⓪ Superposition of plane-flow sols.

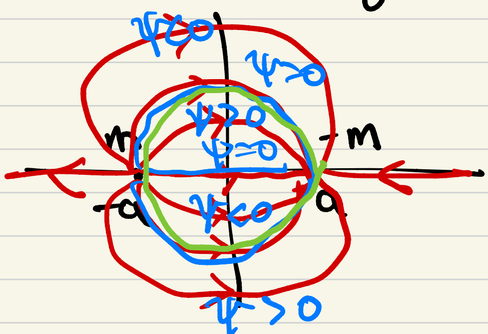
$$\left\{\begin{array}{l}\nabla^2\phi=0\\ \nabla^2\psi=0\end{array}\right\} \rightarrow \text{linear eq.} \rightarrow \text{superposition is possible.}$$

$\phi_1, \phi_2 \rightarrow c_1\phi_1 + c_2\phi_2$ $\quad \left(u_1=\frac{\partial\phi_1}{\partial x}, \; u_2=\frac{\partial\phi_2}{\partial x} \right)$

$\psi_1, \psi_2 \rightarrow c_1\psi_1 + c_2\psi_2$

$p_1, p_2 \rightarrow c_1 p_1 + c_2 p_2$ X $\quad \frac{V^2}{2}+\frac{p}{\rho}=\text{const}$

① source + an equal sink



$\psi = \psi_{source} + \psi_{sink}$ $\quad \left(\begin{array}{l}\psi = m\theta = m\tan^{-1}\frac{y}{x}\\ \phi = m\ln r\end{array}\right.$

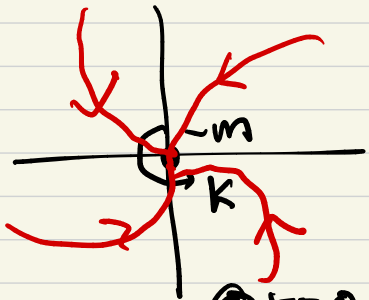
$$= m\tan^{-1}\frac{y}{x+a} - m\tan^{-1}\frac{y}{x-a}$$

$$= -m\tan^{-1}\frac{2ay}{x^2+y^2-a^2}$$

$$\phi = \phi_{source}+\phi_{sink} = \frac{1}{2}m\ln[(x+a)^2+y^2] - \frac{1}{2}m\ln[(x-a)^2+y^2]$$

$u=\frac{\partial\psi}{\partial y}, \; v=-\frac{\partial\psi}{\partial x} \rightarrow p$ from Bernouille eq.

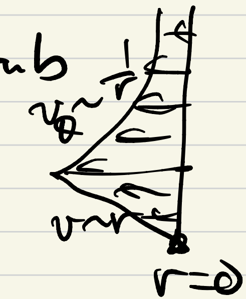
② sink + vortex at the origin



$\psi = -m\theta - k\ln r$

$\phi = -m\ln r + k\theta$

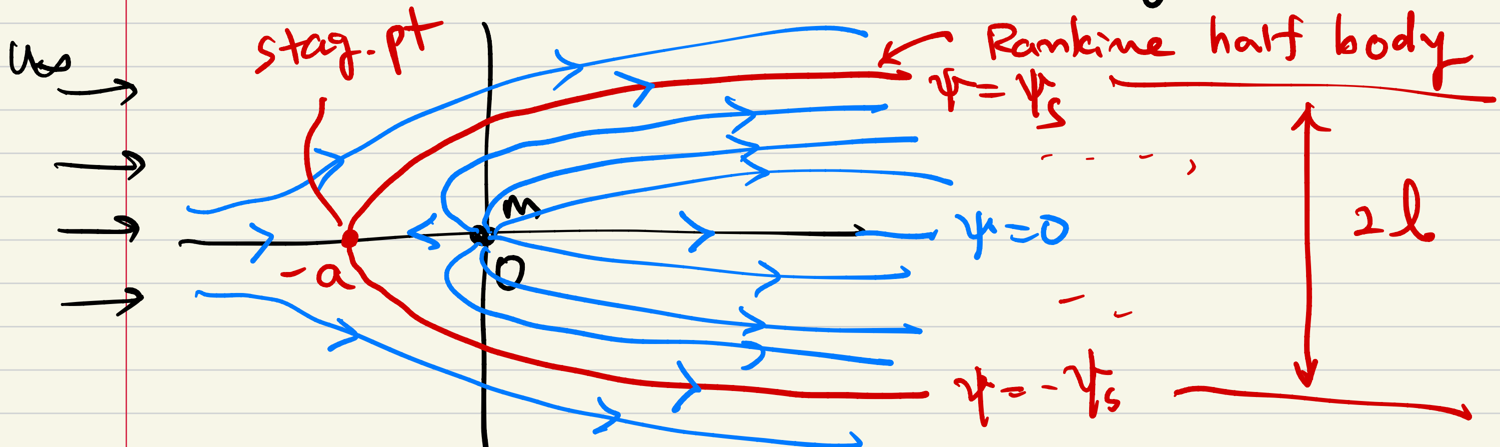
tornado, rapidly draining bathtub



@ $r=0$, $v_\theta \to \infty$

In reality, near $r=0$, solid body rotating

③ uniform stream + a source at the origin



$\psi = u_\infty r \sin\theta + m\theta \rightarrow u = \frac{\partial \psi}{\partial y} = u_\infty + \frac{m}{r}\cos\theta$

$\phi = u_\infty r\cos\theta + m\ln r \qquad v = -\frac{\partial \psi}{\partial x} = \frac{m}{r}\sin\theta$

stag. pt : $u = v = 0$

$\theta = \cancel{0}, \pi \rightarrow u = u_\infty - \frac{m}{r} \Rightarrow \boxed{a = \frac{m}{u_\infty}}$

$m = \frac{Q}{2\pi b} \rightarrow \frac{Q}{b} = 2\pi m$: flow rate from source

$= 2l u_\infty \rightarrow l = \frac{\pi m}{u_\infty} = \pi a$

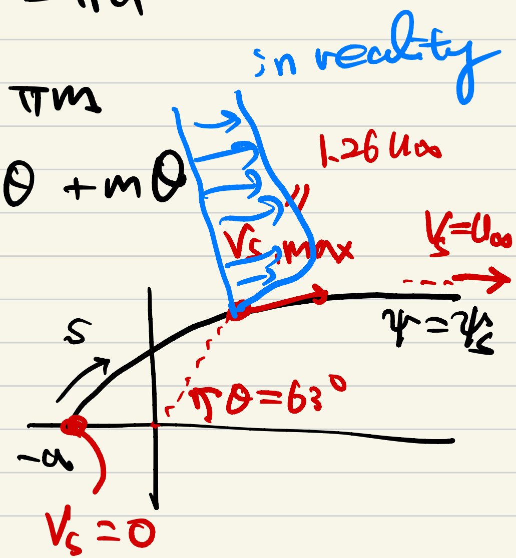
$\frac{Q}{b} = \int d\psi = 2\psi_s = 2\pi m \rightarrow \psi_s = \pi m$

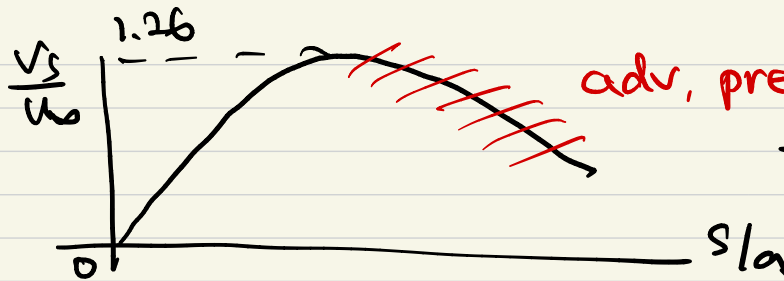
upper surface : $\psi_s = \pi m = u_\infty r\sin\theta + m\theta$

$\rightarrow r = \frac{m(\pi - \theta)}{u_\infty \sin\theta}$

$V_s^2 = u^2 + v^2 = u_\infty^2\left(1 + \frac{a^2}{r^2} + \frac{2a}{r}\cos\theta\right)$

$\frac{\partial V_s^2}{\partial \theta} = 0$: $V_{s,max} = 1.26\,u_\infty$ @ $\theta = 63^\circ$



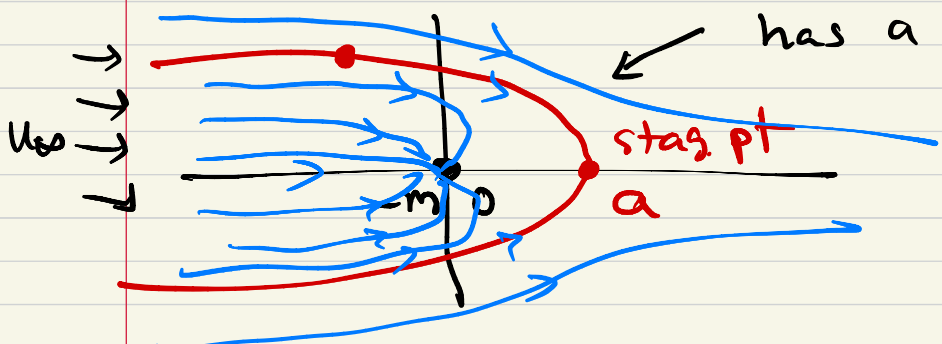


adv, press_grad

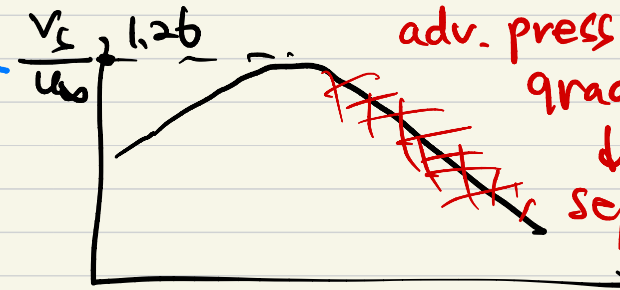
→ possibility of flow sep.

→ in reality, no sep.

④ uniform stream + a sink



has a mirror image of U_∞ + source

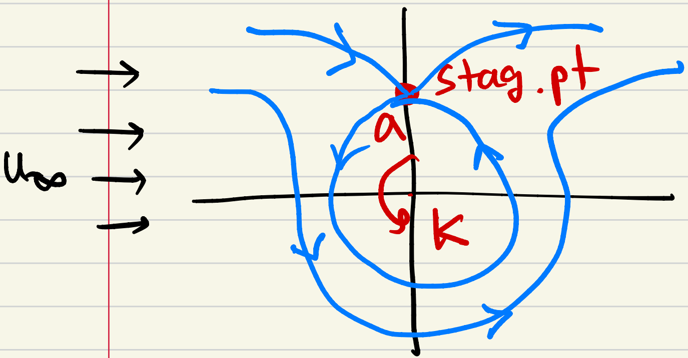


adv. press grad

↓

separation

⑤ Flow past a vortex = unif stream + vortex



$$\psi = u_\infty r\sin\theta - K\ln r$$

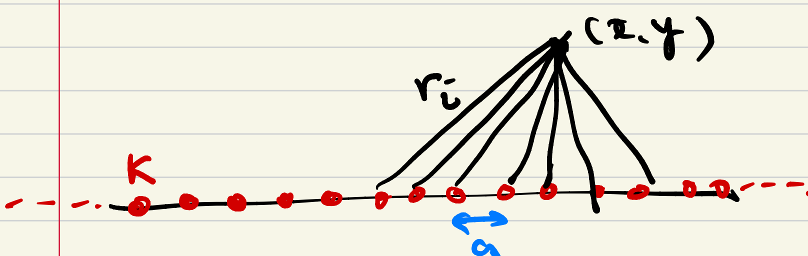
$$\phi = u_\infty r\cos\theta + K\theta$$

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = u_\infty\cos\theta$$

$$v_\theta = -\frac{\partial\psi}{\partial r} = -u_\infty\sin\theta + \frac{K}{r}$$

Stag. pt. $v_r = v_\theta = 0$: $\theta = \frac{\pi}{2}$, $r = a = \frac{K}{u_\infty}$

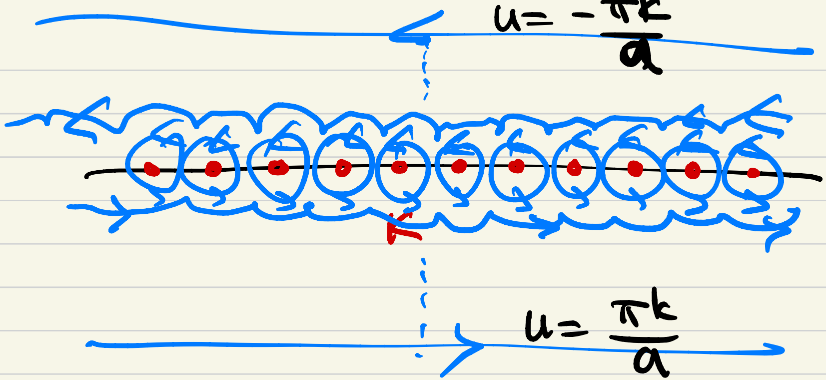
⑥ Infinite row of vortices



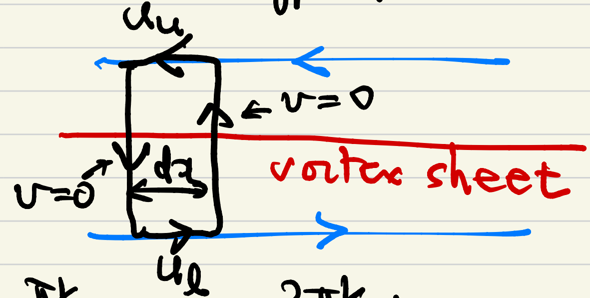
$$\psi = -K\sum_{i=1}^{\infty}\ln r_i$$

(complex variable)

$$= -\frac{1}{2}K\ln\left[\frac{1}{2}\left(\cosh\frac{2\pi y}{a} - \cos\frac{2\pi x}{a}\right)\right]$$



$$u = \left.\frac{\partial \psi}{\partial y}\right|_{|y| \gg a} = \pm \frac{\pi k}{a}$$



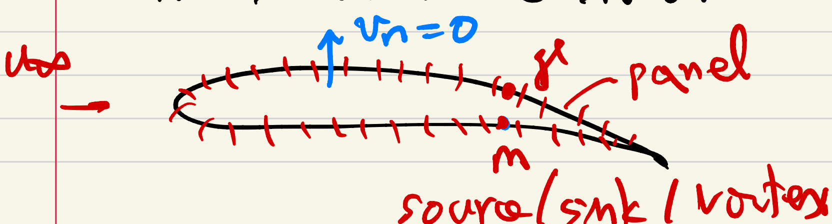
$$d\Gamma = \oint \underline{u} \cdot d\underline{l} = -\frac{\pi k}{a} \cdot (-dx) + \frac{\pi k}{a}(dx) = \frac{2\pi k}{a} dx$$

$$\boxed{\frac{d\Gamma}{dx} = \frac{2\pi k}{a} = \gamma}$$

: strength of vortex sheet
circulation per unit length of the vortex sheet

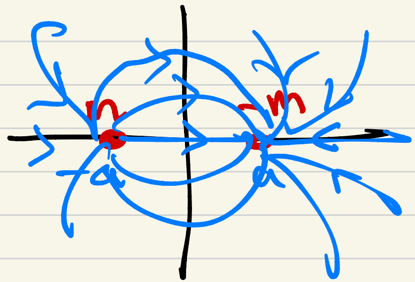
↓

This is used to simulate a thin-body shape like airfoil, flat plate ...



⇒ panel method

⑰ Doublet : source + sink pair w/ a vanishingly small distance a



$a \to 0$

$2am = \text{const} = \lambda$: strength of doublet

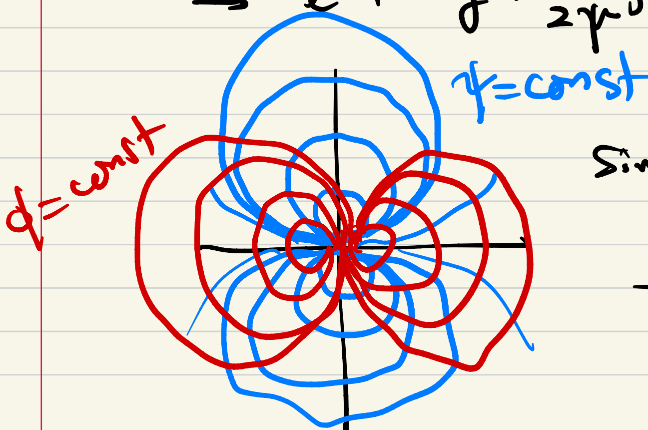
$$\psi = \lim_{\substack{a\to 0\\ 2am=\lambda}} \left(-m \tan^{-1} \frac{2ay}{x^2+y^2-a}\right)$$

$$\left(\tan^{-1} x = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 \cdots\right)$$

$$\simeq -\frac{2amy}{x^2+y^2} = -\frac{\lambda y}{x^2+y^2} = -\frac{\lambda \sin\theta}{r}$$

$$\to x^2 + \left(y + \frac{\lambda}{2\psi}\right)^2 = \left(\frac{\lambda}{2\psi}\right)^2$$: streamlines are circles. $(\psi = c)$

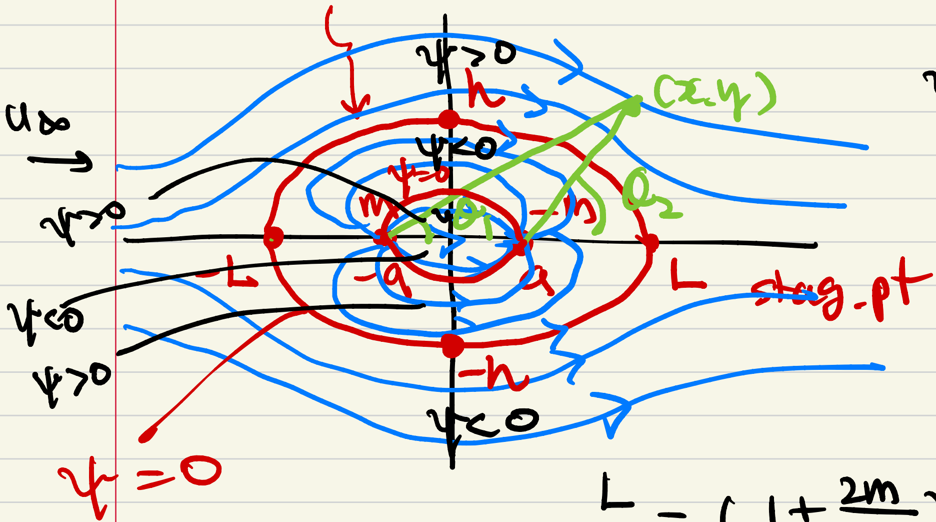
$\left[0, -\frac{\lambda}{2\psi}\right]$



Similarly $\phi = \dfrac{\lambda x}{x^2+y^2}$

$$\to \left(x - \frac{\lambda}{2\phi}\right)^2 + y^2 = \left(\frac{\lambda}{2\phi}\right)^2$$

⑧ Rankine oval : uniform stream + source + sink



$$\psi = u_\infty y - m\tan^{-1}\frac{2ay}{x^2+y^2-a^2}$$

$$= u_\infty r\sin\theta + m(\theta_1 - \theta_2)$$

$$\left.\begin{aligned} u &= \frac{\partial\psi}{\partial y} = \cdots \\ v &= -\frac{\partial\psi}{\partial x} = \cdots \end{aligned}\right\} \quad u = v = 0 \ \text{(stag. pt.)}$$

$$\frac{L}{a} = \left(1 + \frac{2m}{u_\infty a}\right)^{\frac{1}{2}}$$

@ $y = h,\ x = 0$: $\psi = u_\infty h - m\tan^{-1}\dfrac{2ah}{h^2 - a^2} = 0 \rightarrow \dfrac{h}{a} = \cot\dfrac{h/a}{\frac{2m}{u_\infty a}}$

$V_s^2 = u^2 + v^2$ on $\psi = 0 \rightarrow \dfrac{\partial V_s^2}{\partial\theta} = 0$

$\rightarrow \dfrac{u_{max}}{u_\infty} = 1 + \dfrac{2m/u_\infty a}{1 + h^2/a^2}$ @ $\theta = 90^\circ$

For $m/u_\infty a = 1$,

$u_{max} = 1.74\,u_\infty$