Chapter 15: Hardening

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Hardening

General principle for strengthening

- How to increase strength of metals?
  - Place obstacles in the path of dislocations, which inhibits the free movement of dislocations until the stress is increased to move them forward.

\[
\tau \approx \frac{Gb}{L'} \cos\left(\frac{\phi_c}{2}\right)
\]

- Effective particle spacing \( L' \)
- Critical angle to which the dislocation bends to breaking away from the obstacle \( \phi_c' \)

\[
\phi_c' = 180^\circ - \phi_c
\]
Hardening

Work hardening from physical metallurgy

- During plastic deformation, there is an increase in dislocation density. It is this increase in dislocation density which ultimately leads to work hardening.

- Dislocation interact with each other and assume configurations that restrict the movement of other dislocations.

- The dislocations can be either “strong” or “weak” obstacles depending on the types of interactions that occurs between moving dislocations.
Hardening

Work (strain) hardening of single crystal

- Stage I: the stress fields interact during the early stages of deformation resulting in “weak” drag effects
- In stage I, dislocations are “weak” obstacles
Hardening

Work (strain) hardening of single crystal

- Stage II: Linear hardening

- In stage II, work hardening depends strongly on dislocation density

Interactions produce immobile dislocation configurations like jogs.
Hardening

Work (strain) hardening of single crystal

- Stage II: Linear hardening
- In stage II, work hardening depends strongly on dislocation density
- Lomer-Cottrell locks (sessile dislocation)

Interactions produce immobile dislocation configurations like jogs.
Elastic Plasticity Models

Elastic-plasticity models in sheet metal forming

1) Yield function
2) Hardening model
3) Elastic modulus

Accurate predictions for stress, deformation
Yield function

Elasticity models for FEM

- Yield functions
  - Elastic vs. Plastic
  - Rate of plastic strain

\[ \sigma_Y = \varepsilon^e + \varepsilon^p \]

1D

\[ d\varepsilon^p = d\lambda \frac{\partial f}{\partial \sigma} \]

3D (6D)

- R-value

\[ r = \frac{d^w}{d_t} \]

\[ \varepsilon_t, \varepsilon_w \]

\[ \sigma_1, \sigma_2, \sigma_3 \]

von Mises Yield Curve

Hydrostatic Axis

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Yield function

Yield functions

1-D  2-D  3-D (plane stress)
Yield function

Plane stress

I. Stretching
II. Stretch flanging
III. Drawing
IV. Bending

From Kuwabara
Yield function

- Uniaxial tension
- Biaxial tension (small strain range)
- Uniaxial compression

Von Mises
Hill 1948
Barlat 2000-2d

Principal stress plane

From Kuwabara
Hardening law: importance

Hardening behavior

Sidewall region

(Upper) Tension – Compression – Unloading
(Bottom) Compression – Tension – Unloading

Stress-strain behavior under reverse loading

Monotonic tension
Permanent softening
Transient hardening
Bauschinger effect

C-T test (6% compression)

DP590-1.4mm
Hardening law: importance

Hardening models

Isotropic hardening

\[ \sigma_T = |\sigma_C| \]

Bauschinger effect can be reproduced by kinematic hardening

\[ \sigma_T > |\sigma_C| \]

kinematic hardening

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Hardening law: basic

Uniaxial stress-strain curve of typical metals

- Yield point $Y$
- Elastic unload
- Perfectly-plastic
- Hardening
Hardening law: basic

Initial yield surface

\[ f(\sigma_{ij}) = 0 \]

For perfect plasticity, the yield surface remains unchanged. But, in general case, the size, position, shape of the yield surface change

\[ f(\sigma_{ij}, K_i) = 0 \]

where K represents hardening parameters, which evolves during the plastic deformation.
Hardening law: basic

Isotropic hardening

Yield function takes the following form: no change in position, shape, but size changes

\[ f(\sigma_{ij}, K_i) = f(\sigma_{ij}) - K = 0 \]

Von Mises

\[ f(\sigma_{ij}, K_i) = f(\sigma_{ij}) - K = \sqrt{\frac{1}{2} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]} - Y = 0 \]
Hardening law: basic

Isotropic hardening

Size of yield surface = from a uniaxial tensile test

\[ K = C\left(\varepsilon^p\right)^n \]  
Power law hardening

\[ K = Y_0 + C\left(\varepsilon^p\right)^n \]  
Ludwik model

\[ K = C\left(\varepsilon_0 + \varepsilon^p\right)^n \]  
Swift hardening model

\[ K = Y_0 + Y_1\left(1 - \exp\left(-C\varepsilon^p\right)\right) \]  
Voce hardening model
Hardening law: basic

Kinematic hardening

- Isotropic hardening: tensile yield stress = |compressive yield stress|
- No Bauschinger effect
- Kinematic hardening: Softening in the compression direction during the loading in tensile direction
Hardening law: basic

**Kinematic hardening**

\[ f(\sigma_{ij}, K_i) = f(\sigma_{ij} - \alpha_{ij}) = 0 \]

The hardening parameter is called “back stress”

\[ f(\sigma_{ij}, K_i) = f(\sigma_{ij} - \alpha_{ij}) = \sqrt{\frac{1}{2} \left( (\sigma_1 - \alpha_1 - \sigma_2 - \alpha_2)^2 + (\sigma_2 - \alpha_2 - \sigma_3 - \alpha_3)^2 + (\sigma_3 - \alpha_3 - \sigma_1 - \alpha_1)^2 \right)} \]

Von Mises
Hardening law: basic

Flow curve

To model three dimensional deformation behavior, a concept of effective variables, such as effective stress, effective plastic strain is introduced to control the size or position of the yield surface.

\[ \sigma = h(\varepsilon^p) \]

\[ H \equiv \frac{d\sigma}{d\varepsilon^p} \]

\[ \hat{\sigma} = h(\hat{\varepsilon}^p) \]

\[ H \equiv \frac{d\hat{\sigma}}{d\hat{\varepsilon}^p} \]
Hardening law: basic

Kinematic hardening: how to define the movement?

\[ d\alpha_{ij} \sim \sigma_{ij} - \alpha_{ij} \]

\[ d\alpha_{ij} \sim d\varepsilon_{ij}^p \]

Ziegler model

Prager model
**Hardening law: basic**

**Linear kinematic hardening**

\[ f = \sqrt{\frac{3}{2}} (s_{ij} - \alpha_{ij}) : (s_{ij} - \alpha_{ij}) - K = 0 \]

\[ d\alpha_{ij} = \frac{2}{3} C d\varepsilon_{ij}^p \quad \text{or} \quad \Delta\alpha_{ij} = \frac{2}{3} C \Delta\varepsilon_{ij}^p \]

“Size of yield function, K=constant”

“Translation of yield function (back stress) – linearly proportional to the plastic strain rate”

![Diagram showing yield surfaces and strain rates](image)
Hardening law: advanced

- Classical isotropic hardening
  - Not so effective for non-monotonic straining
  - No Bauschinger effect and transient behavior

- Classical combination type of iso-kinematic hardening by Prager and Ziegler
  - Bauschinger effect only
  - No transient behavior

- Combination type of the isotropic and kinematic hardening
  - Chaboche, Krieg and Dafalias/Popov
  - YU model
  - Bauschinger effect and transient behavior

- Distortional hardening
  - Without kinematic hardening (No back stress)
Hardening law: advanced

Nonlinear Kinematic Hardening Model (NKH)

\[ \Delta \alpha = \frac{2}{3} C \Delta \varepsilon^p - \gamma \alpha \Delta \bar{\varepsilon} \]

“Recall” term

\[ f = \sqrt{\frac{3}{2}} (s_{ij} - \alpha_{ij} \cdot (s_{ij} - \alpha_{ij}) - R = 0 \]

![Diagram showing the hardening law and stress-strain relationship in materials science](image)
"Chaboche" Model

\[ \Delta \alpha = \sum_{i=1}^{n} \Delta \alpha_i = \frac{2}{3} \sum_{i=1}^{n} C_i \Delta \varepsilon_i^p - \gamma_i \alpha_i \Delta \overline{\varepsilon} \]

\[ f = \sqrt{\frac{3}{2} (s_{ij} - \alpha_{ij}) : (s_{ij} - \alpha_{ij}) - R = 0} \]
Hardening law: advanced

- Two surfaces are used to represent Bauschinger and transient behaviors

- Inner (loading) and outer (bounding) surfaces

\[ f(\sigma - \alpha) - \bar{\sigma}_iso^m = 0 \]
\[ F(\Sigma - \mathbf{A}) - \bar{\Sigma}_iso^m = 0 \]

- Translation of the inner surface

\[ d\alpha = \frac{d\bar{\alpha}}{\bar{\sigma}_iso(\mathbf{v})} \mathbf{v} \]

\( \mathbf{v} \): normalized quantity of \( d\varepsilon^p \) or \( \sigma - \alpha \)
The hardening curve of the inner surface is newly updated every time unloading occurs, considering the gap $\delta$

The separation of the isotropic and kinematic hardening in the inner surface should be performed considering the gaps
Hardening law: advanced

\[
\frac{d\Sigma}{d\varepsilon} = \sigma
\]

\[
\delta_{in}
\]

\[
\Sigma
\]

\[
\delta
\]

\[
\delta_{in}
\]
Hardening law: advanced

- In order to properly represent the transient behavior, the hardening behavior of the inner surface is prescribed every time reloading occurs.

The stress relation for the 1-D tension test

\[
\frac{d\Sigma}{d\varepsilon^p} = \frac{d\sigma}{d\varepsilon^p} + \frac{d\delta}{d\varepsilon^p}
\]

Example of gap variation between inner and outer surfaces

\[
\frac{d\delta}{d\varepsilon^p} = A(\delta) = -\chi(\delta_{in}) \left( \frac{\delta}{\delta_{in} - \delta} \right)
\]

This satisfies the continuous hardening slope between elastic and plastic ranges

\[
\frac{d\sigma}{d\varepsilon^p} = \infty \text{ for } \delta = \delta_{in}
\]

\[
\frac{d\sigma}{d\varepsilon^p} = \frac{d\Sigma}{d\varepsilon^p} \text{ for } \delta = 0
\]
Hardening model: YU Model

Hardening model: YU Model

\[ F = f(\sigma - \alpha) - Y = 0 \]

Back stress of “active (real)” yield surface

\[ F_1 = f(\sigma - \beta) - (B + R) = 0 \]

Back stress of “Bounding” surface

Isotropic part of “Bounding” surface

Control “global” hardening

\[ \alpha_* = \alpha - \beta \]

\[ \alpha_* = \sqrt{\frac{2}{3}} \, C a d \, e_p \left( n_p - \sqrt{\frac{\alpha_*}{a}} \, n_* \right) \]

\[ a = B + R - Y \]

\[ dR = m \left( R_{sat} - R \right) d\bar{e}^p \]

\[ d\beta = m \left( \frac{2}{3} b d\bar{e}^p - \beta d\bar{e}^p \right) \]
**Hardening model: YU Model**

**Initial yield surface**

\[ F = f(\sigma - \alpha) - Y = 0 \]

**Bounding surface**

\[ F_1 = f(\sigma - \beta) - (B + R) = 0 \]

**The relative motion law**

\[ \alpha_* = \alpha - \beta \]

\[ \alpha_* = \sqrt{\frac{2}{3} C a d \varepsilon_e^p} \left( n_p - \frac{\alpha_*}{a} n_* \right) \]

\[ a = B + R - Y \]

**Isotropic hardening of the bounding surface**

\[ dR = m( R_{sat} - R ) d\bar{\varepsilon}^p \]

**Kinematic hardening of the bounding surface**

\[ d\beta = m \left( \frac{2}{3} b d\varepsilon^p - \beta d\bar{\varepsilon}^p \right) \]

**Yoshida-Uemori Parameters**

Y : Size of loading surface

B : Initial size of bounding surface

C : Parameter for the back-stress evolution

\[ R_{sat} , b , m : \text{Parameters for the size of bounding surface} \]

h : The parameter for work-hardening stagnation or cyclic hardening characteristics.

**Plastic strain dependency of Unloading modulus**

\[ E = E_0 - (E_0 - E_a) \left[ 1 - \exp(-\xi \bar{\varepsilon}^p) \right] \]
Hardening model parameter

- Measuring hardening, Bauschinger and transient behaviors

- Anti-buckling system
  - Prevention of buckling when sheet specimen compressed
  - Use of fork-shaped guides along the side of the sheet
  - Large clamping force causes the fork to bend
  - Difficult to obtain smooth hardening curves

- Modified method using simple rigid plates
  - Smooth hardening curves were obtained
  - Mounted at universal testing machine
Hardening model parameter

Fork device

Modified device

Tension-compression test (Boger et al., 2005)

Simple shear device connected on MTS universal testing machine
Hardening model parameter

- Under-estimate the measured stress value
- For the biaxial stress state

\[
\bar{\sigma} = f^m = \left( \frac{|X'_1 - X'_2|^m + |2X''_2 + X''_1|^m + |X''_2 + 2X''_1|^m}{2} \right)^{1/m}
\]

\[
X'_1 - X'_2 = L'_{11}\sigma_{xx} - L'_{22}\sigma_{zz}
\]

\[
2X''_2 + X''_1 = (2L''_{21} + L''_{11})\sigma_{xx} + (L''_{11} + 2L''_{22})\sigma_{zz}
\]

\[
X''_2 + 2X''_1 = (L''_{21} + 2L''_{11})\sigma_{xx} + (2L''_{12} + L''_{22})\sigma_{zz}
\]

- Using clamping force and contact area, the biaxial effect can be corrected
Hardening model parameter

Tension-compression test
Over-estimate the measured stress value

Indirectly evaluated by comparing the two tensile data: With/without clamping force

Apparent friction coefficient:

\[
\mu = \frac{(F_{unc} - F_{std})}{N} = \frac{(\sigma_{unc} - \sigma_{std})}{N} A_o
\]

Apparent friction coefficient was obtained as one average value since it varies with respect to engineering strain
Hardening model parameter

**Chaboche model**

\[
\frac{d\delta}{d\varepsilon^p} = h(\delta_{in})(\frac{\delta}{\delta_{in} - \delta}) = -(a + b\delta_{in})(\frac{\delta}{\delta_{in} - \delta})
\]

\[
h = \frac{\delta_{in}}{\varepsilon^*}(1 - \ln(\frac{\delta_{in}}{\delta})) - \frac{\delta}{\varepsilon^*}
\]

\[
a = \frac{h_1\delta_{in,2} - h_2\delta_{in,1}}{\delta_{in,2} - \delta_{in,1}}
\]

\[
b = \frac{h_1 - h_2}{\delta_{in,1} - \delta_{in,2}}
\]

**Two-surface model**

![Graph showing stress-strain relationship with hardening models]
Hardening model implementation

Outline of implicit integration scheme

Element equilibrium equation for a static state

\[
[K^e]u^e = [F^e]
\]

- \(K^e\): Element stiffness matrix \textit{(constitutive model)}
- \(F^e\): Surface traction + frictional force on element \textit{(friction model)}
- \(u^e\): Element nodal displacement vector

Global equilibrium equation

\[
[K]u = [F]
\]

Implicit integration scheme (Newton-Raphson method)

- Initially guess \(\Delta u\)
- Estimate the residual force
  \[
  \Phi = [K(U)]u - [F(U)]
  \]
- \(\Phi < \text{Tol}\)?
  - yes: Proceed to next increment
  - no: Adjust \(\Delta u = \Delta u + d\Delta u\)

where

\[
\Phi(U) + \frac{\partial \Phi(U)}{\partial \Delta U} d\Delta U = 0
\]
Hardening model implementation

- **Beginning of Analysis**
- **Define Initial Conditions**
- **Start of Step**
- **Start of Increment**
- **Start of Iteration**
- **Define $K^{el}$**
- **Define Loads $R^\alpha$**

**UEXTERNALDB**
- **UEXTERNALDB**
- **DLOAD, FILM, HETVAL, UWAVE**
- **CREEP, FRIC, UEL, UEXPAN, UGENS, UMAT, USDFLD**

Writing User Subroutine with ABAQUS, ABAQUS
Hardening model implementation

To Start of Increment

Solve $K^{el} c = R^\alpha$

Converged?

Write Output

End of Step?

No

UEXTERNALDB URDFIL

No

Yes

To Start of Iteration

To Start of Step
Hardening model implementation

Integration of global equilibrium equation (Main code)

- Estimate $\Delta U$ for new increment.
- Calculate strain from $\Delta U$.
  - Call UMAT and update material stress.
  - Calculate internal force ($KU$).
- Calculate slip and normal pressure from $\Delta U$.
  - Call FRIC and update frictional stress.
  - Calculate external force ($F$).
- Calculate the residual $\Psi$.
- Is $|\Psi| < Tol$?
  - yes: Accept $\Delta U$ as a solution.
  - no: Modify $\Delta U$.
- Material stress update (Subroutine UMAT)
- External data file for communication between the subroutines
- Frictional stress update (Subroutine FRIC)
Hardening model implementation

Figure 1–2. A More Detailed Flow of ABAQUS/Standard

- Start of Increment
  - Calculate Integration Point Field Variable from Nodal Values
  - Start of Iteration
    - Calculate $\Delta \varepsilon$
      - Calculate $\sigma$, $\frac{\partial \Delta \sigma}{\partial \Delta \varepsilon}$
    - Define Loads $\frac{\partial P}{\partial x}$
    - UEL
    - UMAT
    - USDFLD
    - FILM $\frac{dh}{d\theta}$, $\frac{dr}{d\theta}$
    - HETVAL
    - CREEP: $\Delta \varepsilon^{cr}$, $\Delta \varepsilon^{sw}$
    - UEXPAN: $\Delta \varepsilon^{th}$
    - FRIC: $\partial \Delta \tau / \partial \Delta \gamma$
    - UGENS: $\partial N / \partial E$
    - DLOAD, UWAVE

Writing User Subroutine with ABAQUS, ABAQUS

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Hardening model implementation

Stress integration: predictor-corrector

\[
\Delta \varepsilon = \Delta \varepsilon^e + \Delta \varepsilon^p
\]

\[
\sigma^T = \sigma + C \Delta \varepsilon
\]

Specific Algorithm

\[
\delta(\Delta \bar{\varepsilon})
\]

\[
\Delta \bar{\varepsilon}^{new} = \Delta \bar{\varepsilon}^{old} + \delta(\Delta \bar{\varepsilon})
\]

\[
\sigma^{New} = \sigma^{old} - C : \left( \Delta \bar{\varepsilon} \frac{\partial f}{\partial \sigma} \right)
\]
Hardening model implementation

SUBROUTINE UMAT(STRESS, STATEV, DDSDDE, SSE, SPD, SCD, RPL,
1 DDSSDDT, DRPLDE, DRPLDT, STRAN, DSTRAN, TIME, DTIME, TEMP, DTEMP,
2 PREDEF, DPRED, CMNAME, NDI, NSHR, NTENS, NSTATV, PROPS, NPROPS,
3 COORDS, DROT, PNEWDT, CELENT, DFGRD0, DFGRD1, NOEL, NPT, LAYER,
4 KSPT, KSTEP, KINC)

C

INCLUDE 'ABA_PARAM.INC'

C

CHARACTER*8 CMNAME

C

DIMENSION STRESS(NTENS), STATEV(NSTATV), DDSDDE(NTENS, NTENS),
1 DDSSDDT(NTENS), DRPLDE(NTENS), STRAN(NTENS), DSTRAN(NTENS),
2 PREDEF(1), DPRED(1), PROPS(NPROPS), COORDS(3), DROT(3, 3),
3 DFGRD0(3, 3), DFGRD1(3, 3)

Writing User Subroutine with ABAQUS, ABAQUS
Application: springback
Defect in dimensional accuracy*

Elastic recovery of bending moment by uneven stress in the thickness direction after bending

Elastic recovery of bending moment by uneven stress in the thickness direction after bending & unbending

Elastic recovery of torsion moment by uneven stress in plane

Elastic recovery of bending moment along punch ridgeline by uneven stress in the thickness direction

* Yoshida & Isogai, Nippon Steel Tech Report, 2013
Application: springback

Before (selected from literature)

\[ \frac{R}{t} \]

- Kuwabara (1996)
- Gerdeen (1994)
- Wang (1993)
- Wenner (1982)

\[ \Delta \theta \]

- Friction
- Thickness

- Tool radius

\[ \Delta \theta \]

- Strength, anisotropy
- Hardening exponent

- Lee (1995)

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Application: springback

Idealized model

\[ T = w \int_{-t/2}^{t/2} \sigma dz \]

\[ M = \frac{Yt^2}{4} \left( 1 - \bar{T}^2 \right) \]

\[ \Delta \theta = \frac{1}{R} - \frac{1}{r} = \frac{3Y}{Et} \left( 1 - \bar{T}^2 \right) \]
Application: springback

- 2D draw bending simulations

(Stage 1) Pre-tensile strain 8%

(Stage 2) Draw-bend forming and springback
Application: springback

DP780 1.4t
Pre-strained

Experiment
von Mises
Yld2000-2d

\( \sigma_{yy}/\sigma_0 \)
\( \sigma_{xx}/\sigma_0 \)
von Mises
Yld2000-2d
(\( m=6 \))

\( \sigma_{xy}=0 \)
Application: springback

![Graph showing springback behavior with experimental data and simulations for DP780 1.4t material.](image)

- **Y [mm]**
- **X [mm]**
- DP780 1.4t
- Pre-strained
- Experiment
- IH
- IH-KH
- HAH

![Graph showing true stress versus true strain for DP780 1.4t material.](image)

- True stress [MPa]
- True strain
- EXP
- IH
- IH-KH
- HAH

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Application: issue

Friction coefficient

- The friction coefficient is not uniform due to different contact condition.
- A specific constant coefficient may provide similar predictions, but this value would not work if the forming condition changes.
Application: issue

![Graphs showing comparison between conventional and advanced techniques for punch force and punch stroke with different BHF values.]

TRIP780 1.2t
BHF: 20 kN

- Experiment
- Conventional
- Advanced

TRIP780 1.2t
BHF: 70 kN
Application: issue

Sensitivity in springback

At low BHF
Hardening > Elastic modulus >> friction > yield function

At high BHF
Hardening > friction > elastic modulus >> yield function

Lee et al, IJP, 2015
Thank you.

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