Lecture 12

Ch. 12 Effect of reservoir depletion (생산에 따른 저류층 고갈의 영향)

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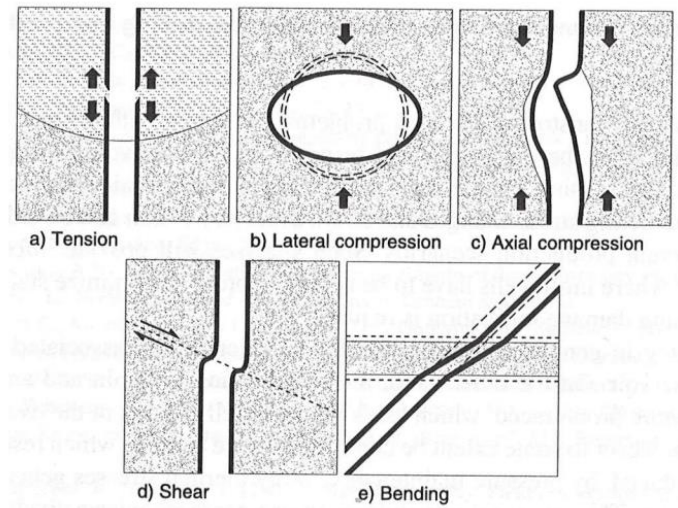
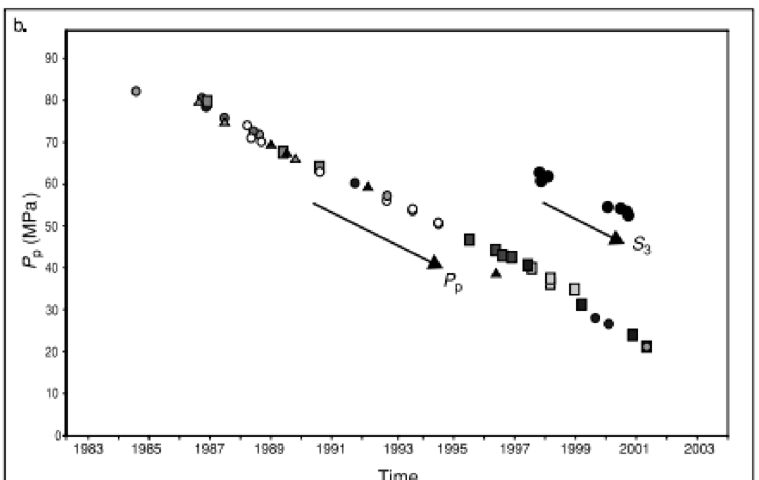
Effects of reservoir depletion (저류층 고갈의 영향)
Importance

- Effect of reservoir depletion
  - Surface subsidence
  - Casing collapse
  - Drilling problem – need to lower the mud weight
  - Depletion induced faulting (seismicity)
  - Hydraulic fracturing performance
  - Compaction drive



Casing damage scenarios



Pore pressure & S_3 decline in Gulf of Mexico (X field)

Zoback MD, 2007, Reservoir Geomechanics, Cambridge University Press

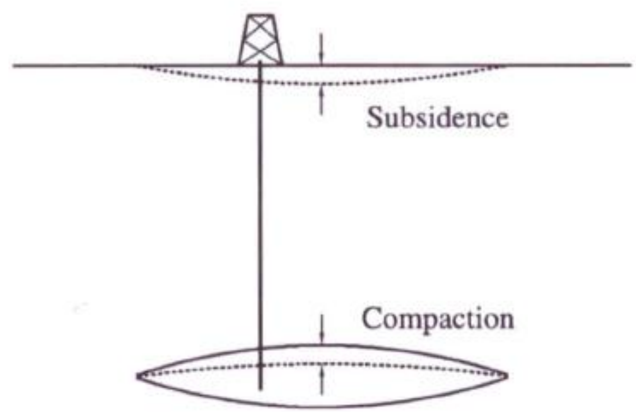


Fig. 12.1. Compaction and subsidence.

Fjaer E et al., 2008, Petroleum-related Rock Mechanics, 2nd Ed., Elsevier

Effects of reservoir depletion(저류층 고갈의 영향)
Topics

- Changes within the reservoir
  - Stress changes within the reservoir – stress path
  - Depletion induced faulting within reservoir
  - Stress rotation by depletion on one side of fault
- Changes outside (around) the reservoir
  - Deformation and stress surrounding the reservoir
  - Induced slip outside the reservoir
- Deformation due to depletion
  - Deformation within a depleting reservoir – DARS (Deformation Analysis in Reservoir Space)
  - Permeability loss and porosity loss
  - Compaction drive

Reservoir stress paths

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- Poroelastic theory to predict the magnitude of stress changes with depletion
  - Isotropic, porous and elastic reservoir with infinite lateral extent
  - With no lateral strain, relationship between vertical effective stress, horizontal effective stress.

$$S_{\mathrm{Hor}} = \left(\frac{\nu}{1-\nu}\right)(S_{\mathrm{v}}) + \alpha P\left(1 - \frac{\nu}{1-\nu}\right) \tag{12.1}$$

where S_{Hor} corresponds to both S_{Hmax} and S_{hmin} (Lorenz, Teufel *et al.* 1991), α is Biot's coefficient and ν is Poisson's ratio. Taking the derivative of both sides with respect to pore pressure and simplifying yields

$$\Delta S_{\mathrm{Hor}} = \alpha\frac{(1-2\nu)}{(1-\nu)}\Delta P_{\mathrm{p}} \tag{12.2}$$

  - Stress path parameter

Rearranging equation (12.2), it is possible to define a *stress path* of a reservoir that corresponds to the change in horizontal stress with changes in production, A, as

often, γ is used

$$\gamma = A = \alpha\frac{(1-2\nu)}{(1-\nu)} = \frac{\Delta S_{\mathrm{Hor}}}{\Delta P_{\mathrm{p}}} \tag{12.3}$$

With α = 1, ν = 0.25 $\Delta S_{\mathrm{Hor}} \sim \frac{2}{3}\Delta P_{\mathrm{p}}$

Zoback MD, 2007, Reservoir Geomechanics, Cambridge University Press

Reservoir stress paths - derivation

Introduction
Prediction of in situ stress

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- Heim's rule
  - Assumption: no lateral deformation

$$
\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{pmatrix}
=
\begin{pmatrix}
\frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\
-\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\
-\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G}
\end{pmatrix}
\begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{pmatrix}
$$

$0 = \frac{1}{E}\sigma_x - \frac{\nu}{E}\sigma_y - \frac{\nu}{E}\sigma_z$

$0 = -\frac{\nu}{E}\sigma_x + \frac{1}{E}\sigma_y - \frac{\nu}{E}\sigma_z$

$\varepsilon_z = -\frac{\nu}{E}\sigma_x - \frac{\nu}{E}\sigma_y + \frac{1}{E}\sigma_z$

$\sigma_x = \nu(\sigma_y + \sigma_z)$

$\sigma_y = \nu(\sigma_x + \sigma_z)$

$\rightarrow \sigma_x = \nu(\nu(\sigma_x + \sigma_z) + \sigma_z)$

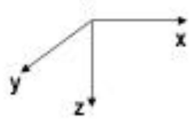
$= \nu^2\sigma_x + \nu(\nu+1)\sigma_z$

$(1-\nu^2)\sigma_x = \nu(\nu+1)\sigma_z$

$(1-\nu)(1+\nu)$

$\boxed{\sigma_x = \frac{\nu}{1-\nu}\sigma_z \quad \text{Similarly } \sigma_y = \frac{\nu}{1-\nu}\sigma_z}$

with $\nu = \sim 0.$[illegible] $\boxed{\sigma_x = \sigma_y = \frac{1}{3}\sigma_z}$



This rule usually works for soils. It does not generally works in rock/reservoir due to tectonic activities, erosion, ...

Δp, ΔS: pressure/stress change

$\sigma_x = \frac{\nu}{1-\nu}\sigma_z \quad , \quad \sigma_y = \frac{\nu}{1-\nu}\sigma_z$

$\sigma_x = \sigma_{Hor} = S_{Hor} - P_p$

$\sigma_z = \sigma_v = S_v - P_p$

$S_{Hor} - \alpha P_p = \frac{\nu}{1-\nu}(S_v - \alpha P_p)$

$S_{Hor} = \frac{\nu}{1-\nu} \cdot S_v + \alpha P_p\left(1 - \frac{\nu}{1-\nu}\right)$

$= \frac{\nu}{1-\nu} S_v + \alpha P_p\left(\frac{1-2\nu}{1-\nu}\right)$

by taking derivatives,

$\Delta S_{Hor} = \alpha \frac{1-2\nu}{1-\nu} \Delta P_p$

if $\Delta p = -10\ MPa \longrightarrow \Delta S_{Hor} = -\alpha \frac{1-2\nu}{1-\nu} \cdot 10\ MPa$

With depletion, S_{Hor} decrease, S_v remains constant

in idealized model with lateral constraints.

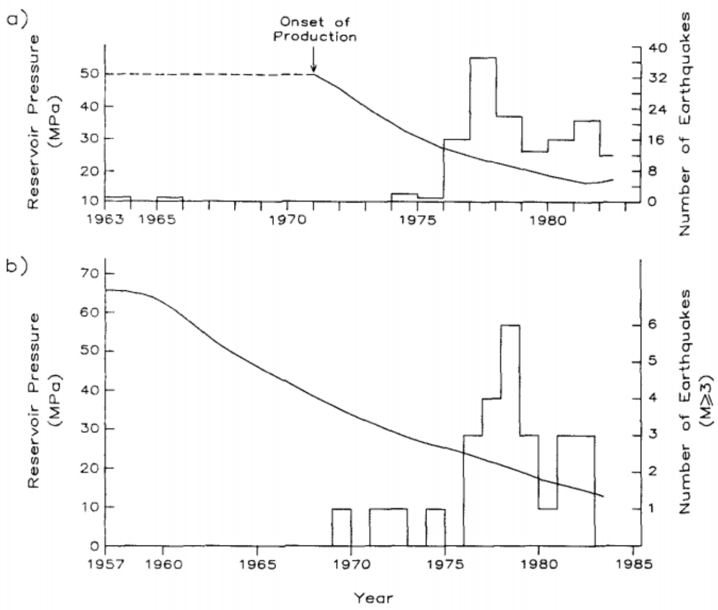
- Some observations

$$\Delta S_{\mathrm{Hor}} = \alpha \frac{(1-2\nu)}{(1-\nu)} \Delta P_{\mathrm{p}}$$

  - No change in vertical stress
  - Lateral to thickness ~ 10:1(= lateral: height), the model works. With smaller than (10:1), some consideration is necessary
  - Inelastic behavior is not taken into account

Production Induced faulting in normal faulting areas

- Both injection and withdrawal of fluid can induce faulting in reservoir



Strachan field in Alberta (Canada). Average static bottomhole pressure at 2.8 km

Pau basin (France). M >3 earthquakes and average reservoir pressure

Number ot earthquakes recorded per year and decline in average reservoir pressure (Segall, 1989)

Segall, 1989, Earthquakes triggered by fluid extraction, Geology, 17:942-946

Stress changes in depleting reservoirs (저류층 고갈에 따른 응력 변화)
Production Induced faulting in normal faulting areas

Normal faulting $\frac{\sigma_1}{\sigma_3} = \frac{S_v - P_p}{S_{hmin} - P_p} \leq [(\mu^2+1)^{1/2} + \mu]^2$

Strik-slip faulting $\frac{\sigma_1}{\sigma_3} = \frac{S_{Hmax} - P_p}{S_{hmin} - P_p} \leq [(\mu^2+1)^{1/2} + \mu]^2$

Reverse faulting $\frac{\sigma_1}{\sigma_3} = \frac{S_{Hmax} - P_p}{S_v - P_p} \leq [(\mu^2+1)^{1/2} + \mu]^2$

$$\frac{[S_V - (P_p - \Delta P_p)]}{[(S_{hmin} - \Delta S_{hmin}) - (P_p - \Delta P_p)]} = f(\mu) \tag{12.4}$$

where

$$f(\mu) = (\sqrt{\mu^2+1} + \mu)^2$$

Simplifying this results in:

$$\frac{S_V - P_p}{S_{hmin} - P_p} = \left[1 - \frac{\Delta S_{hmin} - \Delta P_p}{S_{hmin} - P_p}\right] f(\mu) - \frac{\Delta P_p}{S_{hmin} - P_p} \tag{12.5}$$

In areas where normal faults are in frictional equilibrium, the left-hand side of equation (12.5) is equivalent to $f(\mu)$ such that,

$$f(\mu) = f(\mu) - \frac{\Delta S_{hmin} - \Delta P_p}{S_{hmin} - P_p} f(\mu) - \frac{\Delta P_p}{S_{hmin} - P_p}$$

$$\frac{\Delta S_{hmin} - \Delta P_p}{S_{hmin} - P_p} f(\mu) = -\frac{\Delta P_p}{S_{hmin} - P_p}$$

$$\frac{\Delta S_{hmin} - \Delta P_p}{\Delta P_p} = -\frac{1}{f(\mu)}$$

Substituting $A = \Delta S_{hmin}/\Delta P_p$ yields the stress path, A^*, which if exceeded, can lead to production-induced normal faulting:

$$A^* = 1 - \frac{1}{(\sqrt{\mu^2+1} + \mu)^2} \tag{12.6}$$

μ = 0.6 → A* = 0.67

Zoback MD, 2007, Reservoir Geomechanics, Cambridge University Press

Normal faulting

$$\frac{\sigma_1}{\sigma_3} = \frac{S_v - P_p}{S_{hmin} - P_p} \le \left[\sqrt{\mu^2+1} + \mu\right]^2 = f(\mu)$$

condition to be stable

$P_p \rightarrow P_p - \Delta p$

$S_{hmin} \rightarrow S_{hmin} - \Delta S_h$

$$\frac{S_v - (P_p - \Delta P_p)}{S_{hmin} - \Delta S_{hmin} - (P_p - \Delta P_p)} \le f(\mu)$$

$$S_v - (P_p - \Delta P_p) \le (S_{hmin} - \Delta S_{hmin}) f(\mu) - (P_p - \Delta P_p) f(\mu)$$

$$S_v - P_p \le (S_{hmin} - \Delta S_{hmin}) f(\mu) - (P_p - \Delta P_p) f(\mu) - \Delta P_p$$

$$S_v - P_p \le (S_{hmin} - P_p) f(\mu) - (\Delta S_{hmin} - \Delta P_p) f(\mu) - \Delta P_p$$

Divide by $(S_{hmin} - P_p)$

in frictional equilibrium $f(\mu)$

$$\frac{S_v - P_p}{S_{hmin} - P_p} \le f(\mu) - \frac{(\Delta S_{hmin} - \Delta P_p)}{S_{hmin} - P_p} f(\mu) - \frac{\Delta P_p}{S_{hmin} - P_p}$$

$$\frac{\Delta S_{hmin} - \Delta P_p}{S_{hmin} - P_p} f(\mu) \le - \frac{\Delta P_p}{S_{hmin} - P_p}$$

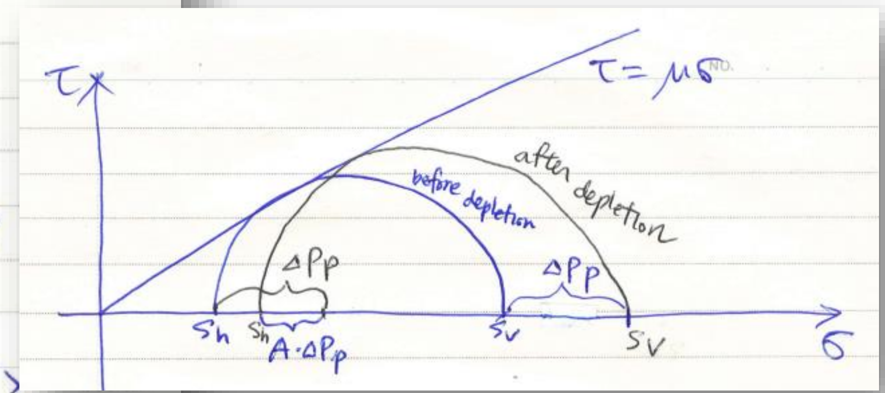
$$\frac{\Delta S_{hmin} - \Delta P_p}{\Delta P_p} \le - \frac{1}{f(\mu)}$$

$$\rightarrow \frac{\Delta S_{hmin}}{\Delta P_p} \le 1 - \frac{1}{f(\mu)} = 1 - \frac{1}{\left[\sqrt{\mu^2+1} + \mu\right]^2} = A^*$$

condition to be stable

$\therefore A > A^* \rightarrow$ slip (normal faulting)

Δp, ΔS: magnitude of <u>decreased</u> pressure/stress

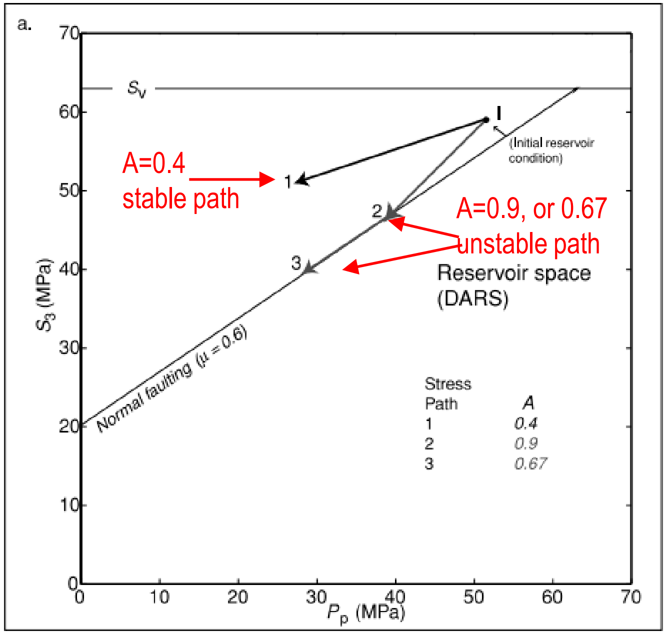


Production Induced faulting in normal faulting areas

- Condition for depletion induced faulting. A > A*

$$A = \alpha \frac{(1-2\nu)}{(1-\nu)} = \frac{\Delta S_{\mathrm{Hor}}}{\Delta P_{\mathrm{p}}} \qquad A^* = 1 - \frac{1}{(\sqrt{\mu^2+1}+\mu)^2}$$

  - Sufficient depletion will eventually result in production induced faulting
  - Vertical stress is assumed to be constant



Schematic stress paths in *reservoir space*

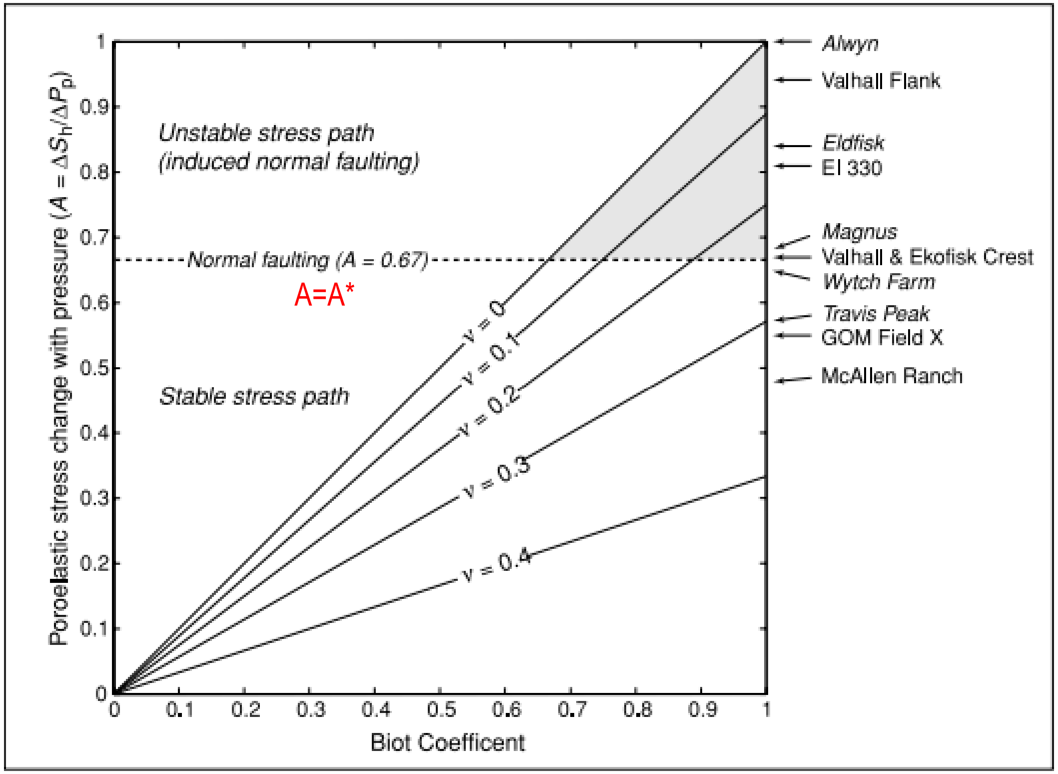
Zoback MD, 2007, Reservoir Geomechanics, Cambridge University Press

- Variation of horizontal stress change with pressure as a function of Biot coefficient, and Poisson's ratio

$$A = \alpha \frac{(1-2\nu)}{(1-\nu)} = \frac{\Delta S_{\mathrm{Hor}}}{\Delta P_{\mathrm{p}}}$$

$$A^* = 1 - \frac{1}{(\sqrt{\mu^2+1}+\mu)^2}$$

μ = 0.6 → A* = 0.67

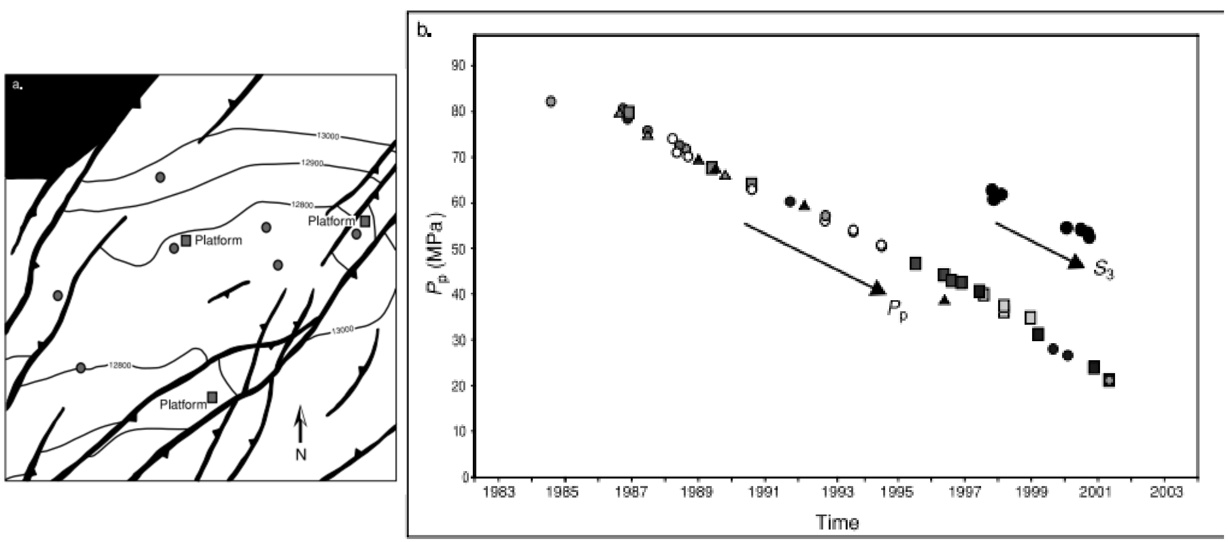


Zoback MD, 2007, Reservoir Geomechanics, Cambridge University Press

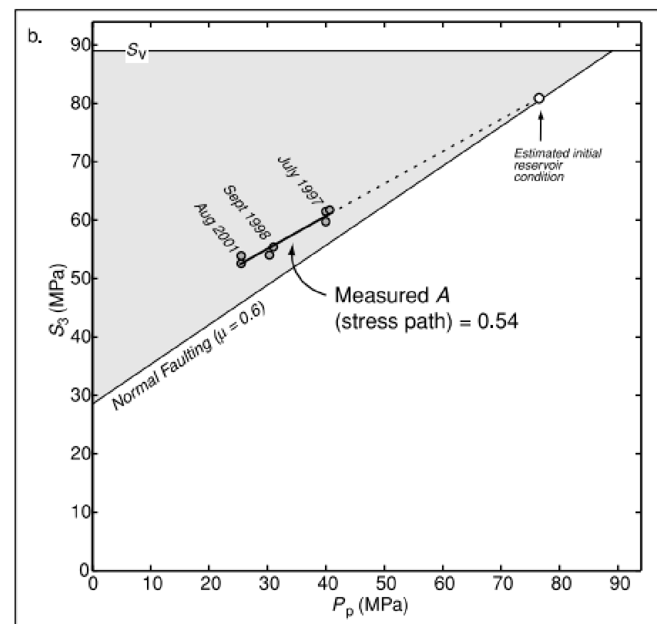
Reservoir stress paths

- Example 1) (Gulf of Mexico)
  - All of the wells depletion follow the same paths
  - S3 change is also measured
  - Considerable depletion from ~80 MPa → ~25 MPa (for 25 years)
  - Stress path parameter ~0.54 → stable
  - Initially, it was in a frictional failure equilibrium (through extrapolation)

Interpolation/extrapolation?



Pore pressure & S_3 decline in Gulf of Mexico



Pore pressure & S_3 decline in reservoir space

Zoback MD, 2007, Reservoir Geomechanics, Cambridge University Press

Stress changes in depleting reservoirs (저류층 고갈에 따른 응력 변화)

Example 2) Valhall oil field in North Sea

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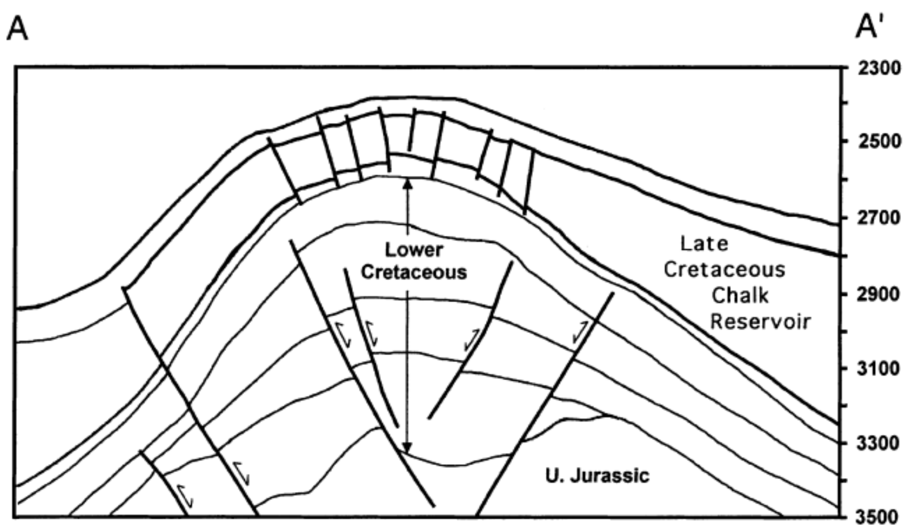
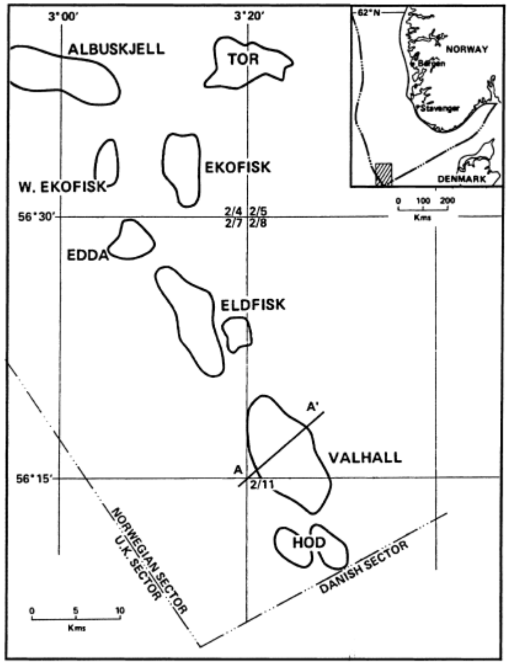
Chalk (백악): A porous marine limestone composed of fine grained remains of microorganisms with calcite shells, coccolithophores, such as the White Cliffs of Dover (UK). The Austin Chalk of the US Gulf coast is a prolific, fractured oil reservoir that spurred widespread horizontal drilling activity.

- Valhall oil field in Norwegian North Sea
  - Fractured chalk reservoir, trend NW-SE an elongated anticline
  - 1975 Discovery well, 1982 production, ~ 2,400 m subsea
  - Porosity ~ 50%, permeability ~1 md → reservoir permeability (150 md) controlled by extensive natural fracture system
  - Concerns: active faulting, numerous casing failure, appreciable gas leakage through shale cap rock



Dome structure of the Valhall oil field (A profile along AA')



Location of Valhall and Ekofisk oil fields

Zoback MD, Zinke JC, 2002, Production induced normal faulting in the Valhall and Ekofisk oil fields, *Pure & Applied Geophysics*, 159:403-420

Stress changes in depleting reservoirs (저류층 고갈에 따른 응력 변화)
Example 2) Valhall oil field in North Sea

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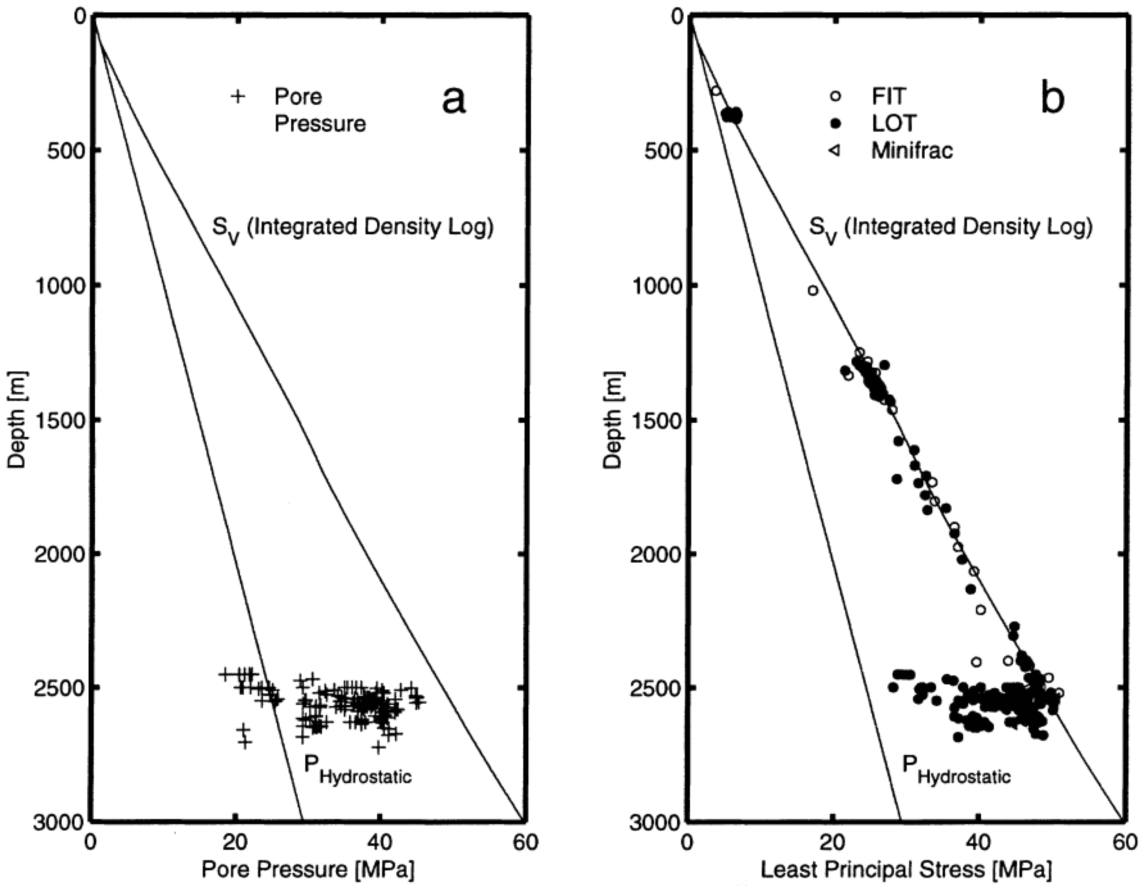
- Pore pressure and S3 with respect to depth
  - Very scattered at the reservoir



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Stress changes in depleting reservoirs (저류층 고갈에 따른 응력 변화)
Example 2) Valhall oil field in North Sea

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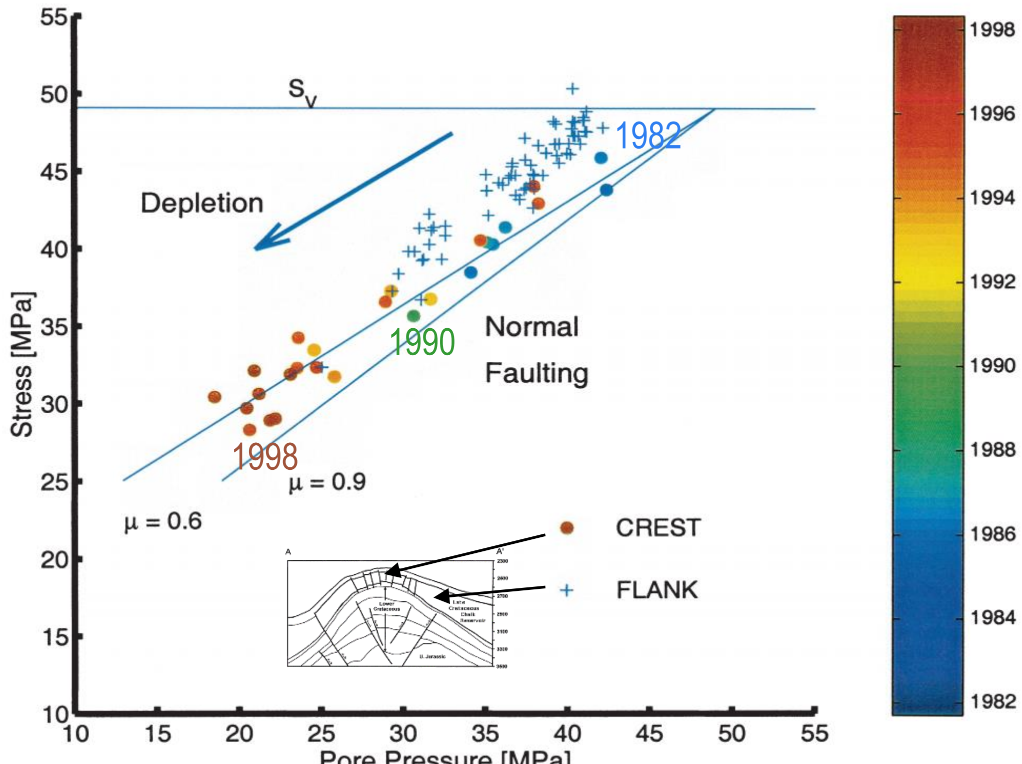
- Pore pressure and S_3 in reservoir space
  - Initially p_p and S_3 were high close to Sv.
  - Clear reduction of p_p and S_3 with time ← Effect of depletion
  - Normal stress condition at crest initially and this was maintained



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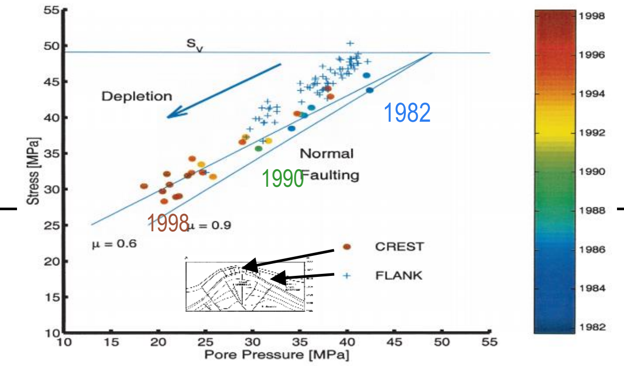
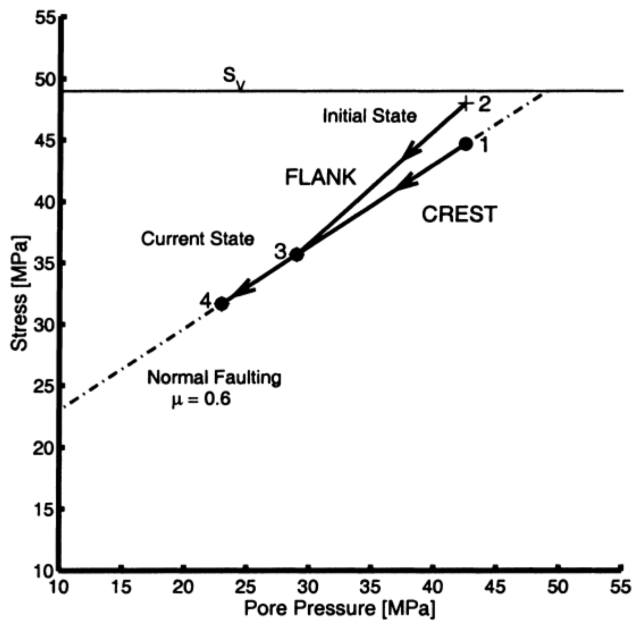
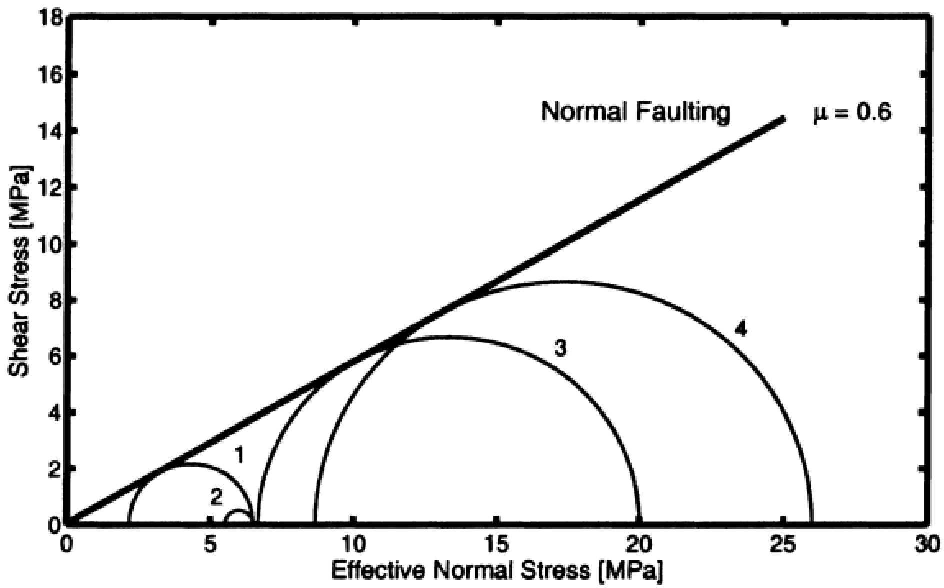
Stress changes in depleting reservoirs
Example 2) Valhall oil field in North Sea



- Example. Ekofisk and Valhal field in North
  - In crest: 1 → 3 → 4
  - In flank: 2 → 3

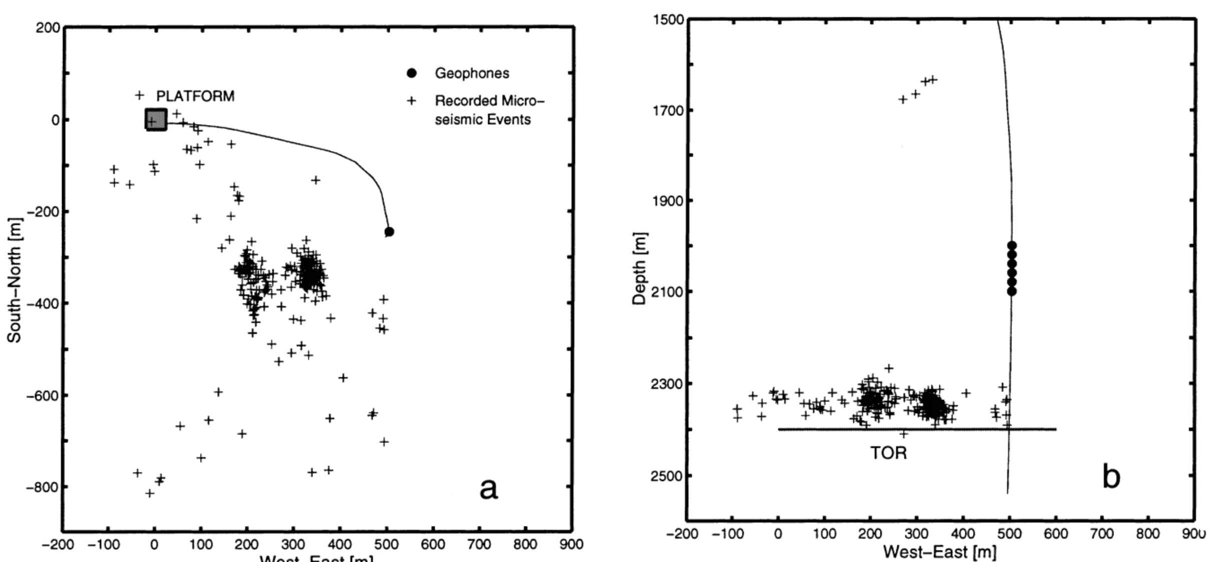


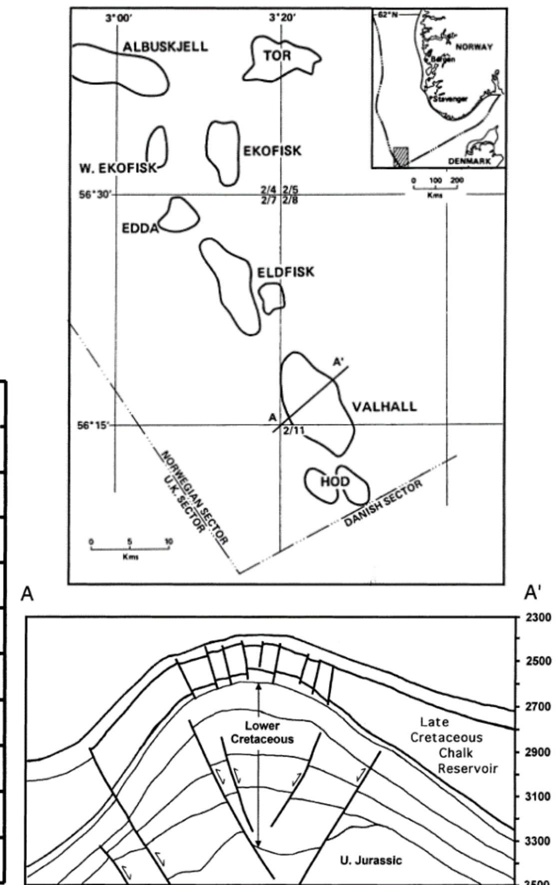


Zoback MD, Zinke JC, 2002, Production induced normal faulting in the Valhall and Ekofisk oil fields, *Pure & Applied Geophysics*, 159:403-420

Stress changes in depleting reservoirs (저류층 고갈에 따른 응력 변화)
Example 2) Valhall oild field in North Sea

- Microseismic monitoring (1998)
  - Vertical section of the well, Six 3-component seismometers with 20 m spacing
  - 328 microseismic events for ~ 7 weeks, mostly 200 m to the west of the well
  - Location: at the top of the reservoir or shale cap rock
  - Focal mechanism was normal faulting confirming the theory
  - Normal faulting propagating up into the cap rock from the reservoir
  - Reverse faulting on the top of reservoir???





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Stress changes in depleting reservoirs (저류층 고갈에 따른 응력 변화)
Example 2) Valhall oil field in North Sea

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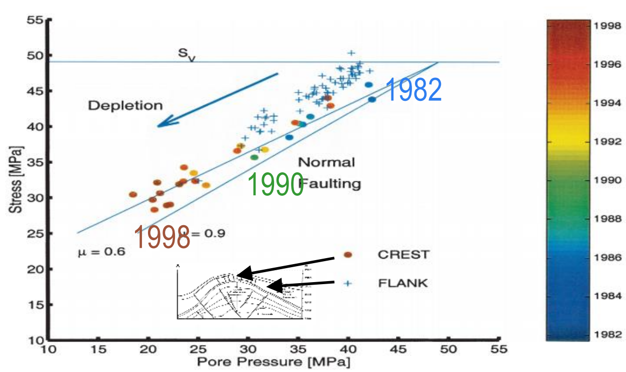
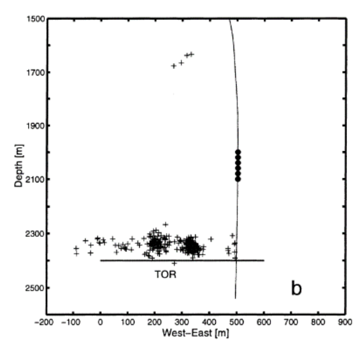
- Summary – depletion induced faulting is a good news and bad news situation
  - Concerns
    - active faulting
    - numerous casing failure
    - appreciable gas leakage through shale cap rock
  - Explained through depletion induced stress path
  - Despite the reservoir compaction accompanying depletion, reservoir productivity remained steady or slightly increased
    - This is explained by active faulting in the reservoir increased the permeability





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Stress changes in depleting reservoirs (저류층 고갈에 따른 응력 변화)

Stress rotation associated with depletion

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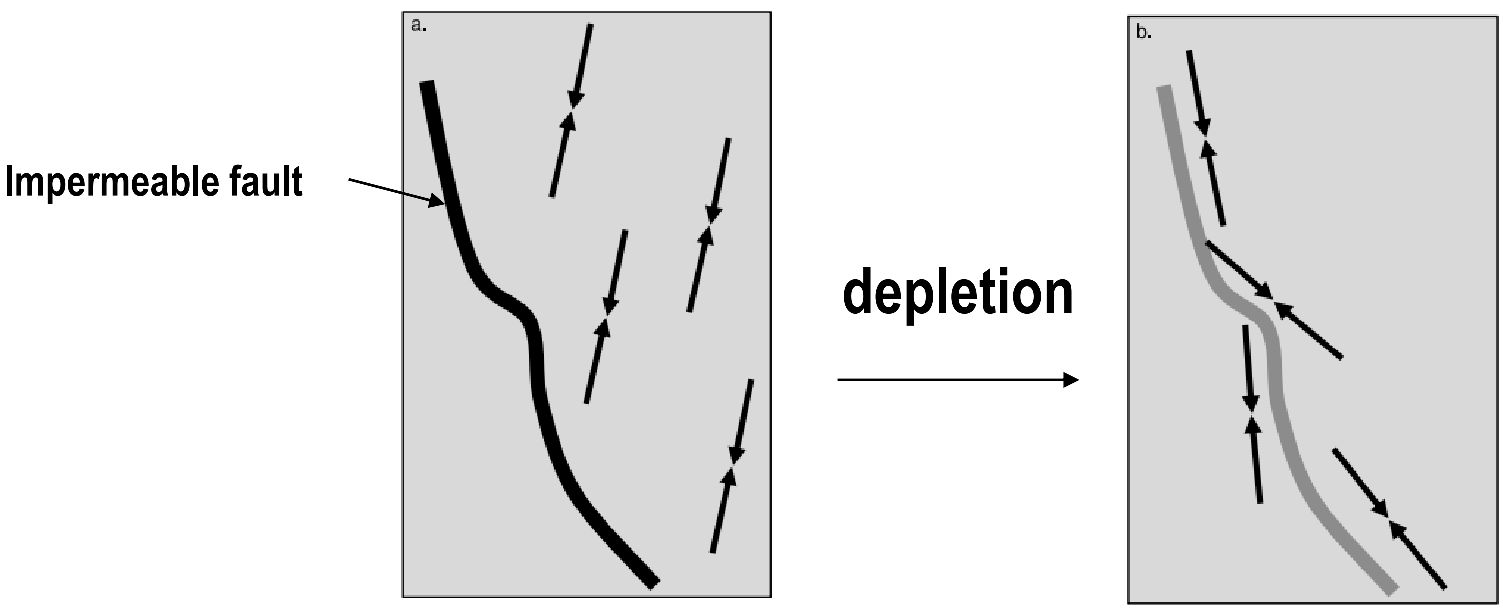
- Stress rotation
  - If a reservoir is bounded by an impermeable fault, stress change is not isotropic
  - In laterally extensive reservoir, Shmin & Shmax decrease by the same amount ← isotropic horizontal stress change



Zoback MD, 2007, Reservoir Geomechanics, Cambridge University Press

- Fault often has a core that is impermeable

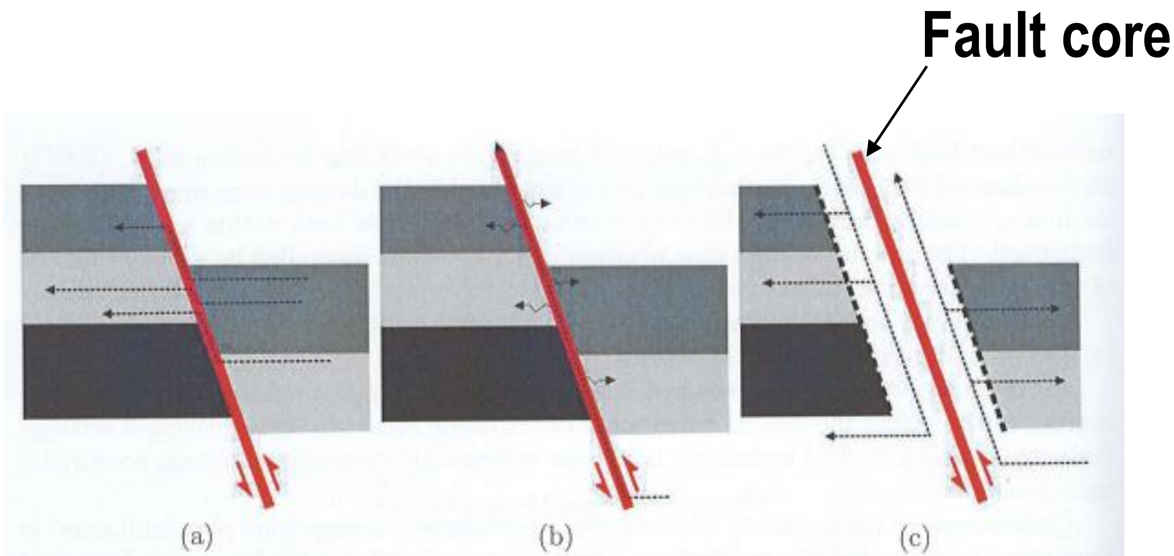
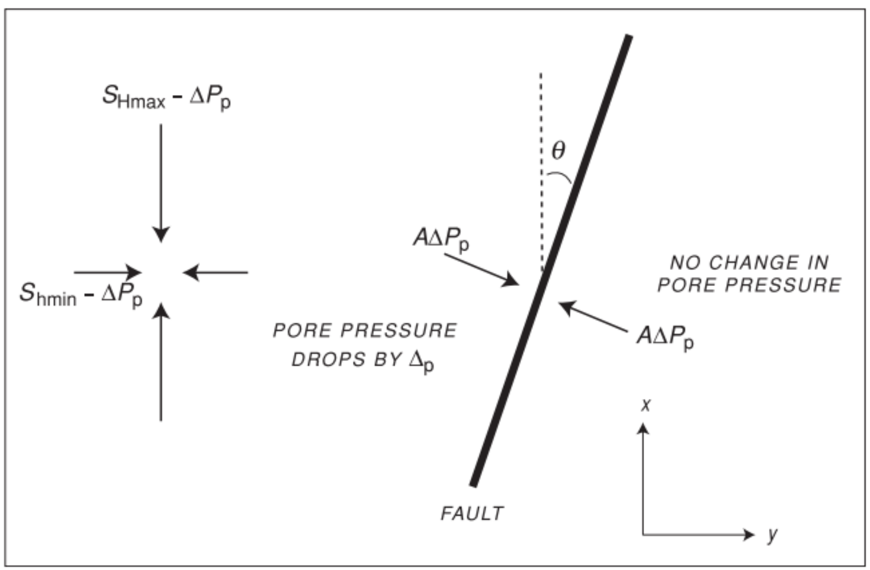


Fig. 5.3: End member hydraulic behavior for a fault zone embedded in a stratified geological layer, modified after Davatzes and Aydin (2005). (a) Fault zone which is transparent to fluid flow along its plane. Only across-flow is possible depending on the permeability of the surrounding rocks. (b) Fault zone which is highly permeable (compared to the host rocks). Due to the permeability contrast between fault and surrounding rock, fluid preferentially flow along the fault, while a minor across-fault flow is still possible. (c) Fault zone consisting of an impermeable fault core and a highly permeable damage zone. Three dimensional flow occurs within the damage zone but not across the fault.

Guido Blöcher, 2020, “The role of fractures and faults in reservoirs – thermal-hydraulic-mechanical characteristics and processes in Enhanced Geothermal Systems (EGS)”, Habilitation Treatise, Submitted to Technical University of Berlin.

- Some explanations about the stress rotations.
  - In a nutshell, stress tends to align with the impermeable faults
  - Rotation is a function of S_{Hmax}, S_{hmin}, Δp and angle with the impermeable fault.



$$S_x = (S_{\mathrm{Hmax}} - A\Delta P_{\mathrm{p}}) - \frac{A\Delta P_{\mathrm{p}}}{2}(1 - \cos 2\theta) \tag{12.7}$$

$$S_y = (S_{\mathrm{hmin}} - A\Delta P_{\mathrm{p}}) - \frac{A\Delta P_{\mathrm{p}}}{2}(1 + \cos 2\theta) \tag{12.8}$$

$$\tau_{xy} = \frac{A\Delta P_{\mathrm{p}}}{2}\sin 2\theta \tag{12.9}$$

The rotation, γ, of the new maximum, principal horizontal stress near the fault relative to the original S_{Hmax} azimuth can be found by

$$\gamma = \frac{1}{2}\tan^{-1}\left[\frac{2\tau_{xy}}{S_x - S_y}\right] = \frac{1}{2}\tan^{-1}\left[\frac{A\Delta P_{\mathrm{p}}\sin 2\theta}{(S_{\mathrm{Hmax}} - S_{\mathrm{hmin}}) + A\Delta P_{\mathrm{p}}\cos 2\theta}\right] \tag{12.10}$$

The sign of γ is the same as the sign of θ. If we define q as the ratio of the pore pressure change (positive for depletion) to the original, horizontal differential stress,

$$q = \frac{\Delta P_{\mathrm{p}}}{(S_{\mathrm{Hmax}} - S_{\mathrm{hmin}})} \tag{12.11}$$

following Zoback, Day-Lewis *et al.* (2007) we can express the stress rotation simply as a function of q, the stress path (A), and the fault orientation (θ):

$$\gamma = \frac{1}{2}\tan^{-1}\left[\frac{Aq\sin 2\theta}{1 + Aq\cos 2\theta}\right] \tag{12.12}$$

Zoback MD, 2007, Reservoir Geomechanics, Cambridge University Press

- Example: Arcabuz-Culebra field in Mexico

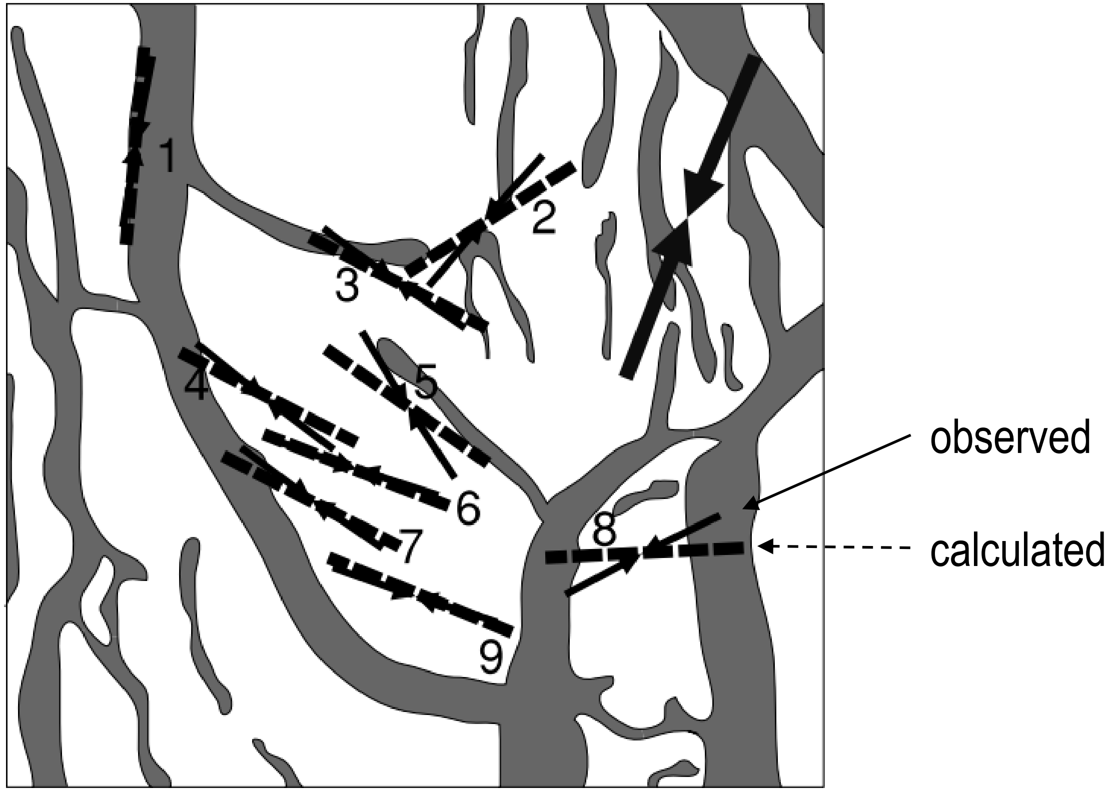


Figure 12.9. Comparison of the calculations presented in Figure 12.8 with data from the field presented in Figure 12.6d. The observed stress orientation is shown by the inward arrows. The dashed lines show the orientation of a sealing fault required to explain the observed rotation. Because there is little specific information available on the amount of depletion or the magnitude of S_{Hmax}, the calculations were done assuming $q = 2$. After Zoback, Day-Lewis *et al.* (2007)

Zoback MD, 2007, Reservoir Geomechanics, Cambridge University Press

Stress changes in depleting reservoirs (저류층 고갈에 따른 응력 변화)
Drilling and hydraulic fracturing in depleted reservoir

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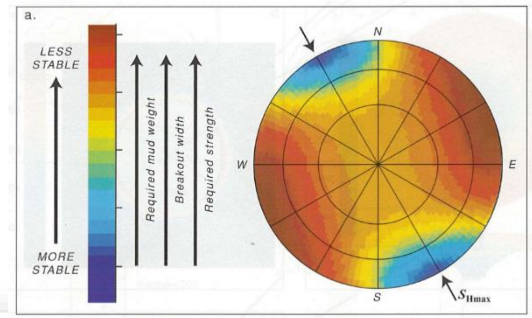
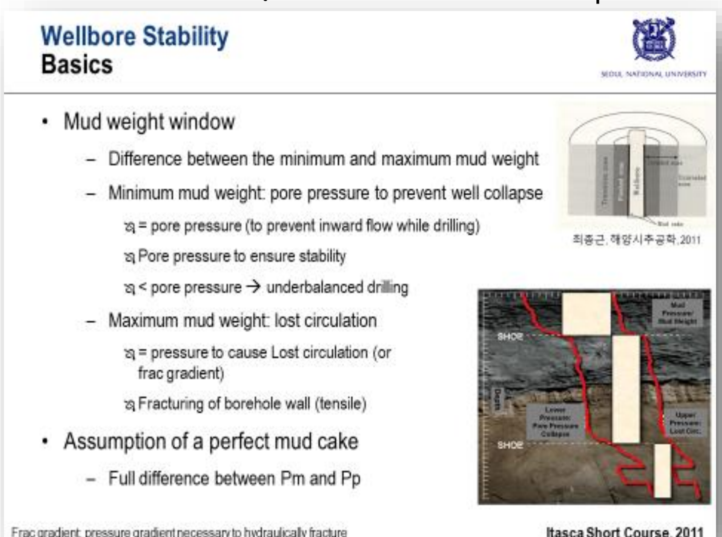
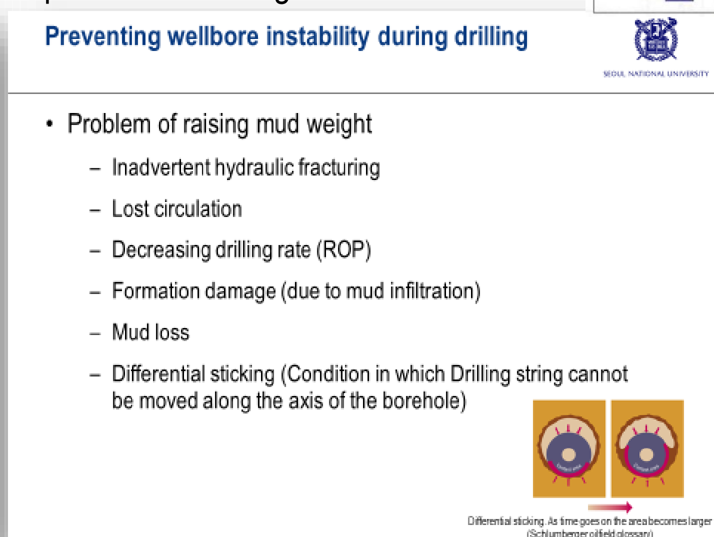
- Drilling
  - Unintentional lost circulation
  - Unintentional hydraulic fracturing due to decreased principal stress
  - Differential sticking
  - Wellbore stability problem
  - Alternative
    - Drilling in optimal direction to avoid hydraulic fracturing and lost circulation
    - Use some additives to present mud penetration/strength the formation



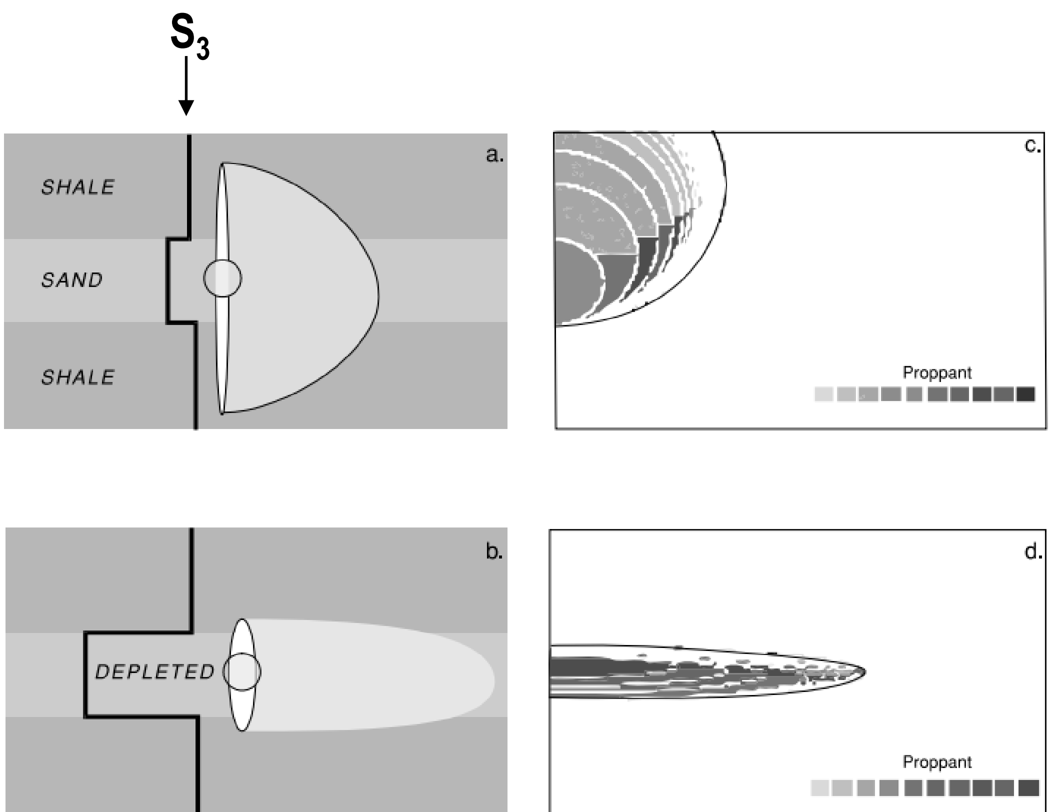


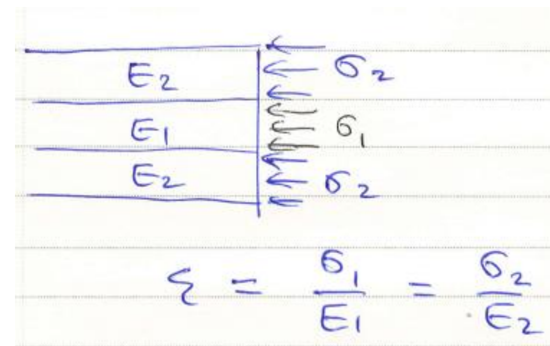


Stress changes in depleting reservoirs (저류층 고갈에 따른 응력 변화)

Drilling and hydraulic fracturing in depleted reservoir

- Hydraulic fracturing
  - In general more advantageous than prior to depletion
  - Due to stress rotation, new fracture to a new azimuth possible
  - Inadvertent vertical fracture growth can be avoided (due to decreased S_3)





Zoback MD, 2007, Reservoir Geomechanics, Cambridge University Press

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- Effect of compaction outside the depleting reservoir
  - Subsidence
  - Induced faulting (seismicity)

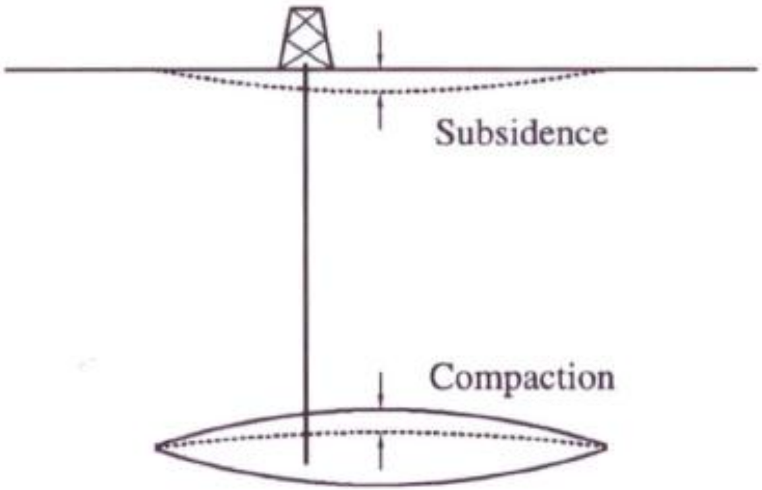

Fig. 12.1. Compaction and subsidence.

Fjaer E et al., 2008, Petroleum-related Rock Mechanics, 2nd Ed., Elsevier

Deformation and stress changes outside of depleting reservoirs (저류층 주변의 변위/응력 변화)

Compaction and subsidence

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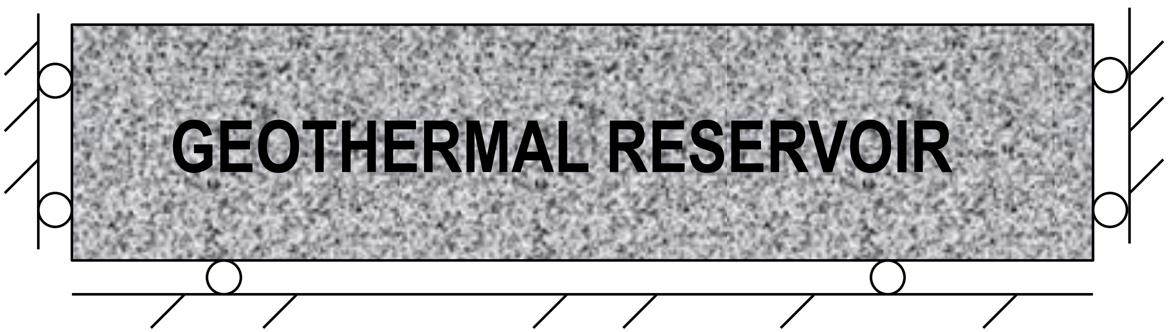
- Compaction
  - Simple model of compaction = subsidence

$$\frac{\Delta h}{h} = -\frac{1}{E}\frac{(1+\nu)(1-2\nu)}{1-\nu}\alpha\Delta p$$



$$0 = \frac{1}{E}\sigma_x - \frac{\nu}{E}\sigma_y - \frac{\nu}{E}\sigma_z$$

$$0 = -\frac{\nu}{E}\sigma_x + \frac{1}{E}\sigma_y - \frac{\nu}{E}\sigma_z$$

$$\varepsilon_z = -\frac{\nu}{E}\sigma_x - \frac{\nu}{E}\sigma_y + \frac{1}{E}\sigma_z$$

$$\sigma_x = \nu(\sigma_y + \sigma_z)$$

$$\sigma_y = \nu(\sigma_x + \sigma_z)$$

$$\rightarrow \sigma_x = \nu(\nu(\sigma_x+\sigma_z)+\sigma_z)$$

$$= \nu^2\sigma_x + \nu(\nu+1)\sigma_z$$

$$(1-\nu^2)\sigma_x = \nu(\nu+1)\sigma_z$$

$$(1-\nu)(1+\nu)$$

$$\sigma_x = \frac{\nu}{1-\nu}\sigma_z$$

Similarly $\sigma_y = \frac{\nu}{1-\nu}\sigma_z$

with $\nu = \sim 0.25$ $\boxed{\sigma_x = \sigma_y = \frac{1}{3}\sigma_z}$

$$\varepsilon_z = -\frac{\nu}{E}\cdot\frac{\nu}{1-\nu}(\sigma_z) - \frac{\nu}{E}\frac{\nu}{1-\nu}\sigma_z + \frac{1}{E}\sigma_z$$

$$= -\frac{\nu^2+\nu^2-(1-\nu)}{E(1-\nu)}(S_v-(p+\Delta p))$$

$$= -\frac{(1+\nu)(1-2\nu)}{E(1-\nu)}\Delta p$$

Deformation and stress changes outside of depleting reservoirs (저류층 주변의 변위/응력 변화)

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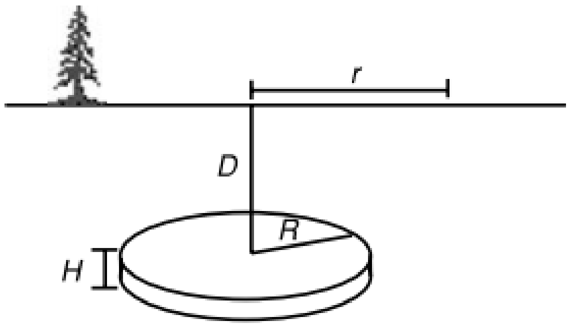
Compaction and subsidence

- Subsidence
  - Disk-shaped reservoir , thickness H, radius R, depth D
  - Effect of compaction on surface subsidence in elastic half-space



- Subsidence in vertical and lateral (radial) direction
  - Subsidence/compaction of the reservoir

$$\Delta H = \int_0^H C_m(z)\Delta p(z)dz$$

$$u_z(r,0) = -2C_m(1-\nu)\Delta pHR\int_0^\infty e^{-D\alpha}J_1(\alpha R)J_0(\alpha r)d\alpha$$

$$u_r(r,0) = 2C_m(1-\nu)\Delta pHR\int_0^\infty e^{-D\alpha}J_1(\alpha R)J_1(\alpha r)d\alpha$$

$$\frac{u_z(r,0)}{\Delta H} = A(\rho,\eta)$$

$$\frac{u_r(r,0)}{\Delta H} = B(\rho,\eta)$$

where A and B are linear combinations of the elliptic integrals of the first and second kind (F_0, E_0) and Heuman's Lambda function (Λ_0)

$$A = \begin{cases} -\dfrac{k\eta}{4\sqrt{\rho}}F_0(m) - \dfrac{1}{2}\Lambda_0(p,k) + 1 & (p < 1) \\ -\dfrac{k\eta}{4}F_0(m) + \dfrac{1}{2} & (p = 1) \\ -\dfrac{k\eta}{4\sqrt{\rho}}F_0(m) + \dfrac{1}{2}\Lambda_0(p,k) & (p > 1) \end{cases}$$

$$B = = \frac{1}{k\sqrt{\rho}}\left[\left(1 - \frac{1}{2}k^2\right)F_0(m) - E_0(m)\right]$$

where

$$m = k^2 = \frac{\rho}{(1-\rho)^2 + \eta^2} \quad \text{and} \quad p = \frac{k^2[(1-\rho)^2 + \eta^2]}{(1-\rho)^2 + k^2}$$

$$\rho = r/R \text{ and } \eta = D/R$$

Zoback MD, 2007, Reservoir Geomechanics, Cambridge University Press

Deformation and stress changes outside of depleting reservoirs (저류층 주변의 변위/응력 변화)

Compaction and subsidence

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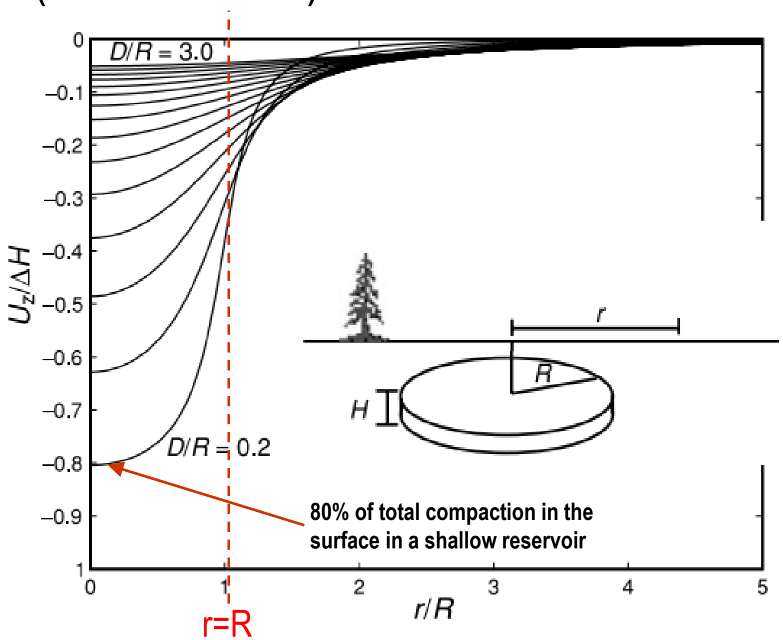
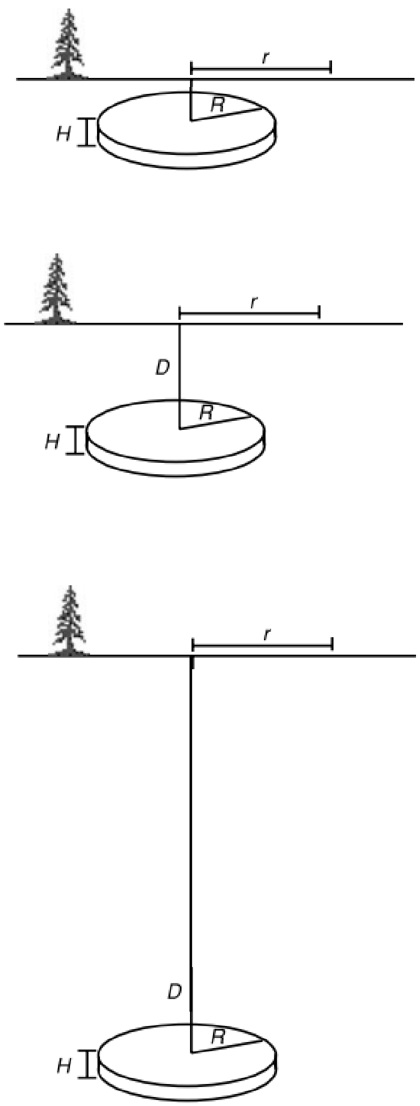
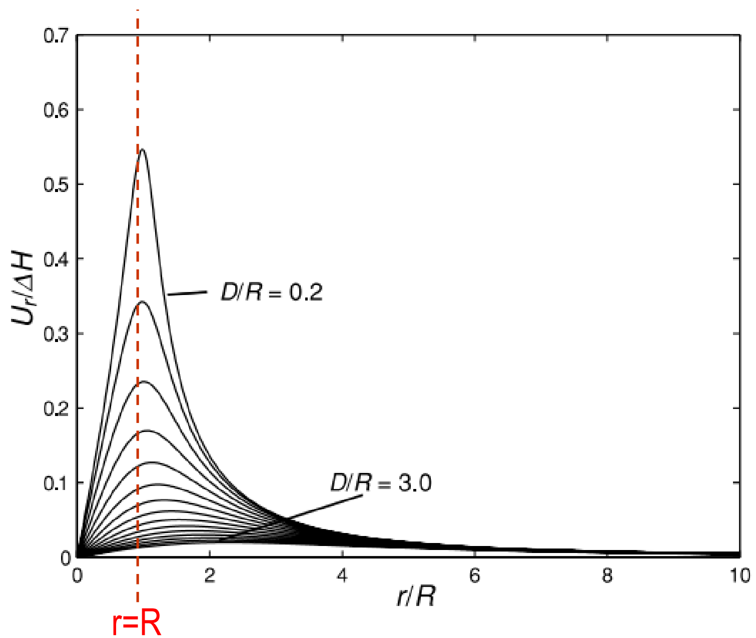
- Normalized subsidence (vertical displacement in the surface)
  - Concentrated on the center
  - Depends a lot on the depth, D/R~0.2 → 80%
  - Wilmington field (California) 9 m from 1 km deep reservoir (18 km x 5 km)

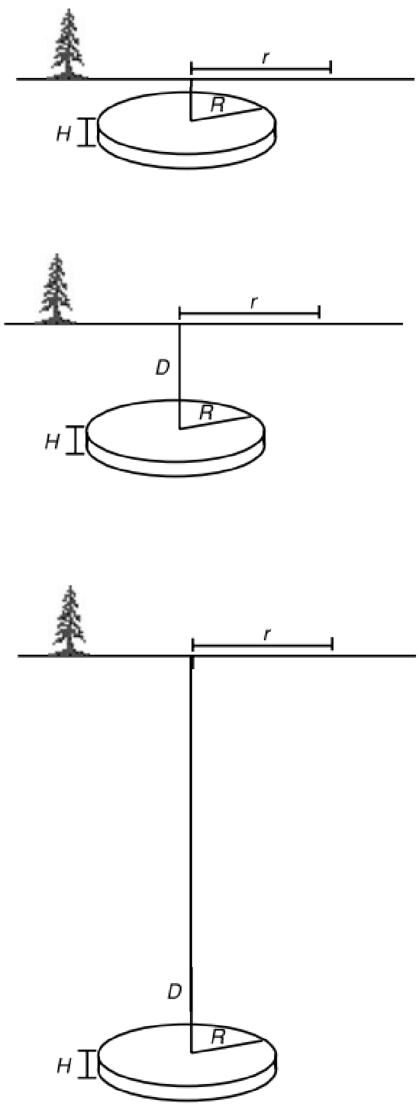




Zoback MD, 2007, Reservoir Geomechanics, Cambridge University Press

- Normalized horizontal displacement
  - Concentrated in the boundary
  - Wilmington field (California) ~3.7 m





Zoback MD, 2007, Reservoir Geomechanics, Cambridge University Press

Deformation and stress changes outside of depleting reservoirs (저류층 주변의 변위/응력 변화)

Compaction and subsidence

- Displacement (and strain) in the surface

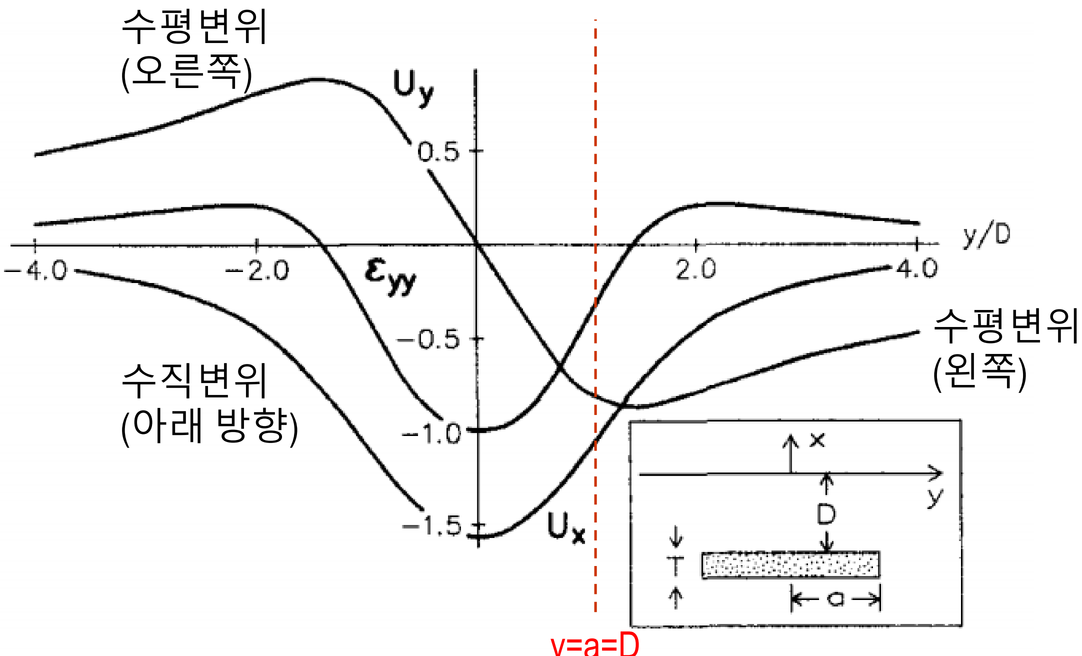


Figure 5. Calculated surface displacements u_x, u_y, and strain ϵ_{yy} (extension positive) for $a/D = 1.0$. Geometry is shown in inset. Displacements are normalized by $2(1 + \nu_u)\, BT\Delta m(t)/3\pi\rho_o$, strains by $2\,(1 + \nu_u)\, BT\Delta m(t)/3\pi\rho_o D$.

Segall, 1989, Earthquakes triggered by fluid extraction, Geology, 17:942-946

Deformation and stress changes outside of depleting reservoirs (저류층 주변의 변위/응력 변화)

Subsidence

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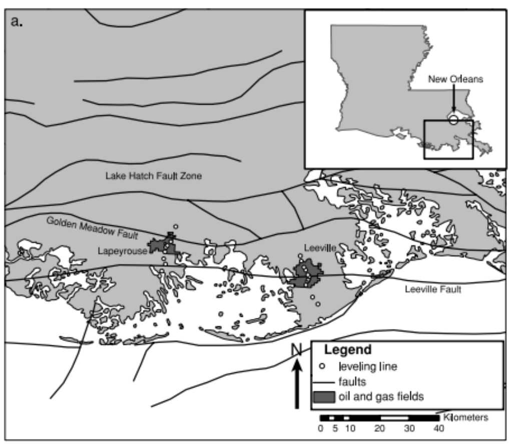
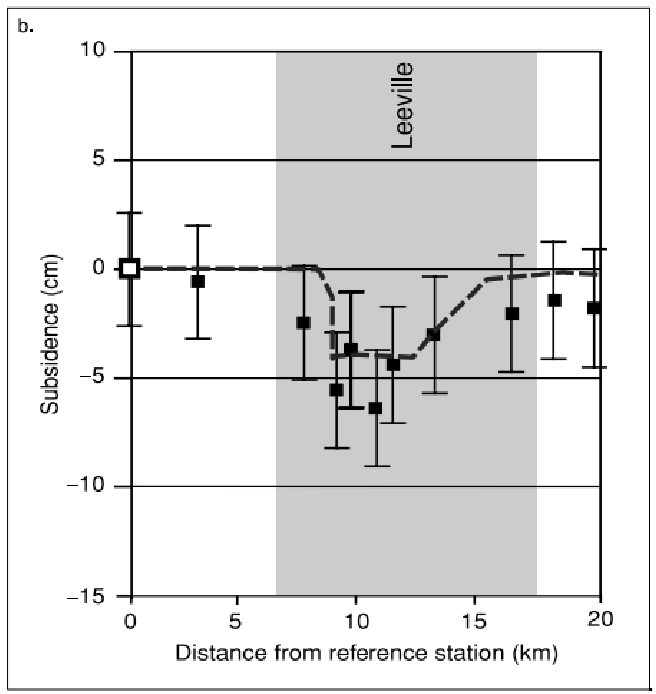
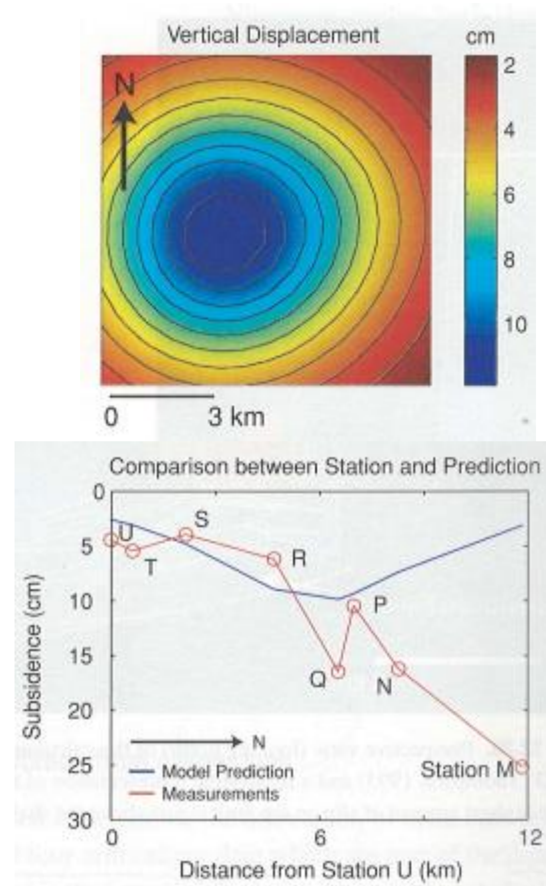
- Example 1. Leeville and Lapeyrouse fields (Louisiana)
  - Maximum subsidence 3~9 cm (Leeville)
  - Lapeyrouse, D=4.5 km, H=10 m, R=1 km ~2.5 km)





Leeville



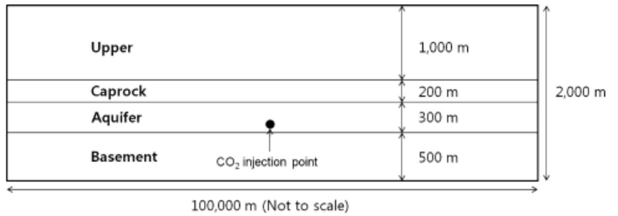
Lapeyrouse

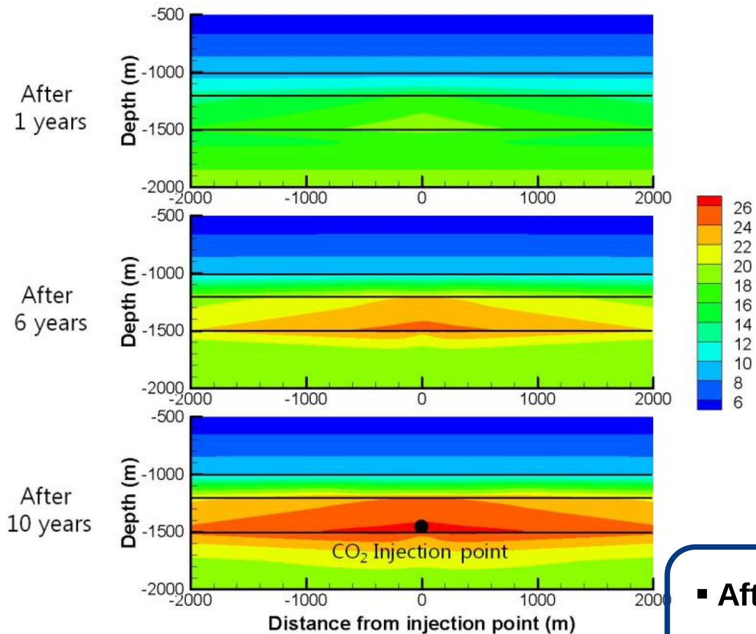
Zoback MD, 2007, Reservoir Geomechanics, Cambridge University Press

Deformation and stress changes outside of depleting reservoirs (저류층 주변의 변위/응력 변화)

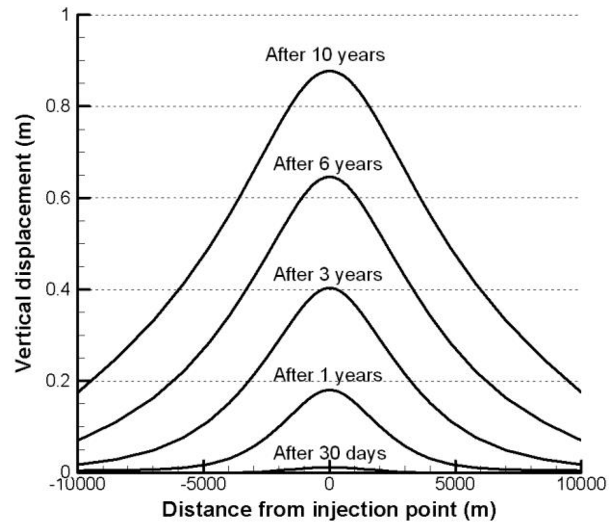
Subsidence

- Example 2. - CO_2 storage (heaving) in hypothetical site
  - Coupled hydromechanical numerical analysis





Pressure change with time near the injection point (Units: MPa)



Vertical displacement profile

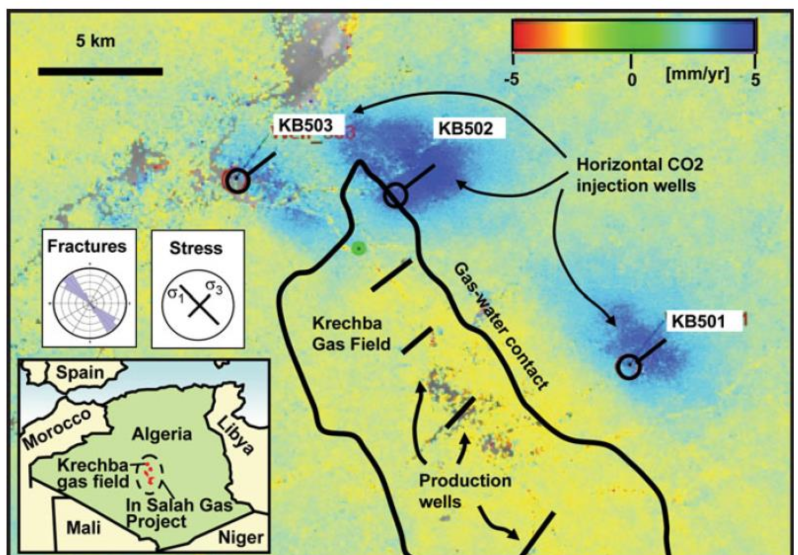
- **After 10 years**
  - **- the pore pressure : about 12 MPa**
  - **- the vertical displacement : about 0.87 m**

Lee, J., Min KB, Rutqvist J (2013). "Probabilistic Analysis of Fracture Reactivation Associated with Deep Underground CO2 Injection." Rock Mechanics and Rock Engineering 46(4): 801-820.

Deformation and stress changes outside of depleting reservoirs (저류층 주변의 변위/응력 변화)

Subsidence

- Example 3. - CO2 storage (heaving) in In Salah - Ground Heaving due to fluid (CO_2) injection
  - Based on InSAR (Interferometric Systhetic Aperture Radar, difference in the phases of waves to Satellite, ~mm scale) data for average distance change (~ vertical displacement) from Aug 2004 to March 2007
  - 5 mm/year (with up to 1 million ton/year)
  - 2 km deep 20 m thick sandstone reservoir



Rutqvist, J. (2012). "The Geomechanics of CO2 Storage in Deep Sedimentary Formations." Geotechnical and Geological Engineering 30(3): 525-551.

Deformation and stress changes outside of depleting reservoirs (저류층 주변의 변위/응력 변화)

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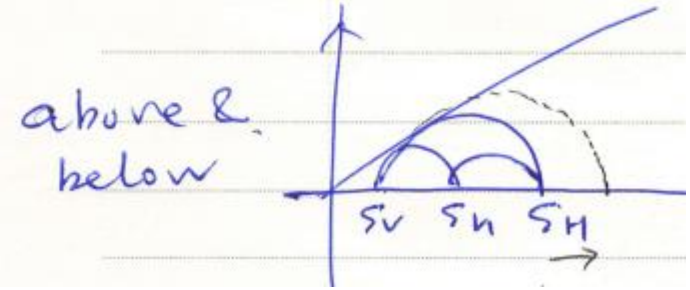
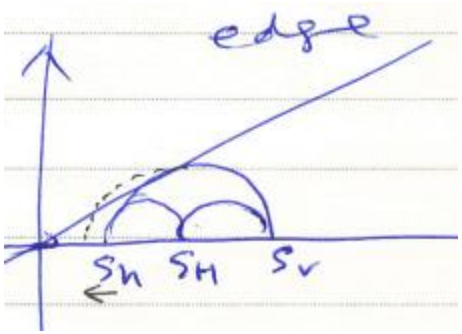
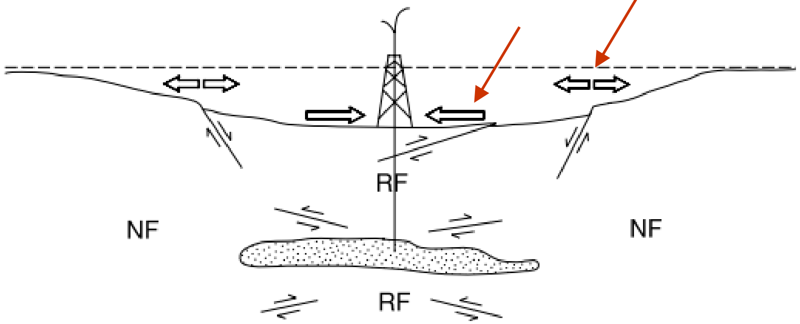
Stress changes

- Stresses around the reservoir
  - Reverse faulting at the top/below
  - Normal faulting at the side
  - (Normal faulting within the reservoir)







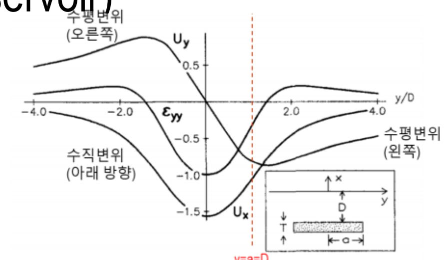
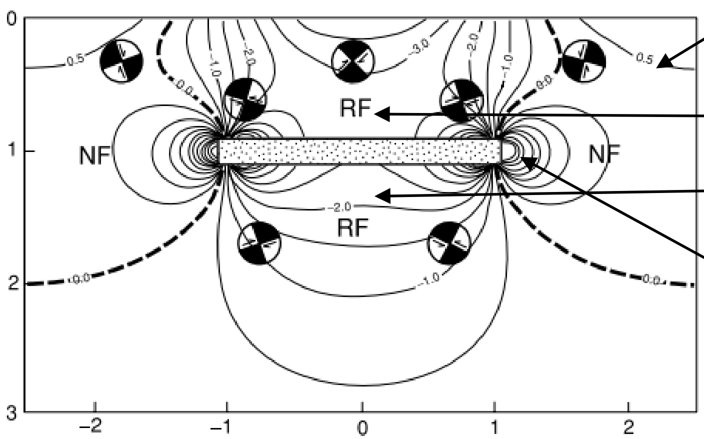


Figure 5. Calculated surface displacements u_x, u_y, and strain ϵ_{yy} (extension positive) for a/D = 1.0. Geometry is shown in inset. Displacements are normalized by $2(1+\nu_u)\ BT\Delta m(t)/3\pi\rho_o$, strains by $2(1+\nu_u)\ BT\Delta m(t)/3\pi\rho_o D$.

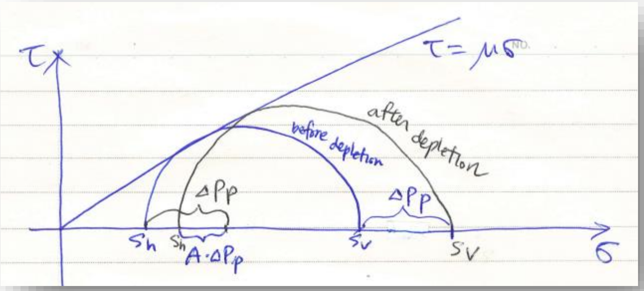


Relative change of horizontal stress. Notice: Tension (+)

Compression

Compression

tension



Segall, 1989, Earthquakes triggered by fluid extraction, Geology, 17:942-946

Deformation and stress changes outside of depleting reservoirs (저류층 주변의 변위/응력 변화)

Stress changes

- Stress changes above a flat-lying ellipsoidal reservoir

$$\frac{\Delta S_{\mathrm{h}}}{\Delta P_{\mathrm{p}}} = \alpha\left(\left(\frac{1-2\nu}{1-\nu}\right)\left(\frac{\pi}{4}\right)\left(\frac{H}{2R}\right)\right) \tag{12.29}$$

  - In Valhall, H/2R~0.03, ~1% → 0.2 MPa
  - Above and below the reservoir
    - Reservoirs located in reverse faulting regime → faulting may happen
  - Edge of the reservoir
    - Reservoirs located in normal faulting regime → faulting may happen

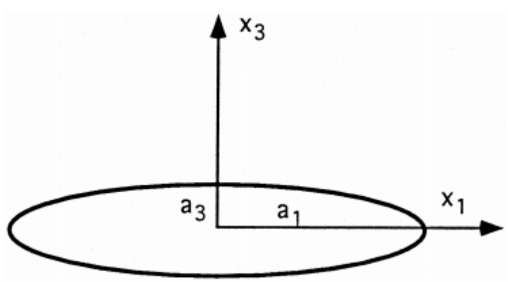


Fig. 3. Model reservoir with semi-major axes a_1, a_2, a_3 where $a_1 = a_2 > a_3$.

Segall & Fitzgerald, 1998, A note on induced stress changes in hydrocarbon and geothermal reservoirs, Tectonophysics 289:117-128

Deformation in depleting reservoirs (고갈 저류층의 변형)
compaction with increased confining pressure

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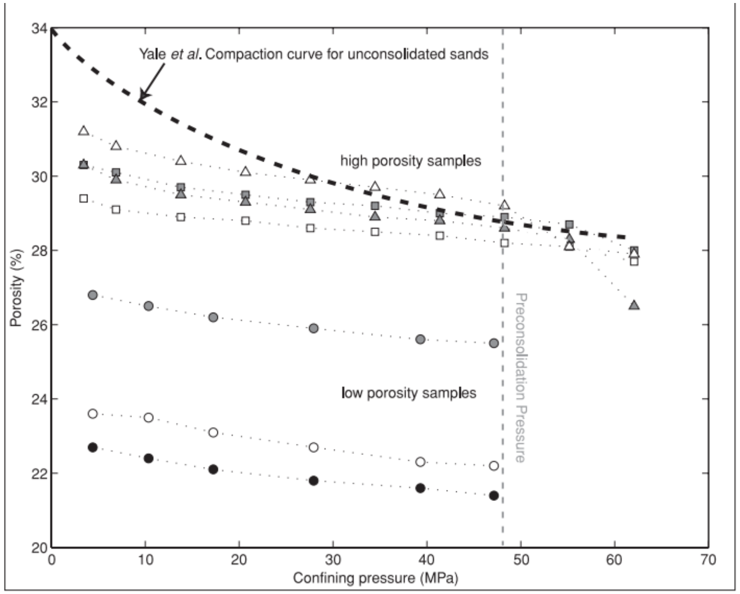
- Porosity tends to decrease with confining pressure

$$C_{\mathrm{f}} = A(\sigma_{\mathrm{lab}} - B)^{C} + D$$

where C_{f} is the formation compressibility and σ_{lab} is the laboratory stress. A, B, C and D are constants derived from laboratory experiments and, in the case of poorly sorted unconsolidated, they have the values of -2.8×10^{-5}, 300, 0.14, and 1.18×10^{-4}, respectively. Given that $C_{\mathrm{f}} = \Delta\phi/\Delta p$, by rearranging equation (12.13), the porosity



Porosity vs. confining pressure from Gulf of Mexico (field X)

Zoback MD, 2007, Reservoir Geomechanics, Cambridge University Press

Deformation in depleting reservoirs (고갈 저류층의 변형)

Depletion-induced permeability loss

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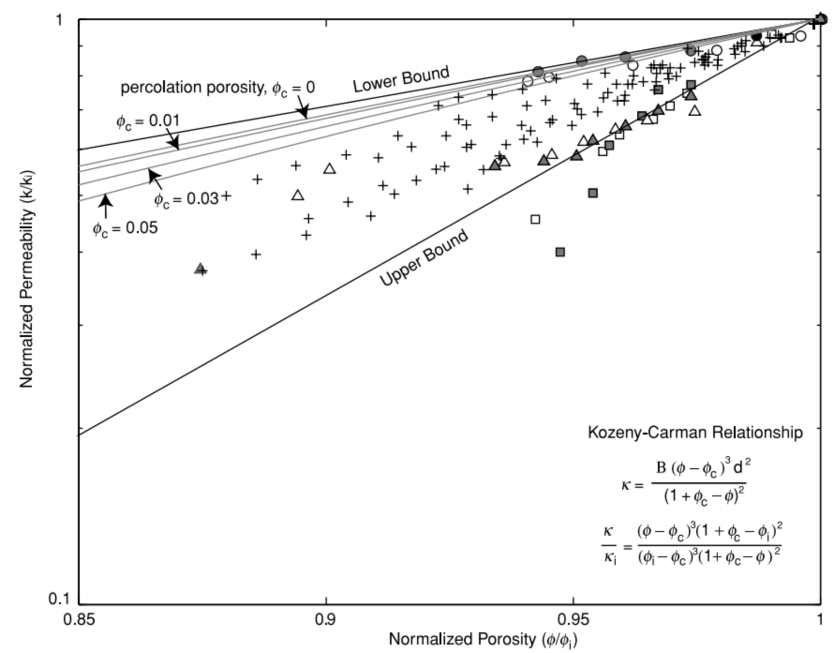
- Permeability loss
  - Permeability tends to decrease with increased confining pressure
  - Various empirical (and semi-analytical) relations exist
  - Permeability-stress or permeability-porosity
  - Kozeny-Carman relation is an example
  - Factors: porosity, percolation porosity, grain crushing

$$\kappa = \frac{B\phi^3}{\tau^2 S^2} = B\phi^3 \frac{d^2}{\tau} \tag{12.17}$$

where k is the permeability, B is a geometric factor, τ is tortuosity and d is the average grain diameter. The porosity, ϕ, and the specific surface area, S, can be expressed by:

$$\phi = \frac{\pi R^2}{A} \quad \text{and} \quad S = \frac{2\pi R}{A} \tag{12.18}$$

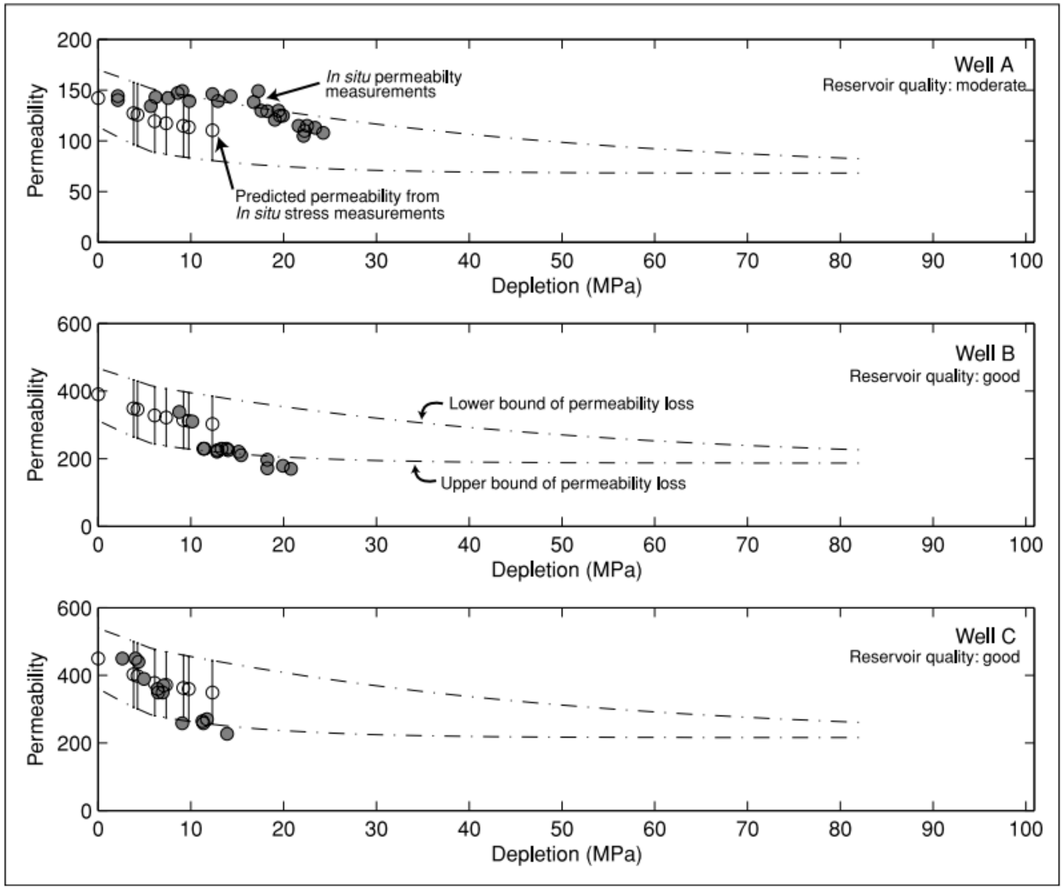
where R and A are the radius and the cross-sectional area of the imaginary pipe.



Zoback MD, 2007, Reservoir Geomechanics, Cambridge University Press

Deformation in depleting reservoirs
(고갈 저류층의 변형)
Depletion-induced permeability loss

- Model of permeability change



Zoback MD, 2007, Reservoir Geomechanics, Cambridge University Press

Deformation in depleting reservoirs (고갈 저류층의 변형)
Depletion-induced permeability loss

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- Compaction drive
  - Expulsion of oil due to large reduction in the pore volume with depletion
  - Reduction in permeability tends to reduce recovery
  - Trade-off between porosity decrease and permeability decrease needs to be evaluated
  - Produced fluid volume as a result of pore pressure decrease

$$\Delta V_{\text{prod}} = -V_{\text{p}}(C_{\text{f}} + C_{\text{p}p}^{\gamma})\Delta p_{\text{f}}$$

$$C_{\text{p}p}^{\gamma} = \frac{1+\nu_{\text{fr}}}{3(1-\nu_{\text{fr}})}\frac{\alpha}{\phi}\frac{1}{K_{\text{fr}}} + \left[\frac{2(1-2\nu_{\text{fr}})\alpha}{3(1-\nu_{\text{fr}})\phi} - 1\right]\frac{1}{K_{\text{s}}}$$

: $K_{\text{fr}} \ll K_{\text{s}}$ the equation simplifies to

$$C_{\text{p}p}^{\gamma} = \frac{1+\nu_{\text{fr}}}{3(1-\nu_{\text{fr}})\phi K_{\text{fr}}} = \frac{C_{\text{m}}}{\phi}$$

Fjaer E et al., 2008, Petroleum-related Rock Mechanics, 2nd Ed., Elsevier

Deformation in depleting reservoirs
(고갈 저류층의 변형)
Depletion-induced permeability loss

- Compaction drive

$$C_{pp}^{\gamma} = \frac{1+\nu_{fr}}{3(1-\nu_{fr})}\frac{\alpha}{\phi}\frac{1}{K_{fr}} + \left[\frac{2(1-2\nu_{fr})\alpha}{3(1-\nu_{fr})\phi} - 1\right]\frac{1}{K_s}$$

: $K_{fr} \ll K_s$ the equation simplifies to

$$C_{pp}^{\gamma} = \frac{1+\nu_{fr}}{3(1-\nu_{fr})\phi K_{fr}} = \frac{C_m}{\phi}$$

$$\Delta V_{prod} = -V_p(C_f + C_{pp}^{\gamma})\Delta p_f$$

pore compressibility due to pore pressure

porosity

bulk modulus of frame

$$C_{pp}^{\gamma} = \frac{1+\nu}{3(1-\nu)\phi K_{fr}}$$

$$\Delta V_{prod} = -V_p(C_f + C_{pp}^{\gamma})\Delta P_f$$

i) K_f of oil $\sim 1\,GPa$

$K_{fr} \sim 1\,GPa$

$\phi \sim 0.25$

$\nu \sim 0.3$

$$C_{pp}^{\gamma} = \frac{1+0.3}{3(1-0.3)\,0.25\times 1\,GPa} \doteq 2.5\,/GPa.$$

$C_f = 1\,/GPa$

ii) if $K \sim 10\,GPa$

$\phi \sim 10\,\% \sim 0.1$

$\nu \sim 0.2$

$$C_{pp}^{\gamma} = \frac{1+0.2}{3(1-0.2)\,0.1\times 10\,GPa} = 0.5/GPa$$

Deformation in depleting reservoirs (고갈 저류층의 변형)
Depletion-induced permeability loss

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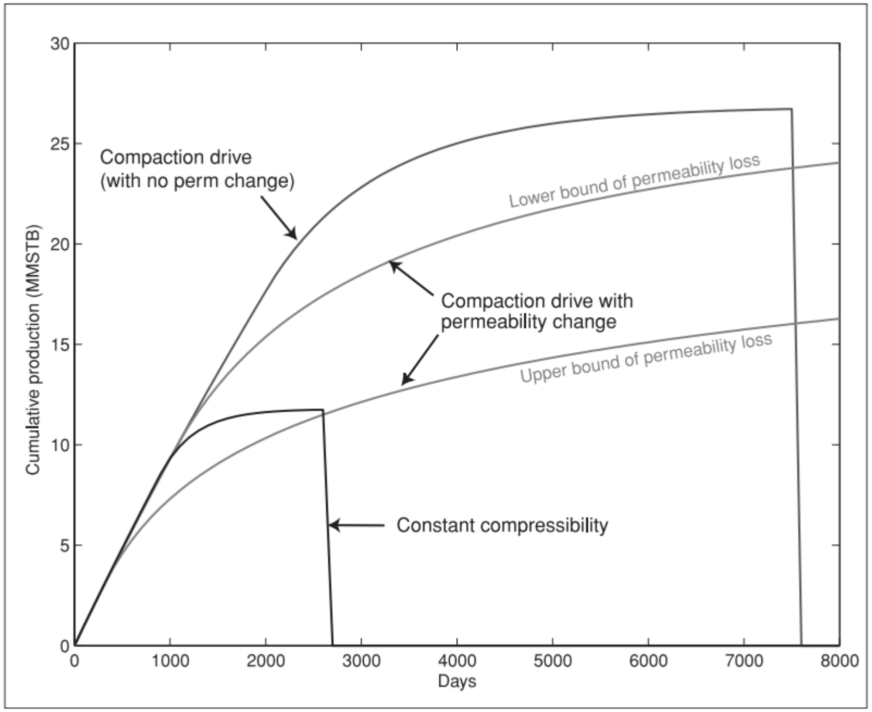
- Production considering compaction drive
  - Compaction drive with no perm change
  - Compaction drive with perm change
  - Constant compressibility



MMSTB: million stock tank barrel

Zoback MD, 2007, Reservoir Geomechanics, Cambridge University Press