Entropy Equation for a Control Volume

(Lecture 10)

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열역학 **(M2794.001100.002)**

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(*) Some texts and figures are borrowed from Sonntag & Borgnakke unless noted otherwise.

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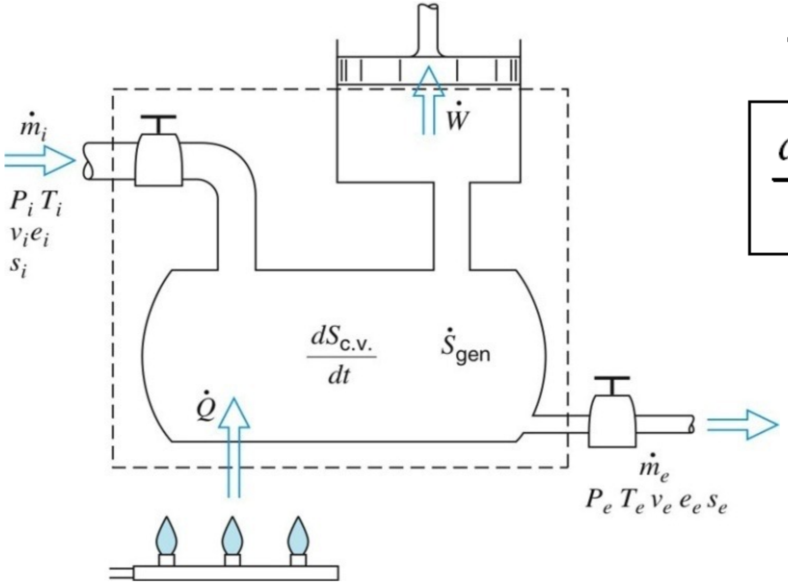
7.1 The Second Law of Thermodynamics for a Control Volume

- For a control volume, we add the entropy contributions from the mass flow in and out of the control volume.



1st law of thermodynamics

$$\frac{dE_{c.v}}{dt} = \dot{Q}_{c.v} - \dot{W}_{c.v} + \sum \dot{m}_i\left(h_i + \frac{1}{2}V_i^2 + gz_i\right) - \sum \dot{m}_e\left(h_e + \frac{1}{2}V_e^2 + gz_e\right)$$

2nd law of thermodynamics

$$\frac{dS_{c.v.}}{dt} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_{c.v.}}{T} + \dot{S}_{gen}$$

7.2 The Steady State Process and The Transient Process

- For the steady-state process, there is no change with time of the CV entropy, or,

$$\frac{dS_{\text{c.v.}}}{dt} = 0 \longrightarrow \boxed{\frac{dS_{\text{c.v.}}}{dt} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_{\text{c.v.}}}{T} + \dot{S}_{\text{gen}}}$$

Then,

$$\sum \dot{m}_e s_e - \sum \dot{m}_i s_i = \sum_{\text{c.v.}} \frac{\dot{Q}_{\text{c.v.}}}{T} + \dot{S}_{\text{gen}}$$

For a single-inlet-and-outlet system, or $\dot{m}_i = \dot{m}_e = \dot{m}$

$$\dot{m}(s_e - s_i) = \sum_{\text{c.v.}} \frac{\dot{Q}_{\text{c.v.}}}{T} + \dot{S}_{\text{gen}}$$

$$s_e = s_i + \sum \frac{q}{T} + s_{\text{gen}} \qquad \text{(per mass basis)}$$

For an adiabatic process,

$$s_e = s_i + s_{\text{gen}} \geq s_i$$

Entropy Equation for a Control Volume

For the transient process,

$$\boxed{\frac{d}{dt}(ms)_{\text{c.v.}} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_{\text{c.v.}}}{T} + \dot{S}_{\text{gen}}}$$

Integrating over the time interval *t* (with constant inlet/outlet properties),

$$\int_0^t \frac{d}{dt}(ms)_{\text{c.v.}}\, dt = (m_2 s_2 - m_1 s_1)_{\text{c.v.}}$$

$$\int_0^t \left(\sum \dot{m}_i s_i\right) dt = \sum m_i s_i, \quad \int_0^t \left(\sum \dot{m}_e s_e\right) dt = \sum m_e s_e, \quad \int_0^t \dot{S}_{\text{gen}}\, dt = {}_1S_{2\text{gen}}$$

Then,

$$(m_2 s_2 - m_1 s_1)_{\text{c.v.}} = \sum m_i s_i - \sum m_e s_e + \int_0^t \sum_{\text{c.v.}} \frac{\dot{Q}_{\text{c.v.}}}{T}\, dt + {}_1S_{2\text{gen}}$$

In case when the temperature is uniform throughout the control surface,

$$\int_0^t \sum_{\text{c.v.}} \frac{\dot{Q}_{\text{c.v.}}}{T}\, dt = \int_0^t \frac{1}{T} \sum_{\text{c.v.}} \dot{Q}_{\text{c.v.}}\, dt = \int_0^t \frac{\dot{Q}_{\text{c.v.}}}{T}\, dt$$

$$\boxed{(m_2 s_2 - m_1 s_1)_{\text{c.v.}} = \sum m_i s_i - \sum m_e s_e + \int_0^t \frac{\dot{Q}_{\text{c.v.}}}{T}\, dt + {}_1S_{2\text{gen}}}$$

7.3 The Steady-State Single-Flow Process

- Consider the shaft work (no boundary work of CV) associated with a steady-state single-flow device. From inlet to outlet states,

1st law $$q + h_i + \frac{1}{2}\mathbf{V}_i^2 + gZ_i = h_e + \frac{1}{2}\mathbf{V}_e^2 + gZ_e + w$$

2nd law $$\delta q = Tds - T\delta s_{gen} = dh - vdP - T\delta s_{gen}$$

$$dS = \frac{\delta Q}{T} + \delta S_{\text{gen}}$$

$$q = \int_i^e \delta q = \int_i^e dh - \int_i^e vdP - \int_i^e T\delta s_{gen} = h_e - h_i - \int_i^e vdP - \int_i^e T\delta s_{gen}$$

Combining the two equations,

$$w = q + h_i - h_e + \frac{1}{2}(\mathbf{V}_i^2 - \mathbf{V}_e^2) + g(Z_i - Z_e)$$

$$= h_e - h_i - \int_i^e v\,dP - \int_i^e T\,\delta s_{\text{gen}} + h_i - h_e + \frac{1}{2}(\mathbf{V}_i^2 - \mathbf{V}_e^2) + g(Z_i - Z_e)$$

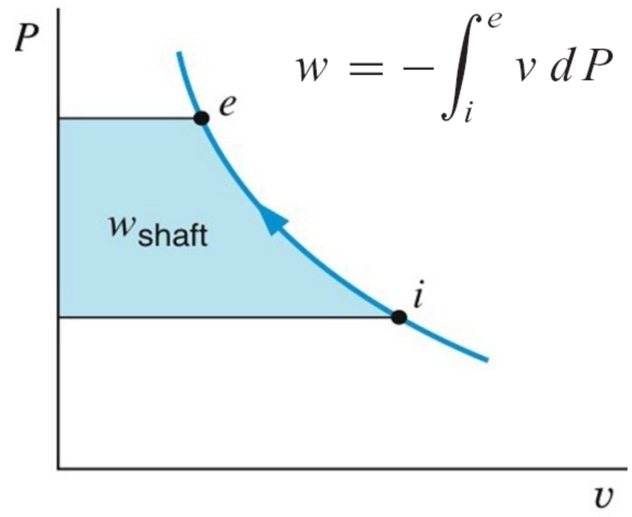
$$\boxed{w = -\int_i^e v\,dP + \frac{1}{2}(\mathbf{V}_i^2 - \mathbf{V}_e^2) + g(Z_i - Z_e) - \int_i^e T\,\delta s_{\text{gen}}}$$

Entropy Equation for a Control Volume

$$w = -\int_i^e v\,dP + \frac{1}{2}(\mathbf{V}_i^2 - \mathbf{V}_e^2) + g(Z_i - Z_e) - \int_i^e T\,\delta s_{\text{gen}}$$

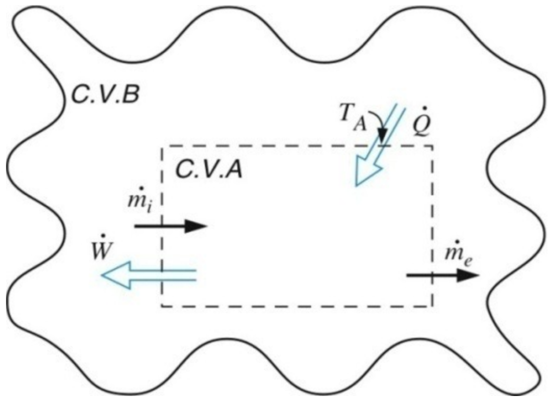
1. The maximum work output occurs for a reversible process.
2. For a reversible process with negligible KE and PE, when the pressure increases (compressor or pump), the work is negative, and vice versa. (cf.) For liquid (*v* is small), pumping is easier than for gas (*v* is large)
3. For negligible KE and PE without work(w=0), irreversibility (e.g. friction) results in pressure drop (or pressure loss).
4. For an incompressible fluid (*v*=constant) and a reversible process.

$$w = -v(P_e - P_i) + \frac{1}{2}(\mathbf{V}_i^2 - \mathbf{V}_e^2) + g(Z_i - Z_e)$$



7.4 Principle of the Increase of Entropy

➔ Consider two C.V.'s exchanging mass, heat and work. Here, we only consider the irreversibility by heat transfer.



$$\frac{dS_{CV\,A}}{dt} = \dot m_i s_i - \dot m_e s_e + \frac{\dot Q}{T_A}$$

$$\frac{dS_{CV\,B}}{dt} = -\dot m_i s_i + \dot m_e s_e - \frac{\dot Q}{T_A} + \dot S_{\text{gen}\ B}$$

$$= -\dot m_i s_i + \dot m_e s_e - \frac{\dot Q}{T_A} + \left\{\frac{\dot Q}{T_A} - \frac{\dot Q}{T_B}\right\} = \dot m_e s_e - \dot m_i s_i - \frac{\dot Q}{T_B}$$

$$\frac{dS_{\text{net}}}{dt} = \frac{dS_{CV\,A}}{dt} + \frac{dS_{CV\,B}}{dt}$$

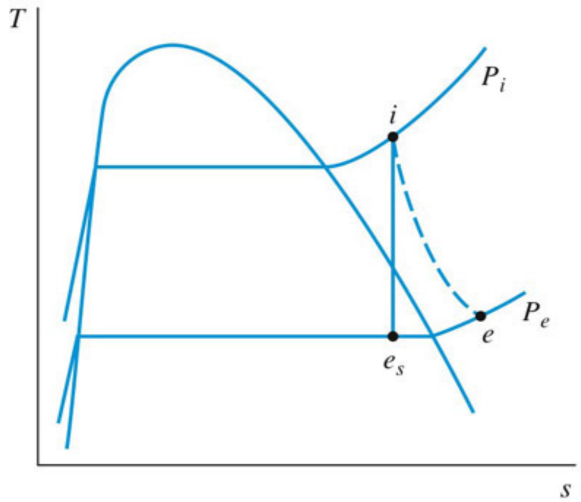
$$= \dot m_i s_i - \dot m_e s_e + \frac{\dot Q}{T_A} - \dot m_i s_i + \dot m_e s_e - \frac{\dot Q}{T_B} = \frac{\dot Q}{T_A} - \frac{\dot Q}{T_B} \geq 0$$

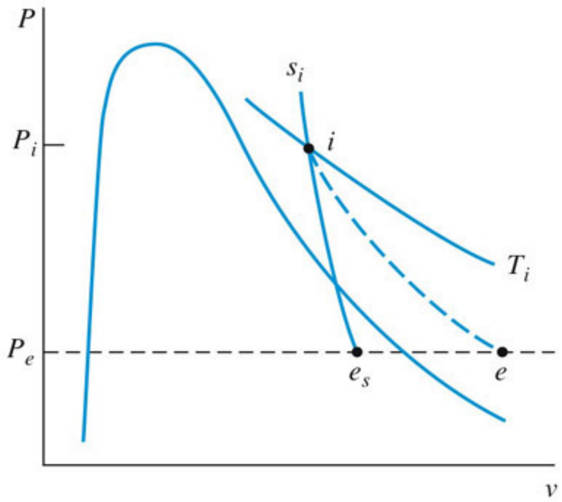
7.5 Engineering Applications – Efficiency

- Actual devices are not reversible, but the ideal reversible models can be used to compare with the real counterparts.

$$\text{Actual machine efficiency} = \frac{\text{Actual}}{\text{Ideal}}$$

- For a turbine with given P_i, T_i, and P_e, the ideal model is a reversible adiabatic process (or isentropic expansion).

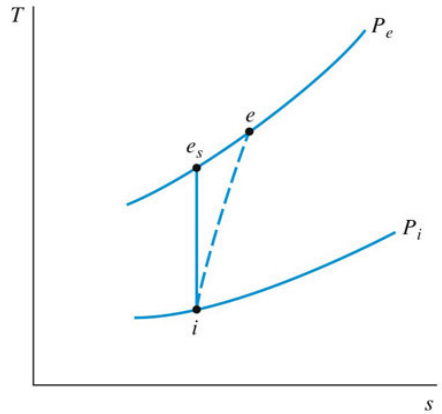


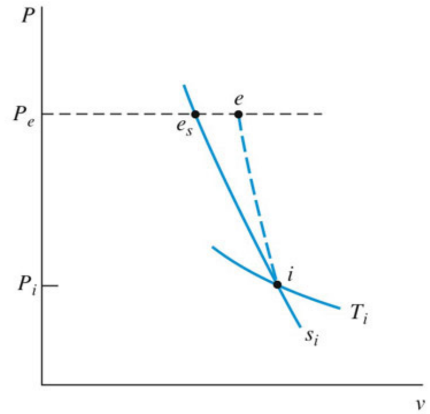


$$\eta_{\text{turbine}} = \frac{w}{w_s} = \frac{h_i - h_e}{h_i - h_{es}}$$

= 0.70~0.88
for typical turbines

- For a compressor with given P_i, T_i, and P_e, the ideal model is a reversible adiabatic process (or isentropic compression).

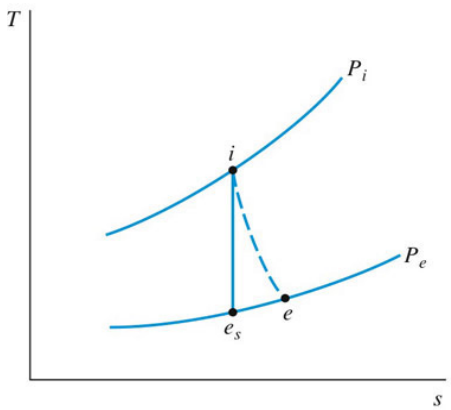


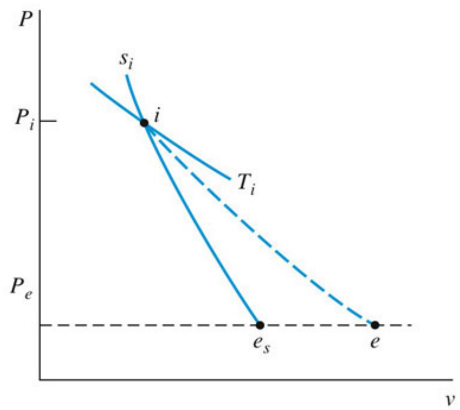


$$\eta_{\text{comp}} = \frac{w_s}{w} = \frac{h_i - h_{es}}{h_i - h_e}$$

= 0.70~0.88
for typical compressors

- For a nozzle with given P_i, T_i, and P_e, the ideal model is a reversible adiabatic process (or isentropic expansion).





$$\eta_{\text{nozz}} = \frac{\mathbf{V}_e^2/2}{\mathbf{V}_{es}^2/2}$$

= 0.90~0.97
for typical nozzles