<Blasius solution>

$$v = -\frac{1}{2}\sqrt{\frac{U\nu}{x}}\,f + \frac{1}{2}Ux^{-1}f'y$$

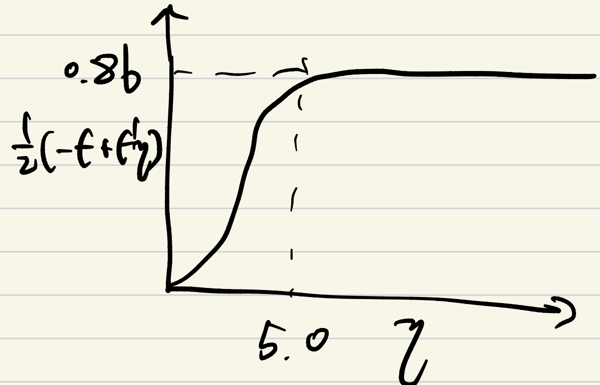
$$= -\frac{1}{2}U\sqrt{\frac{\nu}{Ux}}\,f + \frac{1}{2}Uf'\frac{y}{x}$$

$$= -\frac{1}{2}U\sqrt{\frac{\nu}{Ux}}\,f + \frac{1}{2}Uf'\eta\sqrt{\frac{U}{\nu x}}$$

$$\eta = y\sqrt{\frac{\nu}{Ux}}$$

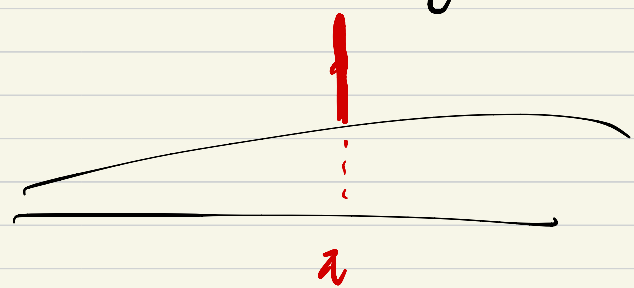
$$= \frac{y}{x}\sqrt{\frac{\nu x}{U}} \rightarrow \frac{y}{x} = \eta\sqrt{\frac{U}{\nu x}}$$

$$\therefore\ \boxed{v = \frac{1}{2}U\sqrt{\frac{\nu}{Ux}}\left(-f + f'\eta\right)}$$



$$v = 0.86\,U/\sqrt{Re_x} \quad \text{as } \eta \to \infty$$

- $$\frac{v}{U} = \frac{0.86}{\sqrt{Re_x}} \quad \text{as } \eta \to \infty$$



$$\frac{v}{U} = \frac{0.5(-f+f'\eta)}{\sqrt{Re_x}}$$

Ch.8

$$\frac{L}{\frac{1}{2}\rho V^2 A_p} = C_L$$

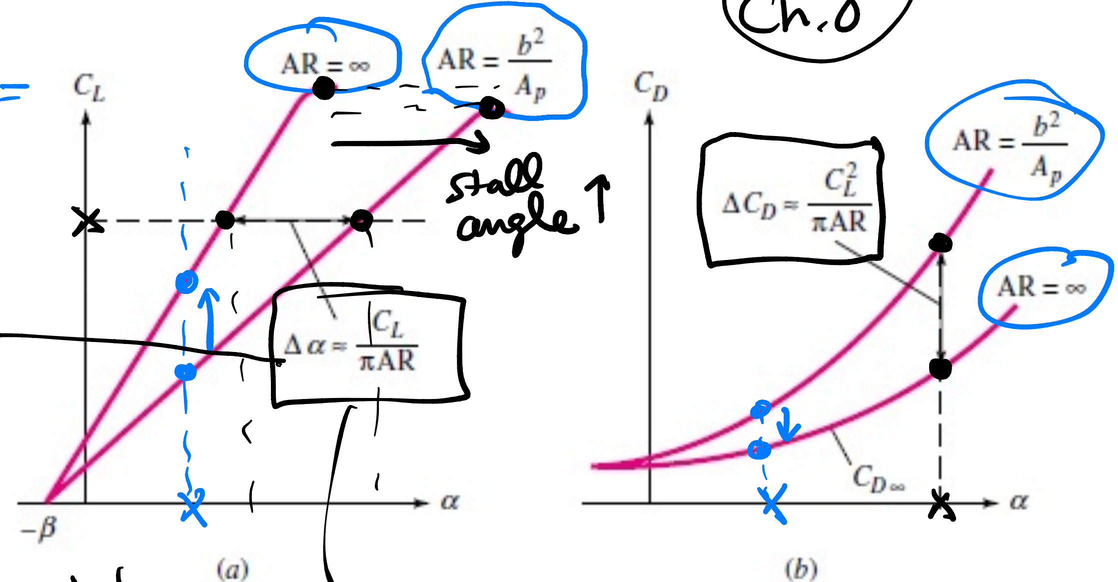
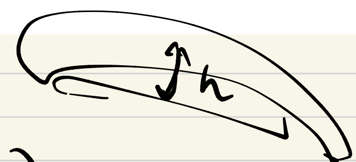


Fig. 7.27 Effect of finite aspect ratio on lift and drag of an airfoil: (*a*) effective angle increase; (*b*) induced drag increase.

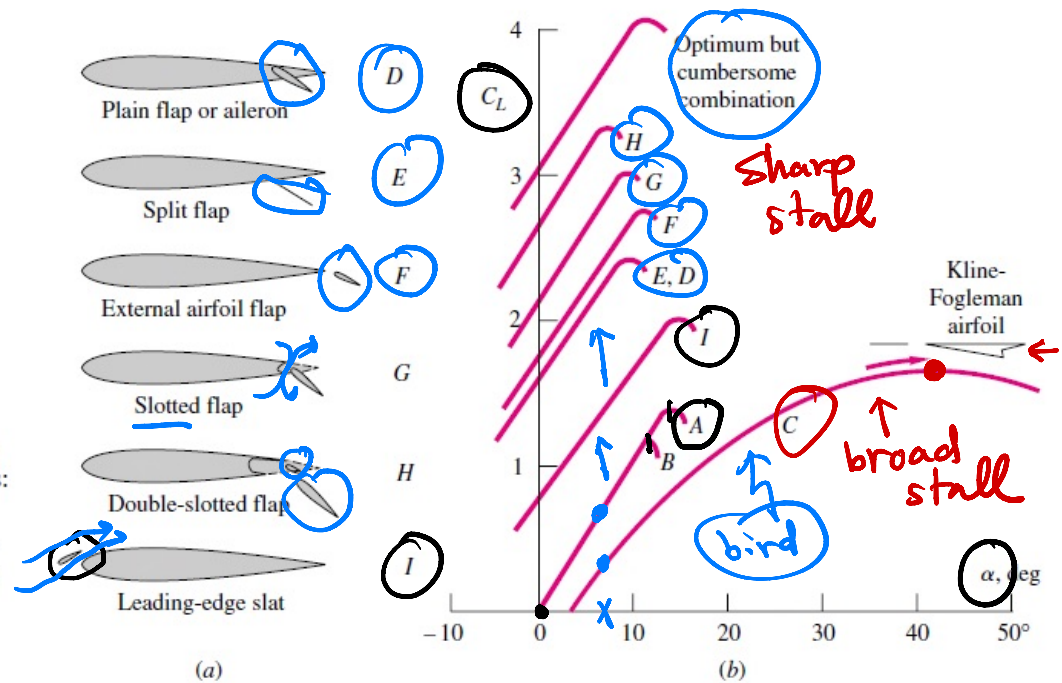
effective angle of attack increases by the amount of $\Delta\alpha$

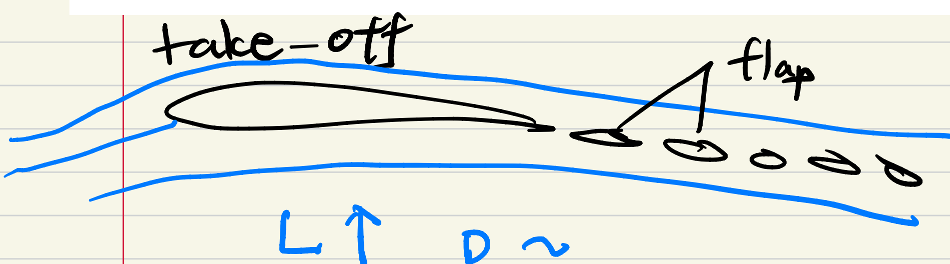
finite-span wing theory



$$C_L \simeq \frac{2\pi \sin(\alpha + 2h/c)}{1 + 2/AR}$$

Fig. 7.28 Performance of airfoils with and without high-lift devices: A = NACA 0009; B = NACA 63-009; C = Kline-Fogleman airfoil (*from Ref. 17*); D to I shown in (a): (a) types of high-lift devices; (b) lift coefficients for various devices.





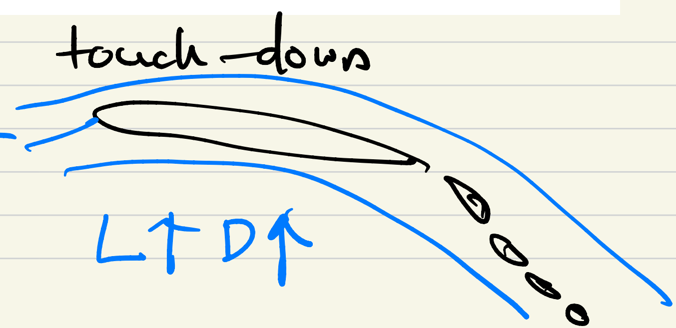
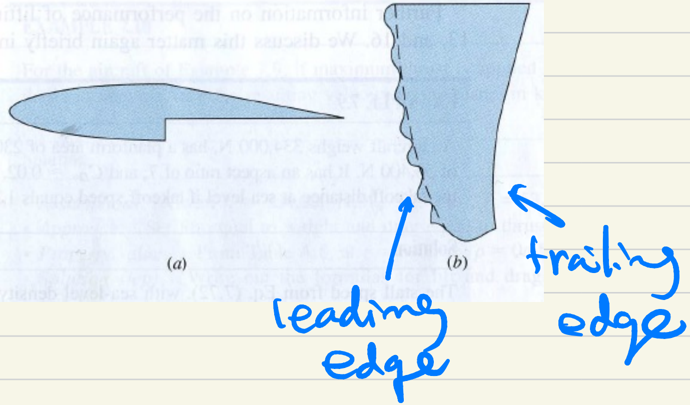
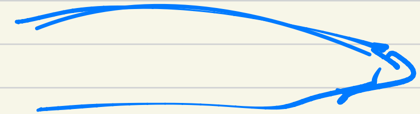
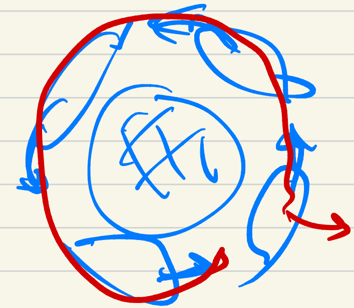


Fig. 7.29 Two new experimental airfoils: (*a*) cross section of the Kline-Fogelman airfoil with a round leading edge and a step cut-out at 50 percent chord [55]; (*b*) plan view of a wing modeled on the humpback whale flipper [57].





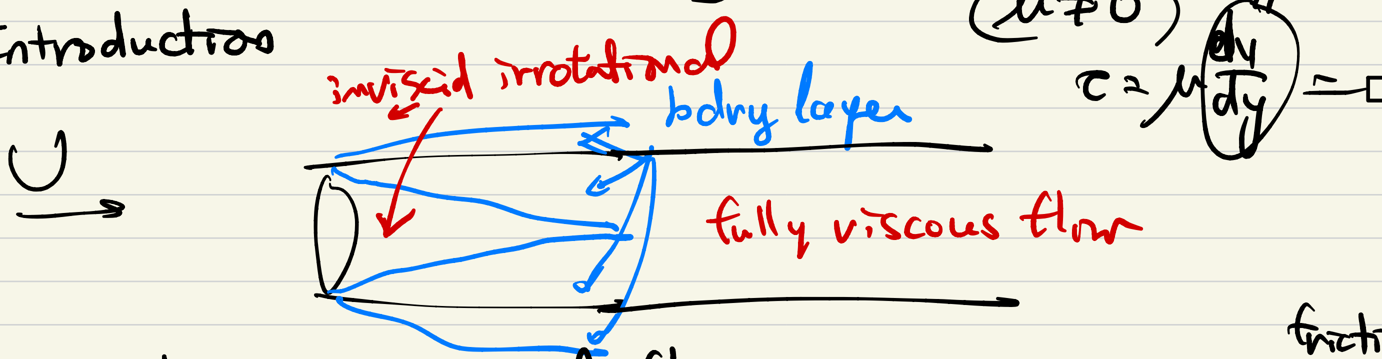


Ch. 8 Potential flow and CFD

= inviscid irrotational flow

$(\mu = 0)$ $(\underline{\omega} = \nabla \times \underline{V} = 0)$

8.1 Introduction



• Frictionless irrotational flow

"

inviscid

$(\mu = 0,\ \tau = 0)$ N-S eqs: $\rho \frac{dV}{dt} = \rho \underline{g} - \nabla p + \nabla \cdot \cancel{\tau}$ ϕ^0 frictionless

$$\rightarrow \rho \frac{\partial V}{\partial t} + \rho \underbrace{(\underline{V} \cdot \nabla)\underline{V}}_{\nabla\left(\frac{V^2}{2}\right) + (\nabla \times \underline{V}) \times \underline{V}} = \rho \underline{g} - \nabla p$$

$\nabla \times \underline{V} = \underline{\omega}$

$$\rightarrow \frac{\partial \underline{V}}{\partial t} + \nabla\left(\frac{V^2}{2}\right) + \underline{\omega} \times \underline{V} + \frac{1}{\rho}\nabla p - \underline{g} = 0$$: frictionless flow

Let $d\underline{r}$ be an arbitrary displacement vector.

$$\star \left[\frac{\partial \underline{V}}{\partial t} + \nabla\left(\frac{V^2}{2}\right) + \underline{\omega} \times \underline{V} + \frac{1}{\rho}\nabla p - \underline{g}\right] \cdot d\underline{r} = 0$$

$\boxed{(\underline{\omega} \times \underline{V}) \cdot d\underline{r} = 0}$ if

① $\underline{V} = 0$ trivial

② $\underline{\omega} = 0 = \nabla \times \underline{V}$ irrotational flow

③ $\underline{\omega} \times \underline{V} \perp d\underline{r}$

④ $\underline{V} \parallel d\underline{r}$: streamline

⑤ $\underline{\omega} \parallel d\underline{r}$: vortex line

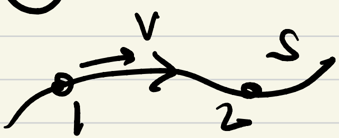
$\downarrow$

with $\underline{g} = -g\hat{k}$,

$$\boxed{\frac{\partial \underline{V}}{\partial t} \cdot d\underline{r} + d\left(\frac{V^2}{2}\right) + \frac{1}{\rho}dp + g\,dz = 0}$$

$$\boxed{\frac{\partial \underline{v}}{\partial t}\cdot d\underline{r} + d\left(\frac{V^2}{2}\right) + \frac{1}{\rho}dp + g\,dz = 0}$$

④ : 1 → 2 along the streamline



$$\boxed{\int_1^2 \frac{\partial V}{\partial t}ds + \frac{1}{2}(V_2^2 - V_1^2) + \int_1^2 \frac{dp}{\rho} + g(z_2 - z_1) = 0}$$

~~frictionless flow~~ along the streamline

② : $\underline{\omega} = \nabla \times \underline{V} = 0$ (irrotational flow)

↳ $\underline{V} = \nabla\phi$ ϕ: velocity potential

(potential line : line of const. ϕ
streamline : " " " ψ)

$$\frac{\partial \underline{V}}{\partial t}\cdot d\underline{r} = \frac{\partial}{\partial t}(\nabla\phi)\cdot d\underline{r} = \nabla\left(\frac{\partial\phi}{\partial t}\right)\cdot d\underline{r} = d\left(\frac{\partial\phi}{\partial t}\right)$$

①

→ $\boxed{\frac{\partial\phi}{\partial t} + \frac{1}{2}|\nabla\phi|^2 + \int\frac{dp}{\rho} + gz = \text{const}}$ unsteady irrotational Bernoulli eq.

If incomp. ($\rho = \text{const}$), $\quad \underline{v} = \nabla\phi$

$$\frac{\partial\phi}{\partial t} + \frac{1}{2}|\nabla\phi|^2 + \frac{p}{\rho} + gz = \text{const}$$

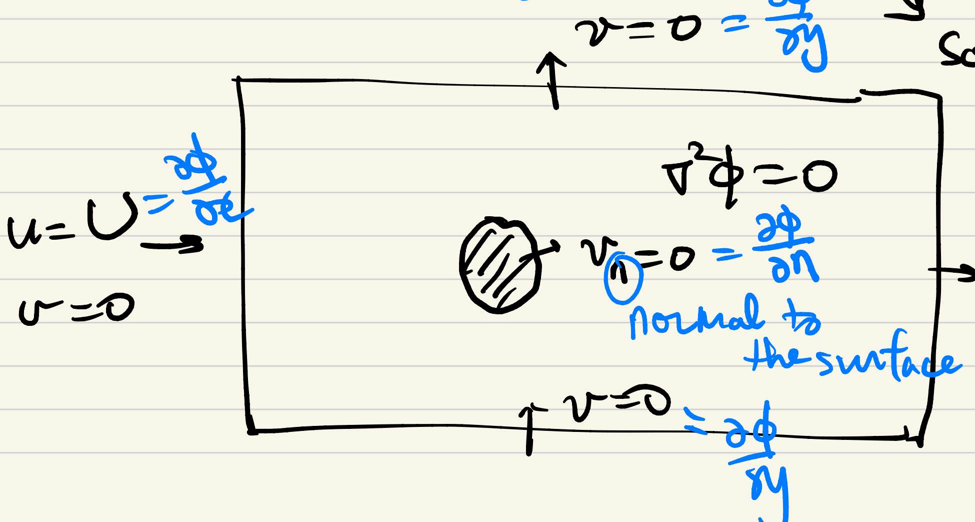
Continuity $\nabla\cdot\underline{v} = 0 \rightarrow \nabla\cdot(\nabla\phi) = 0 \rightarrow \boxed{\nabla^2\phi = 0}$ — ②

Laplace eq.

Potential theory

Solve ② to get ϕ. $\rightarrow \underline{v} = \nabla\phi \rightarrow$ Solve ① to get p.

~~no-slip~~ for $\mu = 0$



$u = U = \frac{\partial\phi}{\partial x}$

$v = 0$

ϕ = sol.

$\phi + c$ = sol.

much easier to solve $\nabla^2\phi=0$ than to solve the N-S eq.

No parameter in governing eq. like Re, Ma, Fr, ...

→ Inviscid flows are kinematically similar without additional parameters.

- Stream ft. ψ

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}\quad (2D) \Leftarrow \nabla\cdot\underline{V}=0$$

$$\underline{\omega}=\nabla\times\underline{V}\quad (2D)\quad \omega_z=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=\frac{\partial}{\partial x}\left(-\frac{\partial\psi}{\partial x}\right)-\frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial y}\right)$$

$$=-\nabla^2\psi$$

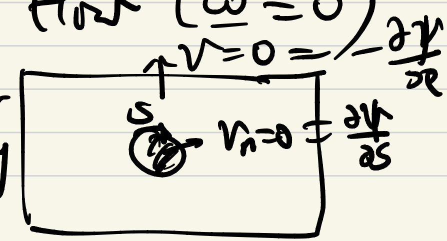
$$\boxed{\nabla^2\psi=-\omega_z}$$

$\boxed{\nabla^2\psi=0}$ for irrotational flow ($\underline{\omega}=0$)

Solve $\nabla^2\psi=0$ to get ψ

→ $u=\frac{\partial\psi}{\partial y}$, $v=-\frac{\partial\psi}{\partial x}$ → solve ① to get p.

$u=U=\frac{\partial\psi}{\partial y}$



- Orthogonality bet. potential lines and streamlines

$$\phi = \text{const} : d\phi = 0 = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy = u\,dx + v\,dy$$

$$\rightarrow \left.\frac{dy}{dx}\right|_{\phi=\text{const}} = -\frac{u}{v}$$

$$\psi = \text{const} : d\psi = 0 = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy = -v\,dx + u\,dy$$

$$\rightarrow \left.\frac{dy}{dx}\right|_{\psi=\text{const}} = +\frac{v}{u}$$

$$\therefore \left.\frac{dy}{dx}\right|_{\phi=\text{const}} \cdot \left.\frac{dy}{dx}\right|_{\psi=\text{const}} = -1 \quad \therefore \text{orthogonal}$$

- Plane polar coord. (r, θ)

$$v_r = \frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$$

$$v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} = 0$$

Laplace eq.