445.204

Introduction to Mechanics of Materials

(재료역학개론)

Chapter 12: Yield and Failure criteria

(Ch. 9 in Shames)

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Yielding and Failure?

- Yielding for ductile materials – yield conditions
- Failure for brittle materials – failure conditions

From one dimensional stress state

1. Max. normal stress
2. Max. shear stress
3. Max. normal strain
4. Max. shear stain
5. Max. distortion strain energy density

Hydrostatic stress

- Three principal stresses are the same
- No shear stress
- Same stress state under deep sea, thus we call this “hydrostatic”

$$\begin{bmatrix} \sigma_p & 0 & 0 \\ 0 & \sigma_p & 0 \\ 0 & 0 & \sigma_p \end{bmatrix}$$

Ductile metals are known to be independent on the “hydrostatic stress state”

Multidimensional stress state – Yield criterion for ductile materials

"Tresca" yield criterion – when max. shear stress meets a critical value

$$\frac{\sigma_{\max} - \sigma_{\min}}{2} = const = \tau_{Tr}$$

- How to determine the constant?

1) Pure shear test – when yielding occurs with a shear stress k

$$\tau_{\max} = \tau_{Tr} = k$$

2) Simple tension (or uniaxial tension test) – when yield stress is σ_Y

$$\tau_{\max} = \tau_{Tr} = \frac{\sigma_Y - 0}{2} = \frac{\sigma_Y}{2}$$

Multidimensional stress state – Yield criterion for ductile materials

"von Mises" or "Maximum distortion strain energy density" yield criterion

$$u_{total} = \frac{1}{2E}\left[\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2\right) - 2\nu\left(\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_2\sigma_3\right)\right]$$

$$u_{hydro} = \frac{3}{2E}\sigma_p^2\left(1-2\nu\right) = \frac{1}{6E}\left(1-2\nu\right)\left(\sigma_1 + \sigma_2 + \sigma_3\right)^2$$

$$u_{distort} = u_{total} - u_{hydro} =$$

$$\frac{1}{6E}\left(1+\nu\right)\left[\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2\right]$$

Multidimensional stress state – Yield criterion for ductile materials

"von Mises" or "Maximum distortion strain energy density" yield criterion

$$u_{distort} = u_{total} - u_{hydro} =$$

$$\frac{1}{6E}(1+\nu)\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]=const$$

- How to determine the constant?

Uniaxial tension test – when yielding occurs with an yield stress σ_Y

$$const=\left(u_{distort}\right)_{u.t}=\frac{1}{3E}(1+\nu)\sigma_Y^2$$

Multidimensional stress state – Yield criterion for ductile materials

"von Mises" or "Maximum distortion strain energy density" yield criterion

$$\frac{\sqrt{2}}{2}\left[\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2\right]^{\frac{1}{2}}=\sigma_Y$$

Multidimensional stress state – Yield criterion for ductile materials; Example

Consider the following state of plane stress:

$$\begin{aligned} \tau_{xx} &= 10{,}000 \text{ psi} \\ \tau_{yy} &= -20{,}000 \text{ psi} \\ \tau_{xy} &= 5000 \text{ psi} \end{aligned} \qquad \text{(a)}$$

If the yield stress τ_Y from a one-dimensional tensile test is 30,000 psi for the materials used, will there be yielding according to the Tresca criterion and according to the maximum distortion strain-energy criterion?

To find the principal stresses in the xy plane we employ Eq. (7.13). Thus,

$$\tan 2\tilde{\theta} = \frac{2\tau_{xy}}{\tau_{xx} - \tau_{yy}} = \frac{10{,}000}{30{,}000} = \frac{1}{3}$$

$$\therefore \tilde{\theta}_{a,b} = 9.22°, 99.22° \qquad \text{(b)}$$

The principal stresses are then

$$\tau_{a,b} = \frac{\tau_{xx} + \tau_{yy}}{2} + \frac{\tau_{xx} - \tau_{yy}}{2} \cos \begin{Bmatrix} 18.43° \\ 198.43° \end{Bmatrix} + \tau_{xy} \sin \begin{Bmatrix} 18.43° \\ 198.43° \end{Bmatrix}$$

$$= \frac{10{,}000 - 20{,}000}{2} + \frac{10{,}000 + 20{,}000}{2} \begin{Bmatrix} .948 \\ -.948 \end{Bmatrix} + (5000) \begin{Bmatrix} .316 \\ -.316 \end{Bmatrix} \qquad \text{(c)}$$

$$= 10{,}790, -20{,}790 \text{ psi}$$

The other principal stress is clearly 0. Hence, for this case we have

$$\begin{aligned} \tau_1 &= 10{,}790 \text{ psi} \\ \tau_2 &= 0 \\ \tau_3 &= -20{,}790 \text{ psi} \end{aligned} \qquad \text{(d)}$$

For the *Tresca* test we now compare

$$\frac{1}{2}(\tau_{max} - \tau_{min}) = \frac{1}{2}[10{,}790 - (-20{,}790)] = 15{,}790 \text{ psi} \qquad \text{(e)}$$

with τ_{Tr} which is $\tau_Y/2 = 15{,}000$ psi. Clearly, the Tresca condition predicts yielding.

As for the *distortional strain-energy* test we compare

$$\begin{aligned} {}^{0}u_{distort} &= \frac{1}{6E}(1+\nu)[(\tau_1 - \tau_2)^2 + (\tau_1 - \tau_3)^2 + (\tau_2 - \tau_3)^2] \\ &= \frac{1}{6E}(1+\nu)[(10{,}790 - 0)^2 + (10{,}790 + 20{,}790)^2 + (0 + 20{,}790)^2] \\ &= \frac{1}{6E}(1+\nu)(116 \times 10^6 + 998 \times 10^6 + 430 \times 10^6) \\ &= \frac{1}{6E}(1+\nu)(1546) \times 10^6 \end{aligned} \qquad \text{(f)}$$

with $({}^{0}u_{distort})_{tens}$, which from the tensile test is

$$(U_{distort})_{tens} = \frac{1}{3E}(1+\nu)\tau_Y^2 = \frac{1}{6E}(1+\nu)\,1800 \times 10^6 \qquad \text{(g)}$$

Multidimensional stress state – failure model for brittle materials

“Max. normal stress” failure criterion

- Fracture in a brittle material such as glass and case iron will occur whenever the max. principal stress equals the ultimate stress from either tension or compression

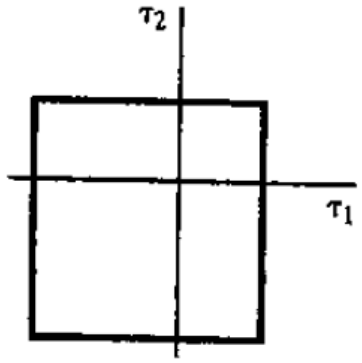

Figure 9.13. Maximum normal stress.

Multidimensional stress state

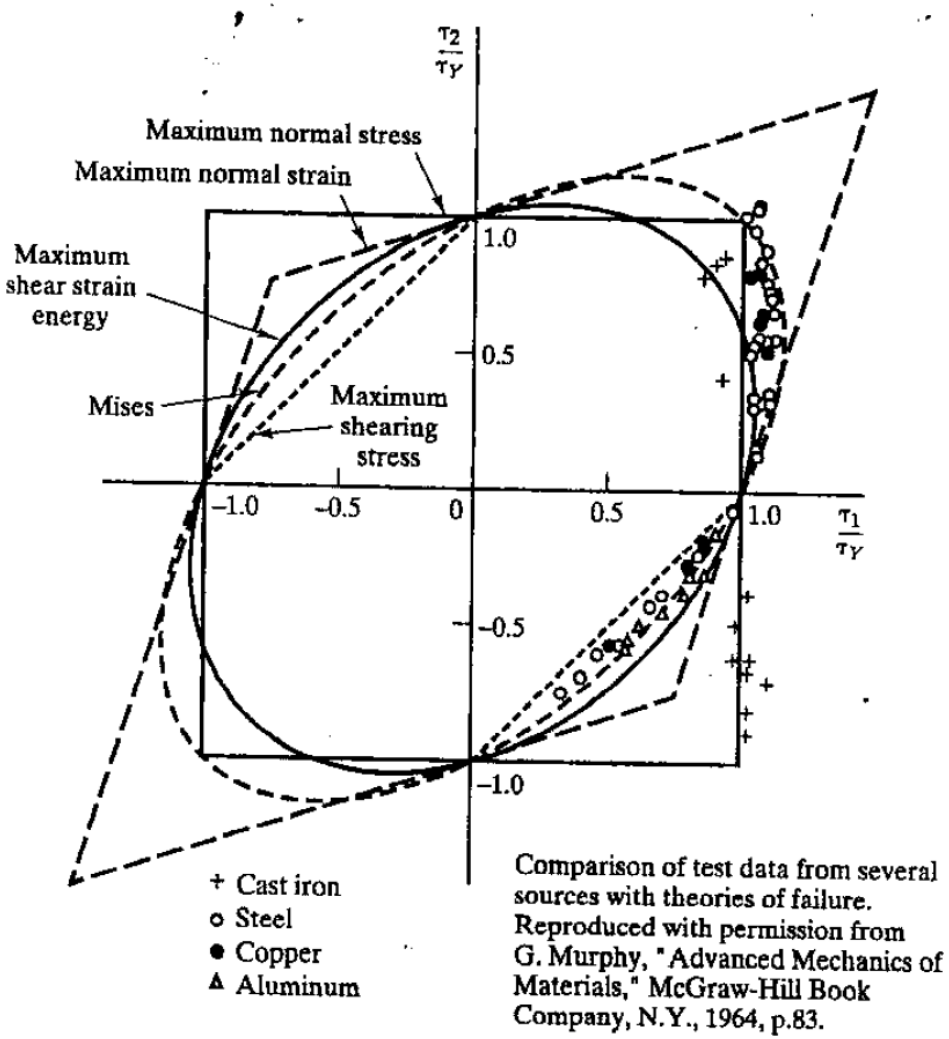

Figure 9.14. Comparison of test data from several sources with theories of failure. [Reproduced with permission from G. Murphy, *Advanced Mechanics of Materials* (McGraw-Hill Book Company, New York, 1964), p. 83.]