Image processing #3

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<u>Contexts</u>

- #1. Introduction and Examples
- #2. Basics of Matlab, Image Processing Toolbox

• <u>#3. 3D input, Segmentation, Edge detection & Transformation</u>

<u>3D data input in Matlab</u>

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🖈 바로 가기

늘 바탕 화면

🐌 다운로드

🔁 문서

🔚 사진

MATLAB

| 강의자료

및 내 PC

🥩 네트워크

11개 항목

📕 숙제 및 시험

공유

사진 도구 MATLAB

<

보기 관리

★ 잘라내기

```
>> clear all; close all
```

- >> listing=dir('CTimage/*ipg');
- >> nFiles=numel(listing);

```
>> ls=imread(['CTimage' '/' listing(1).name]);
```

```
>> [nRows nCols nLvrs] = size(ls);
```

nPix = nBows * nCols;

```
>> resize = 494;
```

```
cBow = round(nBows/2);
```

```
sRow = cRow-resize/2;
```

```
cCol = round(nCols/2);
```

```
sCol = cCol-resize/2;
```

```
>> for k = 1:nFiles;
```

end

```
I = imread(['CTimage' '/' listing(k).name]);
```

```
l2{k} = l(sRow:sRow+resize-1.sCol:sCol+resize-1);
```

🚾 경로 복사 응 선택 안 함 고정 복사 불여별기 👔 바로가기 불여별기 이동 국사 역세 기급 위지 * 위지 * • 바꾸기 풀며 👝 히스토리 🔡 선택 영역 반전 클립보드 구성 구성 새로 만들기 응 김 방 다 ← → < ↑ 📜 > 내 PC > 문서 > MATLAB > CTimage → ~ ↑ 📕 > 내 PC > 문서 > MATLAB 5 ~ MATLAB 검식 🚁 바로 가기 ᡖ 바탕 화면 💺 다운로드 📔 문서 R(A)_rec_RS2_Tra R(A)_rec_RS2_Tra 0000 🔚 사진 CTimac | 11, 12주 (image) 11, 12주 (image) MATLAB | 강의자료 📕 숙제 및 시험 R(A)_rec_RS2_Tra R(A)_rec_RS2_Tra 🗏 🗏 PC 🥩 네트워크 R(A)_rec RS2 Tra 951개 항목 편집기 - fullcode.m 변수 - 12 🕤 × 📝 A SE.Neighborhood SE.Dimensionality X L0 X +5 SE 2 listina 1x950 cell 3 5 2 4 1 494x494 uint8 494x494 ui... 494x494 ui... 494x494 ui... 494x494 ui... 494x494 ui... 2 3 4 5 6 7

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- 모두 선택

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6

- Measuring the size of a lake
 - Google Earth RGB image

Lake.JPG





Level = 0.3608 (Otsu's method)



>> cc=bwconncomp(1-bw,4); >> graindata = regionprops(cc,'basic'); >> grain_areas = [graindata.Area]; >> [max_area, idx]=max(grain_areas) >> grain = false(size(bw)); >> grain(cc.PixelIdxList{idx}) = true; >> figure, imshow(1-grain);title('largest grain')



$Max_area = 41190$

- Measuring the size of a lake
 - Different size depending different value of threshold



- <u>Classifying minerals in thin section</u>
- >> clear all; close all
- >> l=imread('thin.jpg');
- >> nColors=3;
- >> [ind map]=rgb2ind(l, nColors, 'nodither');
- >> figure, imshow(ind, map)

nColors=3



nColors=8





nColors=12



- <u>Classifying minerals in thin section</u>
- Selecting N colors by N mouse clinks (hand)
 - >> figure, imshow(|)
 - >> colors=impixel; save colors;
 - >> nColors=size(colors,1);
 - >> [ind map] = rgb2ind(1, colors/256, 'nodither');
 - >> figure, imshow(ind, map);





Right click after selecting N colors

0	Colors ×	colors 🛛 🛪		
11	x3 double			
	1	2	3	4
1	50	43	33	
2	255	112	132	
3	249	65	119	
4	153	110	0	
5	197	103	29	
6	226	59	147	
7	226	83	17	
8	255	255	232	
9	112	105	61	
0	255	255	253	
1	114	115	83	
12				



- Filtering (basic threshold)
 - >> clear all; close all;
- >> l=imread('lake2.jpg');
- >> figure, imshow(|)

>> [ind map]=rgb2ind(1, 5, 'nodither');
>> figure, imshow(ind, map)

>> level = 4/256; >> bw = im2bw(l,level); >> figure, imshow(bw) >> bw=1-bw;

- >> cc=bwconncomp(bw,4);
- >> graindata = regionprops(cc, 'basic');
- >> grain_areas = [graindata.Area];
- >> [max_area, idx]=max(grain_areas)
- >> grain=false(size(bw));
- >> grain(cc.PixelIdxList{idx}) = true;
- >> figure, imshow(1-grain)









• Filtering (taking average)

>> clear all; close all; >> l=imread('lake2.jpg'); >> figure, imshow(l)



L	변수 - h					
+	colors	× Ir ×	I × ind	× map	× bw ×	h ×
H	5x5 double					
	1	2	3	4	5	6
1	0.0400	0.0400	0.0400	0.0400	0.0400	
2	0.0400	0.0400	0.0400	0.0400	0.0400	
3	0.0400	0.0400	0.0400	0.0400	0.0400	
4	0.0400	0.0400	0.0400	0.0400	0.0400	
5	0.0400	0.0400	0.0400	0.0400	0.0400	
6						

- >> n=5; >> h=ones(n,n)/(n*n); >> l=imfilter(l,h);
- >> figure, imshow(1)



- >> bw=1-bw;
- >> cc=bwconncomp(bw,4);
- >> graindata = regionprops(cc,'basic');
- >> grain_areas = [graindata.Area];
- >> [max_area, idx]=max(grain_areas)
- >> grain=false(size(bw));
- >> grain(cc.PixelIdxList{idx}) = true;
- >> figure, imshow(1-grain)



- <u>Filtering (Entropy filtering)</u>
- >> clear all; close all;
- >> l=imread('lake2.jpg');
- >> figure, imshow(1)



- >> fSize=3; ngood=true(fSize);
 >> G=rgb2gray(|);
- >> E=entropyfilt(G,ngood);
- >> figure, imagesc(E)



>> L=E<0.8;

- >> imshow(L)
- >> cc=bwconncomp(L,4);
- >> graindata=regionprops(cc,'basic').
- >> grain_areas=[graindata.Area];

3474 pixels

- >> max(grain_areas)
- >> imshow(1-L)



- >> cc=bwconncomp(L,4);
- >> graindata=regionprops(cc,'basic');
- >> grain_areas=[graindata.Area];
- >> max(grain_areas)
- >> imshow(1-L)



- Filtering (Entropy filtering)
 - >> clear all, close all
 - >> l=imread('swiss.jpg');
 - >> G=rgb2gray(|);



- >> fSize=5, nhood=true(fSize);
- >> E=entropyfilt(G,nhood);
- >> figure, imagesc(E)



>> L=E<3.5; >> figure, imshow(L)



- <u>Entropy Filter + Gray threshold + Closing (removing)</u>
 - >> L=E<3.5;
 - >> figure, imshow(L)

>> L2=bwareaopen(L, 200); >> figure, imshow(L2) >> L3=bwareaopen(1-L2, 500);
>> figure, imshow(1-L3)







• <u>Entropy Filter + Gray threshold + Closing (removing)</u>

>> L3=bwareaopen(1-L2, 500);
>> figure, imshow(1-L3)

>> G2=G;

>> G2(L3==0)=0;

>> figure, imshow(G2)

Original image

Entropy, Standard Deviation Filter, Gradients





- Matlab likes to operate on objects
- Example of Closing

>> clear all; close all; >> l=imread('lake2.jpg');



n=5; h=ones(n,n)/(n*n); l=imfilter(l,h); figure, imshow(l) level=4/256; bw=im2bw(l,level); figure, imshow(bw)

>> bw=1-bw; >> figure, imshow(bw)



>> SE=strel('disk',10);
>> L1=imclose(bw, SE);
>> figure, imshow(L1)







• Morphological operations (Image Erosion, Image Dilation, Image Opening, Image Closing)



- Does B "fit" in the set X?
- Retain all points (i,j) in X such that when B is centered at (i,j), B is contained in X
- Mathematically:

$$\mathcal{E}_B(X) = \left\{ X \, \middle| \, B_X \subseteq X \right\}$$

- In Matlab: imerode(Img,Elt)

• Morphological operations (Image Erosion, Image Dilation, Image Opening, Image Closing)



- Does B "touch" in the set X?
- Add to X all points (i,j) in B such that when B is centered at (i,j), B is overlapping with X
- Mathematically:

$$\delta_{B}(X) = \bigcup_{b \in B} X _ b$$

- In Matlab: imdilate(Img,Elt)

• Morphological operations (Image Erosion, Image Dilation, Image Opening, Image Closing)



- Does B "fit" in the set X?
- Like with erosion, retain all points (i,j) in X such that when B is centered at (i,j), B is contained in X
- When the previous is true, also keep all of B
- Mathematically:

$$\gamma_B(X) = \bigcup_X \left\{ B_X \, \middle| \, B_X \subseteq X \right\}$$

- In Matlab: imopen (Img,Elt)

• Morphological operations (Image Erosion, Image Dilation, Image Opening, Image Closing)



- Does B "fit" in the backround of the set X?
- When the previous is true, all of B belongs to the background
- The complement of the new background define X
- Mathematically:

$$\phi_B(X) = \left[\bigcup_X \left\{ B_X \mid B_X \subseteq X^c \right\} \right]^c$$

- In Matlab: imclose(Img,Elt)

- Compute the mean and the covariance matrix
- Then similarity implies being with a weighted distance
- The pixels that are within an ellipsoid centered at the mean color are similar



Covariance:

$$\operatorname{cov}(A, B) = \frac{1}{N-1} \sum_{i=1}^{N} (A_i - \mu_A)^* (B_i - \mu_B)$$

Covariance Matrix: $C = \begin{pmatrix} \operatorname{cov}(A, A) & \operatorname{cov}(A, B) \\ \operatorname{cov}(B, A) & \operatorname{cov}(B, B) \end{pmatrix}$.

Variance: $V = \frac{1}{N-1} \sum_{i=1}^{N} |A_i - \mu|^2$ Average: $\mu = \frac{1}{N} \sum_{i=1}^{N} A_i$.

Mahalanobis distance

✓ Mahalanobis Distance

The Mahalanobis distance is a measure between a sample point and a distribution.

The Mahalanobis distance from a vector y to a distribution with mean μ and covariance Σ is

 $d = \sqrt{(y-\mu)\sum^{-1} (y-\mu)'} \,.$

This distance represents how far y is from the mean in number of standard deviations.

mahal returns the squared Mahalanobis distance d^2 from an observation in Y to the reference samples in X. In the **mahal** function, μ and Σ are the sample mean and covariance of the reference samples, respectively.



(From Matlab Help)

- <u>Compute Forest area in the Photo</u>
- Input & Smoothening

```
clear all; close all;
imName = 'naip1.jpg';
rgblmg = imread(imName);
[nRows nCols nLyrs] = size(rgblmg);
nPix = nRows * nCols;
figure; imshow(rgblmg); title(['RGB image ' imName]);
```

%Smooth

filtSize = 10; %set filter size
myFilt = ones(filtSize) ./ filtSize^2;
smoothlmg = imfilter(rgblmg, myFilt);
figure; imshow(smoothlmg); title('smoothed RGB image');



• <u>Use roipoly() for selecting Region of Interest (ROI)</u>

```
figure;
roiMask = roipoly(rgblmg);
%right clink on the polygon and select the option "Create Mask"
figure; imshow(roiMask); title('Mask');
```



```
red=immultiply(roiMask, smoothlmg(:,:,1));
green=immultiply(roiMask, smoothlmg(:,:,2));
blue=immultiply(roiMask, smoothlmg(:,:,3));
roilmg = cat(3, red, green, blue);
figure; imshow(roilmg); title('Color sample');
```



<u>Compute Mahalanobis distance for all pixels</u>

%compute mean patch color and std

imgPix = double(reshape(smoothlmg, nPix, 3)); roiPix = double(reshape(roilmg, nPix, 3)); roildx = find(roiMask); roiPix = roiPix(roildx, 1:3); myMean = mean(roiPix); myStd = max(std(roiPix));

%compute Mahalanobis dist.

mahDist = mahal(imgPix, roiPix); myDist = reshape(mahDist, nRows, nCols); figure; imagesc(myDist); title('Mahalanobis distance from region color'); colorbar; impixelinfo





<u>Segmentation & Histogram</u>

%segmentation
kThresh = 1;
mySegm = false(nRows, nCols);
mySegm(myDist <= kThresh * myStd) = true;</pre>

lr=rgblmg(:,:,1); lg=rgblmg(:,:,2); lb=rgblmg(:,:,3);

Ir(mySegm==1) = 255; Ib(mySegm==1) = 0; Ig(mySegm==1) = 0;

l=cat(3,lr,lg,lb); figure;imshow(l);title('Segmented forest')

%histogram
myCount=sum(mySegm(:));
myDistr = [myCount nPix-myCount] ./ nPix;
figure; bar(myDistr);
axis([0.5 2.5 0 1]);
set(gca, 'XTickLabel', {'Forest', 'Other'});
title('Forest percentage');





• Watershed Transformation



10 -

20 -

30 -

40 -

50 -



- <u>Watershed Transformation</u>
 - >> clear all; close all; >> l=imread('dune.jpg'); >> figure, imshow(l)



>> G=rgb2gray(1);
>> figure; imshow(G)



- >> L=watershed(G);
- >> G2=G;
- >> G2(L==0)=255;
- >> figure; imshow(G2)



• **Smoothed** + Watershed Transformation

>> fSize = 6; >> h=ones(fSize,fSize)/fSize^2; Sm=imfilter(G,h); figure;imshow(Sm)



- >> L=watershed(Sm);
- >> LO=(L==0);
- >> G2=Sm;
- >> G2(LO)=255;
- >> figure, imshow(G2);



- >> 12=1(:,:,2);12(L0)=0;
- >> |1=|(:,:,1);|1(L0)=0;
- >> 12=1(:,:,2);12(L0)=0;
- >> |3=|(:,:,3);|3(L0)=0;
- >> ||=cat(3,|1,|2,|3);
- >> figure, imshow(||)



• Median Filter + Watershed Transformation

>> fSize = 6; >> Sm=medfilt2(G, [fSize fSize]); >> figure; imshow(Sm)



>> L=watershed(Sm); >> LO = (L==O); >> G2 = Sm; >> G2(LO) = 255; >> figure; imshow(G2)



- >> |1=|(:,:,1);|1(L0)=0;
- >> 12=1(:,:,2);12(L0)=0;
- >> |3=|(:,:,3);|3(LO)=0;
- >> ||=cat(3,|1,|2,|3);
- >> figure; imshow(11)



<u>Regional Extended Minima + Watershed Transformation</u>

- >> thresh=3;
- >> Imin=imextendedmin(Sm, thresh);
- >> figure; imshow(lmin)



>> G2=imimposemin(G, Imin); >> figure; imshow(G2)

- >> L2 = watershed(G2);
- >> L0 = (L2==0);
- >> G3=Sm;
- >> G3(LO)=255;
- >> figure; imshow(G3)

- >> |1=|(:,:,1);|1(L0)=0;
- >> 12=1(:,:,2);12(L0)=0;
- >> |3=|(:,:,3);|3(LO)=0;
- >> ||=cat(3,|1,|2,|3);
- >> figure; imshow(11)



- 1. <u>Canny -> Dilation -> Skeletonization</u>
- 2. <u>Bottom-hat -> Distance -> Watershed</u>
- 3. <u>gPb -> Distance -> Watershed</u>
- 4. <u>gPb -> OWT -> UCM</u>



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Automated Sizing of Coarse-Grained Sediments: Image-Processing Procedures¹

David J. Graham,² Ian Reid,² and Stephen P. Rice²

This is the first in a pair of papers in which we present image-processing-based procedures for the measurement of fluvial gravels. The spatial and temporal resolution of surface grain-size characterization is constrained by the time-consuming and costly nature of traditional measurement techniques. Several groups have developed image-processing-based procedures, but none have demonstrated the transferability of these techniques between sites with different lithological, clast form and textural characteristics. Here we focus on image-processing procedures for identifying and measuring image objects (i.e. grains); the second paper examines the application of such procedures to the measurement of fluvially deposited gravels. Four image-segmentation procedures are developed, each having several internal parameters, giving a total of 416 permutations. These are executed on 39 images from three field sites at which the clasts have contrasting physical properties. The performance of each procedure is evaluated against a sample of manually digitized grains in the same images, by comparing three derived statistics. The results demonstrate that it is relatively straightforward to develop procedures that satisfactorily identify objects in any single image or a set of images with similar sedimentary characteristics. However, the optimal procedure is that which gives consistently good results across sites with dissimilar sedimentary characteristics. We show that neighborhood-based operations are the most powerful, and a morphological bottom-hat transform with a double threshold is optimal. It is demonstrated that its performance approaches that of the procedures giving the best results for individual sites. Overall, it out-performs previously published, or improvements to previously published, methods.

KEY WORDS: grain-size analysis, mathematical morphology, segmentation, bottom-hat transform, rivers, fluvial.

1. <u>Canny -> Dilation -> Skeletonization</u>



1. <u>Canny -> Dilation -> Skeletonization</u>



1. <u>Canny -> Dilation -> Skeletonization</u>



- >> E5=bwmorph(E4, 'shrink', Inf);
 >> E6=bwmorph(E5, 'spur',Inf);
 >> E7=bwmorph(E6, 'clearn',Inf);
 >> E7=bwmorph(E6, 'clearn',Inf);
- >> E7=bwmorph(E6, 'clean',Inf);
- >> E8=bwmorph(E7,'diag',Inf);
- >> figure; imshow(E8);

- >> |1=|(:,:,1); |1(E8==1)=255;
- >> l2=l(:,:,2); l2(E8==1)=0;
- >> |3=|(:,:,3); |3(E8==1)=255;
- >> ||=cat(3, |1, |2, |3);
- >> figure; imshow(11)





1. <u>Canny -> Dilation -> Skeletonization</u>



>> C=imcomplement(E8);

- >> [L numCC]=bwlabel(C);
- >> map=rand(numCC, 3);
- >> transp=0.5;
- >> mask=zeros(size(L))+transp;
- >> figure; imshow(1);
- >> hold on
- >> h=imshow(L,map);
- >> set(h, 'AlphaData',mask);
 >> hold off



2. Bottom-hat -> Distance -> Watershed



clear all; close all; l=imread('pebbles.png'); imshow(l); G=rgb2gray(l); figure; imshow(G); fSize = 7; S=medfilt2(G, [fSize fSize]); figure; imshow(S);



2. Bottom-hat -> Distance -> Watershed



bh_radius = 5; SE = strel('disk',bh_radius); S2 = imbothat(S, SE); figure, imshow(imadjust(S2)); BH=imsubtract(S,S2); figure, imshow(BH);



2. Bottom-hat -> Distance -> Watershed



II=BH; low_thresh = 3; [counts, intensity] = imhist(II); cumcounts = cumsum(counts); cumpercent = ... cumcounts/max(cumcounts)*100; cum1 = cumpercent(1:end-1); cum2 = cumpercent(2:end); if cumpercent(1) > low_thresh lowindex = 1; else

> lowindex = find(cum1<=low_thresh & ... cum2 > low_thresh);

end

LB = (|| <= intensity(lowindex));
figure; imshow(LB);</pre>





2. Bottom-hat -> Distance -> Watershed



high_thresh = 35; if cumpercent(1) > high_thresh highindex = 1;

else

highindex = find(cum1 <= high_thresh & ...</pre>

cum2 > high_thresh);

end

figure; imshow(HB)





2. Bottom-hat -> Distance -> Watershed





2. Bottom-hat -> Distance -> Watershed



L=watershed(DT3); L0 = (L==0); G2 = G; G2(L0) = 255; figure; imshow(G2); l1=l(:,:,1); l1(L0)=255; l2=l(:,:,2); l2(L0)=0; l3=l(:,:,3); l3(L0)=255; l1=cat(3,l1,l2,l3); figure; imshow(l1)



2. Bottom-hat -> Distance -> Watershed





1. <u>Canny -> Dilation -> Skeletonization</u>

2. Bottom-hat -> Distance -> Watershed



3. gPb -> Distance -> Watershed

4. gPb -> OWT -> UCM



Transformation

<u> </u>

Wavelet Transform

Hough Transform

- FT is a particular type of integral transform that enables us to view imaging and image processing from an alternative viewpoint by transforming the problem to another space.

- We are usually concerning with 2-D spatial distributions of intensity or color which exist in **real space** (i.e., 2-D Cartesian space in which the axes define units of length)

- The FT operates on such a function to produce an entirely equivalent form which lies in an abstract space called **frequency space**.





Harmonic content of signals. Any signal can be expressed as a weighted linear combination of harmonic (i.e., sine and cosine) functions having different periods or frequencies. These are called the (spatial) frequency components of the signal.

Time series

$$V(t) = \sum_{n=0}^{\infty} a_n \cos\left(\frac{2\pi nt}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nt}{T}\right)$$
$$= \sum_{n=0}^{\infty} a_n \cos(\omega_n t) + \sum_{n=1}^{\infty} b_n \sin(\omega_n t)$$

Note: n-> infinite for exact reproducibility

Fourier basis functions $sin(k_n x) cos(k_n x)$

Spatial series

$$V(x) = \sum_{n=0}^{\infty} a_n \cos\left(\frac{2\pi nx}{\lambda}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nx}{\lambda}\right)$$
$$= \sum_{n=0}^{\infty} a_n \cos(k_n x) + \sum_{n=1}^{\infty} b_n \sin(k_n x)$$

Spatial frequency $k_n = 2\pi n/\lambda$

Fourier or frequency spectrum $\{a_n \text{ and } b_n\}$

<u>Transformation (Fourier Transform</u>) $V(x) = \sum_{n=0}^{\infty} a_n \cos(k_n x) + \sum_{n=1}^{\infty} b_n \sin(k_n x)$

Fourier representation of a signal as a weighted combination of harmonic functions of different frequencies, the assigned weights constitute the **Fourier spectrum**. It is a complete and valid, alternative representation of the signal.

The space domain and the Fourier domain are reciprocal.

Harmonic terms with high frequencies (short periods) are needed to construct small-scale (i.e. sharp or rapid) changes in the signal.

Conversely, smooth features in the signal can be represented by harmonic terms with low frequencies (long periods). The two domains are thus reciprocal.

Fourier series expansion and Fourier transform

The difference is that the Fourier series breaks down a periodic signal into harmonic functions of discrete frequencies. Whereas, the Fourier transform breaks down a nonperiodic signal into harmonic functions of continuously varying frequencies.



 $\sin(k_n x) = \sin(2\pi n x/\lambda)$

<u>Calculation of the Fourier spectrum</u> $\{a_n \text{ and } b_n\}$

Harmonic function form

$$V(x) = \sum_{n=0}^{\infty} a_n \cos\left(\frac{2\pi nx}{\lambda}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nx}{\lambda}\right)$$
$$= \sum_{n=0}^{\infty} a_n \cos(k_n x) + \sum_{n=1}^{\infty} b_n \sin(k_n x)$$

$$a_n = \frac{2}{\lambda} \int_{\lambda/2}^{\lambda/2} f(x) \cos(k_n x) \, \mathrm{d}x$$

$$b_n = \frac{2}{\lambda} \int_{\lambda/2}^{\lambda/2} f(x) \sin(k_n x) \, \mathrm{d}x$$

where $k_n = 2\pi n/\lambda$.

Complex exponential form

$$f(x) = \sum_{n=-\infty}^{\infty} c_n \exp\left(\frac{i2\pi nx}{\lambda}\right)$$
$$= \sum_{n=-\infty}^{\infty} c_n \exp(ik_n x)$$

$$c_n = \frac{1}{\lambda} \int_{\lambda/2}^{\lambda/2} f(x) \exp(ik_n x) dx$$

where $c_k = a_k + ib_k$

Fourier transform of f(x):

Inverse Fourier transform:

$$F(k_x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) \exp(-ik_x x) dx \qquad \qquad f(x) = \int_{-\infty}^{\infty} F(k_x) \exp(ik_x x) dk_x$$

Specific weight (contribution) assigned to the harmonic (complex exponential) functions are given by the function $F(k_x)$

Fourier transform

$$F(k_x) = |F(k_x)| \exp i\varphi(k_x)$$

where $|F(k_x)|^2 = [\operatorname{Re}\{F(k_x)\}]^2 + [\operatorname{Im}\{F(k_x)\}]^2 \qquad \varphi(k_x) = \tan^{-1}\left[\frac{\operatorname{Im}\{F(k_x)\}}{\operatorname{Re}\{F(k_x)\}}\right]$

Fourier method in Image processing

(1) Images are not, in general, periodic functions

(2) Images are typically 2-D spatial functions of finite support



Fourier method in Image processing

$$F(f_x, f_y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \exp\left[-2\pi i (f_x x + f_y y)\right] dx dy$$





Frequency domain

 k_{y}

Original image

FT of image

Original image

FT of image

Discrete Fourier Transform (DFT)



Discrete Fourier Transform (DFT)



Sampling intervals related by



(Images from Fundamentals of Digital Image Processing, Solomon and Breckon 2010)

Discrete Fourier Transform (DFT)

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) \exp\left[-2\pi i \left(\frac{xu}{M} + \frac{yv}{N}\right)\right]$$





(Images from Fundamentals of Digital Image Processing, Solomon and Breckon 2010)

Examples



Image with periodic Noise



Notch Filter



DFT of Image



Result after notch filter applied then inverted



Images from

Examples

#7 Fourier Transformation of TEM image









Transformation

close all; clear all; l=imread('many balls.jpg'); figure; imshow(l); xg = rgb2gray(l); figure; imshow(xg); XG = fft2(xg, 1000, 1000); XGM=(abs(XG)); figure; imshow(XGM*0.00005); XGMS = fftshift(XGM); figure; imshow(XGMS*0.00005);







