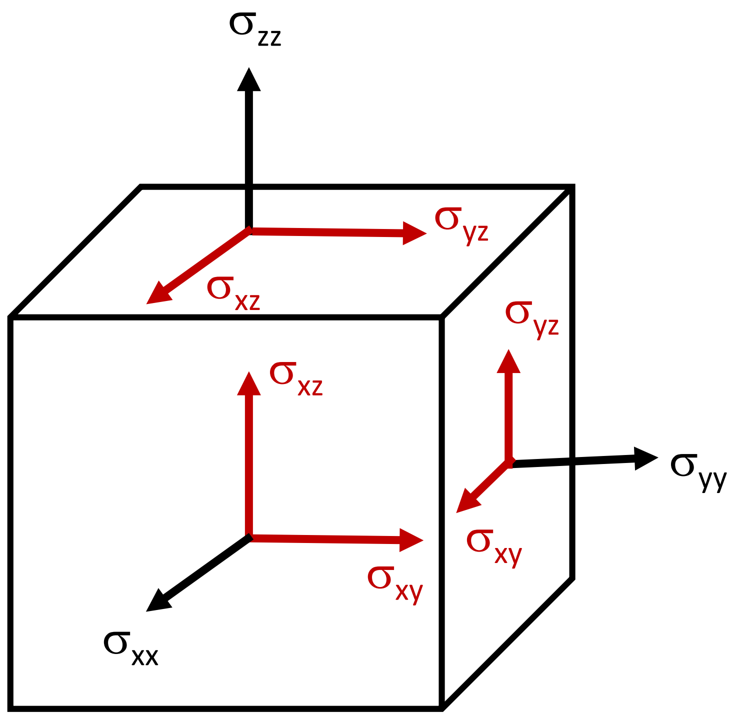
$$\boxed{\sigma_{xy} = \sigma_{yx}}$$

Likewise,

Therefore, only 6 components are independent !

Stress tensor

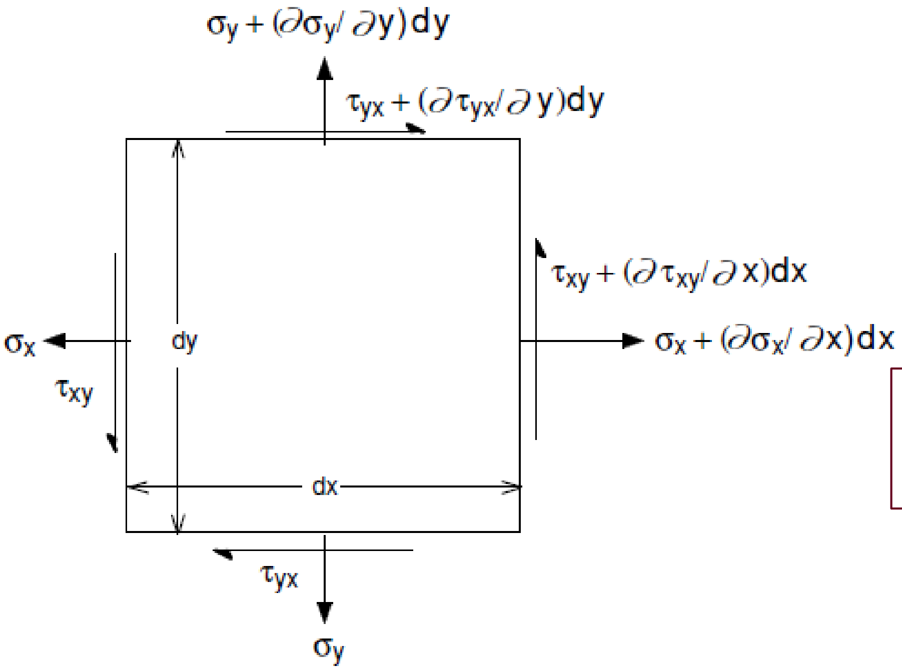
$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix}$$



Stress tensor is symmetric with 6 independent components

1. Three normal components are placed in the diagonal terms (Normal stresses)
2. Three shear components are placed in the off-diagonal terms and they are symmetric (Shear stresses)

Equilibrium equations



Force balance for x-direction

$$\sigma_{xx}+\tau_{yx} = \sigma_x+\frac{\partial\sigma_{xx}}{\partial x}dx + \tau_{yx}+\frac{\partial\tau_{yx}}{\partial y}dx$$

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y} = 0$$

In 3-D (including body force **B=**(B_x, B_y, B_z)

$$\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}+\frac{\partial\tau_{xz}}{\partial z}+Bx = 0$$
$$\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\tau_{yz}}{\partial z}+By = 0$$
$$\frac{\partial\sigma_{zz}}{\partial z}+\frac{\partial\tau_{zy}}{\partial y}+\frac{\partial\tau_{xz}}{\partial x}+Bz = 0$$

Questions ?